Resonant two-photon ionization of atoms by twisted and plane-wave light

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We study the resonant two-photon ionization of neutral atoms by a combination of twisted and plane-wave light within a fully relativistic framework. In particular, the ionization of an isotropic ensemble of neutral sodium atoms ($Z = 11$) from their ground $3\,^2S_{1/2}$ state via the $3\,^2P_{3/2}$ level is considered. We investigate in details the influence of the kinematic parameters of incoming twisted radiation on the photoelectron angular distribution and the circular dichroism. Moreover, we study the influence of the geometry of the process on these quantities. This is done by changing the propagation directions of the incoming twisted and plane-wave light. It is found that the dependence on the kinematic parameters of the twisted photon is the strongest if the plane-wave and twisted light beams are perpendicular to each other.

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I. INTRODUCTION

The interaction of twisted (or vortex) light beams with matter has become an important research topic with extensive applications. These beams can carry a nonzero projection of the orbital angular momentum (OAM) onto the propagation direction. This projection, being an additional degree of freedom, provides a unique possibility to gain a deeper insight into the role of the OAM in light-matter interactions. Moreover, the Poynting vector of the vortex light beams rotates in a corkscrew manner around the propagation direction, and the intensity profile exhibits a ring structure. Therefore, in processes involving vortex photons, the position and structure of the target play a prominent role, in contrast to the plane-wave case.

Twisted photons are currently applied in, e.g., nanotechnology [1], astronomy [2,3], metrology [4], condensed-matter physics [5], and quantum information [6]. Most of these and many other applications rely on knowledge about the interaction of twisted light with ions and atoms. That, in turn, has stimulated investigations of fundamental processes involving twisted light beams and atomic or ionic systems. Up to now, theoretical studies of the excitation [7–12], ionization [13–20], and scattering [21] processes have been presented. Experimental investigations were performed, e.g., for the excitation of a single Ca^+ ion [22] and the ionization of a gas target consisting of helium atoms [23]. For a more in-depth discussion of the possible applications utilizing twisted light, see the reviews [24,25] and references therein.

In the present paper, we perform a fully relativistic investigation of the resonant two-photon ionization of alkali-like ions (or atoms) by a combination of plane-wave and twisted light. We carry out our study in an example of neutral sodium atoms ($Z = 11$). For this system, the resonant two-photon ionization proceeds as follows. In a first step, the plane-wave photon excites the valence electron from $3s$ to $3p_{3/2}$ state, and in the second step it is ionized by a vortex photon. As a target, we consider an isotropic ensemble of atoms as it can be readily performed using present-day techniques. In this case, we show that both the photoelectron angular distribution and the circular dichroism depend on the ratio of the transversal and longitudinal components of the momentum of twisted light, which is defined by the so-called opening angle. We propose to enhance this dependency through an appropriate choice of geometry, i.e., via adjusting the angle between the plane-wave and twisted photons. It is found that the sensitivity to the opening angle is the strongest if the incident (plane-wave and twisted) beams are perpendicular to each other. Moreover, this dependency is stronger than in the case of single-photon ionization by twisted light beams [24].

The paper is organized as follows: In Sec. II A the basic equations for resonant two-photon ionization by plane-wave light are briefly recalled. In Sec. II B the theoretical description of this process involving a combination of twisted and plane-wave light is presented. In Sec. III we investigate the angular distribution and the circular dichroism for different opening angles of the twisted photon. The dependence of the "twistedness"-induced effects on the angle between the first, plane-wave, and the second, twisted, photon is also presented in Sec. III. Finally, a summary and outlook are given in Sec. IV.

Relativistic units, $m_e = \hbar = c = 1$, and the Heaviside charge unit $e^2 = 4\pi\alpha$ (where α is the fine-structure constant) are used in the paper.

II. BASIC FORMALISM

In the present paper, we focus on an investigation of resonant two-photon ionization of alkali-metal atoms using a combination of plane-wave and twisted photons. The resonant two-photon ionization is a two-step process. In the first step, the photon excites the target atom from the ground i state to the excited d one, while the second photon ionizes the atom. Here, we study the scenario in which the first photon is a conventional (plane-wave) one and the second photon is twisted. Our analysis is restricted to the case of continuous-wave lasers. The investigation of the influence of time duration of the pulses and the time delay between them on the process is beyond the scope of the present study. A discussion of the temporal effects in resonant two-photon ionization and similar processes can be found, e.g., in Refs. [26–30]. We start with the basic expressions for the conventional resonant two-photon ionization since the formulas for the twisted case can be traced back to their plane-wave counterparts.

A. Resonant ionization by two plane-wave photons

The probability of two-photon ionization in the resonance approximation (see Ref. [31] for further details) is given by

$$\frac{dW^{(\mathrm{pl})}_{\mathbf{k}_2\lambda_2,\mathbf{k}_1\lambda_1}}{d\Omega_f} = \frac{p_f\varepsilon_f}{\Gamma_d^2}\frac{4(2\pi)^7}{2j_i+1}\sum_{\mu_f m_i}\left|\sum_{m_d}\tau^{(\mathrm{ion,pl})}_{\mathbf{p}_f\mu_f;\mathbf{k}_2\lambda_2,dm_d}\tau^{(\mathrm{exc})}_{dm_d;\mathbf{k}_1\lambda_1,im_i}\right|^2, \quad (1)$$

where ε_f and $\mathbf{p}_f$ are the energy and asymptotic momentum of the emitted electron, respectively, $p_f = |\mathbf{p}_f|$, j_i is the total angular momentum (TAM) of the initial i state, m_d is the TAM projection of the excited d state, and Γ_d is the total width of this state. The first photon is characterized by its momentum $\mathbf{k}_1$ and helicity λ_1, and the second photon by $\mathbf{k}_2$ and λ_2, accordingly. In Eq. (1), averaging over the TAM projection of the initial state m_i and the summation over the helicity of the outgoing electron μ_f are performed. In the present work, we consider alkali-metal atoms. Therefore, we can apply the single active electron approximation for the description of resonant two-photon ionization. In the framework of this approximation, the excitation amplitude is given by

$$\tau^{(\mathrm{exc})}_{dm_d;\mathbf{k}_1\lambda_1,im_i} = -\int d\mathbf{r}\Psi^{\dagger}_{dm_d}(\mathbf{r})R^{(\mathrm{pl})}_{\mathbf{k}_1\lambda_1}(\mathbf{r})\Psi_{im_i}(\mathbf{r}). \quad (2)$$

Here $\Psi_{im_i}(\mathbf{r})$ and $\Psi_{dm_d}(\mathbf{r})$ are the wave functions of the initial and intermediate atomic states, respectively, and $R^{(\mathrm{pl})}_{\mathbf{k}\lambda} = -e\boldsymbol{\alpha}\cdot\mathbf{A}^{(\mathrm{pl})}_{\mathbf{k}\lambda}$ is the photon absorption operator in a Coulomb gauge with the vector of Dirac matrices $\boldsymbol{\alpha}$ and the vector potential of the plane-wave photon:

$$\mathbf{A}^{(\mathrm{pl})}_{\mathbf{k}\lambda}(\mathbf{r}) = \frac{\boldsymbol{\epsilon}_{\mathbf{k}\lambda}e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{2\omega(2\pi)^3}}. \quad (3)$$

The spherical-wave decomposition of the photon field is [32]

$$\boldsymbol{\epsilon}_{\mathbf{k}\lambda}e^{i\mathbf{k}\cdot\mathbf{r}} = \sqrt{2\pi}\sum_{LM} i^L\sqrt{2L+1}D^L_{M\lambda}(\varphi_k,\theta_k,0) \times \sum_{p=0,1}(i\lambda)^p\mathbf{a}^{(p)}_{LM}(\mathbf{r}), \quad (4)$$

where $C^{JM}_{j_1m_1\ j_2m_2}$ is the Clebsch-Gordan coefficient, $D^J_{MM'}$ is the Wigner matrix [32,33], and $\mathbf{a}^{(p)}_{LM}$ denote the magnetic ($p=0$) and electric ($p=1$) multipole fields defined by

$$\mathbf{a}^{(0)}_{LM}(\mathbf{r}) = j_L(kr)\mathbf{Y}_{LLM}(\hat{\mathbf{r}}), \quad (5)$$

$$\mathbf{a}^{(1)}_{LM}(\mathbf{r}) = j_{L-1}(kr)\sqrt{\frac{L+1}{2L+1}}\mathbf{Y}_{LL-1M}(\hat{\mathbf{r}}) - j_{L+1}(kr)\sqrt{\frac{L}{2L+1}}\mathbf{Y}_{LL+1M}(\hat{\mathbf{r}}). \quad (6)$$

Here $\mathbf{Y}_{JLM}$ [33] is the vector spherical harmonics. Utilizing Eqs. (3) and (4) and making use of the Wigner-Eckart theorem, one obtains

$$\tau^{(\mathrm{exc})}_{dm_d;\mathbf{k}_1\lambda_1,im_i} = -\frac{e}{\sqrt{2\omega(2\pi)^2}}\sum_{LM} i^L\sqrt{2L+1}D^L_{M\lambda_1}(\varphi_{k_1},\theta_{k_1},0) \times \sum_{p=0,1}(i\lambda_1)^p\sqrt{\frac{2L+1}{2j_d+1}} \times C^{j_dm_d}_{j_im_i\ LM}\langle\Psi_d\|\boldsymbol{\alpha}\cdot\mathbf{a}^{(p)}_L\|\Psi_i\rangle. \quad (7)$$

The explicit form of the reduced matrix elements $\langle\|\cdots\|\rangle$ can be found in Ref. [34].

The ionization amplitude is given by

$$\tau^{(\mathrm{ion,pl})}_{\mathbf{p}_f\mu_f;\mathbf{k}_2\lambda_2,dm_d} = -\int d\mathbf{r}\,\Psi^{(-)\dagger}_{\mathbf{p}_f\mu_f}(\mathbf{r})R^{(\mathrm{pl})}_{\mathbf{k}_2\lambda_2}(\mathbf{r})\Psi_{dm_d}(\mathbf{r}), \quad (8)$$

where $\Psi^{(-)}_{\mathbf{p}_f\mu_f}$ is the wave function of the outgoing electron with a definite asymptotic momentum [35–37]:

$$\Psi^{(-)}_{\mathbf{p}_f\mu_f}(\mathbf{r}) = \frac{1}{\sqrt{4\pi\varepsilon_fp_f}}\sum_{\kappa m_j}C^{j\mu_f}_{l0\ 1/2\mu_f}i^l\sqrt{2l+1}e^{-i\delta_\kappa}D^j_{m_j\mu_f}(\varphi_f,\theta_f,0)\Psi_{\varepsilon\kappa m_j}(\mathbf{r}). \quad (9)$$

Here $\kappa = (-1)^{l+j+1/2}(j+1/2)$ is the Dirac quantum number determined by the total angular momentum j and the parity l, δ_κ is the phase shift induced by the scattering potential, and $\Psi_{\varepsilon\kappa m_j}(\mathbf{r})$ is the partial-wave solution of the Dirac equation in the scattering field [38]. Substituting Eqs. (3), (4), and (9) into (2), we arrive at

$$\tau^{(\mathrm{ion,pl})}_{\mathbf{p}_f\mu_f;\mathbf{k}_2\lambda_2,dm_d} = -\frac{e}{\sqrt{4\omega(2\pi)^3\varepsilon_fp_f}}\sum_{\kappa m_j}\sum_{LM} i^L\sqrt{2L+1}D^L_{M\lambda}(\varphi_k,\theta_k,0)\sum_{p=0,1}(i\lambda)^p\sqrt{\frac{2L+1}{2j+1}} \times C^{jm_j}_{j_dm_d\ LM}C^{j\mu_f}_{l0\ 1/2\mu_f}i^l\sqrt{2l+1}e^{-i\delta_{j,l}}D^j_{m_j\mu_f}(\varphi_f,\theta_f,0)\langle\Psi_{\varepsilon\kappa}\|\boldsymbol{\alpha}\cdot\mathbf{a}^{(p)}_L\|\Psi_d\rangle. \quad (10)$$

The amplitudes (2) and (8) uniquely define the probability (1), and thus all the properties of the resonant two-photon ionization process.

B. Resonant ionization by plane-wave and twisted photons

In the present work, we restrict ourselves to the case of Bessel-wave twisted photons. These waves possess a well-defined energy ω, helicity λ, as well as projections of the linear k_z and total angular m momenta onto the propagation direction. We fix the z axis along this direction. The Bessel-wave twisted photon is described by the vector potential [14,39,40]:

$$\mathbf{A}^{(\mathrm{tw})}_{\varkappa m k_z \lambda}(\mathbf{r}) = i^{\lambda-m}\int \frac{e^{im\varphi_k}}{2\pi k_\perp}\delta(k_\perp-\varkappa)\delta(k_\parallel-k_z)\mathbf{A}^{(\mathrm{pl})}_{\mathbf{k}\lambda}(\mathbf{r})d\mathbf{k}, \tag{11}$$

where $k_\parallel$ and $k_\perp$ are the longitudinal and transversal components of the momentum $\mathbf{k}$, respectively, and $\varkappa = \sqrt{\omega^2 - k_z^2}$ is the well-defined transversal momentum of the Bessel photon. From Eq. (11), it is seen that in momentum space, Bessel states represent a cone with the opening angle $\theta_k = \arctan(\varkappa/k_z)$. In the coordinate space, the intensity profile and the flux density of the twisted photon are not homogeneous functions but exhibit complex internal structures. As an example, the intensity profile reads

$$\begin{aligned} I^{(\mathrm{tw})}_\perp(r_\perp) &= \left|\mathbf{P}^{(\mathrm{tw})}_z(r_\perp)\right| \\ &= \frac{\omega\lambda}{(4\pi)^3}\left[J^2_{m-1}(\varkappa r_\perp)c^2_{+1} - J^2_{m+1}(\varkappa r_\perp)c^2_{-1}\right] \end{aligned} \tag{12}$$

with

$$c_{\pm 1} = 1 \pm \lambda\cos\theta_k.$$

Here J_n is the Bessel function of the first kind [41,42], $r_\perp$ is the perpendicular component of r, and $\mathbf{P}^{(\mathrm{tw})}_z(r_\perp)$ is the z component of the time-averaged Poynting vector [17]:

$$\mathbf{P}^{(\mathrm{tw})}(\mathbf{r}) = \tfrac{1}{2}\mathrm{Re}\{i\mathbf{A}^{(\mathrm{tw})}(\mathbf{r}) \times [\nabla \times \mathbf{A}^{(\mathrm{tw})}(\mathbf{r})]^*\}. \tag{13}$$

Figure 1 displays a characteristic intensity profile that has a ringlike structure. This distinguishing feature makes the ionization amplitude-dependent on the relative position of the twisted photon and the target. To investigate the resonant two-photon ionization of a single alkali-metal atom by a combination of plane-wave and Bessel photons, we first need to discuss the geometry of this process, which is schematically depicted in Fig. 2. As was mentioned before, the z axis is directed along the propagation direction of the second, Bessel, photon. The reaction x-z plane is formed by the z axis and the wave vector of the first, plane-wave, photon $\mathbf{k}_1$. The position of the target atom is given by the impact parameter $\mathbf{b} = (b, \varphi_b, 0)$ in cylindrical coordinates. The probability of the process under investigation in the resonance approximation is given by

$$\begin{aligned} &\frac{dW^{(\mathrm{tw})}_{\varkappa m k_z \lambda_2, \mathbf{k}_1\lambda_1}(\mathbf{b})}{d\Omega_f} \\ &= \frac{p_f\varepsilon_f}{\Gamma_d^2}\frac{4(2\pi)^7}{2j_i+1}\sum_{\mu_f m_i}\left|\sum_{m_d}\tau^{(\mathrm{ion,tw})}_{\mathbf{p}_f\mu_f;\varkappa m k_z\lambda_2, d m_d}(\mathbf{b})\,\tau^{(\mathrm{exc})}_{d m_d;\mathbf{k}_1\lambda_1, i m_i}\right|^2, \end{aligned} \tag{14}$$

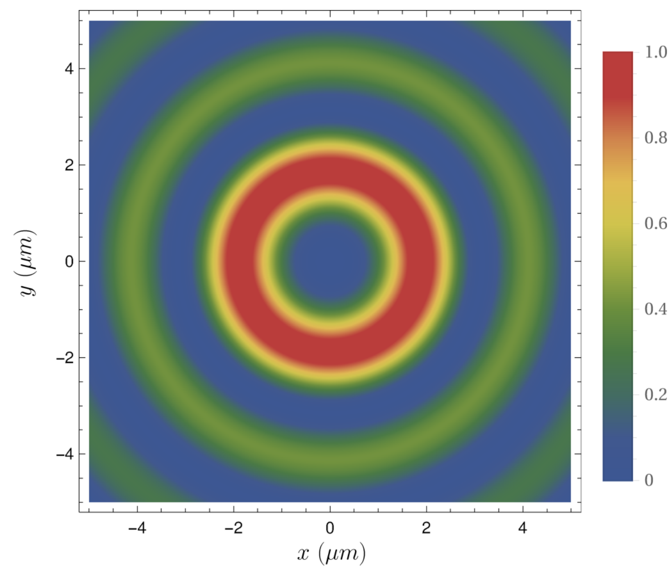

FIG. 1. The transverse intensity profile of a Bessel beam $I_\perp$ is characterized by a vanishing intensity on the beam axis ($x = y = 0$) and an infinite number of concentric rings. It is shown in units of the maximum intensity for a beam with $\omega = 3.677\,96$ eV, $\theta_k = 5^\circ$, $\lambda = 1$, and $m = 3$.

where the amplitude of the ionization by the twisted photon has the following form:

$$\begin{aligned} &\tau^{(\mathrm{ion,tw})}_{\mathbf{p}_f\mu_f;\varkappa m k_z\lambda_2, d m_d}(\mathbf{b}) \\ &= -\int d\mathbf{r}\,\Psi^{(-)\dagger}_{p_f\mu_f}(\mathbf{r}-\mathbf{b})R^{(\mathrm{tw})}_{\varkappa m k_z\lambda_2}(\mathbf{r})\Psi_{d m_d}(\mathbf{r}-\mathbf{b}). \end{aligned} \tag{15}$$

Here $R^{(\mathrm{tw})}_{\varkappa m k_z\lambda}$ is the photon absorption operator:

$$R^{(\mathrm{tw})}_{\varkappa m k_z\lambda} = -e\boldsymbol{\alpha}\cdot\mathbf{A}^{(\mathrm{tw})}_{\varkappa m k_z\lambda}. \tag{16}$$

Substituting Eq. (11) into Eq. (15) and changing the integration variable $\mathbf{r}$ to $\mathbf{r} - \mathbf{b}$, one can express the amplitude of the ionization by a twisted photon through the one obtained in the plane-wave case (8):

$$\begin{aligned} \tau^{(\mathrm{ion,tw})}_{\mathbf{p}_f\mu_f;\varkappa m k_z\lambda_2, d m_d}(\mathbf{b}) = \int &\frac{e^{im\varphi_k}}{2\pi k_\perp} i^{\lambda-m}\delta(k_\perp-\varkappa) \\ &\times \delta(k_\parallel - k_z)e^{i\mathbf{k}\cdot\mathbf{b}}\,\tau^{(\mathrm{ion,pl})}_{\mathbf{p}_f\mu_f;\mathbf{k}\lambda_2, d m_d}d\mathbf{k}. \end{aligned} \tag{17}$$

So far we have discussed the resonant ionization of a single atom by the combination of the plane-wave and Bessel photons. This process is interesting from a theoretical viewpoint, but most photoionization experiments deal with extended (macroscopic) targets. Therefore, we focus below on the macroscopic target, and we describe such a target as an incoherent superposition of atoms randomly and homogeneously distributed. The probability of resonant two-photon ionization in this case is given by

$$\begin{aligned} \frac{dW^{(\mathrm{mac,tw})}_{\varkappa k_z\lambda_2, \mathbf{k}_1\lambda_1}}{d\Omega_f} &= \int \frac{d\mathbf{b}}{\pi R^2}\frac{dW^{(\mathrm{tw})}_{\varkappa m k_z\lambda_2, \mathbf{k}_1\lambda_1}(\mathbf{b})}{d\Omega_f} \\ &= \frac{2}{\pi R\varkappa}\int_0^{2\pi}\frac{d\varphi_k}{2\pi}\frac{dW^{(\mathrm{pl})}_{\mathbf{k}\lambda_2, \mathbf{k}_1\lambda_1}}{d\Omega_f}, \end{aligned} \tag{18}$$

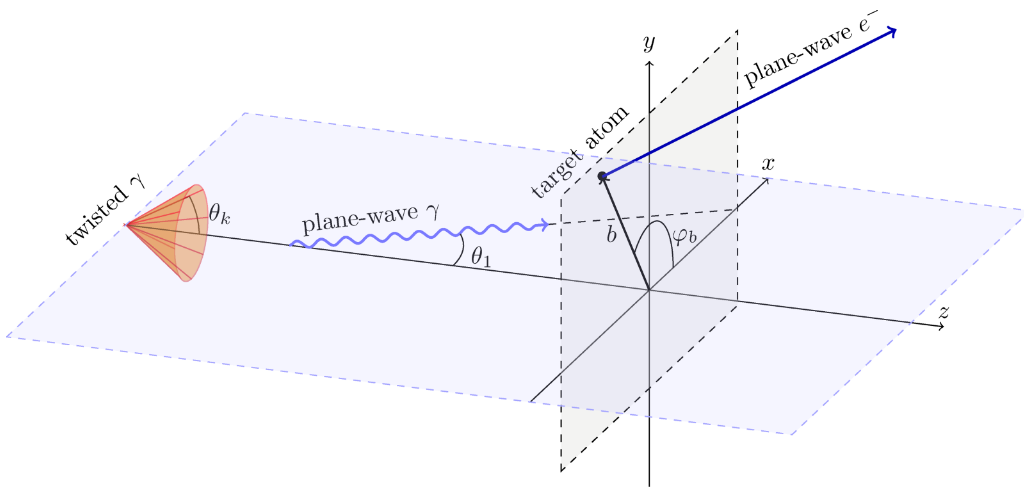

FIG. 2. Geometry of the resonant ionization of a single atom by the combination of plane-wave and twisted light.

where the vector $\mathbf{k}$ is defined by cylindrical coordinates $(\varkappa, \varphi_k, k_z)$, $1/\pi R^2$ is the cross-section area, with R being the radius of the cylindrical box. Note that for macroscopic targets, the probability of the resonant two-photon ionization does not depend on the TAM projection m but is still sensitive to the opening angle of the incoming twisted photon.

In the present investigation, we restrict ourselves to the normalized probability

$$\frac{dW^{(\mathrm{norm,tw})}_{\lambda_2,\lambda_1}}{d\Omega_f} = \frac{1}{W^{(\mathrm{avr})}_{\lambda_2,\lambda_1}} \frac{dW^{(\mathrm{mac,tw})}_{\varkappa k_z \lambda_2, \mathbf{k}_1 \lambda_1}}{d\Omega_f}, \tag{19}$$

where

$$W^{(\mathrm{avr})}_{\lambda_2,\lambda_1} = \frac{1}{4\pi} \int d\Omega_f \frac{dW^{(\mathrm{mac,tw})}_{\lambda_2,\lambda_1}}{d\Omega_f}. \tag{20}$$

III. RESULTS AND DISCUSSIONS

Let us proceed to the investigation of the effects of the “twistedness” and explore the possibilities of their enhancement. Since in the present study we consider only the scenario of the macroscopic target, these effects are constituted in the dependence of the measurable quantities on the opening angle of the vortex light beam. In the present work, we consider two-photon ionization of a valence electron of neutral sodium ($Z = 11$). We describe this process within the framework of the single active electron approximation. The active electron in the ground $3s$, excited $3p_{3/2}$, and continuum states is described by the wave functions being the solutions of the Dirac equation with the effective potential describing the electric field of the nucleus and the spectator electrons. Here we utilize the so-called $X\alpha$ central potential whose parameters are adjusted in such a way as to reproduce the energy of the $3\,^2S_{1/2}$-$3\,^2P_{3/2}$ transition, namely 2.104 43 eV [43]. The radial Dirac equation with the effective potential is solved by the modified RADIAL package [44]. The energy of the twisted photon $\omega_2 = 3.677\,96$ eV is chosen to be the same as in the experiment [45] where the resonant two-photon ionization of the neutral sodium atoms by two plane-wave photons was studied. We note that the fine-structure levels $3\,^2P_{1/2}$ and $3\,^2P_{3/2}$ in neutral sodium are separated by 0.002 eV [43] and their width is approximately 0.000 26 meV [46]. The technique of the exclusive population of these nonoverlapping levels is well-established [47].

A. Ionization probability

We start with the analysis of the probability of the resonant ionization of the macroscopic sodium target by two photons with $\lambda_1 = \lambda_2 = 1$. Except for the right panel of the last row, Fig. 3 presents the normalized probability (19) as a function of the polar angle of the ionized electron θ_f for different values of the angle between the two photons θ_1. For reference, we show the normalized probability of ordinary two-photon ionization (solid black line). From the upper three rows of this figure, it is seen that the ionization probability changes significantly when the angle θ_1 is changed to $180^\circ - \theta_1$. This phenomenon can be understood from the two-photon ionization by co- and counterpropagating plane-wave light (first row in Fig. 3). In this scenario, the process is invariant under rotations around the z axis, and, as a result, the TAM projection onto this axis is conserved. Therefore, only the following transitions can take place:

$$\theta_1 = 0^\circ: \quad \left|3s_{1/2}\; m_i = \pm\frac{1}{2}\right\rangle \to \left|3p_{3/2}\; m_d = +\frac{1}{2}, +\frac{3}{2}\right\rangle \to \left|\mathbf{p}_f\; m_f = +\frac{3}{2}, +\frac{5}{2}\right\rangle, \tag{21}$$

$$\theta_1 = 180^\circ: \quad \left|3s_{1/2}\; m_i = \pm\frac{1}{2}\right\rangle \to \left|3p_{3/2}\; m_d = -\frac{1}{2}, -\frac{3}{2}\right\rangle \to \left|\mathbf{p}_f\; m_f = \pm\frac{1}{2}\right\rangle, \tag{22}$$

where κ_f is the Dirac quantum number of the electron in the continuum, and m_f is its TAM projection onto the z axis. The TAM projection of the intermediate state is $m_d = m_i + \lambda_1$ for copropagating photons and $m_d = m_i - \lambda_1$ for

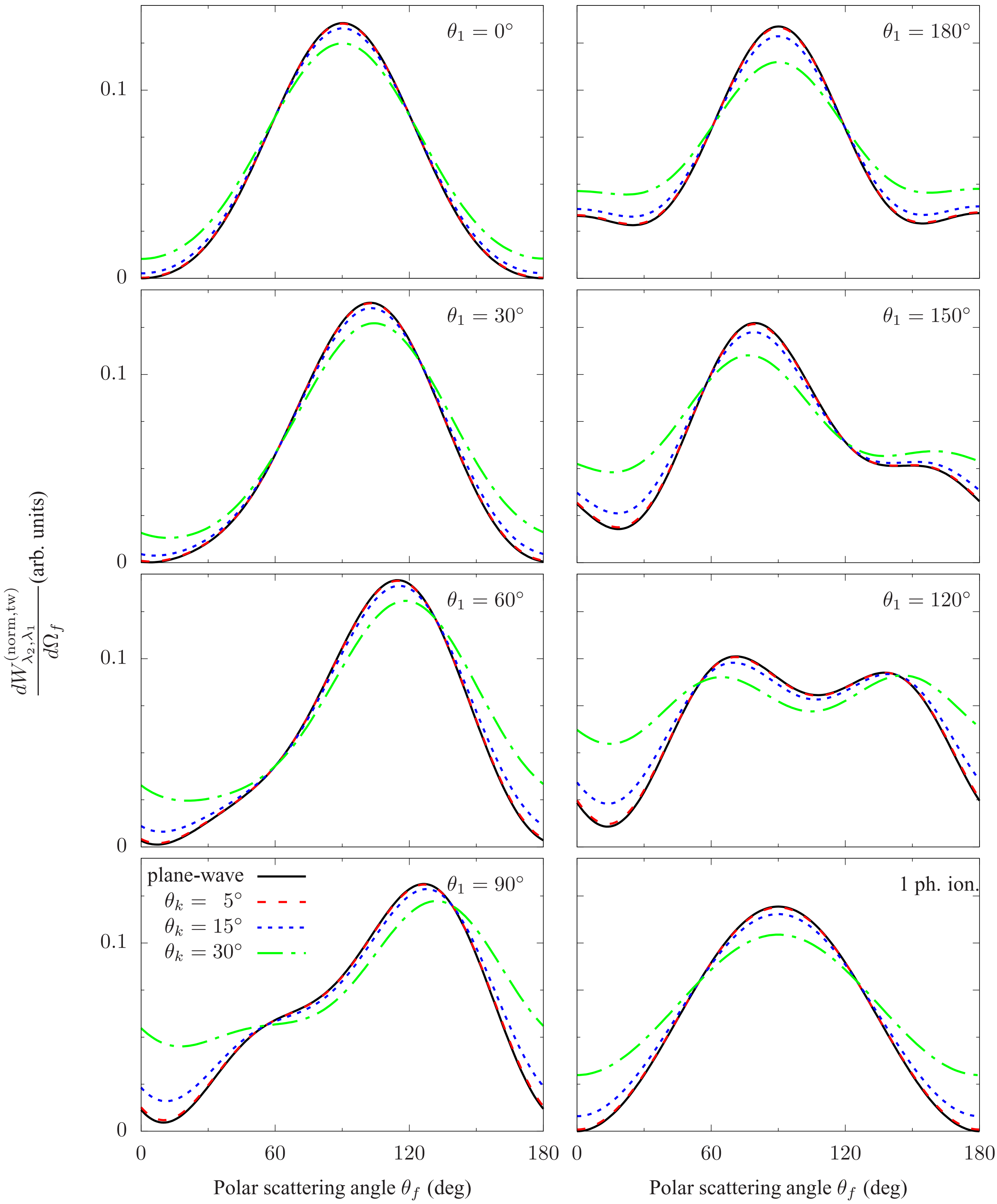

FIG. 3. Except for the right panel of the last row, the normalized probability (19) of the resonant two-photon ionization of a macroscopic sodium target as a function of the polar angle of the ionized electron θ_f is depicted. The results are presented for different values of the angle between the first, plane-wave, and the second, Bessel, photon θ_1 (see Fig. 2). The Bessel photon is characterized by its opening angle θ_k. In the right panel of the last row, the normalized probability (23) of single-photon ionization by twisted light is depicted.

counterpropagating beams. From the scheme (21), it is seen that $s_{1/2}$ and $p_{1/2}$ waves are absent in the final electron state for the scenario with $\theta_1 = 0°$. This can, in principle, lead to the significant differences between the cases of co- and counterpropagating plane-wave light beams. Although for other geometries these simple selection rules are not valid, the contribution of the $s_{1/2}$ and $p_{1/2}$ waves still remains suppressed for the scenarios with $\theta_1 < 90^\circ$, which is supported by numerical results.

From Fig. 3, it is also seen that the ionization probability depends strongly on the opening angle θ_k of the twisted photon. As an example, in the case of ionization by two copropagating plane-wave photons (the upper left panel of Fig. 3), the photoelectrons are not emitted under the angles 0° and 180°. But for the ionization by a combination of plane-wave and twisted light beams, the probability of forward and backward emission is nonzero. From Fig. 3 it is also seen that the closer θ_1 is to 90°, the stronger is the dependence of the ionization probability on the opening angle of the ionizing twisted photon θ_k. For $\theta_1 = 90^\circ$, the dependence becomes the most pronounced, as seen from the left panel of the last row in Fig. 3. In this case, the probability changes significantly, in comparison to the plane-wave one, even for relatively small opening angles. That makes the scenario with $\theta_1 = 90^\circ$ the most promising for the detection of the kinematic effects in two-photon ionization. In addition, the dependence of the probability on the opening angle of the ionizing twisted photon is larger for angles $\theta_1 > 90^\circ$ in comparison to the case in which $\theta_1 \to 180^\circ - \theta_1$.

It is instructive to compare our results with those presented in Ref. [24], where the ionization from the ground state of hydrogenlike ions by a twisted photon has been considered. This comparison should serve only as a qualitative indication of the enhancement of the kinematic effects. Within the nonrelativistic formalism, which was used in Ref. [24], the single-photon ionization probability is given by

$$\frac{dW_{1\text{ph}}^{(\text{norm,tw})}}{d\Omega_f} = \frac{1}{4\pi}[1 - P_2(\cos\theta_k)P_2(\cos\theta_f)]. \tag{23}$$

We note that this ionization probability does not depend either on the nuclear charge Z or on the energy of the ionizing photon. To restore these dependences, one should consider the relativistic formalism (see, for details, Ref. [48], where the time-reversed process of photoionization, namely radiative recombination, was considered). To the best of our knowledge, a fully relativistic description of photoionization by twisted photons has not yet been presented in the literature. In the right panel of the last row of Fig. 3, the normalized probability of single-photon ionization by twisted light is depicted. From Fig. 3 it is seen that by choosing properly the geometry in the process of the ionization by the combination of twisted and plane-wave photons, one can significantly enhance the kinematic effects in comparison to the single-photon case.

B. Circular dichroism

The two-photon ionization by a combination of plane-wave and twisted light can be additionally characterized by dichroism. As was shown before, for macroscopic targets the angular distribution of photoelectrons does not depend on the TAM projection of the vortex photon. Therefore, the dichroism signal in our case can appear only due to a flip of the helicity of the first or second incoming photon. Such a signal is commonly known as circular dichroism (CD):

$$\text{CD} = \frac{dW_{1,1} - dW_{1,-1}}{dW_{1,1} + dW_{1,-1}}, \tag{24}$$

where $dW_{\lambda_2,\lambda_1} \equiv \frac{dW_{\lambda_2,\lambda_1}^{(\text{norm,tw})}}{d\Omega_f}$. Before proceeding to the numerical results for the circular dichroism, we present the following properties of the ionization probabilities:

$$dW_{\lambda_2,\lambda_1} = dW_{-\lambda_2,-\lambda_1} \tag{25}$$

and

$$dW_{\lambda_2,\lambda_1} \xrightarrow[\substack{\theta_f \to 180^\circ - \theta_f \\ \theta_1 \to 180^\circ - \theta_1}]{} dW_{-\lambda_2,\lambda_1}. \tag{26}$$

With the use of these expressions and the results presented in Fig. 3, one can calculate the CD. For the sake of visualization, we present the CD (24) for the scenarios with $\theta_1 < 90^\circ$ in Fig. 4. Let us note that from Eqs. (25) and (26) one can notice that the CD is an antisymmetric function with respect to the simultaneous replacement $\theta_1 \to 180^\circ - \theta_1$ and $\theta_f \to 180^\circ - \theta_f$. Therefore, for $\theta_1 = \theta_f = 90^\circ$ the CD equals zero, which can be seen in the right panel of the second row in Fig. 4. From this graph, one can also see that in the case when the electrons are emitted in the forward or backward directions, the CD tends to zero. One can conclude that in this scenario, the ionization probability does not depend on the polarization of the photons. In general, from Fig. 4 it is seen that, as in the case of the normalized probability, the dependence of the CD on the opening angle θ_k increases while θ_1 approaches 90°. In the case of $\theta_1 = 90^\circ$, the kinematic effects become the most pronounced.

We want to note that while our calculations are performed for the continuous-wave laser case, they can also (at least qualitatively) be used to understand the experimental results obtained in a case of pulsed lasers. Thus, the theoretical description of resonant two-photon ionization of neutral sodium by two laser pulses is presented in a seminal paper by Hansen and coauthors [27]. In that work, the modification of the photoelectron angular distribution due to the evolution of the intermediate $3\,^2P_{3/2}$ state during the time delay between exciting and ionizing laser pulses was studied. These modifications, however, are of minor importance when compared with those induced by the vortex structure of the second photon. This means that for the experimental study of the process considered here, it is sufficient to make the time delay between the laser pulses much smaller than the lifetime of the $3\,^2P_{3/2}$ intermediate state, which is 16.254(22) ns [46].

IV. CONCLUSION

In the present work, we studied the resonant ionization by a combination of plane-wave and twisted light within the fully relativistic formalism. The study is performed for the ionization of a valence $3s$ electron of a neutral sodium atom ($Z = 11$) via the $3p_{3/2}$ state. We consider a scenario in which the plane-wave and Bessel beam collide with a macroscopic target. In particular, we investigate the photoelectron angular distribution and the circular dichroism for different opening angles of the twisted photon θ_k and different angles between the first, plane-wave, and the second, Bessel, photons θ_1. It was found that the kinematic effects do increase with θ_1 approaching 90°. That makes the case of $\theta_1 = 90^\circ$ the most promising scenario for the observation of these effects in an experimental realization of the investigated process.

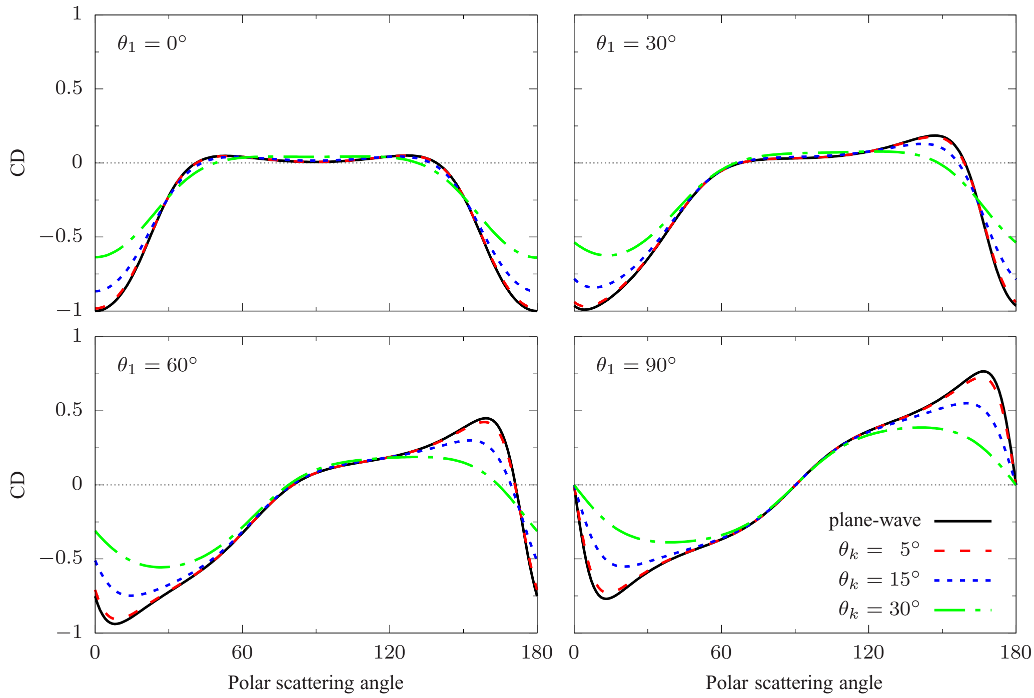

FIG. 4. The circular dichroism CD (24) for the resonant two-photon ionization of a macroscopic sodium target as a function of the polar angle of the ionized electron θ_f is shown. The results are presented for different values of the angle between the first, plane-wave, and the second, Bessel, photon, θ_1 (see Fig. 2). The Bessel photon is characterized by its opening angle θ_k.

We also performed a qualitative comparison between our and the nonrelativistic results presented in Ref. [24] for the single-photon ionization from the ground state of hydrogen-like ions by the twisted light. From this comparison, we found that the presence of the plane-wave photon in the two-photon ionization significantly increases the kinematic effects in comparison with those in the single-photon ionization.

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