Accelerated adiabatic quantum search algorithm via pulse control in a non-Markovian environment

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Adiabatic quantum computation requires that the system remains in its ground state. However, an adiabatic process often loses partly or entirely its quantumness, such as entanglement in its long runtime, due to the system-environment interactions. Here we put forward an effective quantum control technique to realize the adiabatic quantum search algorithm in a nonadiabatic regime and in the presence of environment. Using the non-Markovian quantum state diffusion equation approach, we numerically study the system dynamics characterized by the success probability. The results show that the probability increases with decreasing coupling strength and temperature. In particular, non-Markovianity from the environment can help to enhance the success probability. By choosing a suitable pulse intensity and period, a high success probability can be obtained in a short runtime for weak system-bath coupling, low temperature, and strong non-Markovian heat baths.

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I. INTRODUCTION

Adiabatic quantum computation (AQC) [1,2], polynomially equivalent [3] to the corresponding circuit quantum computation [3–5], uses a network of quantum nodes (such as in superconducting [6,7], photonic [8], and atomic quantum device [9,10]) that can be configured to represent a complicated computational problem. The solution to the problem is encoded in the ground state of the node system. Starting from the ground state of an easily prepared node Hamiltonian, the node system is gradually switched to a final Hamiltonian, whose ground state encodes the solution of a complicated computational problem. The dynamical process requires that the system remains in its ground state, as stated in the adiabatic theorem [11,12] that holds if a time-dependent Hamiltonian $H(t)$ changes slowly enough such that the eigenstate is sufficiently separated from neighboring eigenstates. The total evolution time required by the adiabatic conditions could be used to measure the computational cost of an adiabatic algorithm [13,14].

When performing AQC, the node system will be immersed in its environment. As a result, the influence of the environment is not negligible and decoherence will be present [15]. For such open systems, when the memory effects can be completely neglected, a standard Lindblad equation can be derived under Markovian approximation [16,17]. The influence of the environment on AQC has been investigated under the Markovian approximations, e.g., using master equations [18], superoperators [19], or Markovian noises [20,21]. The effects of the thermal environment [22] or a structured environment [14] may improve the performance of a quantum adiabatic algorithm. When the memory effects of the environment cannot be neglected, a non-Markovian description is required [23]. Non-Markovianity could be very different; e.g., it could regenerate entanglement in the system [24]. However, it is often a daunting task to solve the non-Markovian dynamics of the system. The quantum state diffusion (QSD) equation provides an effective approach [25,26]. For some interesting systems, the analytical or exact solutions can be obtained. Non-Markovian decoherence in the adiabatic search algorithm has been investigated and the scalability depends on the infrared behavior of the environment [27].

When dealing with a real system, decoherence is inevitable due to the existence of the system-bath interactions. Both theoretical and experimental efforts have been made to combat the decoherence problems in quantum computation. The strategies include using decoherence-free subspace [28], dynamical decouplings [29–31], quantum error corrections [32,33], or robust quantum control [34]. Because of its requirement for a long coherence time, an AQC is often ruined in a noisy environment. Recently, the speedup of adiabaticity by applying a sequence of fast pulses has been studied [35–37], as a result the detrimental effects of the environmental noise are suppressed [36]. By knowing the adiabatic basis, the speedup can be made by counterdiabatic driving [38–42]. In this paper, we focus on the adiabatic quantum Grover's search algorithm [43] in an environment [44]. The model is that each qubit is surrounded by its individual finite-temperature heat bath [45,46], and the system dynamics is governed by the QSD equation. As expected, our calculation results show that the success probability of this algorithm decreases with increasing temperature and system-bath interaction strength. Specifically, we study the effects of the bath non-Markovianity on the system and find that it can be

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helpful in boosting the success probability. Since an adiabatic process requires a long runtime, decoherence could proceed long enough to break down the system quantumness. To suppress the decoherence, we propose a protocol to speed up the adiabatic search algorithm in a nonadiabatic regime of the system. The strategy is to add a sequence of pulses on the system. The required pulse conditions have been obtained in a time-independent frame [47] or time-dependent frame [48] when the energy gap is a constant. However, in the adiabatic quantum search algorithm problem, the energy gap between the nearest neighbor energy level is time dependent. We derive pulse control conditions for a time-dependent energy gap. The calculation results show that once the required pulse conditions are satisfied, the success probability becomes almost perfect in weak system-bath coupling, low temperature, and strong non-Markovian heat baths.

II. ADIABATIC GROVER'S ALGORITHM

The adiabatic version of quantum Grover's algorithm is formulated by the Hamiltonian [1]

$$H_s(t) = [1 - s(t)]H_0 + s(t)H_m, \tag{1}$$

where $H_0 = (1 - |\psi_0\rangle\langle\psi_0|)$ is the initial Hamiltonian with $|\psi_0\rangle = \frac{1}{\sqrt{N}}\sum_{x=0}^{N-1}|x\rangle$ as the ground state, and $H_m = (1 - |m\rangle\langle m|)$ is the final Hamiltonian with $|m\rangle \in \{|x\rangle\}_{x=0}^{N-1}$ as the ground state and also the solution of the search algorithm. The time-dependent Hamiltonian $H_s(t)$ is constructed by a smoothly interpolation function $s(t)$ between H_0 and H_m. $s(t) \in [0, 1]$ is a dimensionless parameter. It was used to solve the problem of finding a marked item $|m\rangle$ in an unstructured database with length N. The state $|x\rangle$ which labels the items is represented by the n qubit basis states in a Hilbert space with dimension $N = 2^n$.

Now let $s(0) = 0$, $s(S) = 1$, where S is the runtime of the algorithm. If the adiabatic condition is satisfied, the final state will be $|\Psi_f\rangle = |m\rangle$, so $s(t)$ represents the adiabatic parameter and it controls how quickly the Hamiltonian changes. For example, $s(t) = 1 - t/S$ corresponds to a constant velocity $\dot{s}(t) = 1/S$ and the scaling $S = O(N)$ is obtained as the classical case. When $\dot{s} \propto (\Delta E)^2$ or $\dot{s} \propto \Delta E$, the quadratic speedup $S = O(\sqrt{N})$ or $S = O(\sqrt{N}lnN)$ will be recovered, respectively [13]. This is due to adjusting the velocity of change of the Hamiltonian to the instantaneous gap; the interpolating function with $\dot{s} \propto (\Delta E)^2$ is [13]

$$s(t) = \frac{1}{2}\left(1 + \frac{\tan[2\varepsilon t(\sqrt{N-1})/N - \arctan\sqrt{N-1}]}{\sqrt{N-1}}\right), \tag{2}$$

where $\varepsilon \ll 1$, and the final overlap with the solution is $1 - \varepsilon^2$. The total running time of the algorithm is $S_{\rm opt} \simeq \pi\sqrt{N}/2\epsilon$ for $N \gg 1$ [14]. Recently, quantum adiabatic algorithm design using reinforcement learning has been investigated, where the qubit number from $n = 1$ to $n = 16$ is discussed [49]. Quantitatively, the reinforcement-learning-designed algorithm can automatically produce an adiabatic quantum algorithm that has the quadratic speedup. With the same amount of time, it has a better performance in comparison with the analytical nonlinear path in Eq. (2).

III. ADIABATIC EVOLUTION OF THE SYSTEM COUPLED TO NON-MARKOVIAN BATHS

Now suppose that the adiabatic quantum computer is immersed in an environment. The total Hamiltonian has the form

$$H_{\rm tot} = H_s + H_b + H_{\rm int}, \tag{3}$$

where H_s is the time-dependent Hamiltonian (1) of the quantum computer and $H_b = \sum_{j=1}^{N} H_b^j$ are the N-independent bath Hamiltonian with $H_b^j = \sum_k \omega_k^j b_k^{j\dagger} b_k^j$. ω_k^j is the frequency of the kth bosonic mode and $b_k^{j\dagger}$, b_k^j are the bosonic creation and annihilation operators. This bosonic bath could consist of phonons in a solid lattice or the radiation field at finite temperature.

The system-bath interaction is

$$H_{\rm int} = \sum_{j,k}\left(g_k^{j*} L_j^\dagger b_k^j + g_k^j L_j b_k^{j\dagger}\right). \tag{4}$$

The Lindblad operator L_j describes the coupling between the system and the jth bath and g_k^j is the coupling constant between the system and kth mode of the jth bath.

Initially the system's Hamiltonian has to be prepared in the ground state $|\psi_0\rangle$ with energy $E_0 = 0$. Assume that all baths are initially in a thermal equilibrium state at temperature T_j with the density operator $\rho_j(0) = e^{-\beta H_b^j}/Z_j$, where $Z_j = {\rm Tr}[e^{-\beta H_b^j}]$ is the partition function, $\beta = 1/(K_B T_j)$. The evolution equation of the system within the weak system-bath coupling approximation is given by (see the Appendix for further details)

$$\frac{\partial}{\partial t}\rho_s = -i[H_s, \rho_s] + \sum_j \Big\{\big[L_j, \rho_s \overline{O}_z^{j\dagger}(t)\big] - \big[L_j^\dagger, \overline{O}_z^j(t)\rho_s\big] + \big[L_j^\dagger, \rho_s \overline{O}_w^{j\dagger}(t)\big] - \big[L_j, \overline{O}_w^j(t)\rho_s\big]\Big\}, \tag{5}$$

with $\overline{O}_{z,(w)}^j = \int_0^t ds\alpha_{z,(w)}^j(t-s)O_z^j$ and $\alpha_{z,(w)}^j(t-s)$ is the correlation function. The operator O is an ansatz (see, for instance, Refs. [50,51]).

To calculate the correlation function, we need to know the spectral density of the bath. Here we use the ohmic type with a Lorentz-Drude cutoff as an example, whose spectrum density is given by $J(\omega) = \frac{\Gamma}{\pi}\frac{\omega}{1+(\frac{\omega}{\gamma})^2}$ [52–54], where Γ, γ is real parameters. γ is the characteristic frequency of the bath and Γ represents the strength of the system-bath coupling. For weak system-bath coupling, $\Gamma \ll 1$ [27]. Using the above approximations, we have

$$\alpha_z^j(t-s) = \Gamma_j T_j \Lambda_j(t,s) + i\dot{\Gamma}_j \Lambda_j(t,s), \tag{6}$$

$$\alpha_w^j(t-s) = \Gamma_j T_j \Lambda_j(t,s), \tag{7}$$

where $\Lambda_j(t,s) = \frac{\gamma_j}{2}e^{-\gamma_j|t-s|}$ is an Ornstein-Uhlenbeck correlation function. The parameter γ_j controls the correlation time of the bath and it decays as $1/\gamma_j$. The larger γ_j corresponds to the smoother spectral function, and thus the shorter time the

bath takes to relax to equilibrium and a more Markovian bath. The operator $\overline{O}^j_{z,(w)}$ satisfies

$$\frac{\partial \overline{O}^j_z}{\partial t} = \left(\frac{\Gamma_j T_j \gamma_j}{2} - \frac{i\Gamma_j \gamma_j^2}{2}\right) L_j - \gamma_j \overline{O}^j_z + \left[-iH_s - \sum_j \left(L_j^\dagger \overline{O}^j_z + L_j \overline{O}^j_w\right), \overline{O}^j_z\right], \quad (8)$$

$$\frac{\partial \overline{O}^j_w}{\partial t} = \frac{\Gamma_j T_j \gamma_j}{2} L_j^\dagger - \gamma_j \overline{O}^j_w + \left[-iH_s - \sum_j \left(L_j^\dagger \overline{O}^j_z + L_j \overline{O}^j_w\right), \overline{O}^j_w\right]. \quad (9)$$

Now by using Eqs. (8) and (9), the master equation in Eq. (5) can be numerically solved.

We now analyze the effects of the heat baths on the performance of the adiabatic quantum search algorithm, where as an example the number of sites is taken as $n = 6$ in the simulations. We calculate the final success probability of the algorithm under different types of interactions. The Lindblad operators are taken as $L_j = \sigma_j^z, \sigma_j^-, \sigma_j^x$, which represent three types of one-qubit errors [27]. $\sigma_j^- = (\sigma_j^x - i\sigma_j^y)/2$ is the lowering operator for a qubit.

First we consider $L = \sigma^z$. Note that for the N individual baths, the parameters for each bath in this paper are taken as the same, $\Gamma_i = \Gamma, T_i = T, \gamma_i = \gamma$ for $i = 1, 2, \ldots, N$. In Figs. 1(a), 1(b) and 1(c), we plot the final success probability P as a function of the normalized time t/S for different parameters Γ, T, and γ. The function $s(t)$ takes a nonlinear path as in Eq. (2), the total evolution time $S = 0.81 S_{\rm opt} \approx 102$. In Fig. 1(a), the success probability decreases with parameters Γ for fixed parameters $\gamma = 0.1$ and $T = 20$. In the absence of baths, the final success probability $P(s = 1) = 0.96$, while with bath $\Gamma = 0.001$, $P(s = 1) = 0.32$. Figure 1(b) plots the effects of the temperature on the performance of the algorithm. Obviously P decreases with increasing T, where $\Gamma = 10^{-4}, \gamma = 0.2$. Higher temperature correlates with worse performance of the algorithm. Figure 1(c) plots P versus the normalized time t/S for different γ with $\Gamma = 10^{-4}$ and $T = 20$. Larger γ correlates with lower P. The lowest $P(s = 1) = 0.23$ corresponds to the Markovian case. Non-Markovianity or the memory effect of the baths plays an important role on enhancement of the success probability.

Next we consider $L = \sigma^-$ and $L = \sigma^x$. In Figs. 2(a) and 2(b), we plot the time evolution of the success probability for different parameters γ ($L = \sigma^-$) and T ($L = \sigma^x$), respectively. The results show again that P decreases with increasing T or γ. Then with the same parameters Γ, T, and γ, how do the three types of interactions affect the success probability? Figure 2(c) plots the comparison of evolutions of the success probability for three models, where $\Gamma = 10^{-4}, \gamma = 1.0$, and $T = 30$. It shows that the detrimental effect of the case $L = \sigma^z$ is the most severe, and $L = \sigma^-$ is at the last. It is interesting to note that the result is different from that in the state transfer [55], where the case $L = \sigma^z$ affects the least.

AQC requires the system to remain in its adiabatic regime, and the *theoretically* required evolution time is infinitely long. On the other hand, the existence of the environment ruins

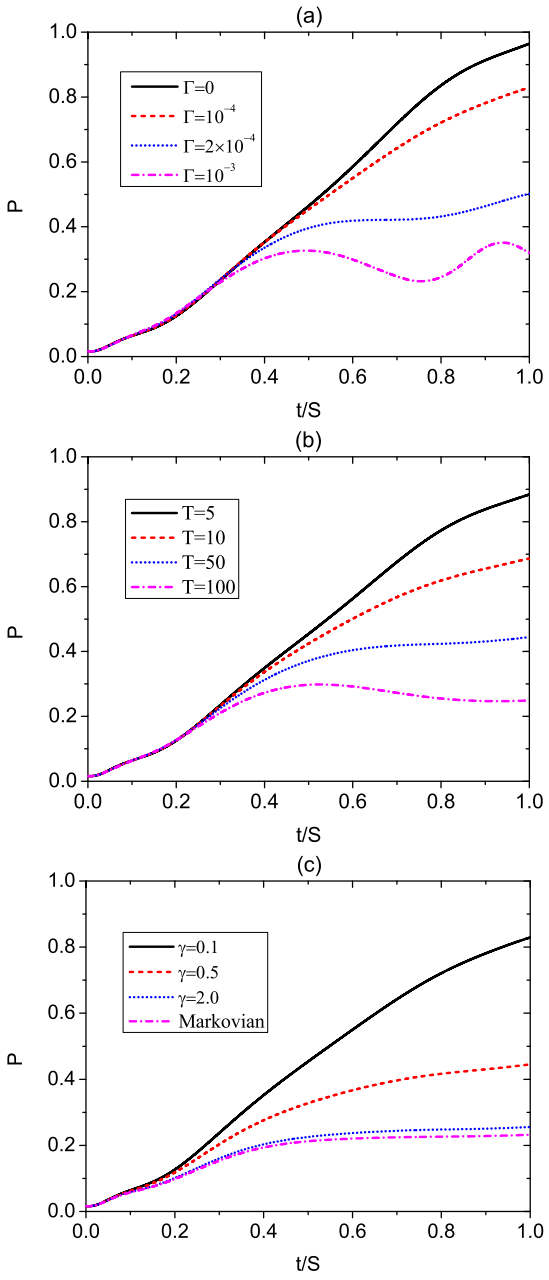

FIG. 1. $L = \sigma^z$: The success probability of the Grover algorithm vs the normalized time t/S for (a) different coupling constants Γ, $\gamma = 0.1$, $T = 20$; (b) different temperature T, $\Gamma = 10^{-4}$, $\gamma = 0.2$; and (c) different parameters γ, $\Gamma = 10^{-4}$, $T = 20$. Results from adiabatic algorithms using a tailored nonlinear path [13]. The total adiabatic time is chosen to be $S = 102$ for qubit number $n = 6$, following $S = 0.81 S_{\rm opt} = 102$ with $\varepsilon = 0.1$ [14].

adiabatic passages. This detrimental effects are accumulated with the runtime [36]. In Fig. 3, we plot the evolution time $S = 10, 100, 400$ with and without environment, where the number of qubits $n = 7$, $s(t) = 1 - t/S$ is taken as a linear path, $\Gamma = 10^{-3}, \gamma = 1.0, T = 10$, and $L = \sigma_-$. Figure 3 shows that without environment the survival probability P will increase with S. $P(1) > 0.99$ when $S = 400$ and the system can be regarded as in the adiabatic regime. Now in the presence of environment, P will decrease. For a shorter

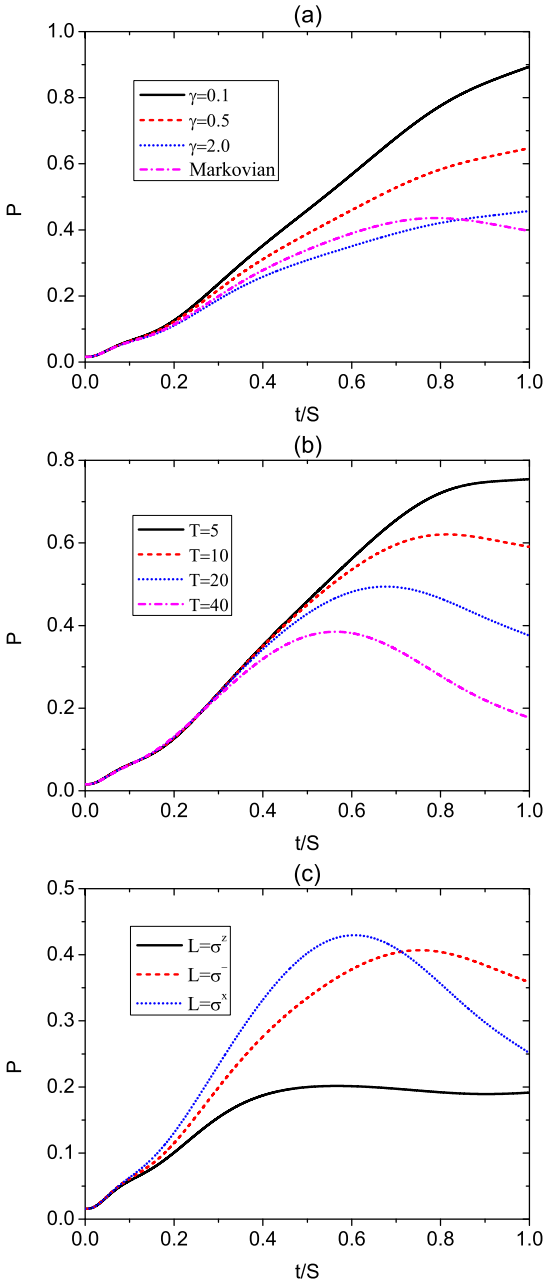

FIG. 2. The success probability of the Grover algorithm vs the normalized time t/S for (a) different parameters γ, $\Gamma = 10^{-4}$, $T = 10$, $L = \sigma^-$; (b) different temperature T, $\Gamma = 10^{-4}$, $\gamma = 1.0$, $L = \sigma^x$; and (c) different Lindblad operators, $\Gamma = 10^{-4}$, $\gamma = 1.0$. $T = 30$. We use a tailored nonlinear path and $S = 102$ as in Fig. 1.

$S = 10$, environment does not affect the survival probability very much; however, for a longer $S = 100$, it reduces the probability considerably and P becomes much lower in the long-time limit. When $S = 400$, the survival probability $P = 0.09$ almost vanishes. Because of the existence of environment, the runtime of an algorithm is required to be as short as possible to avoid accumulation of the detrimental effects, which is contradictory to the adiabatic conditions. As it is not possible to isolate the node system from the system, here we propose a strategy to speedup the adiabatic process using

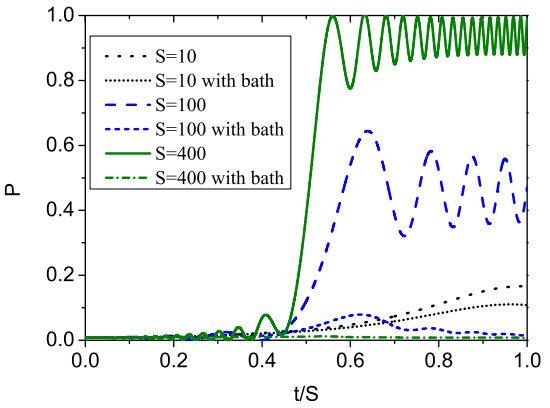

FIG. 3. The success probability of the Grover algorithm vs the normalized time t/S for different S with and without environment; with environment $\Gamma = 10^{-3}$, $\gamma = 1.0$, $T = 10$, $L = \sigma_-$. Results from adiabatic algorithm using a linear path $s(t) = 1 - t/S$. The number of qubits $n = 7$.

external control. We show that the external control helps to shorten the adiabatic evolution time and meanwhile the environmental detrimental effects are reduced [55]. As a result, a high success probability of the algorithm can be reached in a nonadiabatic regime.

IV. ADIABATIC QUANTUM SEARCH ALGORITHM UNDER EXTERNAL CONTROL

In what follows, we will apply a sequence of pulses on the system to speed up the algorithm. We consider a *dressed* Hamiltonian,

$$H_d(t) = [1 + c(t)]H_s(t), \tag{10}$$

where $c(t)$ is a control function, and it can be realized by a sequence of fast pulses in this paper. We here use the so-called zero-energy-change pulses to realize the effective control. The pulses take alternating positive and negative values. For example, for the rectangular pulses $c(t) = \pm I$ (+ for first half period, − for another half period), it was proved that when the energy gaps $\Delta E_{kl} = E_k - E_l$ are constant, the pulse condition is $I_0\tau = 2m\pi (m = \pm 1, \pm 2, ...)$ [47], where I_0 is the pulse intensity and τ is the half pulse period. The control functions such as sine and triangular have also been studied to obtain effective control [48,56,57]. Now we focus on developing a control technique for the adiabatic search algorithm, where the energy gap ΔE_{10} between the ground state and the first excited state is time dependent. It has the analytic expression [13]

$$\Delta E_{10} = \sqrt{1 - 4(1 - 1/N)t(1 - t/S)/S}. \tag{11}$$

This implies that the pulse condition derived by a constant energy gap cannot be directly used. The new condition with time-dependent energy gaps will be [47]

$$\int_t^{t+\tau} I(t')\Delta E_{10}(t')dt' = 2\pi m, \quad m = \pm 1, \pm 2, \ldots. \tag{12}$$

If the pulse intensity is time dependent, $I(t) = I_0/\Delta E_{10}(t)$, the pulse condition becomes

$$I_0\tau = 2m\pi, \quad m = \pm 1, \pm 2, \ldots. \tag{13}$$

Note that the alternating positive-negative pulse sequence can be equivalent to the always-positive pulse sequence where the pulse intensity profile is smooth [36]. This can also be intuitively seen from Eq. (13) that changing I_0 to $-I_0$ does not affect the control conditions. The advantage of the positive-negative pulse sequence is that the average value of the energy is almost zero after a complete control cycle. We will use this positive-negative pulse sequence to speed up the quantum adiabatic process and discuss the success probability. Dynamical decoupling control (B-B control) has been shown to be an effective way to control a quantum dynamical process, where the pulse duration is short and the intensity is ultrastrong [58]. For our nonperturbative control pulse, the pulse intensity and duration are finite and tuneable. Experimentally, time-dependent intensity-modulated [59] or phase-modulated [60] continuous dynamical decoupling has been realized to overcome decoherence against fluctuations in a dense ensemble of nitrogen-vacancy centers in diamond. The additional Hamiltonian can be generated by a time-dependent modulation of the amplitude or phase of the original driving. For implementing our protocol, the energy scales should have I times bigger than the original driving with a pulse duration τ. Furthermore, the pulse imperfections should be considered in a practical experiment. Theoretically the fluctuations of pulse intensity or duration has been considered in the adiabatic speedup via pulse control in a spin system [56]. It shows that the adiabatic speedup is fault tolerant against noise in the pulse intensity or duration.

We consider $L = \sigma^z$, which has the most severe destruction on the system's adiabatic dynamics in our case. Figure 4(a) plots the success probability versus the normalized time t/S with $(\Gamma = 0, 0.001, 0.005)$ and without $(\Gamma = 0.001)$ control. We set $\gamma = 1.0$ and $T = 10$. The control pulses satisfy Eq. (13) with $I_0 = 100$, $\tau = \pi/50$, and $m = 1$. The total evolution time $S = \pi$, which is in a nonadiabatic regime without control $[P(s = 1) = 0.07]$. With control, we obtain a high success probability $[P(s = 1) = 0.88]$ even when there exists a strong system-bath interaction $\Gamma = 0.005$. Figure 4(b) plots the time evolution of success probability for various parameters γ, where $\Gamma = 0.001$ and $T = 10$. S and I_0 are the same as in Fig. 4(a). Figure 4(b) shows that the success probability decreases with increasing γ. It can be dramatically enhanced in the non-Markovian cases, or in other words non-Markovianity helps in enabling the pulse controllability [55]. As for the Markovian cases, it is well known that a control pulse sequence, say, a σ_j^z pulse sequence, is effective mainly for its "orthogonal" component or direction $L_j = \sigma_j^x$ in the system-bath interaction (4). However, our control "rotates" along all directions, so that those directions that do not interact with the bath, for example, σ_j^z, benefit. As a whole, the Markovian cases benefit from the control pulses that are not "perpendicular" to the Markovian baths. We also plot the pulse intensity $I(t)$ versus the normalized time t/S in the inset of Fig. 4(b). The pulse intensity is time dependent and tuned by the energy gap. Narrower energy gaps require bigger pulse intensity to attain the speedup. Figure 4(c) plots the time evolution of success probability for different temperature T, where $\Gamma = 0.002$, $\gamma = 0.5$, $I_0 = 80$, and $\tau = \pi/20$. Figure 4(c) shows that the control is more effective in a lower temperature. The physics is that the transitions between the ground state and the first excited state take place more easily in a higher temperature bath and as a result the adiabaticity

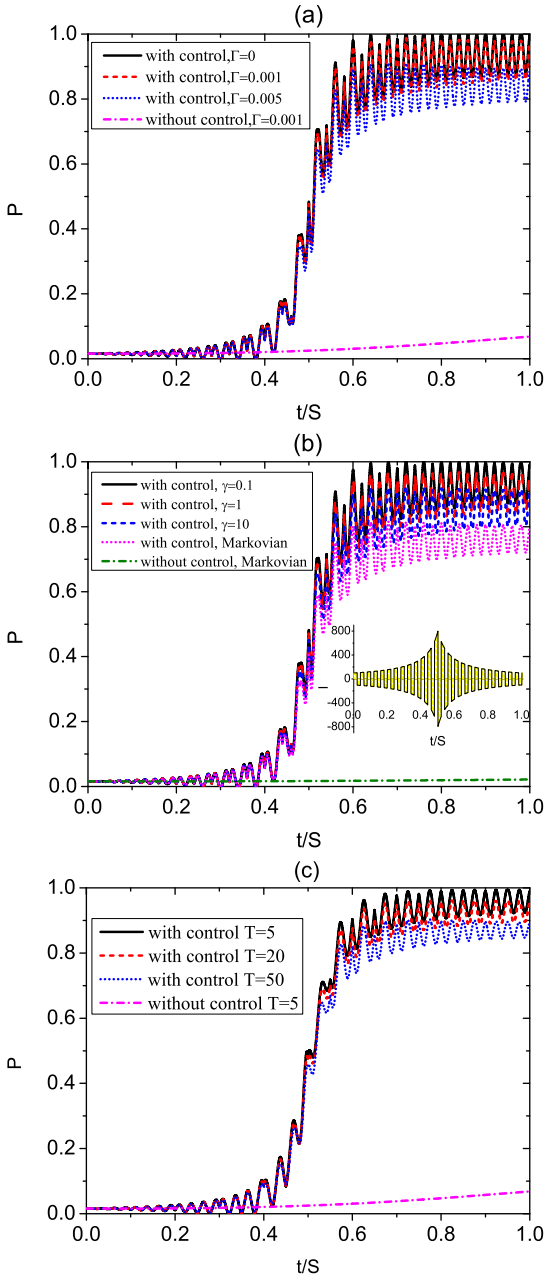

FIG. 4. The success probability of the Grover algorithm vs the normalized time t/S with and without control for (a) different coupling constants Γ, $\gamma = 1.0$, $T = 10$; and (b) different parameters γ, $\Gamma = 0.001$, $T = 10$. To clearly show the pulses we applied, in the right bottom of Fig. 4(b) we plot the pulse intensity vs time. (c) Different temperature T, $\Gamma = 0.002$, $\gamma = 0.5$. Results from adiabatic algorithm using a linear path $s(t) = 1 - t/S$. $S = \pi$, $L = \sigma^x$. For panels (a) and (b), the half-pulse period is taken to be $\tau = \pi/50$, which satisfies the condition $I_0\tau = 2m\pi$, $m = 1$. For panel (c), $\tau = \pi/20$, $m = 2$.

is ruined more than in the case in a lower temperature bath. Again, the effective pulse control can be realized even for a high temperature $T = 50$ [$P(s = 1) = 0.88$] as long as the bath is non-Markovian.

V. CONCLUSIONS

When performing a AQC task, the computational system inevitably suffers from environmental decoherence due to the system-bath interaction. In this paper, we have studied the performance of the adiabatic quantum search algorithm in finite-temperature and non-Markovian heat baths. By using the QSD technique, we are able to study the dynamics of the search algorithm in terms of the success probability. We have focused on the influence of baths on the system for three types of one-qubit error. The results show that the success probability always decreases due to the presence of the environment; in particular the probability decreases with increasing coupling strength Γ and temperature T. Specifically, we have analyzed the effects of non-Markovianity parameter γ on the algorithm. We find that non-Markovianity is crucial in enabling the pulse controllability and enhancing the success probability. To combat the detrimental effects of the environment, we propose an effective pulse control scheme to speed up the adiabatic process, where we add a sequence of zero-energy-change pulses on the system. The pulse conditions which include suitable pulse intensity and period are obtained for a time-dependent energy gap. Once the pulse conditions are satisfied, enhancement of the success probability can be obtained in a much shorter runtime in a nonadiabatic regime for both Markovian and non-Markovian cases. However, it is found that this control strategy can be more effective for a more non-Markovian environment.

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APPENDIX: DERIVATION OF THE NON-MARKOVIAN MASTER EQUATION

The finite-temperature problem can be transformed into the zero-temperature one by adding N fictitious heat baths to the original baths [50,51]. The corresponding QSD equation is [51]

$$\frac{\partial|\Psi_t\rangle}{\partial t} = \left\{-iH_s + \sum_j \left[L_j z_t^{j*} - L_j^\dagger \int ds \alpha_z^j(t-s)\frac{\delta}{\delta z_s^{j*}} + L_j^\dagger w_t^{j*} - L_j \int ds \alpha_w^j(t-s)\frac{\delta}{\delta w_s^{j*}}\right]\right\}|\Psi_t\rangle, \quad \text{(A1)}$$

where

$$\alpha_z^j(t-s) = \sum_k \left(\bar{n}_k^j + 1\right)\left|g_k^j\right|^2 e^{-i\omega_k^j(t-s)}, \quad \text{(A2)}$$

$$\alpha_w^j(t-s) = \sum_k \bar{n}_k^j \left|g_k^j\right|^2 e^{i\omega_k^j(t-s)}, \quad \text{(A3)}$$

are the correlation functions. $\bar{n}_{kj} = \frac{1}{\exp(\hbar\omega_k^j/k_BT)-1}$ is the mean thermal occupation number of quanta in mode ω_k^j;

$$z_t^{j*} = -i\sum_k \sqrt{\bar{n}_k^j + 1} g_k^j z_k^{j*} e^{i\omega_k^j t}, \quad \text{(A4)}$$

$$w_t^{j*} = -i\sum_k \sqrt{\bar{n}_k^j} g_k^{j*} w_k^{j*} e^{i\omega_k^j t}, \quad \text{(A5)}$$

are the jth independent, complex Gaussian noises satisfying

$$M\left[z_t^j z_s^{j*}\right] = \alpha_z^j(t-s), M\left[w_t^j w_s^{j*}\right] = \alpha_w^j(t-s). \quad \text{(A6)}$$

Equation (A1) can have time-local forms by introducing an O operator by using the ansatz [50,51]

$$\frac{\delta}{\delta z_s^{j*}}|\Psi_t\rangle = O_z^j(t, s, z_1^*, w_1^*, ..., z_N^*, w_N^*)|\Psi_t\rangle, \quad \text{(A7)}$$

$$\frac{\delta}{\delta w_s^{j*}}|\Psi_t\rangle = O_w^j(t, s, z_1^*, w_1^*, ..., z_N^*, w_N^*)|\Psi_t\rangle. \quad \text{(A8)}$$

Then Eq. (A1) can be rewritten as

$$\frac{\partial}{\partial t}|\Psi_t\rangle = \left[-iH_s + \sum_j \left(L_j z_t^{j*} - L_j^\dagger \overline{O}_z^j + L_j^\dagger w_t^{j*} - L_j \overline{O}_w^j\right)\right]|\Psi_t\rangle, \quad \text{(A9)}$$

where we have defined $\overline{O}_z^j = \int_0^t ds \alpha_z^j(t-s) O_z^j$, $\overline{O}_w^j = \int_0^t ds \alpha_w^j(t-s) O_w^j$.

The O operators satisfies

$$\frac{\partial O_z^j}{\partial t} = \left[-iH_s + \sum_j \left(L_j z_t^{j*} - L_j^\dagger \overline{O}_z^j + L_j^\dagger w_t^{j*} - L_j \overline{O}_w^j\right), O_z^j\right] - \sum_j \left(L_j^\dagger \frac{\delta \overline{O}_z^j}{\delta z_s^{j*}} + L_j \frac{\delta \overline{O}_w^j}{\delta z_s^{j*}}\right), \quad \text{(A10)}$$

$$\frac{\partial O_w^j}{\partial t} = \left[-iH_s + \sum_j \left(L_j z_t^{j*} - L_j^\dagger \overline{O}_z^j + L_j^\dagger w_t^{j*} - L_j \overline{O}_w^j\right), O_w^j\right] - \sum_j \left(L_j^\dagger \frac{\delta \overline{O}_z^j}{\delta w_s^{j*}} + L_j \frac{\delta \overline{O}_w^j}{\delta w_s^{j*}}\right). \quad \text{(A11)}$$

For a weak system-bath coupling, noise-dependent $\overline{O}(t, z_1^*, w_1^*, ..., z_N^*, w_N^*)$ operators can be approximated well [61] by noise-independent operators $\overline{O}_z^j(t, z_1^*, w_1^*, ..., z_N^*, w_N^*) = \overline{O}_z^j(t)$ and $\overline{O}_w^j(t, z_1^*, w_1^*, ..., z_N^*, w_N^*) = \overline{O}_w^j(t)$. Then the evolution equation (5) is obtained.

When we introduce the spectral density $J(\omega_j)$, the correlation functions in Eqs. (A2) and (A3) are

$$\alpha_z^j(t-s) = \int d\omega_j J(\omega_j)\left(\bar{n}_k^j + 1\right)e^{-i\omega_j(t-s)}, \tag{A12}$$

$$\alpha_w^j(t-s) = \int d\omega_j J(\omega_j)\bar{n}_k^j e^{i\omega_j(t-s)}. \tag{A13}$$

Now in the high-temperature or low-frequency limit, let $k_B = 1.0 = \hbar$, and we have the approximations $\frac{1}{\bar{n}_k^j - 1} \approx T_j/\omega_j$. For the two correlation functions in Eqs. (6) and (7), we both have the relations

$$\frac{\partial \alpha_{z(w)}^j(t-s)}{\partial t} = -\gamma_j \alpha_{z(w)}^j(t-s). \tag{A14}$$

Using the above relation and Eqs. (A10) and (A11), finally we obtain Eqs. (8) and (9).

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