Rapid Communications

Acceleration noise constraints on gravity-induced entanglement

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It has been proposed that quantum features of the gravitational field can be exposed experimentally by employing gravity as a mediator of entanglement. We show that in order to witness this type of entanglement experimentally, strong limits on acceleration noise, which has been neglected in previous work, must be overcome. In the case of two particles of similar mass, Casimir-Polder forces lead to a fundamental limit of tenths of $\mathrm{fm\,s^{-2}/\sqrt{Hz}}$. Limits are between three and six orders of magnitude less strict for two particles of unequal mass, depending on collisional decoherence.

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Despite tremendous efforts in quantum gravity research, there is no empirical evidence, to date, as to whether or not the gravitational field must be quantized [1,2]. Indirect arguments for the necessity of its quantization [3,4] are generally considered inconclusive [2,5]. Proposals for experimental tests [6–8] focus on the specific semiclassical model where curvature of a classical space-time is sourced by the modulus squared of the quantum state [1,9,10]. On the other hand, experiments to test classical gravitational forces in micromechanical systems [11] are still a long way from probing gravitational fields sourced by nonclassical states, leaving a large gap between systems with observed quantum features on the one hand and systems whose gravitational fields have been measured on the other.

Quantum entanglement, which is often considered the most characteristic feature that separates quantum systems from the classical world, may serve as a means to close this gap. For two quantum particles interacting only gravitationally, it is expected that a quantized gravitational field can yield an entangled state, whereas classical space-time curvature cannot. In a recent letter, Bose *et al.* [12] propose an idea how to use spin as a witness for this type of gravitationally induced entanglement. Two spin-$\frac{1}{2}$ particles are each put into a spatial superposition state, where one part of the superposition of each particle experiences a gravitational pull depending on the state of the other particle. This results in a conditional phase shift, which can yield nonclassical spin correlations.

As a concrete realization, Bose *et al.* propose to use micrometer-sized diamonds, initially separated by 450 μm. In a magnetic field gradient of 10^6 T/m these are split up for half a second to yield a superposition of 250 μm each, such that the two closer parts of the superposition approach each other at 200 μm distance. After moving parallelly for 2.5 s, a reversed magnetic field gradient rejoins both particle states.

These parameters are carefully chosen: distances between particles must remain large enough for Casimir-Polder forces not to supercede gravitational ones, flight times shorter than relevant decoherence times, and field strengths technologically feasible, with the gravitational potential still yielding a sufficiently large phase shift.

There is, however, an obvious caveat: as the gravitational acceleration scales with R^3/L^2, R being the source mass radius and L its distance, the acceleration resulting from the micrometer particle at 200 μm distance is matched by the gravitational acceleration of a centimeter particle in kilometer distance. Hence, one should ask why an experiment sensitive to the former should not be influenced by the latter.

As long as both the particles and the experimental setup, including the magnetic field gradient, are in perfect free fall, the equivalence principle prevents any observable effect of external homogeneous gravitational fields. The proposed experiment, therefore, is ideally performed in a zero gravity environment. If, however, external forces act on the particles and the rest of the experiment differently, such that either the particles or the magnetic fields experience an acceleration relative to the geodesic motion of the center of gravity of the entire experiment, one ends up with a residual observable phase, similar to the famous Colella-Overhauser-Werner experiment [13,14].

Residual acceleration cannot be entirely avoided even in zero gravity, where it can be expressed in the form of noise spectra that for subhertz frequencies resemble white noise. On Earth, typical residual accelerations are micro-g; Selig *et al.* [15] describe an approximately frequency-independent noise spectrum around $\sqrt{S_0} \sim 10^{-7}\mathrm{m\,s^{-2}/\sqrt{Hz}}$ for frequencies below 10 Hz in drop tower experiments. In space, the LISA pathfinder mission [16] minimized acceleration noise as a main objective. The acceleration noise spectrum shows a frequency-independent value around $\sqrt{S_0} \sim 5.6\ \mathrm{fm\,s^{-2}/\sqrt{Hz}}$ in the subhertz range, with a significant increase for frequencies above 0.1 Hz. We will use these two values as a reference for feasible noise levels on Earth and in space, respectively.

a. Gravitational phase in spatial superpositions. A quantum particle in a gravitational potential experiences a phase shift [13] which can be derived as a perturbative effect around the quasiclassical trajectory. As in Ref. [12], we consider two spin-$\frac{1}{2}$ particles at positions $\boldsymbol{r}(t)$ and $\boldsymbol{s}(t)$, respectively,

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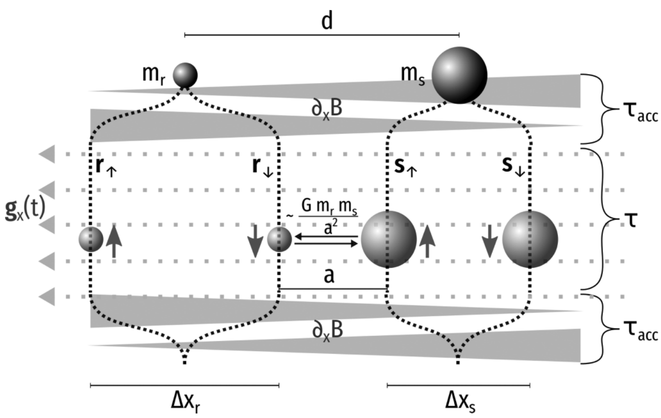

FIG. 1. Depiction of the experimental scenario: spin-$\frac{1}{2}$ particles with masses m_r and m_s are put in spatial superposition states with trajectories $\boldsymbol{r}_{\uparrow\downarrow}$ and $\boldsymbol{s}_{\uparrow\downarrow}$, respectively, in a magnetic field gradient (with a spin flip after $\tau_{\rm acc}/2$ effectively inverting the field). This acceleration phase of duration $\tau_{\rm acc}$ is followed by a free flight phase for time τ, followed by the deceleration (again for $\tau_{\rm acc}$). The final state will exhibit a phase that depends on both the gravitational interaction between both particles and the external acceleration $g_x(t)$.

which for the remainder of this Rapid Communication will be labeled by r and s. The initially separable state is subject to a magnetic field gradient for a time $\tau_{\rm acc}$, entangling spin and position. The time τ of free flight is followed by an opposite field gradient, again for time $\tau_{\rm acc}$. If $\boldsymbol{r}_\uparrow(t)$, $\boldsymbol{r}_\downarrow(t)$, $\boldsymbol{s}_\uparrow(t)$, and $\boldsymbol{s}_\downarrow(t)$ denote the quasiclassical trajectories of the spin eigenstates, as depicted in Fig. 1, the final state will be

$$|\Psi\rangle = \frac{e^{i\phi_{\uparrow\uparrow}}}{2}\,|\uparrow\uparrow\rangle + \frac{e^{i\phi_{\uparrow\downarrow}}}{2}\,|\uparrow\downarrow\rangle + \frac{e^{i\phi_{\downarrow\uparrow}}}{2}\,|\downarrow\uparrow\rangle + \frac{e^{i\phi_{\downarrow\downarrow}}}{2}\,|\downarrow\downarrow\rangle. \tag{1}$$

In an accelerated frame with time-dependent acceleration $\boldsymbol{g}(t)$, the phases are (cf. Supplemental Material S1 [17]):

$$\phi_{\uparrow\uparrow} = \frac{Gm_rm_s}{\hbar}\int_0^t \frac{dt'}{|\mathbf{r}_\uparrow(t')-\mathbf{s}_\uparrow(t')|} + \frac{1}{\hbar}\int_0^t dt'\,\mathbf{g}(t')[m_r\mathbf{r}_\uparrow(t')+m_s\mathbf{s}_\uparrow(t')], \tag{2}$$

and accordingly for $\phi_{\uparrow\downarrow}$, $\phi_{\downarrow\uparrow}$, and $\phi_{\downarrow\downarrow}$.

We choose as a reference frame the initial rest frame of the two particles, with the x axis defined by the particle positions at time $t=0$, i.e., $\boldsymbol{r}_\uparrow(0)=\boldsymbol{r}_\downarrow(0)=(-d/2,0,0)$ and $\boldsymbol{s}_\uparrow(0)=\boldsymbol{s}_\downarrow(0)=(d/2,0,0)$.

We split up each of the phases into the contributions from the mutual gravitational interaction during the free flight time τ, during the initial and final acceleration periods, as well as the phases for each trajectory due to the external acceleration: $\phi_{\uparrow\uparrow}=\phi^\tau_{\uparrow\uparrow}+\phi^{\rm acc}_{\uparrow\uparrow}+\phi^{\rm ext}_{r\uparrow}+\phi^{\rm ext}_{s\uparrow}$, and accordingly for the other three phases.

With $\Delta x=(\Delta x_r+\Delta x_s)/2$ and $\delta x=(\Delta x_r-\Delta x_s)/2$, we find that these phases are

$$\begin{aligned}
\phi^\tau_{\uparrow\uparrow} &= \frac{Gm_rm_s\tau}{\hbar(d+\delta x)}, & \phi^{\rm ext}_{r\uparrow} &= -\frac{m_rd}{2\hbar}v_x+\frac{1}{2}\chi,\\
\phi^\tau_{\uparrow\downarrow} &= \frac{Gm_rm_s\tau}{\hbar(d+\Delta x)}, & \phi^{\rm ext}_{r\downarrow} &= -\frac{m_rd}{2\hbar}v_x-\frac{1}{2}\chi,\\
\phi^\tau_{\downarrow\uparrow} &= \frac{Gm_rm_s\tau}{\hbar(d-\Delta x)}, & \phi^{\rm ext}_{s\uparrow} &= \frac{m_sd}{2\hbar}v_x+\frac{1}{2}\chi,\\
\phi^\tau_{\downarrow\downarrow} &= \frac{Gm_rm_s\tau}{\hbar(d-\delta x)}, & \phi^{\rm ext}_{s\downarrow} &= \frac{m_sd}{2\hbar}v_x-\frac{1}{2}\chi
\end{aligned} \tag{3}$$

with the velocity

$$v_x = \int_0^{\tau+2\tau_{\rm acc}} dt\; g_x(t), \quad \text{as well as} \tag{4}$$

$$\chi \approx \frac{\mu_B\,\partial_xB\,\tau_{\rm acc}^2}{2\hbar}\int_{\tau_{\rm acc}}^{\tau+\tau_{\rm acc}} dt\; g_x(t). \tag{5}$$

Detailed calculations, including phases for the acceleration periods, can be found in the Supplemental Material S1 [17].

b. Effect of random external acceleration. Let us now address the phase χ due to the external acceleration. Evidently, only the component g_x of the acceleration parallel to the x axis (defined by the particle positions) affects the phase. The experimental setup will generally be chosen in such a way that the time average is $\langle g_x(t)\rangle \approx 0$, for instance by aligning the field gradient and the particles parallel to the surface of the Earth. However, there will be fluctuations of g_x in time which can be associated with a noise spectrum $S(\omega)$ through the correlation functions

$$\langle g_x(0)g_x(t)\rangle = \int \frac{d\omega}{2\pi} S(\omega)e^{-i\omega t}. \tag{6}$$

We assume that the acceleration noise is well approximated by white Gaussian noise, i.e., $S(\omega)\approx S_0$. To obtain the variance of the phase χ over the averaging time τ, according to Eq. (5), one can apply a low-pass filter with bandwidth $1/\tau$ [18] yielding

$$\begin{aligned}
\Delta\chi^2 &= \left(\frac{\mu_B\,\partial_xB\,\tau_{\rm acc}^2}{2\hbar}\right)^2\tau^2\int\frac{d\omega}{2\pi}\frac{S_0}{1+\omega^2\tau^2}\\
&= \frac{m_rm_s\Delta x^2}{4\hbar^2}S_0\tau.
\end{aligned} \tag{7}$$

We find that in repeated measurements the phase will be distributed around $\chi=0$ with a probability density

$$P(\chi) = (2\pi\,\Delta\chi^2)^{-1/2}\exp\left(-\frac{\chi^2}{2\Delta\chi^2}\right). \tag{8}$$

c. Constraints on witnessing entanglement. Focusing, for now, on the situation where both particles have similar masses, $m_r\approx m_s$, we have $\Delta x\approx\Delta x_r\approx\Delta x_s$ and $\delta x\ll\Delta x<d$. If we extract from the phases (2) the global phase ϕ we can write the final state as

$$\frac{e^{i\phi}}{2}(e^{i\tilde{\chi}}\,|\uparrow\uparrow\rangle + e^{i\delta\phi}\,|\uparrow\downarrow\rangle + e^{i\Delta\phi}\,|\downarrow\uparrow\rangle + e^{-i\tilde{\chi}}\,|\downarrow\downarrow\rangle), \tag{9}$$

with $\widetilde{\chi} = \chi - \delta\chi$ (cf. Supplemental Material S2 [17]) and

$$\Delta\phi = \frac{Gm_r m_s \tau}{\hbar(d-\Delta x)} - \frac{Gm_r m_s \tau}{\hbar d} + \Delta\phi_-^{\text{acc}}, \tag{10a}$$

$$\delta\phi = \frac{Gm_r m_s \tau}{\hbar(d+\Delta x)} - \frac{Gm_r m_s \tau}{\hbar d} - \Delta\phi_+^{\text{acc}}. \tag{10b}$$

Repeated measurements yield a density matrix [using Eq. (8) in the basis $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$]:

$$\hat{\rho} = \int d\chi\, P(\chi)\,|\,\Psi(\chi)\rangle\langle\Psi(\chi)| = \frac{1}{4}\begin{pmatrix} 1 & e^{-\gamma-i(\delta\phi+\delta\chi)} & e^{-\gamma-i(\Delta\phi+\delta\chi)} & e^{-4\gamma-2i\delta\chi} \\ e^{-\gamma+i(\delta\phi+\delta\chi)} & 1 & e^{-i(\Delta\phi-\delta\phi)} & e^{-\gamma+i(\delta\phi-\delta\chi)} \\ e^{-\gamma+i(\Delta\phi+\delta\chi)} & e^{i(\Delta\phi-\delta\phi)} & 1 & e^{-\gamma+i(\Delta\phi-\delta\chi)} \\ e^{-4\gamma+2i\delta\chi} & e^{-\gamma-i(\delta\phi-\delta\chi)} & e^{-\gamma-i(\Delta\phi-\delta\chi)} & 1 \end{pmatrix}. \tag{11}$$

With $\gamma = \Delta\chi^2/2$ and the spin correlations

$$\langle\sigma_x \otimes \sigma_z\rangle = \frac{e^{-\gamma}}{2}[\cos(\Delta\phi + \delta\chi) - \cos(\delta\phi - \delta\chi)], \tag{12a}$$

$$\langle\sigma_y \otimes \sigma_y\rangle = \frac{1}{2}[\cos(\Delta\phi - \delta\phi) - e^{-4\gamma}\cos(2\delta\chi)], \tag{12b}$$

one finds for the entanglement witness $\mathcal{W}$ as in Ref. [12],

$$\mathcal{W} = |\langle\sigma_x \otimes \sigma_z\rangle + \langle\sigma_y \otimes \sigma_y\rangle| \leqslant \frac{1}{2} + e^{-\gamma} + \frac{e^{-4\gamma}}{2}. \tag{13}$$

Evidently, for $\gamma = 0$ one recovers the result by Bose *et al.*, that $0 \leqslant \mathcal{W} \leqslant 2$. However, for finite γ, in order to find $\mathcal{W} > 1$ and, therefore, evidence for nonclassical behavior, one requires $\gamma \lesssim \gamma_{\max} \approx 0.75$ or

$$S_0 \lesssim \frac{8\gamma_{\max}\hbar^2}{m_r m_s \tau\,\Delta x^2} \approx \frac{6\hbar^2}{m_r m_s \tau\,\Delta x^2}. \tag{14}$$

Equation (14) puts a limit on witnessing entanglement. To stay below the limit where acceleration noise constraints the entanglement witness to values below unity, either the masses, or flight time τ, or the superposition size Δx must be sufficiently small. On the other hand, those exact parameters need to be sufficiently large for the gravitational phase to be significantly different from zero.

d. Closest approach and Casimir-Polder forces. For the gravitational interaction of the two particles to dominate, it must be stronger than all other interactions between the two particles. Otherwise, the gravitationally induced phase difference will be obfuscated. For neutral particles, the most long-range forces stem from the Casimir-Polder interaction. Hence, we require, as has been required in Ref. [12], that the gravitational potential must be significantly stronger than the Casimir-Polder energy [19,20]:

$$\frac{Gm_r m_s}{a} \gg \frac{23}{4\pi}\left(\frac{3}{4\pi}\right)^2 \frac{\hbar c \alpha_r \alpha_s m_r m_s}{\rho_r \rho_s a^7}, \tag{15}$$

where $\rho_{r,s}$ are the densities of the two particles (still assuming that both particles are of almost equal size). The polarizability $\alpha = (\varepsilon^2 - 1)/(\varepsilon^2 + 2)$ can be derived from the static relative permittivity ε (for nonferromagnetic materials with relative permeability $\mu \approx 1$). In the limit $\varepsilon \to \infty$ (metals), one finds $\alpha = 1$, whereas the lowest naturally occurring permittivities for dielectrics are around 2.6 for lead(II) acetate [21], limiting the polarizability to values between $0.35 \lesssim \alpha \leqslant 1$.

Assuming $\rho_r = \rho_s = \rho$ and $\alpha_r = \alpha_s = \alpha$, Eq. (15) results in a limit on the distance a between particles:

$$a \gg \frac{1}{2\sqrt{\pi}}\left(\frac{3\alpha}{\rho}\sqrt{\frac{23\hbar c}{G}}\right)^{1/3}. \tag{16}$$

For $a \ll \Delta x$, we find the phases $\Delta\phi \approx Gm_r m_s \tau/(\hbar a)$ and $\delta\phi \approx 0$ (cf. Supplemental Material S2 [17]). According to Eq. (12), entanglement occurs if $\Delta\phi$ is close to an odd multiple of π, which together with Eqs. (16) and (14) yields a limit on the acceleration noise:

$$S_0 \ll \frac{4}{\sqrt{\pi}\Delta x^2}\left(\frac{81\hbar^5 G^7 \rho^2}{23c\alpha^2}\right)^{1/6}. \tag{17}$$

This is an interesting result, showing that for given mass density and permittivity (which are both limited by material choices) the *only* way to overcome acceleration noise is to *decrease* the size of the superposition Δx. If this may sound unintuitive at first, remember that the phase uncertainty $\Delta\chi$ scales with Δx.

It is intuitively clear (and we show in Supplemental Material S2 [17]) that for $\Delta x \ll a$ no entanglement can be witnessed due to the closeness of the classical trajectories. Hence, Eq. (17) poses the least strict constraint in the situation where $a \approx \Delta x$. The requirement for a detectable gravitational phase is then

$$\Delta\phi - \delta\phi \approx \frac{2}{3}\frac{Gm_r m_s \tau}{\hbar a} \approx (2n+1)\pi \quad (n \in \mathbb{N}). \tag{18}$$

For $\mathcal{W} > 1$ one then requires $\gamma \lesssim 0.5$ from Eqs. (12), which together with Eqs. (18) and (14) yields

$$S_0 \lesssim \frac{8\hbar G}{3\pi a^3} \ll \frac{64\rho}{9\alpha}\sqrt{\frac{\pi\hbar G^3}{23c}}. \tag{19}$$

We found an absolute limit for the acceleration noise, depending only on the material properties (density and polarizability). Essentially, the Casimir-Polder force puts an absolute limit on the particle distance a; the requirement to have a detectable gravitational phase shift then requires $\Delta x \gtrsim a$ as well as $m^2\tau$ above some limit. Hence, the phase uncertainty $\Delta\chi \sim m^2 \Delta x^2 \tau S_0$ is limited from below by the noise S_0 only, yielding the absolute limit for said noise.

With the values for diamond (as used in [12]), $\varepsilon = 5.7$ and $\rho = 3.5\ \text{g/cm}^3$, Eq. (19) yields $\sqrt{S_0} \ll 0.07\ \text{fm}\,\text{s}^{-2}/\sqrt{\text{Hz}}$. If instead we take into account that for realistic materials

$\alpha > 0.35$ and the element with the largest density is osmium with $\rho = 23$ g/cm^3, we get an absolute limit of $\sqrt{S_0} \ll 0.24$ fm s$^{-2}/\sqrt{\text{Hz}}$, which is more than an order of magnitude below the acceleration noise achieved by the LISA Pathfinder mission [16].

It should be stressed, that this represents a best case scenario, which cannot be superceded by any choice of materials, particle mass, distances, magnetic field gradients, etc. The only major assumption entering our considerations which could fundamentally change this result is the similarity of the masses $m_r \approx m_s$.

e. Particles of different mass. Let us now consider the case where the two particles are of considerably different masses, $m_r \ll m_s$. Firstly, note that in the situation where a single magnetic field gradient is used to create the superpositions we have $\Delta x_r \gg \Delta x_s$ and the smaller superposition Δx_s will be too small to allow for significant entanglement (cf. Supplemental Material S3 [17]).

Hence, we assume two different field gradients, chosen such that $\Delta x_r \approx \Delta x_s$ despite the largely different masses $m_r \ll m_s$. Then only the mass m_s contributes to the phase χ, which will be half as large with otherwise identical results as in the case $m_r \approx m_s$ before. Instead of Eq. (14), we then have

$$S_0 \lesssim \frac{32\gamma_{\max}\hbar^2}{m_s^2 \tau \Delta x^2}. \tag{20}$$

Rather than by Casimi-Polder forces, the minimal approach distance is determined by the radius R of the larger particle. As before, observable entanglement with the largest possible acceleration noise is achieved in the case $\Delta x \approx a \approx R \approx d/2$, requiring $\gamma \lesssim 0.5$. Hence, Eq. (20) leads to

$$\sqrt{S_0} \lesssim \frac{3\hbar}{\pi \rho R^4 \sqrt{\tau}}, \tag{21}$$

implying that smaller radius R and flight time τ allow for larger acceleration noise. However, since the second particle must be smaller and $m_r m_s \tau$ sufficiently large for an observable phase, $R^4\sqrt{\tau}$ cannot be arbitrarily small.

Firstly, the time τ must be smaller than the decoherence time from collisional decoherence [22],

$$\tau < \frac{\sqrt{k_B T m_{\text{gas}}}}{16\sqrt{3}\zeta(3/2) P R^2}, \tag{22}$$

where we assume a gas environment with particles of mass m_{gas} at pressure P and temperature T, k_B being the Boltzmann constant and ζ the Riemann zeta function. In combination with the requirement (18) for a detectable phase we then find a limit on the smaller mass m_r:

$$m_r \geqslant \frac{9\hbar}{8G\rho R^2 \tau} > \frac{18\sqrt{3}\zeta(3/2)\hbar P}{G\rho\sqrt{k_B T m_{\text{gas}}}}. \tag{23}$$

On the other hand, Eq. (23) also implies

$$R^4\sqrt{\tau} = \frac{3}{4\pi\rho} m_s R \sqrt{\tau} \geqslant \frac{9\sqrt{\hbar} m_s}{8\pi\sqrt{2G\rho^3 m_r}} \gg \frac{27\hbar\sqrt{\zeta(3/2)P}}{8\pi G\rho^2}\left(\frac{1}{3}k_B T m_{\text{gas}}\right)^{-1/4}, \tag{24}$$

where we used $m_s \gg m_r$ together with (23) in the last step. Inserting this result into Eq. (21), we obtain a limit for the acceleration noise:

$$\sqrt{S_0} \ll \frac{8G\rho}{9\sqrt{\zeta(3/2) n_{\text{gas}}}}\left(\frac{m_{\text{gas}}}{3k_B T}\right)^{1/4}, \tag{25}$$

where we used the ideal gas equation $P = n_{\text{gas}} k_B T$ with particle density n_{gas}.

For the parameters assumed by Bose *et al.* [12], diamond at 10^{-15} Pa and 150 mK, with m_{gas} for nitrogen we obtain a limit of 1.4 pm s$^{-2}/\sqrt{\text{Hz}}$.

f. Discussion. Although Eq. (25) poses a weaker limit than (19), noise requirements are still orders of magnitude below what is usually achieved on Earth. Contrary to the limit (19) for particles of similar mass, the constraint (25) is not limited in an absolute sense by fundamental parameters and material properties. Nonetheless, even at the vacuum quality of the interstellar medium with $n_{\text{gas}} \sim 1/\text{cm}^3$ and microkelvin temperatures one would require acceleration noise below nm s$^{-2}/\sqrt{\text{Hz}}$. Verifying gravitationally induced entanglement with an acceleration noise background above this value, which includes typical experiments on Earth, seems extremely challenging, if not infeasible.

As far as potential loopholes in our arguments are concerned, neither the assumption of white noise, nor imperfections of the field gradient indicate any obvious path towards overcoming the limitations posed by acceleration noise. Quite to the contrary, it seems reasonable that loosening these assumptions will only result in additional noise. Stochastic fluctuations in the preparation of the experiment have been studied by Nguyen and Bernards [23]. Inverting the requirement that the acceleration period be short compared to the free flight, $\tau_{\text{acc}} \ll \tau$, dominant contributions to both the gravitational phase $\Delta\phi$ and the noise phase χ will stem from the acceleration period rather than the free flight, and decoherence will restrict τ_{acc} rather than τ. Although calculations are more tedious for this case, it seems evident that our considerations remain valid, at least as far as the orders of magnitude of relevant effects are concerned. Similarly, a deviation from the assumption that $\Delta x_r \approx \Delta x_s$ in the scenario of unequal particle masses will not result in significantly different bounds. The proposal to screen Casimir forces with a thin, perfectly conducting plate [24] can avoid the stronger limit (17); however, the limitation (25) remains in effect even in the case of similar particle masses.

One could think about a setup where one gains statistical data from an arrangement of identical and simultaneous experiments, rather than in a series of repeated measurements. A time-dependent external acceleration $\boldsymbol{g}(t)$ would only contribute to an overall phase which is the same for all measurements, although spatial fluctuations of $\boldsymbol{g}$ would still be required to be sufficiently small. It is beyond the scope of this

Rapid Communication to judge the feasibility of such an idea. However, relative acceleration between the different copies would pose problems and creating copies of the experiment that are almost perfectly identical regarding particle masses, distances, and magnetic fields appears to be a tremendous challenge.

A slightly more promising workaround could be to precisely monitor the acceleration noise and actively correct for it when calculating correlations. At least for the pm $s^{-2}/\sqrt{Hz}$ noise limit for realistic pressures and temperatures, this would, however, push the capability limits of state-of-the-art accelerometers. Therefore, a detailed discussion of a feasible mitigation mechanism is still pending and should be far up on the list of challenges to be faced in order to test gravitationally induced entanglement on Earth.

Our analysis focused on the specific scenario outlined by Bose *et al.* [12], where spin is used as an entanglement witness for gravitational interactions. The main results, however, are quite generally applicable. The precise mechanism used to create spatial superposition states is irrelevant, as long as different parts of the superposition are subject to different gravitational potentials. The decision to use spin as an entanglement witness is also merely a practical consideration: essentially the entanglement occurs purely due to the position superposition and could potentially be witnessed in any way. Our results, therefore, show with rather general applicability that experimental attempts to witness the entanglement between two massive particles due to their gravitational interaction can only be successful in an environment with very low acceleration noise. There seems no obvious route towards conducting such an experiment on Earth. Acceleration noise should play a crucial role in the evaluation of the feasibility of any possible scenario, including space missions.

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