Capturing nonexponential dynamics in the presence of two decay channels

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The most unstable quantum states and elementary particles possess more than a single decay channel. At the same time, it is well known that typically the decay law is not simply exponential. Therefore, it is natural to ask how to spot the nonexponential decay when (at least) two decay channels are opened. In this work, we study the tunneling phenomenon of an initially localized particle in two spatially opposite directions through two different barriers, mimicking two decay channels. In this framework, through specific quantum mechanical examples which can be accurately solved, we study the general properties of a two-channel decay that apply for various unstable quantum states (including unstable particles). Apart from small deviations at early times, the survival probability and the partial tunneling probability along the chosen direction are very well described by the exponential-decay model. In contrast, the ratios of the decay probabilities and probability currents are evidently not a simple constant (as they would be in the exponential limit), but display time-persisting oscillations. Hence, these ratios are optimal witnesses of deviations from the exponential-decay law.

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I. INTRODUCTION

The fact that the decay law in quantum mechanics (QM) is not described by an exponential function is well established [1–13]. In particular, decaying systems very often exhibit the *Zeno period* at short initial times, in which the nondecay probability, i.e., the probability $p(t)$ that the unstable particle prepared at the initial time $t = 0$ has not decayed yet at a later time $t > 0$, is quadratic in time, $p(t) - 1 \propto -t^2$. On the other hand, for very long times (typically several orders of magnitude larger than the lifetime [2]), the nondecay probability is typically governed by a power law. From the experimental point of view, the deviations from the exponential decay have been verified at short times in the study of tunneling of sodium atoms in an optical potential [14] and, more recently, in the study of decays of unstable molecules via the emission of photons [15]. Even if ubiquitous from a theoretical point of view, in physical systems the deviations from the exponential case are typically very small, making them very difficult to be measured.

Quite remarkably, the nonexponential decay also allows one to influence the decay rate by changing the way the measurement is performed. As examples, the famous Quantum Zeno Effect (QZE) and the Inverse Zeno Effect (IZE) are direct consequences of the peculiarity of the decay law [16–27] . Indeed, experimental confirmation of both the QZE and the IZE was achieved in experiments in which electrons undergo a Rabi transition between atomic energy levels [28–30]. In these cases, the nondecay probability oscillates in time as $\sim\cos^2(\Omega t)$ and is evidently nonexponential. Even if this is not a real unstable system, the slowdown of the quantum transition by frequent measurements could be seen in these experiments. Even more interestingly, these effects were also confirmed in the tunneling of sodium atoms, which represent a genuine irreversible quantum decay [31]. Finally, the QZE and IZE are also related to the quantum computation and quantum control, which are important elements in this flourishing research field [32,33].

Deviations from the exponential-decay law are indeed expected also in quantum field theory (QFT), which is the ultimate correct framework to study the creation and annihilation of particles, and hence the decay of unstable particles [10,34,35]. Namely, even if a perturbative treatment is not capable to capture such deviations [36], the spectral function in QFT is not a Breit-Wigner [37–39] and, in some cases, it can be very different from it [40]. As a consequence, the decay law is also not a simple exponential. Unfortunately, a direct experimental proof of the nonexponential decay of unstable elementary particles is still missing. Nonetheless, the Zeno effect confirmed recently in cavity QED [41] suggests that different dynamical features of the simplest QM systems may also have their counterparts in different purely QFT situations.

An interesting case is realized when an unstable quantum state (or particle) can decay in (at least) two channels. Indeed, this situation takes place very often in Nature. For instance, in the realm of particle physics, most unstable particles possess multiple decay channels [42]. Similarly, electrons in excited atoms can decay in more than a single energy level [43].

As expected, in the exponential limit, the ratio of the decay probabilities into the first and the second channel is a constant. A detailed study of the nonexponential decay when two (or more) decay channels are present is described in [10]. In QM, this ratio is not a constant, but shows some peculiar and irregular oscillations, which in [10] were discussed in the framework of the so-called Lee model [44,45] (also called the Friedrichs model or the Jaynes-Cummings model [43,46]), which captures the most salient features of QFT (for details, see [10,47–50]). Moreover, qualitatively similar results for the ratio of the partial decay probability currents were obtained in [10], also in a quantum field theoretical model. Yet, the topic of nonexponential decay in the presence of more decay channels needs novel and different studies that will allow us to understand, in more detail, its features and make an experimental verification (or falsification) possible.

In this work, we explore the two-channel decay problem in a quantum mechanical context. To this aim, we introduce a simple model of a single particle initially confined in a box potential whose walls are suddenly partially released, allowing the particle to tunnel to the open space. In this way, we slightly generalize the celebrated *Winter's model* [3], where only a single box wall is released. The Winter model is recognized as one of the most important workhorses in the theory of nonexponential decays (see, for example, [4–9] and [51] for a general treatment). In our work, we want to mimic two different channels of a decay and therefore we focus on situations of essentially different barriers. In contrast to the symmetric situation of identical barriers [52–54], in this case the exact analytical solution is known only for the scattering problem of external wave packets [55–59] and it does not provide a straightforward solution for the decay scenario studied here [60]. Specifically, using (in numerical means) the corresponding time-dependent Schrödinger equation, we check how to capture deviations from the exponential-decay law. In agreement with Ref. [10], but with a different method, we find that the ratio of the decay probability currents shows time-persisting deviations from the exponential-decay law predictions. The main advantage of the approach presented here is its complete transparency of all successive steps and its feasibility in physical experiments in which the tunneling in different directions can be obtained by asymmetric potentials. Moreover, as discussed in the summary, the qualitative features of the obtained results are expected to be quite general and can be used not only to describe the generic tunneling processes of particles to the open space, but also to understand decays of unstable relativistic particles in the QFT language.

II. THE MODEL

In this paper, we consider a single particle moving in a one-dimensional space subjected to two separated δ potential barriers. The system is described by the following Hamiltonian:

$$\mathcal{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_L\delta(x+R) + V_R\delta(x-R), \quad (1)$$

where R is the half distance between the two barriers and their height is controlled by the independent parameters V_L and V_R. Our aim is to find the decay properties of a particle that is initially located between the barriers. To this aim, at the initial moment ($t = 0$), the wave function is taken as

$$\Psi(x, t = 0) = \Psi_0(x) = \begin{cases} \frac{1}{\sqrt{R}}\cos\left(\frac{\pi x}{2R}\right), & |x| \leqslant R \\ 0, & |x| > R, \end{cases} \quad (2)$$

which corresponds to the ground state in the limit of barriers of infinite heights. This choice is quite natural, but of course one could use other initial wave functions without changing the qualitative results that we are going to present.

The properties of the studied system are controlled by only two independent dimensionless parameters. It is clearly visible that all quantities can be expressed in units fixed by the half distance R. Namely, if all distances are measured in units of R, energies in units of $\hbar^2/(mR^2)$, and time intervals in units of $mR^2/\hbar$, then the properly rescaled (dimensionless) Hamiltonian takes the form

$$\mathcal{H} = -\frac{1}{2}\frac{d^2}{dx^2} + V_0[\delta(x+1) + \kappa\delta(x-1)], \quad (3)$$

where $V_0 = \frac{mR}{\hbar^2}V_L$ and $\kappa = V_R/V_L$ are two independent dimensionless parameters controlling the heights of the left barrier and the ratio between the right and the left heights, respectively. In these units, we solve the time-dependent Schrödinger equation,

$$(i\partial_t - \mathcal{H})\Psi(x, t) = 0, \quad (4)$$

with the initial wave function (2). Notice that in the chosen units, the initial energy of the system (in the limit $V_0 \to \infty$, and $\kappa > 0$) is $E_0 = \pi^2/8$, which is of the order of 1. Clearly, due to the mirror symmetry of the problem, without losing generality, one can restrict to $0 < \kappa \leqslant 1$.

To quantify the dynamics of the system, we focus our attention on *the nondecay probability* defined as

$$P_0(t) = \int_{-1}^{+1} dx\, |\Psi(x, t)|^2, \quad (5)$$

i.e., the probability that the particle is remaining in the region $x \in (-1, 1)$ at the time t. Note that this quantity is interchangeably also called *the survival probability*, but then some attention is needed [61]. Moreover, we also consider the left and the right decay probabilities defined as

$$P_L(t) = \int_{-\infty}^{-1} dx\, |\Psi(x, t)|^2, \quad (6a)$$

$$P_R(t) = \int_{+1}^{+\infty} dx\, |\Psi(x, t)|^2, \quad (6b)$$

where $P_L(t)$ ($P_R(t)$) is the probability that at the time t, the particle can be found to the left (right) of the well, i.e., it is the probability that the tunneling to the left (right) has occurred in the time interval between 0 and t. Obviously, at any instant t, these probabilities are not independent and must obey the normalization condition

$$P_0(t) + P_L(t) + P_R(t) = 1. \quad (7)$$

It is also extremely useful to consider the probability currents (the time derivatives of the probabilities) describing the speed of their temporal change,

$$p_0(t) = -\frac{dP_0(t)}{dt}, \quad p_L(t) = \frac{dP_L(t)}{dt}, \quad p_R(t) = \frac{dP_R(t)}{dt}. \quad (8)$$

Notice that the definition of $p_0(t)$ takes into account that the nondecay probability decreases with time. Temporal changes of $p_0(t)$ are often measured in experiments since it corresponds to the number of decay products per unit of time (for instance, the lifetime measurement of the neutron by the beam method [62] or the decay of H-like ions via electron capture and neutrino emission [63]). Note that a simple interpretation holds: $p_{L(R)}(t)dt$ is the probability that the decay occurs to the right (left) between t and $t + dt$. Clearly, from the relation (7), one finds that

$$p_0(t) = p_L(t) + p_R(t). \tag{9}$$

The central quantities that we focus on in the following are the right-to-left ratio of probabilities,

$$\boldsymbol{\Pi}(t) = \frac{P_R(t)}{P_L(t)}, \tag{10}$$

and its counterpart, the right-to-left ratio of probability currents,

$$\boldsymbol{\pi}(t) = \frac{p_R(t)}{p_L(t)}. \tag{11}$$

It will turn out that the time dependence of both ratios plays a crucial role in capturing the nonexponential-decay behavior of the system.

Finally, let us recall the explicit forms of all these functions when the exponential Breit-Wigner (BW) limit [64–66] holds. In this limit, the nondecay probability reads

$$P_0(t) \xrightarrow{\text{BW}} e^{-\Gamma t}, \tag{12}$$

where Γ is the decay rate. As argued in [2], the exponential dependence of the nondecay probability is a direct consequence of the Breit-Wigner energy distribution of the unstable state. The decay rate Γ can also be decomposed to partial decay rates to the "left" Γ_L and to the "right" Γ_R associated with these two distinguished decay channels, $\Gamma = \Gamma_L + \Gamma_R$. Then, the partial decay probabilities have the form

$$P_L(t) \xrightarrow{\text{BW}} \frac{\Gamma_L}{\Gamma}(1 - e^{-\Gamma t}), \tag{13a}$$

$$P_R(t) \xrightarrow{\text{BW}} \frac{\Gamma_R}{\Gamma}(1 - e^{-\Gamma t}). \tag{13b}$$

Obviously, the partial decay probability currents read

$$p_L(t) \xrightarrow{\text{BW}} \Gamma_L e^{-\Gamma t}, \quad p_R(t) \xrightarrow{\text{BW}} \Gamma_R e^{-\Gamma t}. \tag{14}$$

For future convenience, we introduce the ratio of the partial decay widths,

$$\beta = \Gamma_R/\Gamma_L, \tag{15}$$

which, in the BW limit, remains constant and directly connects the right-to-left ratios (10) and (11),

$$\boldsymbol{\Pi}(t) = \frac{P_R(t)}{P_L(t)} \xrightarrow{\text{BW}} \beta \xleftarrow{\text{BW}} \frac{p_R(t)}{p_L(t)} = \boldsymbol{\pi}(t). \tag{16}$$

To show that the exponential-decay law is violated, it is sufficient to expose deviations from the constant value of $\beta = \Gamma_R/\Gamma_L$. This is why the right-to-left ratios (10) and (11) are of special interest.

III. RESULTS

We solve the Schrödinger equation (4) by expressing the time-dependent wave function in terms of eigenstates of the dimensionless Hamiltonian (3). In practice, due to a lack of convenient exact analytical solutions, we diagonalize it on a finite spatial interval with closed boundaries at $x = \pm L$ with $L/R \gg 1$ (for more technical details, see the Appendix). We then calculate the nondecay probability $P_0(t)$, the partial decay probabilities $P_L(t)$ and $P_R(t)$, and, finally, the two ratios $\boldsymbol{\Pi}(t)$ and $\boldsymbol{\pi}(t)$.

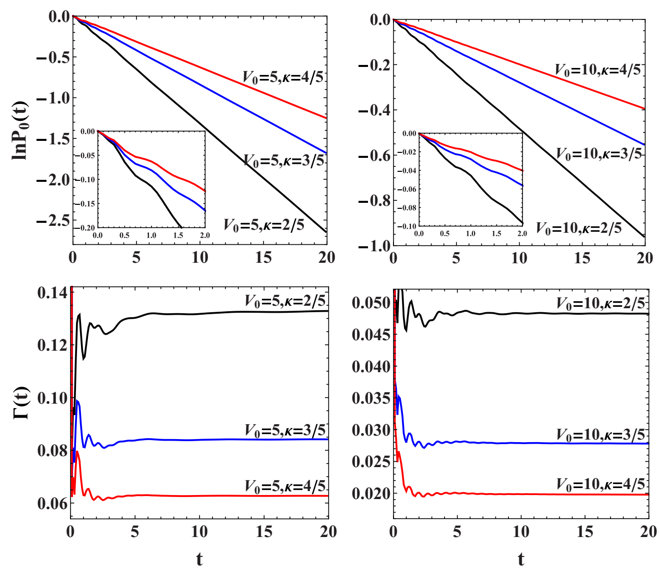

FIG. 1. Upper panels: The nondecay probability $P_0(t)$ as a function of time for some chosen values of κ and V_0. The insets highlight the behavior at short times. Bottom panels: The corresponding results for the decay rate $\Gamma(t) = -\ln P_0(t)/t$.

In the upper panel of Fig. 1, we show the nondecay probability $P_0(t)$ as a function of time for some chosen values of V_0 and κ (the insets highlight the changes for small t). It is clearly seen that after a short initial period, $P_0(t)$ exhibits an exponential decay. It is even more evident when the decay rate $\Gamma(t) = -\ln P_0(t)/t$ is plotted (bottom panel in Fig. 1)—after some small initial wiggles, it reaches a constant value, indicating a quite fast transition to the BW regime. These results suggest that in the regime of exponential decay, the approximation (12) should be applied. It turns out that in this regime, the nondecay probability almost ideally fits the relation

$$P_0(t) \approx e^{-\Gamma(t-t_0)}, \tag{17}$$

manifesting the correctness of the BW limit predictions. Note that in general the additional "time shift" t_0 is nonzero and its inverse is directly related to the initial period of nonexponential decay. In fact, the sign of t_0 indicates if, for small times, the dynamics is sub- or supexponential (see [22] and [46] for detailed discussions of this point). In the cases studied here, this parameter is very close to 0 and, due to numerical uncertainty, we are not able to determine its sign. To gain a deeper insight into the validity of the BW approximation, we additionally check how the ratio of partial decay rates β depends on κ and V_0 (see Fig. 2). It turns out that the ratio β becomes insensitive to changes in V_0 when V_0 is large enough. In fact, for a considered range of κ, the changes in V_0 do not affect the value of β when V_0 exceeds a value

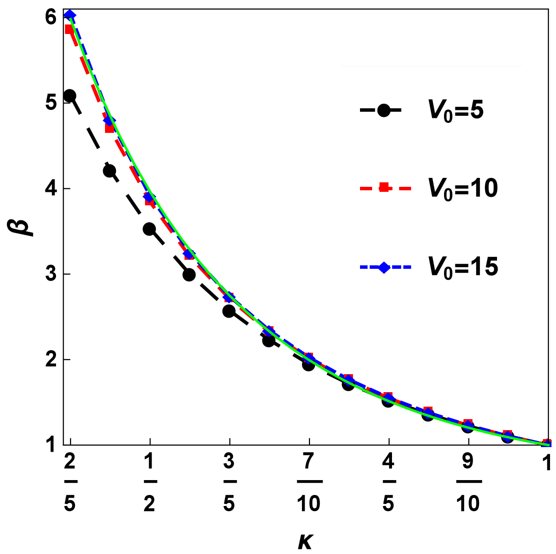

FIG. 2. The ratio of partial decay rates β calculated in the BW limit as a function of the asymmetry parameter κ for different values of V_0. The green solid line indicates a phenomenological relation $\beta = \kappa^{-2}$ justified in the limit of large V_0.

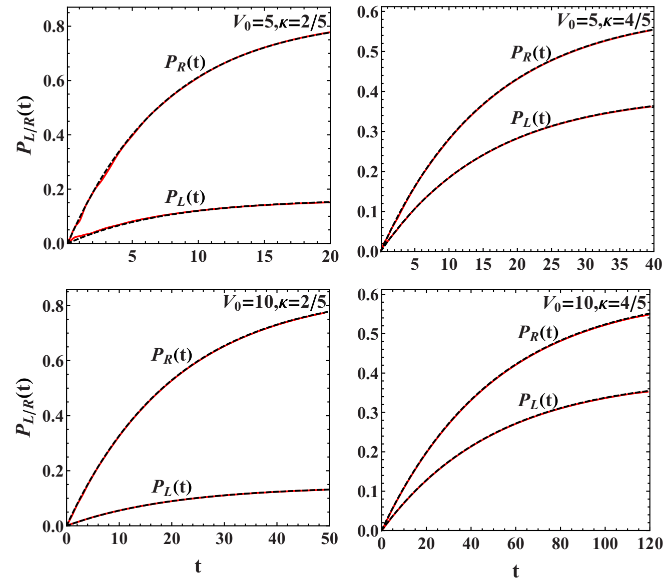

FIG. 3. Partial decay probabilities $P_R(t)$ and $P_L(t)$ as a function of time. Note that accurate numerical results (continuous black lines) coincidence with predictions of the BW limit (13) (red dashed lines). See the main text for details.

of about 15. Moreover, in this regime, the ratio β, when treated as a function of κ, almost perfectly follow the simple relation $\beta(\kappa) \approx \kappa^{-2}$ (green line in Fig. 2). This relation has a direct intuitive phenomenological explanation. For large V_0, tunnelings in opposite directions become almost independent and therefore the ratio of tunneling amplitudes is simply given by the ratio of the barrier heights, κ^{-1}. It means that the ratio of probabilities is controlled solely by κ^{-2}.

The discussion above means that the exponential formula provides a very good approximation for large enough (but not too large) times. Clear deviations are visible only for initial moments (for the cases studied, $t \lesssim 5$). Of course, the deviations become larger for smaller V_0. However, we focus on the cases in which $P_0(t)$ is almost exponential since this is the typically realized scenario in Nature.

The situation is very similar when partial decay probabilities (6) are considered. In this case, after fitting to appropriate exponential functions of the form

$$P_{L/R}(t) \approx \frac{\Gamma_{L/R}}{\Gamma}[1 - e^{-\Gamma(t-t_0)}], \tag{18}$$

we see full agreement of the BW limit predictions with accurate numerical results (see Fig. 3 for comparison).

All three results presented for probabilities $P_0(t)$, $P_R(t)$, and $P_L(t)$ suggest that any discrepancies from the exponential behavior are poorly captured by these quantities. We checked that this is also the case when the probability currents (8), i.e., the temporal derivatives of the probabilities, are considered. However, the situation changes dramatically when, instead of pure probabilities (probability currents), the properties of their temporal ratios $\mathbf{\Pi}(t)$ and $\boldsymbol{\pi}(t)$ are investigated. In Fig. 4, we present accurate numerical results for these ratios as a function of time for the same set of parameters as in Fig. 1. One can see that the ratios $\mathbf{\Pi}(t)$ and $\boldsymbol{\pi}(t)$ have rather complex behavior, especially for the initial period. More importantly, the deviations from the constant value obtained in the exponential BW limit are clearly visible. Both functions eventually reach the expected constant value of β in the limit of large times. Note, however, that here we do not consider very large times in which the decay is again nonexponential due to the onset of a power law. In our studies, when referring to intermediate and large times, we mean periods in which the decay is almost ideally exponential.

In fact, our results allow us to conclude that partial probabilities $P_L(t)$ and $P_R(t)$ are generally linearly independent functions since, if $\mathbf{\Pi}(t)$ and $\boldsymbol{\pi}(t)$ are not identically equal, then the Wronskian $W(t) = P_L(t)p_R(t) - P_R(t)p_L(t)$ is not singular. [Note that for $\kappa = 1$, symmetric tunneling to the left and to the right occurs: $\mathbf{\Pi}(t) = \boldsymbol{\pi}(t) = 1$]. Only for a very large time, when both ratios reach an almost constant value β, one finds that $\mathbf{\Pi}(t) - \boldsymbol{\pi}(t) \approx 0$, which means that

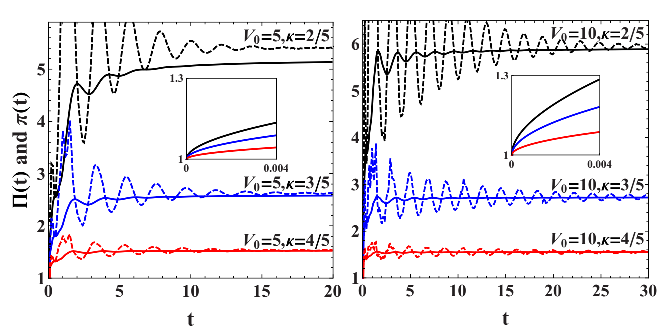

FIG. 4. Temporal ratio of partial probabilities $\mathbf{\Pi}(t)$ (continuous lines) and partial probability currents $\boldsymbol{\pi}(t)$ (dashed lines) as functions of time for the same set of parameters as in Fig. 1. The insets highlight the short-time behavior. Both quantities oscillate at intermediate times but the ratio $\boldsymbol{\pi}(t)$ shows evident deviations from the BW limit predictions even for very long times.

partial probabilities $P_R(t)$ and $P_L(t)$ behave as nearly linear-dependent functions.

In particular, the right-to-left probability currents ratio $\boldsymbol{\pi}(t)$ shows evident oscillations persisting for a very long time. It means that it is an appropriate quantity to exhibit deviations from the exponential BW limit predictions, even in moments when the standard nondecay probability $P_0(t)$, the partial decay probabilities $P_R(t)$ and $P_L(t)$, or even their ratio $\boldsymbol{\Pi}(t)$ are not able to capture this behavior. Let us also recall that the ratio $\boldsymbol{\pi}(t)$ has a straightforward physical meaning. For the time intervals in which $\boldsymbol{\pi}(t) > \beta$ [$\boldsymbol{\pi}(t) < \beta$], the particle decay to the right is more [less] probable than naively expected from the exponential law. Then, the value of β has only an appropriate interpretation as an average ratio. Closer inspection of Fig. 4 shows additional interesting insights for the function $\boldsymbol{\pi}(t)$. Namely, the amplitude of oscillations does not decrease in the limit of large V_0 as long as κ is sufficiently different from unity. Namely, when it approaches 1, the ratio $\boldsymbol{\pi}(t)$ rapidly flattens around the expected value 1. Consequently, in these cases, the deviations from the expected constant limit become very small.

The above analysis shows that the ratios $\boldsymbol{\Pi}(t)$ and $\boldsymbol{\pi}(t)$ can be regarded as appropriate quantities capturing nonexponential decay in the presence of two decay channels. However, as we argued, the ratio of the time derivatives $\boldsymbol{\pi}(t)$ is much more sensitive to nonexponential features of the system than the direct ratio of probabilities $\boldsymbol{\Pi}(t)$. Therefore, from the experimental point of view, if one aims to validate exponential decay, the largest effort should be put toward accurate determination of the quantity $\boldsymbol{\pi}(t)$ rather than $\boldsymbol{\Pi}(t)$.

It is interesting to note that for a given asymmetry of the barriers κ, the amplitude of the oscillations is not strongly dependent on V_0. For example, as presented in Fig. 4, the amplitudes for $V_0 = 5$ and $V_0 = 10$ are not much different when the same value of $\kappa = 2/5$. In contrast, the frequency of the oscillations is essentially affected by the choice of V_0 and it is larger for stronger V_0. The latter observation implies that for very large V_0, experimental detection of oscillations will be very challenging due to the finite resolution of time probes. Simply, to have any realistic chance to detect the effect, a period of the oscillation should not be smaller than the experimental time resolution.

Importantly, it should be pointed out here that in our work, we do not consider deviations from the exponential decay occurring always for very large times, i.e., when the decay is characterized by the power law rather than the exponential one [1,3,15]. In fact, this regime is not well captured in our analysis due to the numerical simplification of the model described in the Appendix. Although going beyond this approximation is straightforward, it highly increases the numerical complexity without changing the results in the time ranges that we are interested in. Therefore, the discussion of properties of the ratios $\boldsymbol{\Pi}(t)$ and $\boldsymbol{\pi}(t)$ for very long times is beyond the scope of this work.

One can expect that the qualitative features of the results obtained do not significantly depend on the details of the employed decay model. This conviction is justified since the origin of the different behavior of $\boldsymbol{\pi}(t)$ and $\boldsymbol{\Pi}(t)$ is ingrained in the fundamental properties of the two-channel decay, rather than a particular physical realization. Note that both quantities are described by the same decay width β only in the BW limit independently in the underlying model. It means that any deviation from this prediction is a direct manifestation of the nonexponential decay. In other words, as long as the probabilities for the two partial decay channels are not equal, the corresponding functions $P_L(t)$ and $P_R(t)$ approach the respective exponential limits in a slightly different way. Consequently, ratios $\boldsymbol{\Pi}(t)$ and $\boldsymbol{\pi}(t)$ are characterized by slightly different and time-dependent parameters. This is the intuitive reason why the ratios enhance the differences quite independently of the details of a model. This is also one of the reasons why very similar results were obtained in a completely different context in [10] in the framework of the Lee model [44] containing essential simplification when compared to the generalized Winter's model considered here. In contrast to the case studied, in the Lee model it is assumed that there exists the unique *unstable* state $|\psi_0\rangle$ decaying to two different subspaces (channels L and R) spanned by states $|k, L\rangle$ and $|k, R\rangle$ having the same dispersion relation $\omega(k)$. In such a case, the Hamiltonian of the system can be written explicitly in the basis of these states as

$$\mathcal{H}_{\mathrm{Lee}} = E_0|\psi_0\rangle\langle\psi_0| + \sum_{\sigma\in\{L,R\}} \int_0^\infty dk\, \omega(k)|k,\sigma\rangle\langle k,\sigma| + \sum_{\sigma\in\{L,R\}} \int_0^\infty dk\, [f_\sigma(k)|k,\sigma\rangle\langle\psi_0| + f_\sigma^*(k)|\psi_0\rangle\langle k,\sigma|], \tag{19}$$

where $f_\sigma(k) = \langle k,\sigma|H|\psi_0\rangle$ are transition amplitudes controlling tunneling through the barriers. The nonexponential decay observed in these two, essentially different models suggests once more that our findings on properties of ratios $\boldsymbol{\pi}(t)$ and $\boldsymbol{\Pi}(t)$ persist model independently.

IV. CONCLUSIONS

In this work, we analyzed the general problem of capturing nonexponential properties in the presence of the two-channel decay process. Taking as a working horse a very simple dynamical problem of a single particle flowing out from a leaky box, we examined direct relations between the probabilities of tunneling to the right and the left as functions of the control parameters. In this way, we studied relations between partial decays into two distinct channels in a relatively simple system, which allows for a very accurate numerical treatment. Since the multiple channel decay of an unstable quantum state is a very frequent problem in QM and QFT, the results can be important for our understanding of a broad range of physical phenomena.

The results obtained confirm that in the presence of two decay channels, the system exhibits a remarkable nonexponential behavior on long timescales. Even in cases where the simplest quantities do not reveal any nonexponential signatures, the interchannel ratio of probability currents $\boldsymbol{\pi}(t)$ directly exposes these features. Importantly, this quantity, although not the simplest property of the system, is almost directly measurable in experiments [67–69]. Therefore, it can be viewed as a possible *smoking gun* of nonexponential-decay behavior.

It is worthwhile to point out that the model discussed in this work, although seemingly oversimplified, to some extent can be realized experimentally and gives prospects for direct verification of our predictions. State-of-the-art experiments [70–73] with ultracold atoms confined in optical traps allow one to prepare quasi-one-dimensional uniform box traps where particles are confined. Moreover, the outside walls of these traps can be controlled independently and released almost on-demand, opening direct routes to realize our model. Another interesting direction of experimental realization is to analyze different nuclei with nonsymmetric few-channel decays, for instance, the decay of α particle in large nonspherical nuclei.

From a theoretical point of view, one can easily extend the present work to more complicated (and more realistic) forms of asymmetric potentials. While any qualitative differences from the results obtained are not expected, such studies would help to establish a closer relevance to upcoming experimental schemes. From the conceptual side, extensions of the results to higher dimensions are also straightforward. Another promising route for further explorations is to study analogous systems containing several interacting particles [74–85] and pin down the role of the quantum statistics. Furthermore, the topic should also be reinvestigated in the realm of QFT to shed some fresh light on the problem of multichannel decays of elementary particles and composite hadrons.

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APPENDIX: NUMERICAL APPROACH

Numerical calculations are performed in the basis of the eigenstates of the Hamiltonian (3) diagonalized numerically on a finite spatial interval with closed boundary conditions at $x = \pm L$. Everywhere besides the points $x = \pm 1$, the Hamiltonian is equivalent to the Hamiltonian of a free particle. Therefore, any of its eigenstates can be expressed as follows:

$$\psi(x) = \begin{cases} A\sin[p(L+x)] & \text{if } x < -1 \\ B\sin[p(L-x)] & \text{if } x > 1 \\ C\sin(px) + D\cos(px) & \text{if } |x| \leqslant 1, \end{cases} \tag{A1}$$

where parameters A, B, C, and D are established in such a way that the wave function fulfills continuity conditions at positions of the left and the right barrier. These four conditions read

$$\lim_{\epsilon\to 0}\,[\psi(-1+\epsilon) - \psi(-1-\epsilon)] = 0, \tag{A2a}$$

$$\lim_{\epsilon\to 0}\left[\frac{d}{dx}\psi(x)\bigg|_{-1+\epsilon} - \frac{d}{dx}\psi(x)\bigg|_{-1-\epsilon}\right] = 2V_L\psi(-1), \tag{A2b}$$

$$\lim_{\epsilon\to 0}\,[\psi(1+\epsilon) - \psi(1-\epsilon)] = 0, \tag{A2c}$$

$$\lim_{\epsilon\to 0}\left[\frac{d}{dx}\psi(x)\bigg|_{1+\epsilon} - \frac{d}{dx}\psi(x)\bigg|_{1-\epsilon}\right] = 2V_R\psi(1), \tag{A2d}$$

and they lead to the homogenous system of linear equations of the form $\mathcal{M}\cdot\vec{v} = 0$, where $\vec{v} = (A, B, C, D)^T$ and

$$\mathcal{M} = \begin{pmatrix} \frac{1}{2}p\cos[(L-1)p] & 0 & -\frac{1}{2}p\cos(p) - V_L\sin(p) & V_L\cos(p) - \frac{1}{2}p\sin(p) \\ 0 & -\frac{1}{2}p\cos[(L-1)p] & \frac{1}{2}p\cos(p) + V_R\sin(p) & V_R\cos(p) - \frac{1}{2}p\sin(p) \\ \sin[(L-1)p] & 0 & \sin(p) & -\cos(p) \\ 0 & -\sin[(L-1)p] & -\sin(p) & -\cos(p) \end{pmatrix}.$$

In this way, the allowed momenta p_i and the corresponding coefficients $\vec{v}_i$ are determined. Then, the the time-dependent wave function is simply given as

$$\Psi(x,t) = \sum_i \alpha_i \exp\left(-itp_i^2/2\right)\psi_i(x), \tag{A3}$$

where the expansion coefficients α_i are determined by the initial wave function (2). The accuracy of the final results is easily controlled (and, if needed, may be straightforwardly improved) by changing the number of terms in the expansion (A3). Typically, in our calculations, we use 3000 terms and $L = 400$–600, which is sufficient to achieve well-converged results avoiding reflections at the walls at $x = \pm L$ for large t. The method used assures a full control on the accuracy of the final results.

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[60] The problem of adaptation of the general scattering solutions reported in, e.g., Ref. [55] to the problem of decay studied here originates in the lack of orthogonality between left-to-right and right-to-left scattering solutions. Finding an orthogonal and complete basis is not an easy task since the orthogonalizing transformation is momentum dependent. More precisely, the problem can be, as expected, solved in the symmetric case: The sum and the difference of the scattering wave solutions are even and odd functions, respectively, and hence they are orthogonal and the desired basis can be found. However, in the general (and, for us, crucial) asymmetric case, this is not possible: For each momentum k, a different field transformation should be performed and it is at present unclear how to handle the problem. (Note that the standard Gram-Schmidt method does not apply for states whose norm is infinite.) In view of these difficulties, since the numerical solutions that we provide are correct and since the derivation of the analytic solution is, albeit interesting and desirable, not central for our goals, we decided to leave this task for the future.

[61] There is a subtle but important difference between the nondecay and the survival probability at a given time t. The former is the probability that no decay has occurred at t, and the latter that the particle occupies the same initial state. In this sense, Eq. (5) defines the nondecay probability, while the survival probability is defined as $P_{\rm surv}(t) = |\int_{-1}^{+1} dx \Psi_0(x)\Psi(x,t)|^2$. Clearly, for the system studied, the nondecay probability is the natural quantity to consider.

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