Testing macroscopic local realism using local nonlinear dynamics and time settings

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We show how one may test macroscopic local realism where, differently from conventional Bell tests, all relevant measurements that lead to a violation of a Bell inequality need only distinguish between two macroscopically distinct states of the system being measured. Here, measurements give macroscopically distinct outcomes for some property of the system and do not resolve microscopically (of order $\hbar$). To obtain a quantifiable test, we define N-scopic local realism where the outcomes are separated by an amount $\sim N$. We show for N up to 20 that violations of N-scopic local realism are predicted for entangled systems of N bosons at each of two sites. We further demonstrate for arbitrarily large α the violation of α-scopic local realism, for entangled superpositions of coherent states with amplitudes $\sim\alpha$, infinitely separated in phase space, as $\alpha \rightarrow \infty$. To achieve the Bell violations, the two separated subsystems evolve dynamically according to a local nonlinear unitary interaction. The tests may be understood in two ways. First, they are macroscopic Bell tests involving spacelike separated measurement events, where the choice of measurement setting at each site corresponds to a choice between two different times of local interaction. The interaction may be seen as the unitary stage of a local measurement process. Second, the proposal is a strong version of a Leggett-Garg test of "macrorealism" that uses two trajectories, where the usual assumption of noninvasive measurability as applied to measurements at one site is replaced by that of macroscopic locality. For $N = 1$, tests are feasible and give a way to demonstrate "no classical trajectories" without the assumption of noninvasive measurability, using instead that of locality.

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I. INTRODUCTION

Motivated by Schrödinger's cat paradox [1], much effort has been devoted to testing quantum mechanics at a macroscopic level. Quantum superpositions of macroscopically distinguishable states, so-called cat states, have been created in a number of different physical scenarios [2–6]. However, Leggett and Garg pointed out that a very strong test of macroscopic quantum mechanics would give a method to falsify *all* possible theories satisfying the notion of macroscopic realism [7].

To address this problem, Leggett and Garg formulated inequalities [7], which if violated falsify a type of macroscopic realism called macrorealism. Leggett and Garg's macrorealism combines two classical premises. The first premise is *macroscopic realism* (MR), which asserts that a system which has two macroscopically distinguishable states available to it must at any time be in one or the other of these states. The two states are identified in the context of a "distinguishing measurement" $\hat{M}$ which has just two macroscopically distinguishable outcomes, these outcomes corresponding to the two macroscopic states of the system [8]. In Schrödinger's paradox, the assumption of macroscopic realism implies that the cat is always dead *or* alive, prior to measurement.

The second Leggett-Garg premise is "macroscopic noninvasive measurability." This premise postulates the existence of a measurement that can distinguish which of the macroscopically distinguishable states the system is in, with a negligible effect on the subsequent macroscopic dynamics. There has been significant interest in testing Leggett-Garg inequalities, even for small systems, and experiments have been performed with superconducting qubits, photons and single atoms [9–19]. However, an obvious complication (which Leggett and Garg referred to as "vexing" [7]) is the justification of the second noninvasive-measurability premise for any practical measurement [7,9,12,18,19].

In this paper, we show how macroscopic reality may be tested using Bell inequalities [20]. This represents an advance because here the second Leggett-Garg premise is replaced by the premise of *macroscopic locality* (ML), which leads to a stronger test of macrorealism. Measurements made at different times are at distinct locations, so that locality can be invoked to justify noninvasiveness. Where macroscopic realism is assumed for a system at one location, ML asserts that a measurement made at another location cannot change the predetermined value (determined by which macroscopic state the system is in) for the measurement $\hat{M}$ on the first system. This is provided the two measurement events are spacelike separated. In Schrödinger's paradox, the consequence of macroscopic locality is that a measurement on a second separated system could not (instantly) change the cat from dead to alive, or vice versa [21]. The combined premises of MR and ML constitute the premise of *macroscopic local realism* (MLR) [22].

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Specifically, we explain how the predictions of quantum mechanics are incompatible with those of MLR for systems prepared in certain macroscopic entangled superposition states. To obtain a quantifiable test, we define N-scopic local realism to apply where the outcomes of the distinguishing measurement $\hat{M}$ are separated by an amount of order N. The important feature of the Bell tests presented in this paper is that the outcomes of *all* relevant measurements involved in the Bell inequality correspond (as $N \to \infty$) to macroscopically distinct states of the system being measured; i.e., measurements only need to distinguish between the two extreme states of a macroscopic superposition state (a "cat state"). The measurements thus do not resolve at the level of $\hbar$.

The violations are possible, because we allow nonlinear dynamics at each of the two separated sites where the measurements take place. The traditional Bell choice between two spin settings is replaced by a choice between two different *times* of evolution, at each of two sites. The nonlinear dynamics arises from a local unitary evolution brought about by a nonlinear medium, and can be thought of as modeling the unitary stage of a measurement process, which involves interaction with a local measurement device. The choice of different local time settings is equivalent to a choice between two nonlinearities, and may be interpreted as corresponding to measurements of different system observables.

We consider two cases. In the first, measurements detect either *all* N bosons in one field mode, or *all* N bosons in a second mode. In the second case, the measurements distinguish between coherent states of large amplitude ($\sim\alpha$) and well separated in phase space, by an amount of order α. We determine violation of N-scopic local realism for up to $N \sim 20$ bosons, and of α-scopic local realism for all α including $\alpha \to \infty$, which corresponds to both infinite amplitudes and an infinite separation between the two relevant states in phase space. The Bell tests of this paper thus differ from previous Bell tests for macroscopic systems [23], including those for superpositions of macroscopically distinct states [6,24], which almost invariably require at least one measurement that distinguishes microscopically different outcomes, or else involve a continuous range of outcomes [22,25]. In this paper, as in the Leggett-Garg proposal, one considers only measurements distinguishing the two macroscopically distinct states that form an extreme macroscopic superposition state, so that the separation of outcomes well exceeds the level associated with the standard quantum limit ($\hbar$). The results show that Bell violations can be predicted in this macroscopic regime. To the best of our knowledge, such tests have not been performed for $N > 1$.

The Bell inequalities considered in this paper are similar yet different to the Leggett-Garg inequalities, which are often called "temporal Bell inequalities" [11], or "Bell inequalities in time" [3], because they involve measurements made at successive times on a dynamically evolving system [7]. The Leggett-Garg tests, however, involve only a single system. The violation of a Leggett-Garg inequality for a single propagating particle can be used to negate the concept of a classical trajectory, and hence the violation of a Leggett-Garg inequality has been considered important, even at a microscopic level [12,14]. In this paper, we consider *two* dynamical systems, spatially separated, and thus consider two trajectories. We also examine the microscopic case of $N = 1$, by considering two particles prepared initially in a Bell state. The dynamics may be realized using a sequence of (polarizing) beam splitters. The negation of classical trajectories for the propagating particles is then possible, based on the violation of the Bell inequality where correlations are inferred with only one detection at each site. This gives a strong test of the classical trajectories, since the noninvasiveness of the measurement is justified by the assumption of locality.

Layout of paper. The layout of this paper is as follows. In Sec. II, we consider idealized Bell states involving macroscopic qubit states where all N bosons are in one or the other of two modes. We explain how measurements might be made using a hypothetical nonlinear beam splitter, which either transmits or reflects all N bosons incident on the beam splitter. The transmission coefficient is determined by the time of evolution through the beam splitter. For certain choices of transmission, one violates a Bell inequality, where outcomes for measurements are distinct by N. In Sec. III, we show how such N-scopic Bell violations can be realized, for systems prepared conditionally from two NOON states, and for nonlinear beam splitters realized using a nonlinear Josephson interaction, with a careful choice of parameters.

In Sec. IV, we predict violations of macroscopic local realism for a system based on a two-mode entangled superposition of coherent states. The modes are separated, and each system evolves locally by interacting with a nonlinear Kerr medium. The "spin" outcome is measured as the sign of the outcome X of a quadrature phase amplitude measurement made on the mode at each site. The outcomes become macroscopically distinct as the separations between the coherent states (of order α) in phase space become large. We confirm Bell violations for arbitrary $\alpha > 2$. In Secs. III D and IV D, we also explain how one may violate the standard Leggett-Garg inequality [7,9], using separated sites.

Finally, in Secs. V and VI, we give a discussion of results and of the feasibility of an experiment. For $N = 1$, a strong negation of classical trajectories (replacing the noninvasive measurability premise with the locality assumption) is feasible, using traditional sources of entanglement along with polarizing beam splitters. We also point out that, as with the conventional Bell inequalities, the macroscopic Bell inequality can be derived in two ways [21]. The second method assumes a generalized form of macroscopic reality, allowing for changes stochastically brought about by the local interaction of the system with the measurement apparatus, in this case the nonlinear medium or beam splitter. The violation of the Bell inequality is then seen to be a much stronger result, and suggests that the dynamics is important in the realization of the macroscopic Bell violations.

II. BELL INEQUALITY FOR MACROSCOPIC LOCAL REALISM

For spatially separated sites A and B, we consider the two-qubit Bell states for N bosons

$$|\psi_{\pm,+}\rangle_{AB} = \frac{1}{\sqrt{2}}(|1\rangle_A|\pm 1\rangle_B + |-1\rangle_A|\mp 1\rangle_B),$$
$$|\psi_{\pm,-}\rangle_{AB} = \frac{1}{\sqrt{2}}(|1\rangle_A|\pm 1\rangle_B - |-1\rangle_A|\mp 1\rangle_B), \quad (1)$$

where

$$|1\rangle_A = |N\rangle_a|0\rangle_{a_2},\ |-1\rangle_A = |0\rangle_a|N\rangle_{a_2},$$
$$|1\rangle_B = ||N\rangle_b|0\rangle_{b_2},\ |-1\rangle_B = |0\rangle_b|N\rangle_{b_2}.$$

Here we consider two pairs of modes a and a_2, and b and b_2, at locations A and B, respectively. $|n\rangle_a$ and $|n\rangle_b$ are the associated number states. We refer to the states $|1\rangle_A$ and $|-1\rangle_A$ as generating the "up" and "down" outcomes for a measurement of "spin" S_A at site A, in analogy with the spin-1/2 qubit system. Similarly, a spin S_B with outcome ± 1 is detected for $|\pm 1\rangle_B$ at site B.

We next define the action of a hypothetical nonlinear beam splitter (NBS) at site A. The NBS takes an input of N bosons in the ingoing mode, and places *all* N bosons either in a "reflected" mode or in a "transmitted" mode, at the output. The probability of transmission is determined by the interaction time through the NBS. For an initial state $|N\rangle_a|0\rangle_{a_2}$, the state after the hypothetical NBS interaction is

$$\begin{aligned}|\psi(t_a)\rangle &= \hat{U}_A|N\rangle_a|0\rangle_{a_2}\\ &= e^{i\varphi(t_a)}(\cos t_a|N\rangle_a|0\rangle_{a_2} - i\sin t_a|0\rangle_a|N\rangle_{a_2}),\end{aligned} \tag{2}$$

where we have introduced a unitary operator $\hat{U}_A$ and t_a is the time of interaction in scaled units. Here, $\varphi(t_a)$ is a phase factor. A similar NBS interaction is assumed to take place at site B:

$$\begin{aligned}|\psi(t_b)\rangle &= \hat{U}_B|N\rangle_b|0\rangle_{b_2}\\ &= e^{i\varphi(t_b)}(\cos t_b|N\rangle_b|0\rangle_{b_2} - i\sin t_b|0\rangle_b|N\rangle_{b_2}).\end{aligned} \tag{3}$$

Assuming the incoming state on the two nonlinear beam splitters to be $|\psi_{+,\pm}\rangle_{AB}$, the final state is

$$\hat{U}_A\hat{U}_B|\psi_{AB}\rangle = e^{i\varphi}(\cos t_\pm|\psi_{+,\pm}\rangle - i\sin t_\pm|\psi_{-,\pm}\rangle), \tag{4}$$

where $t_\pm = t_a \pm t_b$ and φ is a phase factor. Defining the "spin" at each site as $+1$ or -1 if the system is detected with the "up" or "down" outcome after the NBS interaction (which constitutes part of the measurement, analogous to an analyzer or polarizer), the expectation value for the product of the spins at each site is

$$E(t_a, t_b) = \cos 2(t_\pm), \tag{5}$$

where $t_\pm = t_a \pm t_b$. Assuming the incoming state to be $|\psi_{-,\pm}\rangle_{AB}$, we obtain in a similar fashion $E(t_a, t_b) = -\cos 2(t_\pm)$.

Where N is large, the assumption of macroscopic local realism (MLR) will imply the Clauser-Horne-Shimony-Holt (CHSH) Bell inequality $B \leqslant 2$ [26], where

$$B = E(t_a, t_b) - E(t_a, t_b') + E(t_a', t_b') + E(t_a', t_b). \tag{6}$$

Here we note there are two choices of interaction times at each location: t_a, t_a' at A, and t_b, t_b' at B. This inequality is derived assuming (MLR): that the system is in one or the other of two states which give a definite outcome of spin $+1$ or -1, at each site (MR); and that there is no nonlocal effect changing the state due to the measurement at the other location (ML). This is justified for sufficiently large separations of the sites, where one may assume spacelike separated measurement events. We note the inequality can also be derived using a more general definition of MLR [8] (refer to discussion in Sec. V B). For N large, the NBS interactions will lead to macroscopically distinct outcomes for all choices of t_a and t_b, and the violation of the Bell inequality will falsify MLR. It is known that the given solution for $E(t_a, t_b)$ will violate the inequality for suitable choices of t_a and t_b [26]. We choose

$$t_a = 0,\ t_a' = \pi/8,\ t_b = \pi/4,\ t_b' = 3\pi/8. \tag{7}$$

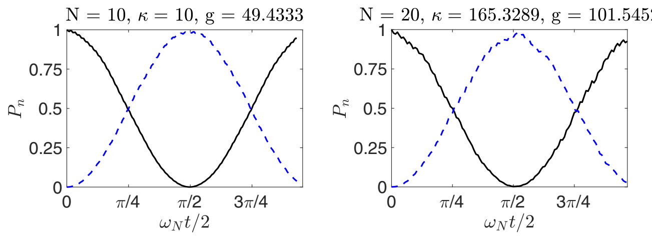

FIG. 1. Realization of a nonlinear beam splitter. Solutions are shown for the Hamiltonian $H_{NL}^{(A)}$ after a time t with initial state $|N\rangle_a|0\rangle_{a_2}$. P_N (black solid line) is the probability for all N bosons to be in mode a; P_0 (blue dashed line) is the probability for all N bosons to be in mode a_2. The parameters identify regimes optimal, or near-optimal, for the nonlinear beam splitter interaction Eq. (2), where $P_N + P_0 \sim 1$ and $P_N \sim \cos^2 \omega_N t/2$. The solutions for $N = 1$ (all κ, g); $N = 2$, $\kappa = 1$, $g = 30$; $N = 5$, $\kappa = 20$, $g = 333.333$; and $N = 7$, $\kappa = 18.23$, $g = 47.85$ are identical to the left plot.

III. N-SCOPIC BELL TESTS FOR N BOSONS

A. Nonlinear beam splitter for N bosons

The above is a straightforward extension of Bell's work, except that it now needs to be shown that the hypothetical NBS interaction can be predicted by quantum mechanics, to a sufficient degree that allows the violation of the Bell inequality. To do this, we consider at A two incoming fields (a and a_2), which interact according to the nonlinear Josephson Hamiltonian [27,28]

$$H_{NL}^{(A)} = \kappa(\hat{a}^\dagger\hat{a}_2 + \hat{a}\hat{a}_2^\dagger) + g\hat{a}^{\dagger 2}\hat{a}^2 + g\hat{a}_2^{\dagger 2}\hat{a}_2^2. \tag{8}$$

Here, $\hat{a}$, $\hat{a}_2$ are the boson operators for the corresponding fields a, a_2, modeled as single modes. A similar interaction $H_{NL}^{(B)}$ is assumed for the fields b and b_2 at B. Such an interaction can be achieved with Bose-Einstein condensates (BECs) or superconducting circuits [17,18,27–34]. For certain choices of g and κ, we find that the interaction (8) indeed acts as a nonlinear beam splitter, where the input $|N\rangle_a|0\rangle_{a_2}$ after a time t gives, to a good approximation, the output of Eq. (2) (Fig. 1). We introduce a scaled time $t_a = \omega_N t$ where $\omega_N = 2g\frac{N}{\hbar(N-1)!}(\frac{\kappa}{g})^N$ [29]. Here $\hat{U}_A$ is the unitary time evolution governed by $H_{NL}^{(A)}$.

B. State preparation using NOON states

It now remains to determine whether the realization of the NBS (which is never exact) can actually allow a violation of the Bell inequality. We first examine a specific method of preparation of (1) and realization of (4) using NOON states [4].

We consider that the two separated modes a and b are prepared at time $t = 0$ in the NOON state

$$|\psi\rangle_{ab} = \frac{1}{\sqrt{2}}(|N\rangle_a|0\rangle_b + e^{i\vartheta}|0\rangle_a|N\rangle_b) \tag{9}$$

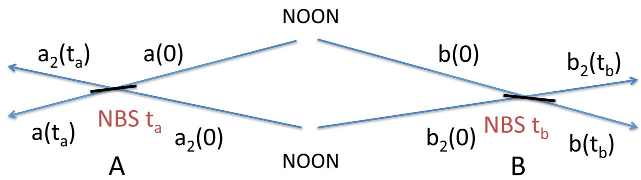

FIG. 2. The macroscopic entangled state (1) can be generated conditionally by interfering two NOON states. The t_a and t_b are time settings selected for the nonlinear beam splitter (NBS) at each site, A and B. For certain choices of parameters κ, g, N of the nonlinear beam splitters, we find that when N bosons are incident at each site A and B, then at each site either all N bosons are detected "up" (modes a and b) or all N bosons are detected "down" (modes a_2 and b_2).

and that the modes a_2 and b_2 are prepared similarly in the NOON state

$$|\psi_+\rangle_{a_2b_2} = \frac{1}{\sqrt{2}}(|N\rangle_{a_2}|0\rangle_{b_2} + e^{i\vartheta}|0\rangle_{a_2}|N\rangle_{b_2}) \tag{10}$$

as depicted in Fig. 2. Alternatively, we might prepare modes a_2 and b_2 in the reversed NOON state,

$$|\psi_-\rangle_{a_2b_2} = \frac{1}{\sqrt{2}}(|0\rangle_{a_2}|N\rangle_{b_2} + e^{i\vartheta}|N\rangle_{a_2}|0\rangle_{b_2}), \tag{11}$$

and choose $\vartheta = -\pi/2$. Such states may be prepared using the ideal nonlinear beam splitter with a 50/50 mixing and the number state inputs $|N\rangle_a|0\rangle_{a_2}|0\rangle_b|N\rangle_{b_2}$. If each mode a and b is then coupled via the NBS interaction to the respective modes a_2 and b_2 with interaction times t_a and t_b, respectively, at each location, then, assuming the optimal NBS parameters, the final state is $|\psi_{f\pm}\rangle = \hat{U}_A\hat{U}_B|\psi\rangle_{ab}|\psi_\pm\rangle_{a_2b_2}$. We find

$$|\psi_{f\pm}\rangle = \frac{e^{i\phi_\pm}}{\sqrt{2}}[\cos(t_\pm)|\psi_{-,\pm}\rangle \mp i\sin(t_\pm)|\psi_{+,\pm}\rangle] + \frac{1}{\sqrt{2}}|\varphi_\pm\rangle_{2N}, \tag{12}$$

where $t_\pm = t_a \pm t_b$ and $\phi_\pm$ is a phase factor. We note $|\varphi_\pm\rangle_{2N}$ are states with either all $2N$ bosons at site A, or all $2N$ bosons at site B. We will see below that these states become irrelevant to the experiment. The total number of bosons at each site can be measured simultaneously with the number of bosons at each mode a and b, since the number operators commute. The state conditioned on there being N bosons at site A and N bosons at site B is given by the first line of Eq. (12). This is a superposition of the Bell states, given by Eq. (1). At time $t_a = t_b = 0$, the conditioned state is the Bell state $|\psi_{-,\pm}\rangle$. The solution (12) thus gives us the required result [similar to (4)].

For $N = 1$, the latter proposal described above that uses the beam splitters to generate the initial NOON states gives a configuration equivalent to that of Ou and Mandel [35,36]. The current proposal is therefore similar to the Ou-Mandel version of a Bell experiment [35] as proposed in Ref. [36], but extended to N quanta. Here, path-entangled cat states are generated conditionally, as suggested in [37]. Contrary to some initial interpretations, in the framework of the Clauser-Horne inequalities [26], this configuration is able to provide a rigorous test of local realism [39] (see discussions in [36,38]). We also note that optical and atomic NOON states may be generated for $N = 2$ using the Hong-Ou-Mandel effect [40].

C. Violation of N-scopic Bell inequalities

To test N-scopic local realism, the mode numbers at the final outputs $a(t_a)$, $a_2(t_a)$, $b(t_b)$, and $b_2(t_b)$ are measured (Fig. 2), for a given setting of the times t_a and t_b at each site. (This may instead be a choice between two different nonlinear interaction values g.) The measurement events at A and B are spacelike separated if the distance between them is sufficiently great, taking into account the times t_a and t_b required for the NBS interactions.

At A, we denote by $+1$ ("up") the outcome of detecting N bosons at location a, *and* 0 bosons at a_2. A similar "up" outcome $+1$ is defined for B. We note that for the calculation, the state $|\varphi_-\rangle_{2N}$ in Eq. (12) thus becomes irrelevant. We define the joint probability P_{++} for the outcome $+1$ at both sites A and B. We also specify P_+^A as the marginal probability for the outcome $+1$ at A, and P_+^B as the marginal probability for the outcome $+$ at B. At each site A and B, observers independently select a time of evolution t_a and t_b for the NBS interaction.

It is evident from the expression (12) that the outcomes of a measurement of mode number at each detector are always one of 0, N, or $2N$, which are macroscopically distinguishable as $N \to \infty$. This is shown in Fig. 3, which plots the probabilities for an optimal choice of nonlinear beam splitter (NBS) parameters. The assumption of N-scopic LR (which becomes MLR as $N \to \infty$) thus implies the validity of a local hidden variable theory, where the system at each site is predetermined to be in one of the states with mode number 0, N, or $2N$. As for the CHSH inequality Eq. (6), the Clauser-Horne (CH) Bell inequality $S \leqslant 1$ [26] is predicted to hold for such a local hidden variable theory, where [39]

$$S = \frac{P_{++}(t_a,t_b) - P_{++}(t_a,t_b') + P_{++}(t_a',t_b) + P_{++}(t_a',t_b')}{P_+^A(t_a') + P_+^B(t_b)}. \tag{13}$$

Assuming *ideal* nonlinear beam splitters, the state $|\psi_{f\pm}\rangle$ gives $P_{++} = \frac{1}{4}\sin^2(t_a \pm t_b)$ and $P_+^A = P_+^B = \frac{1}{4}$. For $t_a = 0$, $t_a' = 2\varphi$, $t_b = \varphi$, and $t_b' = 3\varphi$, the quantum state $|\psi_{f-}\rangle$ of (12) predicts

$$S = \frac{3\sin^2(\varphi) - \sin^2(3\varphi)}{2}. \tag{14}$$

S maximizes at $S = 1.207$ for $\varphi = \pi/16$ (1.96), giving a violation of the CH-Bell inequality.

For $|\psi_{f+}\rangle$, we introduce parameters θ and ϕ, such that $t_a = \theta$, $t_b = 2\pi - \phi$, where $0 < \phi < 2\pi$. Choosing angles $\theta = \theta_0$, $\phi = \theta_0 + \varphi$, $\theta' = \theta_0 + 2\varphi$, and $\phi' = \theta_0 + 3\varphi$, for which $|\theta - \phi| = |\theta' - \phi| = |\theta' - \phi'| = \frac{1}{3}|\theta - \phi'| = \varphi$, the prediction for S becomes that of $|\psi_{f-}\rangle$. Thus, the quantum predictions for both states $|\psi_\pm\rangle$ violate the CH-Bell inequality given by (13).

In practice, the ideal regime giving the precise solution (2) for the nonlinear beam splitters is unattainable, for $N > 1$, since probabilities for other than 0 or N bosons in each mode are not precisely zero. In Figs. 3 and 4, we present *actual* predictions for S, using the Hamiltonian H_{NL}. For large gN/κ, where care is taken to optimize for the NBS regime given

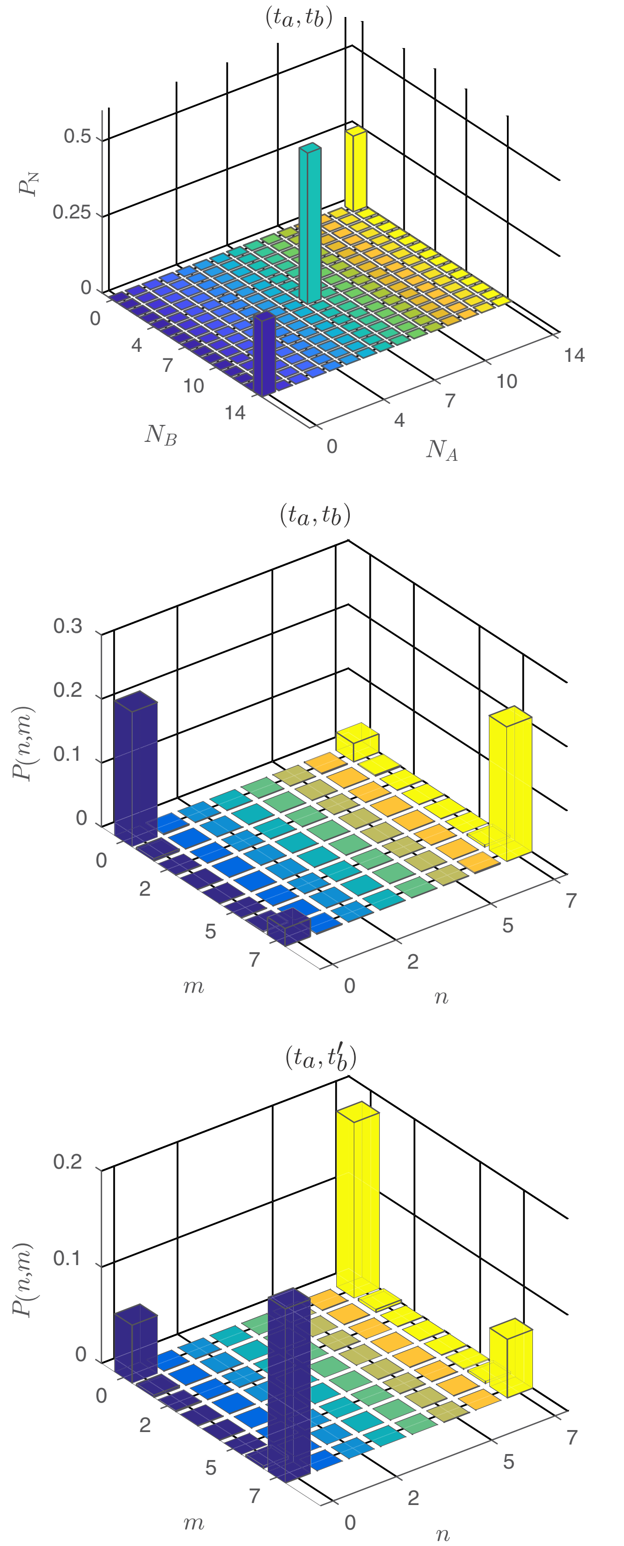

FIG. 3. The outcomes for all the relevant Bell measurements are distinct by N. Top: Probability distribution P_N for the joint detection of a total of N_A and N_B bosons at the respective sites A and B (refer to Fig. 2). Here $N = 7$, $\kappa = 18.23$, $g = 47.85$, and t_a, t_b, t_a', t_b' are specified in the text. The distribution is unchanged for settings (t_a', t_b), (t_a', t_b'), and (t_a, t_b'). Center and Lower: The probability $P(n, m)$ of detecting n bosons in mode a and m bosons in mode b *and* N bosons in total at site A. The figures for settings (t_a', t_b), (t_a', t_b') are identical to that of (t_a, t_b). Similar plots are obtained for all parameters of Fig. 1.

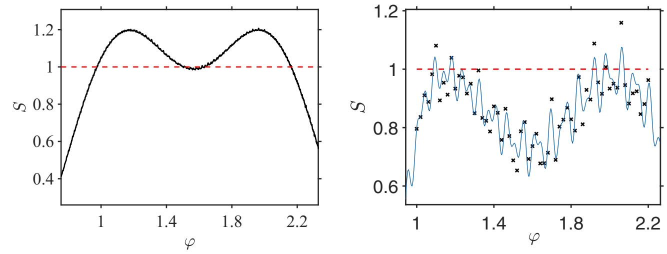

FIG. 4. Violation of N-scopic local realism. Violation of the CH-Bell inequality (13) is obtained when $S > 1$. Where all outcomes are distinct by N, this corresponds to violation of N-scopic local realism. The left graph is for $N = 10$, $g = 49.433$, $\kappa = 10$. Identical plots are obtained for $N = 1$ (all κ and g); $N = 2$, $\kappa = 1$, $g = 30$; $N = 5$, $\kappa = 20$, $g = 333.333$; and $N = 7$, $\kappa = 18.23$, $g = 47.85$ which give ideal two-state oscillatory behavior (Fig. 1). Where the values are not quite optimal for the nonlinear beam splitter interaction (2), rapid oscillations of small amplitude appear. This is shown in the right graph for $N = 20$, $\kappa = 165$, and $g = 101$.

by (2), the Bell violations are predicted, as shown in Fig. 4. To test N-scopic LR, one is required to establish that the outcomes of mode number are distinct by N, for *each* of the joint probabilities $P(t_a, t_b)$ comprising S. Figure 3 highlights the validity of this feature in the optimal parameter space. The probabilities for results other than 0 and N (and $2N$) are negligible (and can rigorously be shown to have no effect on the violation, using the methods of [7,41]). The results given in Fig. 4 are for outcomes verified to be distinct by N bosons, and thus indicate violation of N-scopic local realism.

D. Leggett-Garg version of the macroscopic Bell test

1. For N bosons

In the Bell test, one measures the joint probability for specific outcomes at the two sites A and B, with time settings t_a and t_b, respectively. These measurements are assumed nearly simultaneous in that the two measurement events are spacelike separated. The assumption is made that the measurement (made over the time t_b) at B has no effect on the measurement (made over the time t_a) at A. This is the assumption of locality.

The test may also be carried out in a different way that is more similar to the proposal of Leggett and Garg [7]. We consider that the two separated systems at site A and at site B are prepared in one of the Bell states [Eq. (1)], and that each system then interacts locally from time $t = 0$ with a local nonlinear beam splitter (NBS), as in Fig. 2. The systems evolve continuously and can be measured at the times t_i at the location A, and at the times t_j at the location B. The measurement at time t_i gives a value for the spin $S_A(t_i) = \pm 1$, depending on whether the N bosons at the site A are detected at a (spin "up") or at a_2 (spin "down"). The spin at site B is defined similarly, as $S_B(t_j)$.

Assuming N-scopic realism, each of the two systems A and B is at all times in a state with a definite spin outcome, ± 1, for the spins $S_A(t_i)$ and $S_B(t_j)$. The definite predetermined value for the spins can be denoted by the hidden variables $\lambda_A(t_i)$ and $\lambda_B(t_j)$. Letting $\lambda_1 = \lambda_B(t_1)$, $\lambda_2 = \lambda_A(t_2)$, $\lambda_3 = \lambda_B(t_3)$, we see that $\lambda_1\lambda_2 + \lambda_2\lambda_3 - \lambda_1\lambda_3 \leqslant 1$. Supposing one may construct a noninvasive measurement that can be made at time t in such a way that does not change the state of either system at the future (or past) time t', we arrive at an inequality similar to the three-time Leggett-Garg inequality [7] originally derived by Jordan *et al.* [9]:

$$\langle S_B(t_1)S_A(t_2)\rangle + \langle S_A(t_2)S_B(t_3)\rangle - \langle S_B(t_1)S_B(t_3)\rangle \leqslant 1. \quad (15)$$

Here the $\langle S_A(t_i)S_B(t_j)\rangle$ are two-time correlation functions, the values of which are given in terms of the hidden variables, e.g., $\langle S_A(t_i)S_B(t_j)\rangle = \langle \lambda_A(t_i)\lambda_B(t_j)\rangle$, according to the assumptions of N-scopic realism and noninvasive measurability.

The inequality (15) differs from the Leggett-Garg inequality [9], in that the latter involves three measurements at one site only. Taking $t_1 < t_2 < t_3$, the use of one site makes it challenging to justify that the measurement at time t_2 did not influence the hidden variable λ_3 describing the system prior to the final measurement, made at time t_3.

Looking at the inequality (15), however, which involves two sites, we see that the noninvasive measurability assumption can be justified by locality, as part of the assumption of N-scopic local realism. We assume locality, that the measurements at A do not change the values of the hidden variables for the system at B, and vice versa. This justifies that whether or not the measurement of $S_A(t_2)$ takes place at time t_2 does not change the predetermined value λ_3 and hence has no effect on the value of $\langle S_A(t_2)S_B(t_3)\rangle$, and similarly for $\langle S_B(t_1)S_A(t_2)\rangle$. The measurements of $\langle S_B(t_1)S_A(t_2)\rangle$ and $\langle S_A(t_2)S_B(t_3)\rangle$ are made by measurements performed at the two locations. The measurement of $\langle S_B(t_1)S_B(t_3)\rangle$ is more difficult since it must be justified that the first measurement at time t_1 has no effect on the outcome at the later time t_3, for the same system. Here, we use the knowledge of preparation of the system. We use the fact that for the initial state there is a perfect correlation or anticorrelation between the spin outcomes at A and the spin outcomes at B. We assume that initially, at $t_1 = 0$, the system is prepared in one of the correlated Bell states of Eq. (1). Assuming N-scopic local realism, the predetermined spin at B at time t_1 can then be inferred from the value of the outcome of the measurement at A at time t_1. For the case of $|\psi_{+,-}\rangle$ where the spins are correlated, this allows us to put

$$\langle S_B(t_1)S_B(t_3)\rangle = \langle S_A(t_1)S_B(t_3)\rangle. \quad (16)$$

Now we consider the specific example of a system prepared in the Bell state $|\psi_{+,-}\rangle$ and then undergoing local interactions at each site according to the nonlinear beam splitter (NBS) model. Assuming an ideal NBS at each location (similar to Fig. 2), the two-time correlation is [refer Eq. (5)]

$$\langle S_B(t_j)S_A(t_i)\rangle = E(t_i, t_j) = \cos 2(t_-) = \cos 2(t_i - t_j). \quad (17)$$

Violations of the inequality (15) are predicted, as can be seen by putting $t_1 = 0$, $t_2 = \pi/6$, $t_3 = \pi/3$, in which case $\langle S_B(t_1)S_A(t_2)\rangle = \frac{1}{2}$, $\langle S_A(t_2)S_B(t_3)\rangle = \frac{1}{2}$, and $\langle S_A(t_1)S_B(t_3)\rangle = -\frac{1}{2}$. This implies a violation of the Leggett-Garg-type inequality (15), with a value of 1.5 for the left side of the inequality. Taking $t_3 = 5\pi/12$ also gives a violation.

The proposal using NOON states as given in Fig. 2 generates the Bell states conditionally. At time $t_a = t_b = 0$, the conditioned state is the Bell state $|\psi_{-,\pm}\rangle$. This means that for the reduced ensemble where a total of N bosons are detected at both sites A and B, the observation of spin ± 1 at A implies spin ∓ 1 at B, and vice versa. Moreover, as we show above in Eq. (12), the conditioned evolution will give the two-time correlations of type (5), thus leading to a violation of the inequality. Here, however, possible loopholes are introduced from the postselection of the subensemble, and an additional no-enhancement assumption is needed [26]. The use of the Clauser-Horne inequality for the Bell example in the last section avoids the loophole in that case.

Finally, we note that the Leggett-Garg experiment can be performed using the CHSH form of the inequality, proposed in the original paper by Leggett and Garg [7]. Here, one considers four times $t_1 < t_2 < t_3 < t_4$, and evaluates the CHSH Bell inequality

$$\langle S_A(t_1)S_B(t_2)\rangle + \langle S_A(t_3)S_B(t_2)\rangle + \langle S_A(t_3)S_B(t_4)\rangle - \langle S_A(t_1)S_B(t_4)\rangle \leqslant 2. \quad (18)$$

The inequality is the same as the Bell inequality (6) and is violated for the subensemble conditioned on postselection of N bosons at A and N bosons at B, for the state $|\psi_{-,-}\rangle$ for the choice of times $t_1 = 0, t_2 = \pi/8, t_3 = \pi/4, t_4 = 3\pi/8$. We note that in the Leggett-Garg version of the experiment, one would keep the strict time order. The derivation of this inequality is as given for (6) and is a simple extension of the three-time inequality (15) above. It is assumed that the measurement made at a certain time has no effect on the dynamics at later times, if that measurement is made in a way that ensures spacelike-separated measurement events. Here, all two-time correlations involve separated sites, and so the argument justifying the relation (16) is not necessary.

2. Leggett-Garg test for $N = 1$: No classical trajectories

It is interesting to consider an experiment for $N = 1$. Here, one may prepare a Bell state using standard methods, as, for example, the photonic polarization entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\{|1\rangle_{a_+}|0\rangle_{a_-}|1\rangle_{b_+}|0\rangle_{b_-} + |0\rangle_{a_+}|1\rangle_{a_-}|0\rangle_{b_+}|1\rangle_{b_-}\}. \quad (19)$$

The $a_\pm$ and $b_\pm$ are modes for orthogonal polarization directions, at the sites A and B, respectively. We consider the three-time Leggett-Garg inequality (15). In Fig. 2, the nonlinear beam splitter (NBS) is replaced by a simple polarization beam splitter (PBS) at each site, and the modes a, a_2 and b, b_2 at the sites A and B are now symbolized by $a_\pm$ and $b_\pm$. The choice of time settings t_a, t_a' and t_b, t_b' now becomes the choice of polarization angles θ, θ' at A, and ϕ, ϕ' at B, as in the usual Bell inequality. At each beam splitter, the transformation is given by a unitary evolution. Suppose a measurement is made of the polarization along the direction θ_A of the photon incident at site A, and of the polarization along direction ϕ_B of the photon incident at site B. After the beam splitter interaction, the photon is detected at either the spin "up" or the spin "down" position, at each location. It is well known that for the entangled state, $\langle S_A(\theta_A)S_B(\phi_B)\rangle = \cos 2(\theta_A - \phi_B)$.

Now we consider the following modification of the standard Bell experiment. We suppose that at each site, the incident photon propagates through a series of polarization beam splitters. At time t_0, the photons are prepared in the correlated Bell state. The photons at each site then propagate through a PBS set at angle θ. A measurement $\hat{M}_A(t_1)$ and $\hat{M}_B(t_1)$ consisting of detection at the arms of the PBS at A and B (respectively) could then be made, at time t_1, to give the spin outcomes $S_A(t_1)$ and $S_B(t_1)$. The result would be that the spins are correlated, in accordance with the preparation Eq. (19), since the polarization beam splitters at each site were set similarly at angle θ. Supposing that the measurement $\hat{M}_A(t_1)$ or $\hat{M}_B(t_1)$ (or both) is not made, the photon at A or B then travels though a second PBS orientated at ϕ. After the photon has traveled through the second PBS, a measurement $\hat{M}_A(t_2)$ or $\hat{M}_B(t_2)$ could be made at the PBS at the location A or B, to give the result for the spin $S_A(t_2)$ or $S_B(t_2)$ at the time t_2, respectively. If measurements $\hat{M}_B(t_1)$ and $\hat{M}_A(t_2)$ were made, this would give probabilities such that

$$\langle S_B(t_1)S_A(t_2)\rangle = \cos 2(\phi-\theta). \tag{20}$$

Supposing the measurement $\hat{M}_A(t_2)$ or $\hat{M}_B(t_2)$ does not take place at the given site, the photon at A or B then travels through a third PBS, with angle θ'. After passage through the third PBS, at time t_3, the polarization of the photon can be measured using the detection $\hat{M}_A(t_3)$ or $\hat{M}_B(t_3)$, to give a result for the spin outcome $S_A(t_3)$ or $S_B(t_3)$ at time t_3, respectively.

The Leggett-Garg inequality is satisfied if classical trajectories are valid at each location, and assuming the validity of locality. One considers the three-time Leggett-Garg inequality (15) and the two-time spin correlations: $\langle S_B(t_1)S_A(t_2)\rangle = \cos 2(\phi-\theta)$, $\langle S_A(t_2)S_B(t_3)\rangle = \cos 2(\theta'-\phi)$, and $\langle S_A(t_1)S_B(t_3)\rangle = \cos 2(\theta'-\theta)$. Using that the prepared state has correlated spin, one can use Eq. (16). Choosing $\theta=0, \phi=\pi/6, \theta'=\pi/3$, we find there is violation of the Leggett-Garg inequality. One may also take $\theta=0, \phi=\pi/6, \theta'=5\pi/12$. It is also possible to use the four-time Leggett-Garg inequality given by (18). Here it is necessary to include a fourth PBS set at angle ϕ' and to consider final measurements at a time t_4.

IV. MACROSCOPIC BELL TESTS USING CAT STATES

A. State preparation

To illustrate violations of a Bell inequality in a macroscopic regime, we consider a second example, that of a Bell cat state involving coherent states:

$$|\psi_0\rangle = \mathcal{N}(|+\rangle_a|+\rangle_b - |-\rangle_a|-\rangle_b). \tag{21}$$

Here, we take

$$\begin{aligned}
|+\rangle_a &= -\frac{e^{i\pi/6}}{\sqrt{2}}(|e^{i\pi/3}\alpha\rangle_a + |e^{-i\pi/3}\alpha\rangle_a),\\
|-\rangle_a &= |-\alpha\rangle_a,\\
|+\rangle_b &= -i|\beta\rangle_b,\\
|-\rangle_b &= i\frac{e^{-i\pi/6}}{\sqrt{2}}(|-e^{i\pi/3}\beta\rangle_b + |-e^{-i\pi/3}\beta\rangle_b),
\end{aligned}$$

where $|\alpha\rangle$, $|\beta\rangle$ are coherent states for two modes labeled a and b. We will assume α and β to be real. The normalization constant is $\mathcal{N}$. As α and β become large, the states $|+\rangle_a$ and $|-\rangle_a$, and similarly $|+\rangle_b$ and $|-\rangle_b$, are well separated in phase space, with a separation of order α and β, respectively. In this limit, the states are said to be macroscopically distinct [42]. Since we consider α, β large, the states $|\pm\rangle_a$ become orthogonal, and similarly $|\pm\rangle_b$, and we find that $\mathcal{N}\to\frac{1}{\sqrt{2}}$.

One can perform quadrature phase amplitude measurements $\hat{X}_A = \frac{1}{\sqrt{2}}(\hat{a}+\hat{a}^\dagger)$ and $\hat{X}_B = \frac{1}{\sqrt{2}}(\hat{b}+\hat{b}^\dagger)$ on each field mode. The "spin" result $\hat{s}$ is taken to be $+1$ ("up") if the result for such a measurement is >0, and -1 ("down") otherwise. A state with outcome ±1 is denoted $|\pm\rangle$. We see then that the state (21) corresponds to $|\psi_{+,-}\rangle_{AB}$ of (1), where we substitute $|\pm\rangle_{A/B}$ with $|\pm\rangle_{a/b}$.

B. Nonlinear dynamics via Kerr interactions

Similarly to the situation depicted in Fig. 2, we suppose that the two modes are spatially separated into respective regions labeled A and B, and that at each site there is the choice to interact the system with a nonlinear medium for a certain time. The nonlinear interaction is analogous to the nonlinear beam splitters (NBS) described in the previous sections, except here we consider only one mode at each site. The choice corresponds to a choice between two times t_a or t_a' at A, and an independent choice between two times t_b or t_b' at B. As in the last section, we note this may actually be a choice to switch between two different nonlinear interaction values (two different media) and to interact for the same amount of time. The interaction at site i is represented as a unitary transformation $\hat{U}_i(t)$.

Here, we propose that nonlinear Kerr interactions $H_{NL}^{(A)} = \Omega(\hat{a}^\dagger\hat{a})^2$ and $H_{NL}^{(B)} = \Omega(\hat{b}^\dagger\hat{b})^2$ act locally, for a time t_A and t_B, respectively, on the separated field modes a and b, which are prepared in the cat state (21). For a single mode prepared in a coherent state $|\alpha\rangle$, it is known that the Kerr interaction after certain times leads to the formation of macroscopic superposition states [42–45]. In particular, at time $t_a' = \pi/(3\Omega)$, if the initial state is $|\alpha\rangle$, the state at A is [19,42]

$$|\psi(t_a')\rangle = -i\sqrt{\frac{1}{3}}|-\rangle_a + \sqrt{\frac{2}{3}}|+\rangle_a. \tag{22}$$

For an initial state $|\beta\rangle$, the state at time $t_b' = 2\pi/(3\Omega)$ is

$$|\psi(t_b')\rangle = \sqrt{\frac{1}{3}}|+\rangle_b - i\sqrt{\frac{2}{3}}|-\rangle_b. \tag{23}$$

Now we consider the states that are created by the local nonlinear evolutions $H_{NL}^{(A)}$ and $H_{NL}^{(B)}$ on the systems prepared in the initial state Eq. (21). We specifically select $t_a=0$ and $t_a'=\pi/(3\Omega)$, and $t_b=0$ and $t_b'=2\pi/(3\Omega)$. The full details of the calculation are given in the Appendix, in the limit of large α, β. In particular, we find the final state $|\psi_f\rangle$ for the four combinations of time settings at each location. Solving in the limit of large α, β where orthogonality of the states can be assumed, and limiting to the chosen time settings ($t_A = t_a$ or t_a' and $t_B = t_b$ or t_b') so that we can implement the transformations (A3) and (A6) derived in the Appendix, we find

$$\begin{aligned}
|\psi_f\rangle &= \hat{U}_A(t_A)\hat{U}_B(t_B)|\psi_0\rangle\\
&= \frac{1}{\sqrt{2}}\{(ab+\bar{a}\bar{b})(|+\rangle_{a'}|+\rangle_{b'} - |-\rangle_{a'}|-\rangle_{b'})\\
&\quad + i(\bar{a}b - a\bar{b})(|+\rangle_{a'}|-\rangle_{b'} - |-\rangle_{a'}|+\rangle_{b'})\}.
\end{aligned} \tag{24}$$

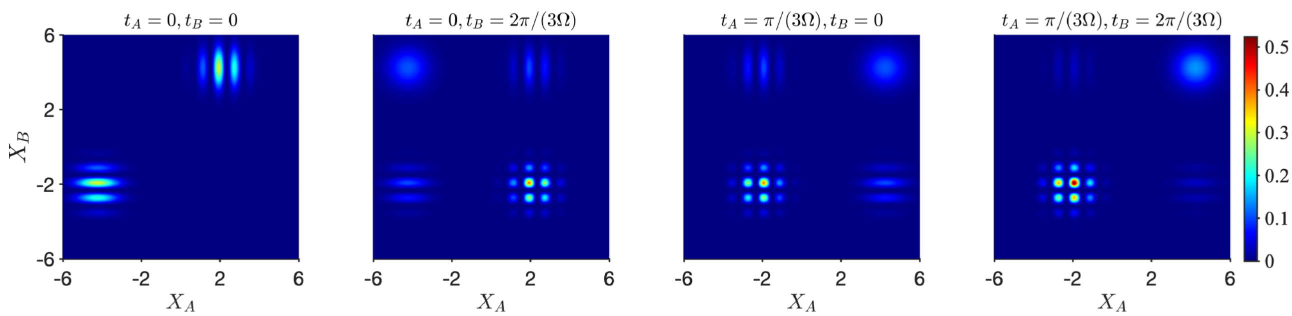

FIG. 5. Macroscopically distinct outcomes in phase space. Contour plots are given for the joint probability distributions $P(X_A, X_B)$ corresponding to the quadrature phase amplitude measurements $\hat{X}_A$ and $\hat{X}_B$ at each site, for each of the time settings relevant to the Bell test. The outcomes for "spin" correspond to the sign of the quadrature phase amplitudes $X_{A/B}$ and hence are directly associated with the quadrant in phase space. For each of the 4 combinations of the time settings given, we see that the outcomes are macroscopically distinct as $\alpha = \beta \to \infty$. Here we give results for $\alpha = \beta = 3$.

Here the a, b when written as part of an equation as above are the coefficients associated with a unitary transformation and we denote $\bar{a} = \sqrt{(1-a^2)}$ and $\bar{b} = \sqrt{(1-b^2)}$. In particular, $a = 1$ if $t_A = t_a = 0$ and $a = \sqrt{\frac{2}{3}}$ if $t_A = t'_a = \pi/(3\Omega)$; $b = 1$ if $t_B = t_b = 0$ and $b = \sqrt{\frac{1}{3}}$ if $t_B = t'_b = 2\pi/(3\Omega)$. The details of the states $|\pm\rangle_{a'}$ and $|\pm\rangle_{b'}$ are given in the Appendix. We do not need to specify those here, except to note that the precise form of $|\pm\rangle_{a'}$ and $|\pm\rangle_{b'}$ depends on the choice of t_A and t_B, respectively, but is fixed for each of those choices. What is important for the remaining calculation is that $|+\rangle_{a'}$ and $|-\rangle_{a'}$ are states giving outcomes $+1$ and -1 for the sign $\hat{s}$ ("spin") associated with the measurement $\hat{X}_A$. Similarly, the states $|\pm\rangle_{b'}$ give outcomes ± 1 for the sign associated with the measurement $\hat{X}_B$.

C. Macroscopic Bell violations

Thus, recalling again that the states $|+\rangle$ and $|-\rangle$ as defined for each mode are orthogonal for large α and β, and correspond to states with a $\pm$ spin outcome, we see that for the final state $|\psi_f\rangle$, the expectation value for the product of spin outcomes at each site is

$$E(t_A, t_B) = |ab + \bar{a}\bar{b}|^2 - |\bar{a}b - a\bar{b}|^2. \quad (25)$$

Using the values for a and b corresponding to each time setting, we evaluate the CHSH-Bell inequality $B = E(t_a, t_b) - E(t_a, t'_b) + E(t'_a, t'_b) + E(t'_a, t_b) \leqslant 2$ referred to in Sec. II. The spin correlations are $E(t_a, t_b) = 1$; $E(t_a, t'_b) = -1/3$; $E(t'_a, t_b) = 1/3$; $E(t'_a, t'_b) = 7/9$. This implies violation of the Bell inequality (6) with $B = 2.44$.

For finite amplitudes, the separated coherent states such as $|\alpha\rangle$ and $|-\alpha\rangle$ are not completely orthogonal. One may calculate the actual joint distributions $P(X_A, X_B)$ for the results (X_A and X_B) of the quadrature phase amplitude measurements $\hat{X}_A$ and $\hat{X}_B$, and confirm the probabilities for the sign of the quadrature phase amplitudes by integration. Hence the spin product $E(t_A, t_B)$ can be evaluated for finite α and β. Figures 5 and 6 give the complete predictions for arbitrary α, β, accounting for the full effect of nonorthogonality of the coherent states. The outcomes of measurements of $\hat{s}$ (the sign of $\hat{X}$) are macroscopically distinct, corresponding to macroscopically distinguishable states in phase space, for *all* of the choices of time settings, as α, β become large. This is seen in Figs. 5 and 6 for $\alpha = \beta = 5$ and $\alpha = \beta = 3$.

One can thus determine the effect of the nonorthogonality on the Bell violation. Where the peaks are well separated (i.e., $\alpha, \beta > 2$), we see from Fig. 6 that there is little effect on the violation of the Bell inequality. Violations of α-scopic local realism are thus predicted for all $\alpha = \beta > 2$, and a violation of macroscopic local realism is obtained as $\alpha, \beta \to \infty$. We note the Gaussian peaks in the distribution for $P(X_A, X_B)$ are well separated for $\alpha = \beta = 2$, and the strong violation predicted for $\alpha = \beta = 2$ is promising that an experiment can be performed.

D. Leggett-Garg version of the macroscopic Bell test

Now it remains to show how to obtain a violation of the Leggett-Garg inequality, given by Eq. (15). With the proposed method of generation using the cat state, the initial state is perfectly correlated in spin with respect to the two modes, and the argument given in Sec. III D 1 for the justification of the measurement of the two-time correlation $\langle S_B(t_1)S_B(t_3)\rangle$ is valid.

Violations of the inequality are predicted, as can be seen by evaluating the two-time correlations, as $\langle S_B(t_i)S_A(t_j)\rangle = E(t_j, t_i)$. We put $t_1 = t_a = t_b = 0$, $t_2 = t'_a = \pi/(3\Omega)$, $t_3 =$

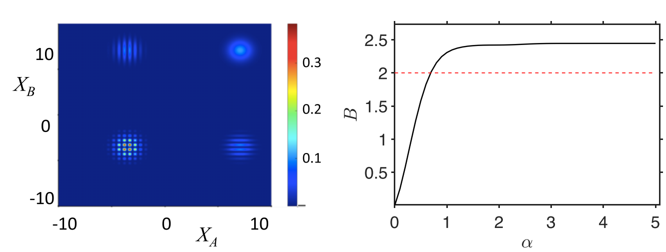

FIG. 6. Violation of macroscopic local realism, for macroscopically distinct states, as $\alpha \to \infty$. Here we allow for finite amplitudes, α and β. Left: A contour plot of the joint probability distribution for the outcomes of the quadrature phase amplitudes $\hat{X}_A$ and $\hat{X}_B$ at times $t_a = \pi/(3\Omega)$ and $t_b = 0$. The four outcomes depicted are macroscopically distinct for α, β large. The outcomes for the remaining three time settings are similarly distinct, as in Fig. 5. Here $\alpha = \beta = 5$. Right: A violation of the CHSH-Bell inequality Eq. (6) for α-scopic local realism is possible when $B > 2$. Violations are obtained for arbitrarily large $\alpha = \beta$.

$t_b' = 2\pi/(3\Omega)$, in which case

$$\langle S_B(t_1)S_A(t_2)\rangle = E(t_b, t_a') = \frac{1}{3},$$

$$\langle S_A(t_2)S_B(t_3)\rangle = E(t_a', t_b') = \frac{7}{9},$$

$$\langle S_A(t_1)S_B(t_3)\rangle = E(t_a, t_b') = -\frac{1}{3}.$$

This implies a violation of the Leggett-Garg inequality, with a value of 13/9 for the left side of the inequality. With the proposed method of generation using the cat state, there is no loophole created through postselection.

V. DISCUSSION

We have argued that the violations of the Bell inequalities presented in this paper falsify macroscopic local realism (where N or α are large) because the outcomes measured at location A (B) are distinct by a macroscopic amount. Small changes to the outcomes cannot change the classification of the "spin" outcome ± 1, at A (B). It is concluded that the violation is due to a failure of macroscopic locality, or else of macroscopic realism (or both).

A. A critic's counterargument

It is interesting to consider counterarguments to the conclusion that macroscopic local realism has been falsified. A new feature of the Bell violations studied here is that of the local dynamics, which occurs over a time frame. A critic C might claim that the violation of the Bell inequality is not due to a macroscopic nonlocal quantum effect, but is a consequence of the known failure of local realism at the microscopic level. A critic thinking this way might argue that there is a microscopic nonlocal quantum effect at some time t, which is then translated by the local dynamics (which occurs over a time t_a or t_b) into a macroscopic effect, which is then registered by the detectors.

This interpretation could be further explored. The predictions for the Bell-inequality violation depend on the values t_a, t_b of the local evolution times at each site, and hence the actual times used at locations A and B can be shifted against a shared clock. The distance between sites A and B is assumed large enough to justify no causal effects between A and B. The observer at each site makes the decision to cease the unitary evolution (e.g., the nonlinear beam splitter interaction) at their location at some time, but the evolution is fully reversible, or can be later continued, up until the time of the final number or quadrature phase amplitude detection, which constitutes the irreversible stage of the measurement process. The times can therefore be adjusted so that the final irreversible detections at each site A and B are made simultaneously. Then the critic would reconsider and likely argue that either there is a sudden large nonlocal effect due to the irreversible detections, or else that nonlocal effects due to the irreversible detections at B (A) act *back* in time, if these effects are to be considered microscopic (and then amplified by the unitary dynamics). By increasing the time delay between turning off the nonlinear (NBS) evolution and making the final irreversible detection at each site, the test could be made stricter. Alternatively, the critic might argue that it is the reversible actions associated with the unitary dynamics that induce (small) nonlocal effects [which then leads to a larger change at A (B) prior to detection].

A second response to the critic is to recognize that in this paper, macroscopic local realism has been defined in the context of spacelike separated measurement events. Macroscopic locality is defined to exclude only a macroscopic change to the separated system over the relevant local time *interval*. In thinking of an analogy to Schrödinger's cat paradox [1], the implication that is seen to be ridiculous in the cat paradox is that the cat can be "simultaneously" both dead and alive. This refers to failure of macroscopic realism, macroscopic realism being the assumption that at a given time t, the result of the distinguishing measurement $\hat{M}$ (as defined in the Introduction) is predetermined [7]. In this paper, we have not shown this type of failure, which relates to a definite time, but rather a paradox over the time span associated with the local dynamics. This motivates a careful analysis of the assumptions behind the Bell derivation as applied to our case, to be given in the next section.

B. Macroscopic realism: Two definitions and two derivations of the Bell inequality

The macroscopic Bell inequalities have been derived from the assumption of *macroscopic local realism* (MLR), as defined consistently with the assumption of macroscopic realism: the system prior to measurement is in a state with a definite predetermined value for the outcome of the distinguishing measurement $\hat{M}$. We might also refer to this as *macroscopic deterministic realism*. This is in line with the original derivation of Bell [20], which assumed hidden variables that have definite values of either $+1$ or -1, for the measurement of Pauli spin: The spin is "up" or "down," prior to measurement.

As with the traditional Bell inequality, we may also derive the macroscopic Bell inequality in a different way, which we refer to as *macroscopic local causality* (MLC). Here, one assumes local hidden variable states, thus allowing for the possibility that the interaction between the system and the local measurement apparatus may stochastically affect the spin outcome [26]. In such a derivation, it is assumed that there exists a probability $P_\theta(\pm|\lambda)$ for a spin outcome of ± 1 at a particular site, given the hidden variable parameters λ and the local choice θ of the measurement setting. Locality is also assumed, meaning that the probability is independent of ϕ, the setting at the remote location. The results of this paper indicate failure of this assumption (referred to as local causality [26]) at the macroscopic level, where the spin outcomes are macroscopically distinct. In this case, the macroscopic local realism premise is generalized: Macroscopic realism (MR) may be generalized to the assumption that the system is in a state which leads (after the interaction with the measurement apparatus) to one or other of the macroscopically distinct outcomes, with a certain probability. Macroscopic locality (ML) then asserts that the measurement at the distant location cannot change the predetermined probabilities of the macroscopic outcomes.

In fact, the results of this paper show that both sets of assumptions (MLR and MLC) are not valid. This helps clarify an argument that may be put forward as to why the first set of assumptions (MLR) does not hold. A common view (given by a critic D) might be that the assumption of definite values cannot be applied simultaneously to both the measurements that occur at a given site, i.e., that prior to the choice of time settings t_a and t'_a (given in this discussion by θ and θ'), the system cannot be regarded as having simultaneous definite values for the outcomes associated with both choices of measurement (t_a and t'_a). This supposed failure of simultaneous *macroscopic deterministic realism* is consistent with the known contextuality of quantum mechanics [46]. The critic D may argue that the measurement choices correspond to different unitary interactions which change the system as it evolves, but in such a way as to give definite macroscopically distinct outcomes *after* the interaction. In this view, macroscopic deterministic reality holds immediately before the final irreversible stage of the measurement, but not before the reversible stage modeled by the unitary evolution. In such a view, the Bell inequalities fail because of a failure of the assumption of a strong macroscopic reality, but a weaker more general form of macroscopic reality holds.

However, such an analysis is not sufficient in itself to explain the violation of the second set of assumptions (MLC). The second set of assumptions allows for an interaction of the localized subsystem with a local measurement apparatus. The local theories account for descriptions where an individual system is in one of two states giving macroscopically distinct outcomes after the unitary evolution, at a definite time t, and just prior to the irreversible final part of the measurement. Yet, the violation of the Bell inequality shows that such a description is not valid, *if* one assumes that the $P_\theta(\pm|\lambda)$ is independent of ϕ [and similarly that $P_\phi(\pm|\lambda)$ is independent of θ]. Thus, the viewpoint (from the perspective where the weaker more general form of macroscopic realism is preserved) would be that the Bell inequalities fail because of a breakdown of the locality assumption. This might occur over the time interval of the unitary dynamics, consistent with arguments put forward by critic C. In summary, one cannot claim that the violations of the macroscopic Bell inequalities arise solely from the failure of the strong form of macroscopic realism (macroscopic deterministic realism) [47].

VI. CONCLUSION

To summarize, we have shown how to violate local realism where measurements are macroscopic, meaning that they do not resolve microscopic details of the system. The measurements thus distinguish between two macroscopically distinct states of the system. We have studied two examples, the first being based on NOON states where there are N bosons at a given site before detection. In the second example, we considered entangled cat states, which are superpositions of coherent states well separated (by of order α) in phase space. To obtain the macroscopic Bell violations, we consider at each of two sites the choice between two measurement settings corresponding to different times of evolution through a nonlinear medium. This could also be implemented by a switch between two devices with a different nonlinearity, allowing the same measurement time. The tests we describe also give a way to falsify Leggett and Garg's macrorealism, except that here the assumption of noninvasive measurability is replaced by that of locality. A discussion is given in Sec. V.

It is interesting to examine the feasibility of an experiment. First, preparing entangled macroscopic superposition states where subsystems are spatially separated is a challenge. This can be addressed if each of the initial NOON states of Fig. 2 has separated modes. For $N = 2$, this might be possible using the Hong-Ou-Mandel effect, which produces a NOON state in the output modes of a beam splitter. In principle, those output modes can be separated [40]. We point out that the observation of the violations for $N = 2$ would be of interest. However, one also needs to achieve the nonlinear beam splitter part of the experiment, which is also challenging. For a Rb BEC, where the number of bosons N is large, the timescales required for the nonlinear beam splitter become inaccessibly long, based on current experiments [18,29,30]. Such a nonlinear beam splitter is however likely achievable using superconducting circuits to obtain high nonlinearities [5,31].

The proposal using cat states may be promising for an experiment. A two-mode cat state similar to (21) has been generated in a microwave setup [6], although without spatial separation of the modes. The dynamics of (22) and (23), which generates multicomponent cat states from coherent states, has been realized for both microwave fields and BECs [44,45]. We note that since macroscopic realism (MR) is suggestive of the validity macroscopic locality (ML) [21], an experimental test, even if without spatial separation, for $\alpha \geqslant 2$, would be of interest.

Finally, we comment that the tests we propose are feasible for $N = 1$. This is discussed in Sec. III D 2. While not macroscopic, such tests are significant for their potential to negate classical trajectories, as with the Leggett-Garg experiments that have been performed with single photons or atoms [10,12,14]. The advantage of the current proposal is a stronger justification of the noninvasive measurability premise. Here, noninvasive measurability is justified by the assumption of locality, for spacelike separations, because we have two particle trajectories (as opposed to one in former tests). The state for $N = 1$ can be prepared by standard methods, as a polarization entangled Bell state. The nonlinear beam splitters are not needed, since the local measurements can be achieved using (polarizing) beam splitters. The proposed test differs from the usual Bell experiment, being constructed similarly to the Leggett-Garg experiment for photons [12], with successive polarizer beam splitters in place, to create two particle trajectories.

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APPENDIX

Continuing from Eq. (23), we note that if initially the system a is in state $|+\rangle_a$, then after a time $t'_a = \pi/(3\Omega)$, the state at A is

$$\hat{U}_A(t'_a)|+\rangle_a = \sqrt{\frac{2}{3}}|+\rangle_{a'} - i\sqrt{\frac{1}{3}}|-\rangle_{a'}, \tag{A1}$$

where

$$|+\rangle_{a'} = -|\alpha\rangle,$$
$$|-\rangle_{a'} = \frac{1}{\sqrt{2}}\exp(-i\pi/6)\{|-e^{i\pi/3}\alpha\rangle + |-e^{-i\pi/3}\alpha\rangle\}.$$

If initially the system a is in state $|-\rangle_a$, then after a time $t'_a = \pi/3\Omega$, the state is

$$\hat{U}_A(t'_a)|-\rangle_a = \sqrt{\frac{2}{3}}|-\rangle_{a'} - i\sqrt{\frac{1}{3}}|+\rangle_{a'}. \quad \text{(A2)}$$

The states $|+\rangle_{a'}$ and $|-\rangle_{a'}$ are orthogonal for large α. In this case, the transformation due to $\hat{U}_A(t'_a)$ can be written as (for large α)

$$\hat{U}_A(t'_a)|+\rangle_a = a|+\rangle_{a'} - i\sqrt{1-a^2}|-\rangle_{a'},$$
$$\hat{U}_A(t'_a)|-\rangle_a = a|-\rangle_{a'} - i\sqrt{1-a^2}|+\rangle_{a'}, \quad \text{(A3)}$$

where $a = \sqrt{\frac{2}{3}}$. We note that $|\pm\rangle_{a'}$ are states with the outcome $\pm$ for the sign of the quadrature measurement $\hat{X}_A$.

Similarly, for site B, we select times $t_b = 0$ and $t'_b = 2\pi/(3\Omega)$. For initial state $|-\rangle_b$, after evolving locally for a time t'_B, the system is given by

$$\hat{U}_B(t'_b)|-\rangle_b = \sqrt{\frac{1}{3}}|-\rangle_{b'} - i\sqrt{\frac{2}{3}}|+\rangle_{b'}, \quad \text{(A4)}$$

where

$$|+\rangle_{b'} = -|\beta\rangle_b,$$
$$|-\rangle_{b'} = e^{i\pi/6}(|-e^{i\pi/3}\beta\rangle_b + |-e^{-i\pi/3}\beta\rangle_b)/\sqrt{2}.$$

Now if initially in state $|+\rangle_b$, then after an interaction time $t'_a = \pi/3\Omega$, the system is in the state

$$\hat{U}_B(t'_b)|+\rangle_b = \sqrt{\frac{1}{3}}|+\rangle_{b'} - i\sqrt{\frac{2}{3}}|-\rangle_{b'}. \quad \text{(A5)}$$

The states $|+\rangle_{b'}$ and $|-\rangle_{b'}$ are orthogonal for large β. Hence the transformation due to $\hat{U}_B(t'_b)$ can be expressed in the form

$$\hat{U}_B(t'_b)|+\rangle_b = b|+\rangle_{b'} - i\sqrt{1-b^2}|-\rangle_{b'},$$
$$\hat{U}_B(t'_b)|-\rangle_b = b|-\rangle_{b'} - i\sqrt{1-b^2}|+\rangle_{b'}, \quad \text{(A6)}$$

where $b = \sqrt{\frac{1}{3}}$. We note that $|\pm\rangle_{b'}$ represent states with the outcome $\pm$ for the sign of the quadrature measurement $\hat{X}_B$.

We also see that for the choice of times $t_a = 0$ and $t_b = 0$, the transformations $\hat{U}_A(t_a)$ and $\hat{U}_B(t_b)$ have a similar form to (A3) and (A6), since we can use $a = 1$ and define $|+\rangle_{a'}$ as $|+\rangle_a$ in the case of $t_A = 0$, and use $b = 1$, defining $|+\rangle_{b'}$ as $|+\rangle_b$, in the case of $t_B = 0$.

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