Spontaneous decay processes in a classical strong low-frequency laser field

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The spontaneous emission of an excited two-level emitter driven by a strong classical coherent low-frequency electromagnetic field is investigated. We find that for relatively strong laser driving, multiphoton processes are induced, thereby opening additional decay channels for the atom. We analyze the interplay between the strong low-frequency driving and the interfering multiphoton decay channels, and discuss its implications for the spontaneous emission dynamics.

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I. INTRODUCTION

Spontaneous emission (SE) is a basic process occurring in excited quantum systems coupled to environments [1–5]. Since it typically competes with coherent processes induced, e.g., by laser fields, its manipulation or even control is of vital importance for many applications. The SE rate of an atom depends on the transition dipole moment and the density of states of the environment [3–5]. Therefore, a first control approach is to suitably modify the environment's density of states, e.g., using cavities [6–10] or photonic crystals [11–15]. An alternative approach is to control the coupling between atom and environment, which typically involves atomic coherence and quantum interference effects [16–19]. For example, slow or fast transition-frequency modulations [20–22] were shown to allow for substantial suppression of the SE of an excited two-level emitter inside a leaking cavity [23–25], and such control schemes can be extended to dc fields [26]. The possible effects of external modulations or perturbations on the SE into potentially structured environments can also be classified on a more general level [27,28]. Another ansatz to control SE facilitates spontaneously generated coherences (SGC) [4,18,19], which may suppress the SE of particular excited states via destructive interference of different decay pathways. Based on this, a broad variety of applications has been proposed, including lasing without inversion [29–31] and the stabilization of coherences in quantum computing [32,33], and SGC have also been observed experimentally [34,35]. Related approaches to control SE are reviewed in [13,14,18,19].

A further ansatz to modify and substantially slow down the usual spontaneous decay of excited atoms was proposed in [36,37], based on the application of a strong low-frequency electromagnetic field (LFF) to the excited emitter. A perturbative analysis in the LFF-atom coupling showed that the LFF induces additional multiphoton decay pathways, in which the atom exchanges photons with the field during the spontaneous decay. These arise since the model includes off-resonant excited auxiliary energy levels as possible intermediate states in the multiphoton processes, in addition to the two energy levels involved in the natural spontaneous decay. Importantly, "low frequency" here refers to driving fields with frequency lower than the spontaneous emission line width, such that the multiphoton pathways are indistinguishable and may interfere, thereby affecting the usual spontaneous decay. The LFF-induced multiphoton pathways were interpreted in [36,37] in terms of an effective upper-state multiplet of energy levels, with the spontaneous-emission modification arising from the interference of the decay amplitudes out of the different multiplet states. However, the initial work triggered further discussions [38–40], in which in particular the role of the multiphoton pathways and the interpretation in terms of an excited-state multiplet was questioned [39]. This invites further investigations on the effect of field-induced multiphoton processes on spontaneous emission and their interpretation.

Motivated by this, here, we investigate the spontaneous emission of an excited two-level quantum emitter interacting with an intense classical LFF. Unlike in the previous work [36,37], we restrict the analysis to a two-level system, and thereby explicitly exclude the possibility to induce interfering multiphoton pathways involving off-resonant auxiliary states. This choice allows us to explore the significance of these processes, but also enables the calculation of higher-order effects in the LFF-atom interaction. We show that despite the absence of intermediate states, the strong LFF still may induce interfering multiphoton evolution. However, the nature of these pathways is very different. They proceed directly from the excited to the ground state, but involve the interaction of the atom with different harmonics of the LFF, which again can be interpreted as the exchange of different numbers of photons of the LFF throughout the atomic transition. As a result of these different pathways, we again find that strong LFF driving may modify the standard exponential spontaneous decay law, either slowing down or accelerating the decay. However, the effect is not as pronounced as that predicted in [36,37]. This suggests that the additional decay pathways via off-resonant auxiliary states are crucial.

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We note that it is well known in general that intense electromagnetic fields may substantially modify the atomic dynamics [41–47]. Relevant to the SE control via multiphoton pathway interference discussed here, the probabilities for relevant multiphoton transitions in two-level systems interacting with a strong and coherent classical electromagnetic field of frequency much lower than the involved transition frequencies were calculated [48], as well as related light emission and absorption processes [49], and the multiphoton resonance-induced fluorescence of strongly driven two-level systems under frequency modulation [50–52]. It was also shown that various superposition states may occur via multiphoton resonant excitations in hydrogenlike atoms [53], and methods were developed to deal with the laser dressing of the atoms [54], or to calculate the relevant transition elements [55]. However, in the above-mentioned works on the quantum dynamics of isolated two-level systems interacting with a low-frequency and strong classical electromagnetic field, "low frequency" typically refers to field frequencies much lower than the involved transition frequencies, but not than the spontaneous emission linewidths. Also, these works do not investigate explicitly the spontaneous decay.

II. ANALYTICAL FRAMEWORK

The Hamiltonian of a two-level emitter interacting with a strong low-frequency field of frequency ω as well as with the environmental vacuum modes of the electromagnetic field reservoir is

$$H = \sum_k \hbar\omega_k a_k^\dagger a_k + \hbar\omega_0 S_z - \hbar\Omega\cos(\omega t+\phi)(S^+ + S^-) + i\sum_k (\vec{g}_k\cdot\vec{d})(a_k^\dagger - a_k)(S^+ + S^-). \quad (1)$$

Here, ω_0 is the transition frequency among the involved states $|2\rangle \leftrightarrow |1\rangle$ with the transition dipole d, whereas Ω is the corresponding Rabi frequency and ϕ is the laser absolute phase. The atom-vacuum coupling strength is $\vec{g}_k = \sqrt{2\pi\hbar\omega_k/V}\vec{e}_\lambda$ where V is the quantization volume while $\vec{e}_\lambda$ is the photon polarization vector with $\lambda = 1, 2$. $a_k^\dagger$ and a_k are the creation and annihilation operators for the photons with the momentum $\hbar k$, energy $\hbar\omega_k$ and polarization λ satisfying standard commutation relations for bosons. Further, $S^+ = |2\rangle\langle 1|$, $S^- = [S^+]^\dagger$, and $S_z = (|2\rangle\langle 2| - |1\rangle\langle 1|)/2$ are the well-known quasispin operators obeying the commutation relations for SU(2) algebra. In the Hamiltonian (1) the first three components are, respectively, the free energies of the environmental electromagnetic vacuum modes and atomic subsystems together with the laser-atom interaction Hamiltonian. The last term accounts for the interaction of a two-level emitter with the surrounding electromagnetic field vacuum modes.

The quantum dynamics of any atomic operator Q is determined by the Heisenberg equation,

$$\frac{d}{dt}Q(t) = \frac{i}{\hbar}[H, Q]. \quad (2)$$

In the following, we perform a spin rotation [21,53], $U(t) = \exp[2i\theta(t)S_y]$, to the entire Hamiltonian which transforms it as follows:

$$\bar{H} = UHU^{-1} - 2(d\theta(t)/dt)US_yU^{-1}. \quad (3)$$

Here, $\theta(t) \equiv \theta = \arctan[(2\Omega/\omega_0)\cos(\omega t+\phi)]/2$, while $S_y = (S^+ - S^-)/(2i)$. Then, the total Hamiltonian reads as follows:

$$\bar{H} = \sum_k \hbar\omega_k a_k^\dagger a_k + 2\hbar\bar{\Omega}(t)R_z + i\hbar\alpha(t)(R^- - R^+) + i\sum_k(\vec{g}_k\cdot\vec{d})(a_k^\dagger - a_k)(\cos 2\theta(R^+ + R^-) - 2\sin 2\theta R_z), \quad (4)$$

where

$$\bar{\Omega}(t) = \sqrt{(\omega_0/2)^2 + \Omega^2\cos^2(\omega t+\phi)}, \quad (5)$$

whereas $\alpha(t) = (\omega/2)\Omega\cos(2\theta)\sin(\omega t+\phi)/\bar{\Omega}(t)$ and $\omega/\omega_0 \ll 1$.

The new quasispin operators, i.e., R_z and $R^\pm$, can be represented via the old ones in the following way:

$$R_z = S_z\cos 2\theta - (S^+ + S^-)\sin 2\theta/2,$$
$$R^+ = S^+\cos^2\theta - S^-\sin^2\theta + S_z\sin 2\theta,$$
$$R^- = [R^+]^\dagger, \quad (6)$$

and obey the commutation relations: $[R^+, R^-] = 2R_z$ and $[R_z, R^\pm] = \pm R^\pm$, similarly to the old-basis ones. The Hamiltonian (4), based on the unitary transformation $U(t)$, will allow us to follow the quantum dynamics of the excited two-level emitter where absorption of the external low-frequency field photons is incorporated naturally. This is not evident if one starts directly with the Hamiltonian (1).

In what follows, we are interested in laser-atom interaction regimes such that $2\Omega/\omega_0 < 1$. On the other side, the Rabi frequency Ω can be smaller, of the same order, or larger than the laser frequency ω, respectively. Consequently, we expand the generalized Rabi frequency $\bar{\Omega}(t)$, in Eq. (5), up to second order in the small parameter $2\Omega/\omega_0$, namely,

$$\bar{\Omega}(t) \approx \frac{\omega_0}{2}\left(1 + \Omega^2/\omega_0^2 + \Omega^2\cos[2(\omega t+\phi)]/\omega_0^2\right). \quad (7)$$

Next, in the Hamiltonian (4), we pass to the interaction picture using the operator,

$$V(t) = \exp\left[2i\int_0^t dt'\bar{\Omega}(t')R_z\right],$$

with Eq. (7), and write down the formal solution of the Heisenberg equation for the field operator $a_k^\dagger(t)$, $a_k(t) = [a_k^\dagger(t)]^\dagger$, that is,

$$a_k^\dagger(t) = a_k^\dagger(0)e^{i\omega_k t} + \frac{(\vec{g}_k\cdot\vec{d})}{\hbar}\int_0^t dt' e^{i\omega_k(t-t')}\left\{\sum_{m=-\infty}^{\infty} J_m(\eta)\left(R^+(t')e^{i(\bar{\omega}_0 t' - \eta\sin 2\phi)}e^{2im(\omega t'+\phi)} + \text{H.c.}\right)\cos 2\theta - 2\sin 2\theta R_z(t')\right\}, \quad (8)$$

where

$$\cos 2\theta \approx 1 - (2\Omega/\omega_0)^2 \cos^2(\omega t' + \phi)/2,$$
$$\sin 2\theta \approx (2\Omega/\omega_0)\cos(\omega t' + \phi),$$

and

$$\bar{\omega}_0 = \omega_0\big(1 + \Omega^2/\omega_0^2\big). \tag{9}$$

Here, we used the expansion via the mth-order Bessel function of the first kind, i.e.,

$$e^{\pm i\eta \sin(2\omega t + 2\phi)} = \sum_{m=-\infty}^{\infty} J_m(\eta) e^{\pm 2im(\omega t + \phi)},$$

with $J_m(\eta)$ being the corresponding ordinary Bessel function, whereas

$$\eta = \frac{\Omega^2}{2\omega\omega_0} \tag{10}$$

stands as a control parameter. In the Markov approximation, we identify the following emission processes based on Eq. (8):

$$\int_0^\infty d\tau e^{i(\omega_k \mp \bar{\omega}_0 \mp 2m\omega)\tau} = \pi\delta(\omega_k \mp \bar{\omega}_0 \mp 2m\omega) + iP_c \frac{1}{\omega_k \mp \bar{\omega}_0 \mp 2m\omega},$$
$$\int_0^\infty d\tau e^{i(\omega_k \pm \omega)\tau} = \pi\delta(\omega_k \pm \omega) + iP_c \frac{1}{\omega_k \pm \omega}, \tag{11}$$

where P_c is the Cauchy principal part. One can observe here that the spontaneous emission processes involve an even laser photon number, i.e., the emission occurs at frequencies: $\omega_k = \bar{\omega}_0 \pm 2m\omega$ or $\omega_k = 2m\omega - \bar{\omega}_0 > 0$. This also means that the pumping field opens additional spontaneous decay channels that may interfere. Actually, the latter emission process implies that the sum frequency of the multiple absorbed photons is larger than the transition frequency—a situation not considered here. Apart from these processes there are also spontaneous transitions around the laser frequency ω, i.e., an induced laser photon absorption is followed by a spontaneously rescattered photon of the same frequency. Thus, the whole quantum dynamics is influenced by the above mentioned processes. Notice the modification of the transition frequency due to the external low-frequency strong coherent electromagnetic pumping field, see expression (9). Also, the contribution of P_c leading to a small frequency Lamb shift compared to the one due to direct photon absorption is ignored here.

The solution (8) has to be introduced in the Heisenberg equation for the mean value of any atomic subsystem's operators Q, namely,

$$\frac{d}{dt}\langle Q(t)\rangle - \frac{i}{\hbar}\langle[\bar{H}_0, Q(t)]\rangle = \sum_k \frac{(\vec{g}_k \cdot \vec{d})}{\hbar}\bigg\langle a_k^\dagger\bigg[2\sin 2\theta R_z - \cos 2\theta \sum_{n=-\infty}^{\infty} J_n(\eta) \big(R^+ e^{i(\bar{\omega}_0 t - \eta \sin 2\phi)} e^{2in(\omega t + \phi)} + \text{H.c.}\big), Q(t)\bigg]\bigg\rangle + \text{H.c.}, \tag{12}$$

where, in general, for the non-Hermitian atomic operators Q, the H.c. terms should be evaluated without conjugating Q, i.e., by replacing Q^+ with Q in the Hermitian conjugate part. The notation $\langle \cdots \rangle$ indicates averaging over the initial state of both the atoms and the vacuum environmental system, respectively. In the master equation (12), the Hamiltonian describing the coherent evolution of the qubit during multiple photon absorption and emission processes is given by

$$\bar{H}_0 = i\hbar\alpha(t) \sum_{n=-\infty}^{\infty} J_n(\eta) R^- e^{-i(\bar{\omega}_0 t - \eta \sin 2\phi)} e^{-2in(\omega t + \phi)} + \text{H.c.}, \tag{13}$$

with $\alpha(t) \approx (\omega\Omega/\omega_0)\sin(\omega t + \phi)$. Contrary to spontaneous emission processes, the coherent evolution involves an odd laser photon number, i.e., resonances occur when $\bar{\omega}_0 + (2n \pm 1)\omega = 0$; see also [56]. The final expression for the master equation in the Born-Markov approximations is somehow cumbersome, however, we have identified those terms given the main contribution to the atom's quantum dynamics. In particular, for $|2m\omega/\omega_0| < 1$ the master equation is

$$\frac{d}{dt}\langle Q(t)\rangle - \frac{i}{\hbar}\langle[\bar{H}_0, Q(t)]\rangle = -\gamma(t)\langle R^+[R^-, Q(t)]\rangle - \gamma^*(t)\langle[Q(t), R^+]R^-\rangle. \tag{14}$$

Here,

$$\gamma(t) = \frac{\gamma}{2}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} e^{2i(m-n)(\omega t + \phi)} \chi_n(x,\eta)\chi_m(x,\eta) \times (1 + x^2/4 + 2m\omega/\omega_0)^3,$$

with γ being the single-atom spontaneous decay rate at the bare transition frequency ω_0, i.e., $\gamma = 4d^2\omega_0^3/(3\hbar c^3)$, whereas

$$x = 2\Omega/\omega_0,$$

and

$$\chi_n(x,\eta) = (1 - x^2(1 + n/\eta)/4) J_n(\eta).$$

Here we have used the relation:

$$J_{n-1}(\eta) + J_{n+1}(\eta) = 2nJ_n(\eta)/\eta.$$

Also, in the numerical simulations we shall truncate the summation range $(-\infty, \infty)$ to $(-n_0, n_0)$ such that for a selected value of η one has $J_{n_0}(\eta) \to 0$ as well as $|2n_0\omega/\omega_0| < 1$. Note that the spontaneous decay processes at the laser frequency ω are too small to influence the whole quantum dynamics and, therefore, are not taken into account.

III. RESULTS AND DISCUSSION

In the following, we shall describe the quantum dynamics of an excited two-level emitter interacting with a classical low-frequency and intense laser field based on transformation (6) and Eq. (14).

A. The case $\eta < 1$

Initially, we begin by investigating the spontaneous emission effect involving only a few or several laser photon processes. This can be achieved when the parameter η is

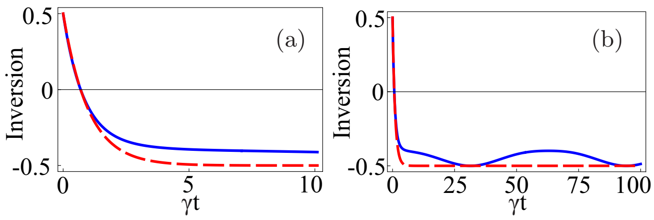

FIG. 1. The spontaneous decay law, given by the mean value of the inversion operator $\langle S_z(t)\rangle$, as a function of time in units of the inverse spontaneous decay rate at the bare transition frequency. Here $(2\Omega/\omega_0)^2 = 0.64$, $\omega_0/\omega = 2\times 10^4$, $\omega/\gamma = 0.05$, and $\phi = 0$. The dashed line depicts the standard spontaneous decay dynamics of an excited two-level emitter in the absence of any coherent driving.

smaller than unity. Let's consider, for instance, that $2\Omega/\omega_0 = 10^{-2}$ while $\omega_0/\omega = 8\times 10^3$ then one has that $\eta = 0.1$. Using the fact that $J_n(\eta) \approx \eta^n(1-\eta^2/[4(1+n)])/(2^n n!)$ if $\eta \ll 1$, then for a $2n_0\omega$ process with $n_0 = 1$ one has $\gamma(t) \approx \gamma(1 - (\eta^2/2)\cos[4(\omega t + \phi)])/2$. Notice here that we have neglected the contributions smaller than $\gamma\eta^2$ in the total decay rate. Under this circumstance, the coherent evolution described by the Hamiltonian $\bar{H}_0$ plays no role and the spontaneous decay process of an excited two-level emitter in a low-frequency strong laser field is characterized by the usual exponential decay law, namely,

$$\begin{aligned}\langle S_z(t)\rangle &\approx -1/2 + \exp\left[-2\int_0^t d\tau\,\gamma(\tau)\right]\\ &\approx -1/2 + \exp[-\gamma t].\end{aligned} \tag{15}$$

The explanation for a $n_0 = 1$ spontaneous decay process is as follows: The decay channels at frequencies $\bar{\omega}_0 \pm 2\omega$ and $\bar{\omega}_0$ lead to mutual cross-correlations such that the extra-induced decay channels cancel each other when $2\omega/\omega_0 \ll \eta < 1$. However, the cross-correlations among the channels $\bar{\omega}_0 + 2\omega$ and $\bar{\omega}_0 - 2\omega$ lead to a small oscillatory contribution, i.e., $\eta^2\cos[4(\omega t + \phi)]$, which does not affect the spontaneous decay. Generalizing in this way, even higher photon number processes, i.e., with $n_0 > 1$, do not modify the standard well-known exponential decay law as long as $2\Omega/\omega_0 \ll \eta < 1$.

B. The case $\eta \geqslant 1$ or $\eta \gg 1$

In this case, i.e., $\eta \geqslant 1$ or $\eta \gg 1$ with $2\Omega/\omega_0 < 1$, the quantum dynamics of an excited two-level emitter interacting with a classical strong low-frequency laser field is determined by multiphoton processes. We have found that there is no deviation of the spontaneous decay from the standard one as long as $\eta \geqslant 1$. However, it is modified for $\eta \gg 1$ and larger values of $2\Omega/\omega_0$, with $2\Omega/\omega_0 < 1$.

In Fig. 1, we show the spontaneous decay law of an excited two-level emitter interacting with a low-frequency and strong classical coherent light source. The standard exponential quantum decay dynamics is clearly modified [compare the dashed and solid curves in Fig. 1(a)]. However, if one checks longer time durations then it can be seen that the decaying emitter starts following the applied field [see Fig. 1(b)]. It looks like we have an interplay among the exponential spontaneous decay and incomplete Rabi oscillations due to the low-frequency coherent driving field. Nevertheless, one can still have a modification of the exponential spontaneous decay because of the quantum interference processes among the induced decay channels. We will return to this issue later. Also, importantly, for $(2\Omega/\omega_0)^2 = 0.64$ as is the case in Fig. 1, we have considered expansion terms up to $(2\Omega/\omega_0)^8$ in expression (5). In this case, the time-dependent spontaneous decay rate in Eq. (14) is given by the following expression,

$$\begin{aligned}\gamma(t) = \frac{\gamma}{2}\sum_{n,n'}\sum_{m,m'}\sum_{s,s'}\sum_{r,r'} & e^{2i(n-n')\phi(t)}e^{-4i(m-m')\phi(t)}e^{6i(s-s')\phi(t)}e^{-8i(r-r')\phi(t)}(1 + x^2/4 - 3x^4/64 + 5x^6/256\\ & - 175x^8/16384 + 2(n-2m+3s-4r)\omega/\omega_0)^3\chi_{nmsr}(x,\bar{\eta},\bar{\xi},\bar{\beta},\rho)\chi_{n'm's'r'}(x,\bar{\eta},\bar{\xi},\bar{\beta},\rho),\end{aligned} \tag{16}$$

where we have assumed that $|2(n-2m+3s-4r)\omega/\omega_0| < 1$, whereas

$$\bar{H}_0 = i\hbar\bar{\alpha}(t)\sum_{n,m,s,r} J_n(\bar{\eta})J_m(\bar{\xi})J_s(\bar{\beta})J_r(\rho)e^{-2i(n-2m)\phi(t)}e^{-i(\bar{\omega}_0 t - \bar{\eta}\sin 2\phi + \bar{\xi}\sin 4\phi - \bar{\beta}\sin 6\phi + \rho\sin 8\phi)}e^{-2i(3s-4r)\phi(t)}R^- + \text{H.c.}.$$

Here $\chi_{nmsr}(x,\bar{\eta},\bar{\xi},\bar{\beta},\rho) = J_n(\bar{\eta})J_m(\bar{\xi})J_s(\bar{\beta})J_r(\rho)\{1 - x^2/4 + 9x^4/64 - 25x^6/256 + 35^2x^8/128^2 - nx^2(1 - 3x^2/4 + 75x^4/128 - 245x^6/512)/(4\bar{\eta}) + 3mx^4(1 - 5x^2/4 + 245x^4/192)/(64\bar{\xi}) - 5sx^6(1 - 7x^2/4)/(512\bar{\beta}) + 35rx^8/(128^2\rho)\}$, with

$$\begin{aligned}\bar{\eta} &= \eta(1 - x^2/4 + 15x^4/128 - 35x^6/512),\\ \bar{\xi} &= \xi(1 - 3x^2/4 + 35x^4/64),\\ \bar{\beta} &= \beta(1 - 5x^2/4),\end{aligned}$$

whereas $\xi = (\omega_0/\omega)(x/4)^4$, $\beta = (\omega_0/\omega)x^6/3072$, and $\rho = 10(\omega_0/\omega)x^8/8^6$. Further, $\bar{\alpha}(t) \approx x\omega\sin\phi(t)(1 - x^2\cos^2\phi(t) + x^4\cos^4\phi(t) - x^6\cos^6\phi(t))/2$, with $\phi(t) = \omega t + \phi$, while

$$\bar{\omega}_0 = \omega_0\left(1 + \frac{x^2}{4} - \frac{3x^4}{64} + \frac{5x^6}{256} - \frac{175x^8}{16384}\right). \tag{17}$$

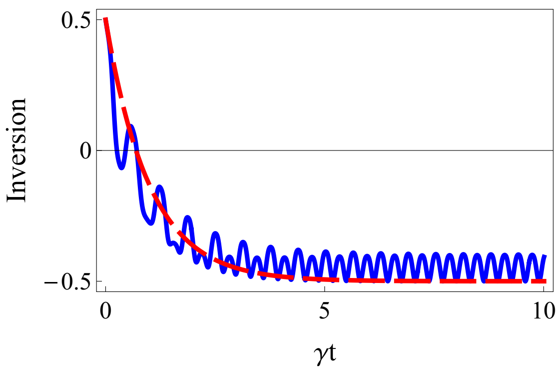

FIG. 2. The same as in Fig. 1(a) but for $\omega/\gamma = 10$.

Note that while we restricted the expansion of Eq. (5) to a certain order in x, in the subsequent calculations we did not. Respectively, one can obtain the time-dependent decay rates for additional expansion terms in Eq. (5). Generally, these decay rates will be proportional with a product of Bessel functions. We have observed that when the argument of one of the Bessel functions is much smaller than unity then the spontaneous quantum dynamics does not change if one adds further expansion terms in (5). Moreover, the modification of the spontaneous decay law is more pronounced for larger values of the ratio $2\Omega/\omega_0 < 1$. As a real system, where this prediction can be checked, may be considered certain solid-state media [57]. Higher decay rates, $\gamma \sim 10^{12}$ Hz, at transition frequencies $\omega_0 \sim 10^{15}$ Hz are proper to such systems. Therefore, for $\omega_0/\omega \sim 2 \times 10^4$ one has $\omega/\gamma \sim 0.05$. In Fig. 1 the Rabi frequency's value corresponds to $\Omega \sim 4 \times 10^{14}$ Hz. In this case, a transition dipole moment $d \sim 2 \times 10^{-29}$ C m would lead to an electric field amplitude of the order of $E_L \sim 10^9$ V/m. The ionization processes can be avoided if the ionization time t_i is larger than $t_i > 10^{-11}$s.

We turn further to Fig. 2 where we show the spontaneous quantum dynamics when the laser frequency is larger than the spontaneous decay rate. At the beginning of the evolution there is a fast population decay which is identified with the strong low-frequency driving rather than to quantum interference effects. Consequently, once the emitter decays to the ground state it will oscillate, in the ground state, due to strong continuous coherent wave driving.

To additionally prove our conclusion, in what follows, we compare our results with those obtained with a standard master equation where the spontaneous emission is introduced in the usual way [4,5,16–19], namely,

$$\frac{d}{dt}\langle Q(t)\rangle = i\langle[\omega_0 S_z - \Omega\cos(\omega t + \phi)(S^+ + S^-), Q]\rangle - \frac{\gamma}{2}(\langle S^+[S^-, Q]\rangle + \langle[Q, S^+]S^-\rangle). \tag{18}$$

We have found that as long as $\omega/\gamma \ll 1$ the results obtained with the analytical formalism described here and the master equation (18) looks somehow similar. This fact does not imply the existence of quantum interference effects. The reason is that in our approach, due to strong laser pumping, the transition frequency is increased by 10% when $x = 0.8$ [see expression (17)], meaning that the spontaneous decay should be faster. However, we obtain almost the same results as those obtained with the master equation (18). This means that the spontaneous decay was slowed down and this is the reason for the correspondence with the master equation (18) which does not contain the modification of the transition frequency due to strong pumping or various induced decay channels. When the frequency of the applied field is of the order of the bare spontaneous decay rate we observe slightly different behaviors; see Fig. 3. The initial time evolution is faster than the standard exponential spontaneous decay law, when it is described by our formalism and, thus, quantum interference is responsible for the rapid decay evolution. In this context, Fig. 4 depicts the time dependence of the scaled decay rate $\bar{\gamma}(t)/\gamma \equiv (\gamma(t) + \gamma(t)^*)/\gamma$ given by Eq. (16). A time-dependent decay rate presented here may help to understand the spontaneous emission dynamics of the excited emitter [although it will enter in that dynamics integrated; see, for instance, the first line of Eq. (15)]. The fact that the magnitude of the decay rate is larger than the single-qubit bare decay rate is due to the frequency shift [see expression (17)], arising from the strongly applied low-frequency coherent driving, i.e., the external field do modify it. Also, when $\bar{\gamma}(t)/\gamma \approx 1$ the spontaneous decay is faster than the usual single-qubit spontaneous decay law obtained in the absence of any coherent pumping; compare Figs. 3(a) and 4, respectively. Notice here that the reference time, i.e., $t = 0$, is taken at $t \sim \Omega^{-1}$, i.e., we have performed the secular approximation. Generalizing in this way, the spontaneous emission is modified because of an interplay among slow classical and strong coherent pumping

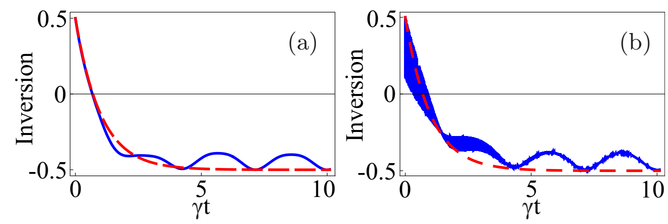

FIG. 3. (a) The population inversion $\langle S_z(t)\rangle$ as a function of γt obtained with the analytical approach developed here, while $\omega/\gamma = 1.1$. (b) The same obtained from the master equation (18). Other parameters are as in Fig. 1.

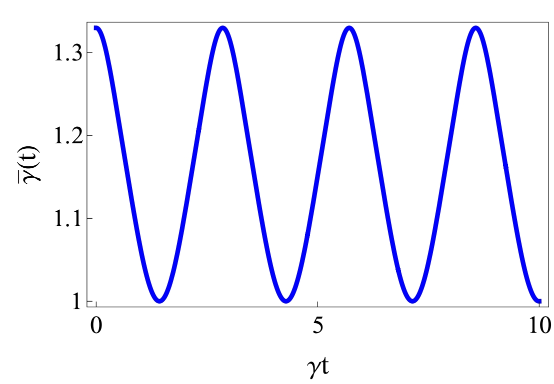

FIG. 4. The time-dependent decay rate, i.e., $\bar{\gamma}(t) = (\gamma(t) + \gamma(t)^*)$ (in units of γ) evaluated with the help of the expression (16) as a function of γt, for $\omega/\gamma = 1.1$. Other parameters are as in Fig. 3(a).

waves and additionally induced spontaneous interfering decay channels.

Finally, we note that there is substantial progress towards control of the spontaneous emission processes. Most of the studies use either near resonant driving or strong low-frequency quantized or classical applied fields [18,19]. In the latter case, the spontaneous emission inhibition occurs via additional energy levels and/or modification of the environmental vacuum reservoir, and based on Markovian or non-Markovian processes [27,28,36,37]. In the present study, however, we focused on an isolated two-level qubit pumped by a strong and low-frequency coherent field, without auxiliary off-resonant atomic states, and coupled to the regular electromagnetic vacuum modes. We find that the spontaneous emission modification is not too drastic, which in part is due to the fact that only the driving field properties remain as control parmeters in our scheme. But comparing this result to those of the model in [36,37], in which the low-frequency field can induce interfering multiphoton decay pathways via additional off-resonant auxiliary energy levels, we may further conclude that these additional multiphoton decay pathways are crucial for the strong spontaneous emission modification found there.

IV. SUMMARY

We have investigated the interaction of an excited two-level emitter with a coherent and strong low-frequency classical electromagnetic field. More precisely, we were interested in the quantum dynamics of the spontaneous emission processes. We have found that the spontaneous emission decay of an initially excited atom is slowed down or accelerated via the action of a strong and coherent classical low-frequency electromagnetic wave. The reasons are the presence of external low-frequency pumping followed by additionally induced decay channels that lead to destructive or constructive quantum interference phenomena and, consequently, to modification of the spontaneous emission. Furthermore, the induced spontaneous decay processes involve an even laser-photon number. Also, the modification of the bare transition frequency due to the strong low-frequency applied field is shown as well. An interesting perspective is to extend the present or related analysis on the effect of intense low-frequency fields beyond atoms, e.g., involving molecules driven by resonant low-frequency laser radiation [58], or multiphoton processes in artificial quantum systems like superconducting quantum circuits [59–61], quantum dot [62,63], or off-resonantly driven solid-state spin systems [64]. This way, more versatile parameter ranges may become possible.

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