Erratum: Spin-averaged effective Hamiltonian of orders $m\alpha^6$ and $m\alpha^6(m/M)$ for hydrogen molecular ions [Phys. Rev. A 98, 032502 (2018)]

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In Eq. (21) of our paper, the term $\vec{r}_a$ in the exponential function should read $\vec{r}_e$ as shown below

$$\langle\phi|\Sigma(E_0)|\phi\rangle = -z_a z_e e^2 \int \frac{d^3k}{(2\pi)^3} G_{\mu\nu}(k_0=0,\vec{k})\langle\phi|j_e^{\mu}(\vec{k})e^{i\vec{k}\cdot(\vec{r}_e-\vec{R}_a)}j_a^{\nu}(-\vec{k})|\phi\rangle\,. \tag{21}$$

In Eq. (26), an exponential factor is missing. The corrected form is

$$\begin{aligned} G_{\mu\nu}(\vec{r}) &= \int \frac{d^3k}{(2\pi)^3} G_{\mu\nu}(\vec{k})e^{i\vec{k}\cdot\vec{r}} \\ &= \frac{1}{4\pi}\begin{cases} -\frac{1}{r}, & \mu=\nu=0, \\ \frac{1}{2r}\left(\delta_{ij}+\frac{r_i r_j}{r^2}\right), & \mu=i, \quad \nu=j. \end{cases} \end{aligned} \tag{26}$$

In Eq. (32), according to Eqs. (28) and (29) of Ref. [1], the sign of the first and second terms should be minus and the factor of $1/2$ in the first and third terms is fixed. The corrected form reads

$$e\mathcal{A}_a^i = -\frac{z_e\alpha}{2r_a}\left(\delta_{ij}+\frac{r_a^i r_a^j}{r_a^2}\right)\frac{p_e^j}{m_e} - \frac{z_e\alpha}{2m_e}\frac{(\vec{\sigma}_e\times\vec{r}_a)^i}{r_a^3} - \frac{z_b\alpha}{2r_{ab}}\left(\delta_{ij}+\frac{r_{ab}^i r_{ab}^j}{r_{ab}^2}\right)\frac{p_b^j}{m_b}. \tag{32}$$

These changes of $\mathcal{A}_a^i$ do not affect the results of our paper.

The commutator identity of Eq. (49) of our paper has a misprint in the first term of the right-hand side. The corrected form should be

$$\begin{aligned} & e^{i\vec{k}\cdot\vec{r}_e}(H_0-E_0)^2 e^{-i\vec{k}\cdot\vec{R}_a} - (E_0-H_0)^2 \\ &= (H_0-E_0)\big(e^{i\vec{k}\cdot\vec{r}_a}-1\big)(H_0-E_0) + (H_0-E_0)\left[\frac{p_a^2}{2m_a}, e^{i\vec{k}\cdot\vec{r}_a}-1\right] \\ &\quad + \left[e^{\vec{k}\cdot\vec{r}_a}-1, \frac{p_e^2}{2m_e}\right](H_0-E_0) + \left[\frac{p_a^2}{2m_a}, \left[e^{i\vec{k}\cdot\vec{r}_a}-1, \frac{p_e^2}{2m_e}\right]\right]. \end{aligned} \tag{49}$$

There is a factor of $z_e z_a$ missing from δH_6 of Eq. (50) and δE_6 of Eq. (63) in our paper. They should be

$$\delta H_6 = \sum_a -\frac{z_e z_a\alpha}{m_e m_a}\left\{\left[p_e^i, V\right]\mathcal{X}^{ij}(r_a)\left[V, p_a^j\right] + p_e^i\left[\mathcal{X}^{ij}(r_a), \frac{p_e^2}{2m_e}\right]\left[V, p_a^j\right]\right\}, \tag{50}$$

$$\delta E_6 = \sum_a -\frac{z_e z_a}{8m_a}\left\{7\left\langle(\vec{p}_e\cdot\vec{r}_a)\frac{V_a^2}{r_a^2}(\vec{r}_a\cdot\vec{p}_e)\right\rangle - 3\left\langle\vec{p}_e V_a^2\vec{p}_e\right\rangle\right\}. \tag{63}$$

There is a term $-V_{12}V^2/2$ missing from δE_1 in Eq. (58) of the original paper. It should be

$$\begin{aligned} \delta E_1 = \frac{1}{16}\Bigg\{ & E_0\left[2\langle p_e^4\rangle - 4\langle(V_1+V_2)p_e^2\rangle + 16\langle V_{12}V\rangle\right] - 8E_0^2\langle V_{12}\rangle + 8\bar{\varepsilon}_1\bar{\varepsilon}_2 + 4\langle(2V_1V_2+V_1V_{12}+V_2V_{12})p_e^2\rangle \\ & - 8V_{12}V^2 + \sum_{a\neq b} 4\left(1-\frac{2}{m_a}\right)\langle\varepsilon_a^2\rangle - 8\left(1-\frac{3}{m_a}\right)\left[\langle V_a^3\rangle + \langle V_a^2(V_b+V_{12})\rangle - E_0\langle V_a^2\rangle\right] \\ & + \frac{4}{m_a}\Big[\langle\varepsilon_{12}^2\rangle + 3(-1)^a\langle\vec{\varepsilon}_a\vec{\varepsilon}_{12}\rangle + E_0\langle(2V+V_1+V_2)p_a^2\rangle + 2E_0\langle V_{12}^2p_a^2\rangle - E_0^2\langle p_a^2\rangle \\ & - 3\langle V_a^2(p_a^2-p_e^2)\rangle - \langle(4V_1V_2+5V_1V_{12}+5V_2V_{12}+3V_b^2+3V_{12}^2)p_a^2\rangle\Big]\Bigg\}. \end{aligned} \tag{58}$$

The fourth term in the curly braces of δE_2 in Eq. (59) of our paper is redundant. The corrected form is

$$\delta E_2 = \frac{1}{128}\sum_{b\neq a}\bigg\{-4\bigg(1-\frac{1}{m_a}\bigg)\langle\varepsilon_a^2\rangle - 4\langle\vec{\varepsilon}_a\vec{\varepsilon}_b\rangle - \frac{4}{m_a}(-1)^a\langle\vec{\varepsilon}_a\vec{\varepsilon}_{12}\rangle + \frac{2}{m_a}\langle V_b V p_a^2\rangle\bigg\}$$
$$+\frac{3\pi}{8}\sum_{b\neq a} z_a z_e\bigg\{2\bigg(1-\frac{1}{m_a}\bigg)[E_0\langle\delta(\vec{r}_a)\rangle - \langle(V_b+V_{12})\delta(\vec{r}_a)\rangle - \langle V_a\delta(\vec{r}_a)\rangle]$$
$$-\frac{1}{m_a}\langle\delta(\vec{r}_a)(p_a^2-p_e^2)\rangle - \frac{1}{m_b}\langle\delta(\vec{r}_a)p_b^2\rangle\bigg\}. \tag{59}$$

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[1] V. Patkóš, V. A. Yerokhin, and K. Pachucki, Higher-order recoil corrections for triplet states of the helium atom, Phys. Rev. A **94**, 052508 (2016).