Universal trapping law induced by an atomic cloud in single-photon cooperative dynamics

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The single-photon cooperative dynamics of an assembly of two-level quantum emitters coupled by a bosonic bath is investigated. The bosonic bath is general and can be anything as long as the exchange of excitations between quantum emitters and bath is present. In these systems it is found that the population on the excited emitter keeps a simple and universal trapping law due to the existence of the system's dark states. Different from the trapping regime caused by photon-emitter dressed states, this type of trapping is only associated with the number of quantum emitters. According to the trapping law, cooperative spontaneous emission at the single-photon level in this kind of system is universally inhibited when the emitter number is large enough.

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I. INTRODUCTION

Cooperative light-matter interaction plays an important role in quantum electrodynamics [1,2] and is useful for various applications of quantum optics such as optical quantum-state storage [3–5], quantum communication [6,7], and quantum information processing [8]. For a single excitation of an ensemble of quantum emitters, the rate and direction of cooperative spontaneous emission can be strongly modified by different light-field environments. While the size and shape of the ensemble have been investigated [9–11], an ensemble of atoms with a single collective excitation also exhibits a dynamics characterized by revivals for different atom numbers in a bosonic bath with a linear dispersion relation [12].

However, almost all the results and conclusions about the single-photon cooperative dynamics provided in the literature have been based on the specific light-field environments and specific coupling coefficient between quantum emitters and the photon [13–28], such as the cooperative dynamics in a waveguide environment, the dynamics in a photonic crystal, the dynamics in vacuum, and the dynamics in a simple cavity. If the light-field environments and coupling coefficient are changed, will these results and conclusions change or remain the same?

Here we focus on the single-photon cooperative dynamics in a system in which the light-field environment and the coupling coefficient are not specific and we investigate the general results and conclusions for the physical system. The emitters are assumed to be placed much closer than the wavelength of the radiation field and thus the emitters are efficiently coupled by the radiation field without retardation effects.

In this paper we report that there is a universal trapping law in the single-photon cooperative dynamics based on an analytical analysis which is beyond the Wigner-Weisskopf and Markovian approximations. A direct conclusion from this law is that the spontaneous emission dynamics in this system is suppressed if the number of emitters is large enough.

II. SINGLE-PHOTON COOPERATIVE DYNAMICS

We begin with a system that contains N two-level atoms coupled to the radiation field in an environment with a general dispersion relation ω_k. The atoms are characterized by the ground state $|g\rangle$ and excited state $|e\rangle$. The Hamiltonian of this system in the rotating-wave approximation takes the form (with $\hbar = 1$)

$$H = \sum_k \omega_k a_k^\dagger a_k + \sum_{j=1}^{N} \Omega_j |e_j\rangle\langle e_j| + \sum_{j,k} V_{k,j}(\sigma_j^+ a_k + \sigma_j^- a_k^\dagger), \tag{1}$$

where the first term on the right-hand side describes the light field and $a_k^\dagger$ (a_k) denotes the creation (annihilation) operator of a photon with momentum k. The second term represents two-level atoms and Ω_j is the atom's transition frequency. Here we set the ground-state energy of the atoms to be zero as reference. The last term represents the interaction between the photon and atoms, where $\sigma_j^+ = |e_j\rangle\langle g_j|$ ($\sigma_j^- = |g_j\rangle\langle e_j|$) is the raising (lowering) operator acting on the jth atom and $V_{k,j}$ is the coupling strength.

To investigate the dynamics of atoms when one of them is excited, we start from the time-dependent Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle, \tag{2}$$

where $|\psi(t)\rangle$ is the state of the system at time t. Since the total excitation number $N_{\rm tot} = \sum_k \omega_k a_k^\dagger a_k + \sum_{j=1}^{N} |e_j\rangle\langle e_j|$ is conserved, the state $|\psi(t)\rangle$ with $N_{\rm tot} = 1$ can be expanded as $|\psi(t)\rangle = \sum_j A_j(t)|g_1 g_2 \cdots e_j \cdots g_N, 0\rangle + \sum_k C_k(t)|g_1 g_2 \cdots g_N, 1_k\rangle$, where $A_j(t)$ is the probability am-

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plitude of the state with the jth atom in the excited state and the other atoms in the ground state and no photon in the environment, while $C_k(t)$ represents the probability amplitude of finding all atoms to be in the ground state and one photon in the environment. Replacing $|\psi(t)\rangle$ in Eq. (2), one obtains the equations for $A_j(t)$ and $C_k(t)$,

$$i\frac{\partial A_j(t)}{\partial t} = \Omega_j A_j(t) + \sum_k V_{k,j} C_k(t), \tag{3}$$

$$i\frac{\partial C_k(t)}{\partial t} = \omega_k C_k(t) + \sum_j V_{k,j} A_j(t). \tag{4}$$

The method based on the Wigner-Weisskopf or Markovian approximation theory is widely used to solve the dynamical equations (3) and (4) with specific ω_k and $V_{k,j}$, which leads to the result that the excited atomic population reveals exponential decay or the population decay is complete. However, it has been pointed out that the population trapping in the decay process will be lost when one of these two kinds of approximation theories is used [29–31].

To go beyond the Wigner-Weisskopf and Markovian approximations, we take a Laplace transform of Eqs. (3) and (4) to obtain

$$i[-A_j(0) + s\tilde{A}_j(s)] = \Omega_j \tilde{A}_j(s) + \sum_k V_{k,j}\tilde{C}_k(s), \tag{5}$$

$$i[C_k(0) + s\tilde{C}_k(s)] = \omega_k \tilde{C}_k(s) + \sum_j V_{k,j}\tilde{A}_j(s). \tag{6}$$

Denoting the initial excited atom by j_0, i.e., the initial amplitudes are $A_{j_0}(0) = 1$, $A_j(0) = 0$ $(j \neq j_0)$, and $C_k(0) = 0$, the expression of $\tilde{A}_{j_0}(s)$ can be acquired

$$\tilde{A}_{j_0}(s) = i\frac{is - \Omega - (N-1)f(s)}{(is - \Omega)[is - \Omega - Nf(s)]}, \tag{7}$$

where $f(s) \equiv \sum_k V_k^2/(is - \omega_k)$. Here it has been assumed that the atoms are identical and thus $\Omega_1 = \Omega_2 = \cdots = \Omega_N = \Omega$ and $V_{k,1} = V_{k,2} = \cdots = V_{k,N} = V_k$. The amplitude $A_{j_0}(t)$ is given by the inverse Laplace transform $A_{j_0}(t) = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \tilde{A}_{j_0}(s)e^{st}ds$, which leads to

$$A_{j_0}(t) = \sum_n \frac{s + i\Omega + i(N-1)f(s)}{[F(s)]'}e^{st}\bigg|_{s = x_n^{(1)}} - \int_C \frac{s + i\Omega + i(N-1)f(s)}{2\pi i F(s)}e^{st}ds, \tag{8}$$

where $F(s) \equiv (s + i\Omega)[s + i\Omega + iNf(s)]$ and $[F(s)]'$ means the derivative of $F(s)$ with respect to s. In addition, $x_n^{(1)}$ is the root of the equation $F(s) = 0$ in the complex plane except the region that ensures that the integrand is a single-valued function. Further, C is the integration contour based on the residue theorem. Generally, C is associated with the specific expression of V_k and ω_k. Different V_k and ω_k lead to a different integration contour C. However, a common conclusion that does not depend on specific V_k and ω_k is that the second term of $A_{j_0}(t)$ in Eq. (8) goes to zero when time t tends to infinity due to the factor e^{st} [32].

The physics in Eq. (8) is not obvious. We transform Eq. (8) into another form, which is the key point for the analysis in

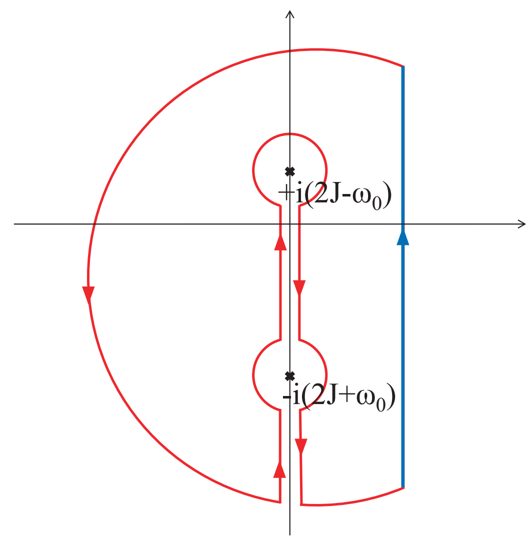

FIG. 1. Integration contours for the calculation of $A_{j_0}(t)$ in the coupled-cavity system. The red line is the integration contour C.

the following,

$$A_{j_0}(t) = \frac{N-1}{N}e^{-i\Omega t} + \sum_m \frac{e^{st}}{N[G(s)]'}\bigg|_{s = x_m^{(2)}} - \int_C \frac{G(s) - if(s)}{2\pi i F(s)}e^{st}ds, \tag{9}$$

where $G(s) \equiv s + i\Omega + iNf(s)$ and $x_m^{(2)}$ is the root of the equation $G(s) = 0$. Here the equation $G(-iE) = 0$ is nothing but the system's eigenenergy equation of the photon-atom dressed state. In fact, the second term on the right-hand side of Eq. (9) comes from the system's photon-atom bound states in which the populations of field modes are not zero and the third term comes from the system's scattering states [29,33]. The first term is only related to the atom's transition frequency and the number of atoms. It comes from the system's dark state with energy Ω in which all the excitation number focuses on the atoms and the populations of field modes are zero [34,35].

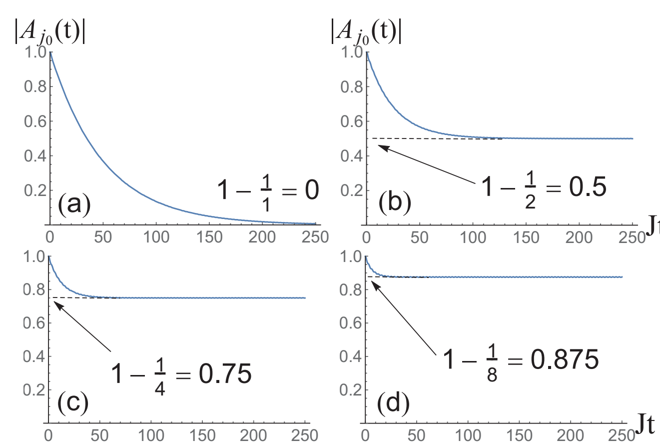

FIG. 2. Time evolution of the population $|A_{j_0}(t)|$ on the excited atom with different atom numbers in the coupled-cavity system: (a) $N = 1$, (b) $N = 2$, (c) $N = 4$, and (d) $N = 8$. The coupling strength $g_0 = 0.2J$ and the detuning $\delta_1 \equiv \Omega - \omega_0 = 0$.

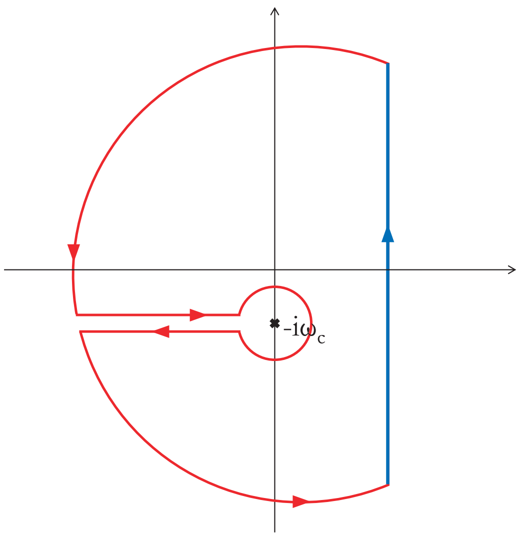

FIG. 3. Integration contours for the calculation of $A_{j_0}(t)$ in the photonic crystal system. The red line is the integration contour C.

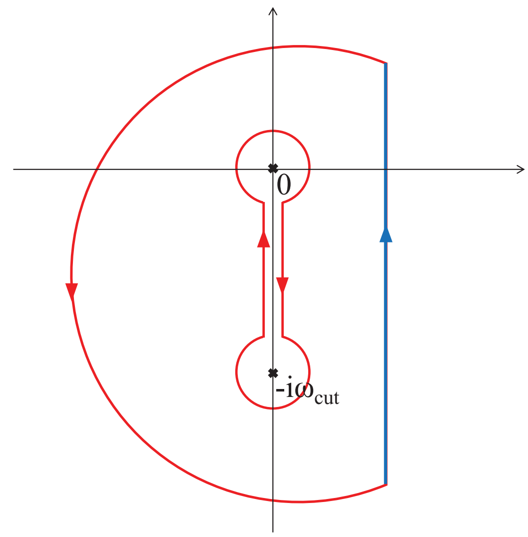

FIG. 5. Integration contours for the calculation of $A_{j_0}(t)$ in the system of the vacuum photonic bath. The red line is the integration contour C.

This kind of dark state is universal in this kind of system. It is caused by the collective coherence of atomic clouds. So the trapping associated with the dark state is universal whether the role of the second term on the right-hand side of Eq. (9) is important or not.

When the equation $G(s) = 0$ has no roots or the system's parameters satisfy the condition $|1/\{N[G(x_m^{(2)})]'\}| \ll 1$, the final result of the amplitude $A_{j_0}(t)$ at $t = \infty$ is

$$|A_{j_0}(\infty)| = 1 - \frac{1}{N}, \tag{10}$$

which is only related to the atom number. This trapping phenomenon takes place when the number $N > 1$.

To check this universal trapping, we now present two examples. One is the system of a one-dimensional coupled-cavity waveguide, in which the dispersion $\omega_k = \omega_0 - 2J\cos(k)$ and the coupling coefficient $V_k = g_0$ [36–38]. Here ω_0 is the on-site energy of each cavity and J represents the hopping energy of the photon between two neighboring cavities. The other is the system of a three-dimensional photonic crystal with $\omega_{\mathbf{k}} = \omega_c + B(\mathbf{k} - \mathbf{k}_0)^2$ and $V_{\mathbf{k}} = \Omega d(2\epsilon_0\omega_{\mathbf{k}}V)^{-1/2}\mathbf{e}_k \cdot \mathbf{u}$ [29,39–41]. Here d and $\mathbf{u}$ are the magnitude and unit vector of the atomic dipole moment, respectively. In addition, V, the volume, and $\mathbf{e}_k$ are the two transverse unit vectors of polarization. Both of the systems have been extensively studied theoretically and experimentally in recent years. For the coupled-cavity system, the integration contour C is shown by the red line in Fig. 1. When $\Omega = \omega_0$, the condition $|1/\{N[G(x_m^{(2)})]'\}| \ll 1$ can be easily satisfied. In Fig. 2 we plot the time evolution of $A_{j_0}(t)$ for different numbers of atoms. We can see that $|A_{j_0}(\infty)|$ meets the value $1 - 1/N$. For the photonic crystal system, the integration contour C is plotted with the red line in Fig. 3. The time evolution of $A_{j_0}(t)$ is shown in Fig. 4. The trapping law $1 - 1/N$ is also obeyed when the condition $|1/\{N[G(x_m^{(2)})]'\}| \ll 1$

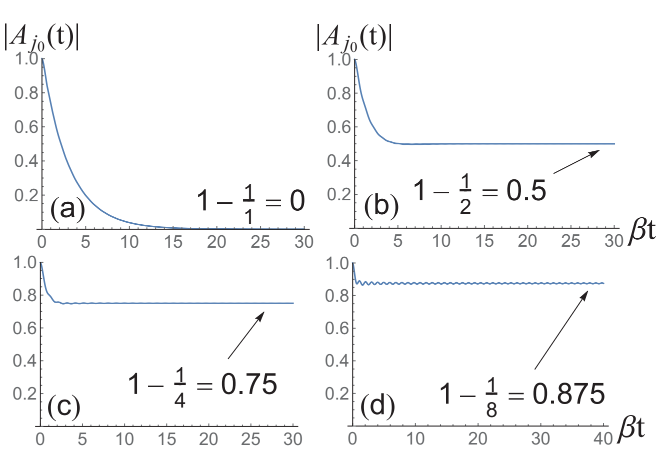

FIG. 4. Time evolution of the population $|A_{j_0}(t)|$ on the excited atom with different atom numbers in the photonic crystal system: (a) $N = 1$, (b) $N = 2$, (c) $N = 4$, and (d) $N = 8$. The detuning $\delta_2 \equiv \Omega - \omega_c = 6.5\beta$ and $\beta^{3/2} \equiv \Omega^2 d^2/6\pi\epsilon_0 B^{3/2}$.

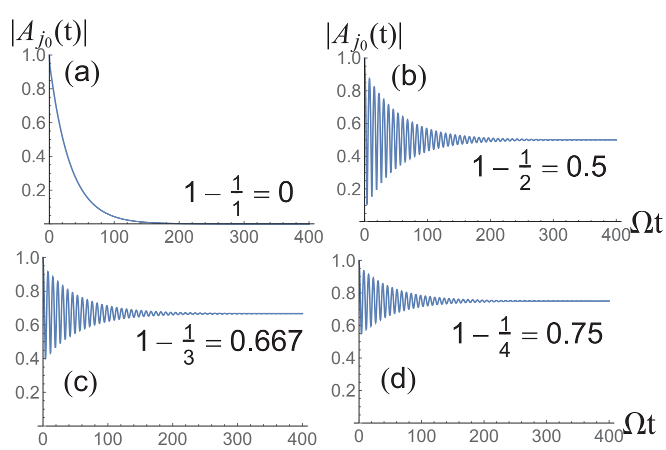

FIG. 6. Time evolution of the population $|A_{j_0}(t)|$ on the excited atom in the system of the vacuum photonic bath for $\omega_{\mathrm{cut}} = 20\Omega$ and (a) $\Gamma = 0.1\Omega$ and $N = 1$, (b) $\Gamma = 0.13\Omega$ and $N = 2$, (c) $\Gamma = 0.083\Omega$ and $N = 3$, and (d) $\Gamma = 0.063\Omega$ and $N = 4$. Here $\Gamma \equiv \Omega^3 d^2/3\pi\varepsilon_0 c^3$.

is satisfied. In Fig. 4(d) we see that $|A_{j_0}(t)|$ decays with a small oscillation. This oscillation is caused by the second term on the right-hand side of Eq. (9), which comes from the contribution of the photon-atom bound states of the system. If the detuning δ_2 ($\delta_2 \equiv \Omega - \omega_c$) becomes larger so that the condition $|1/\{N[G(x_m^{(2)})]'\}| \ll 1$ is better satisfied, the small oscillation will become less obvious.

For the system of the vacuum photonic bath, the dispersion $\omega_{\mathbf{k}} = c|\mathbf{k}|$ and the coupling coefficient $V_{\mathbf{k}} = \Omega d(2\epsilon_0\omega_{\mathbf{k}}V)^{-1/2}\mathbf{e}_k \cdot \mathbf{u}$. In this case, the integration contour C is shown by the red line in Fig. 5. Here ω_{cut} is the cutoff of the photon frequency [42]. In Figs. 6(a)–6(d) we plot the time evolution of $A_{j_0}(t)$ for atom numbers $N = 1, 2, 3, 4$. It can be seen that the trapping law $1 - 1/N$ is satisfied. Further, when time is long enough, there is no oscillation in the evolution curves. Here no root is found in the equation $G(-iE) = 0$, so the second term on the right-hand side of Eq. (9) is zero and the trapping law is exactly obeyed when time goes infinity.

III. CONCLUSION

To sum up, we have explored the single-photon cooperative dynamics in an ensemble of two-level atoms which is coupled to a general bosonic bath. The size of the ensemble is much smaller than the wavelength of the radiation field. The bosonic bath can be a photonic crystal, waveguide, or anything else as long as the exchange of excitations between the atoms and bath can take place. It was found that there is a universal trapping caused by the system's dark state. This kind of trapping obeys a simple law that is only related to the number of atoms. A direct conclusion from this law is that the single-photon cooperative spontaneous emission is suppressed when there are enough atoms. In addition, due to the presence of this trapping, the energy of the radiation field will be less than the initial total energy $E_{\text{tot}} = \Omega$. This trapping law is based on the Hamiltonian (1) with the rotating-wave approximation. It may fail when the system goes beyond the regime of this approximation.

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