Transparency in a chain of disparate quantum emitters strongly coupled to a waveguide

Debsuvra Mukhopadhyay[1,*] and Girish S. Agarwal[1,2,†]

[1]Institute for Quantum Science and Engineering, Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843, USA

[2]Department of Biological and Agricultural Engineering, Texas A&M University, College Station, Texas 77843, USA

(Received 15 January 2020; accepted 19 May 2020; published 8 June 2020)

We demonstrate the emergence of transparent behavior in a chain of periodically spaced nonidentical quantum emitters coupled to a waveguide, in the special case when the nearest neighbor separation is a half-integral multiple of the resonant wavelength, i.e., kL is an integral multiple of π, with k being the wave number and L the spatial periodicity. When equal but opposite frequency detunings are assigned in pairs to a system of even number of atoms, perfect transmission ensues. When the chain size is odd, a similar assignment leads to the disappearance of collective effects as the odd atom determines the spectral behavior. We also manifest the robustness of these features against dissipative effects and show how the spectral behavior hinges significantly on the relative detunings between the atoms as compared to the decay rate. A key distinction from the phenomenon of electromagnetically induced transparency is that in the waveguide case, the presence of an intrinsic waveguide-mediated phase coupling between the atoms strongly affects the transport properties. Furthermore, while reciprocity in single-photon transport does not generally hold due to the phase coupling, we observe an interesting exception for $kL = n\pi$ at which the waveguide demonstrates reciprocal behavior with regard to both the transmission and reflection coefficients.

DOI: 10.1103/PhysRevA.101.063814

I. INTRODUCTION

There is an ever-growing interest in single and few-photon scattering from a one-dimensional (1D) continuum because of their possible applications in quantum information processing. This subject has been studied both theoretically [1–38] and experimentally [39–49] from a variety of perspectives and reviewed quite elaborately in Ref. [50]. Some commonly used 1D waveguides are conducting nanowires [39,40], photonic crystal waveguides [48], and superconducting microwave transmission lines [44,45]. Collective effects emerging from a periodic array of two-level atoms coupled to a waveguide can lead to Fano minima in the reflection spectrum [28,29,38], superradiant decays [48], rearrangement of the optical band structure [51], and realization of Bragg mirrors [52,53]. The role of spatial separation between the atoms in the context of photon scattering from a 1D continuum has been manifested in recent works [5,32,33,52–62]. Very recently, multiple Fano interference channels due to the waveguide-mediated phase coupling between the atoms leading to the appearance of transparency points were demonstrated in [38]. In general, multiemitter waveguide QED has been gaining significant interest among the optics community in recent years owing to the engrossing physics of collective effects and the exciting avenues it opens up [63–70].

Theoretical analysis of photon scattering from a collection of disparately detuned atoms coupled to a waveguide is a hard problem in real space, because of the waveguide-mediated phase coupling between the atoms. Here, we address the collective behavior of such a system by constraining the spatial periodicity to be an integral or half-integral multiple of the resonance wavelength. Working at this phase substantially simplifies the problem and provides insight into the various kinds of collective properties of the output radiation. Since our interest is primarily in the realizability of induced transparency, we demonstrate the emergence of new Fano minima in the reflection spectrum for a system of even number of emitters via appropriate choice of detunings. For an odd chain size, the system manifests an effective reemergence of single-atom behavior. These are effects that were not observed in [38], as the atoms were all supposed to have identical transition frequencies. Our results are strong enough to withstand dissipative effects as long as the dissipative channel is weak compared to the waveguide channel and the atomic detunings are large compared to the decay rate. In other words, much like the phenomenon of electromagnetically induced transparency (EIT) [71,72], while perfect transparency emerges in an ideal dissipation-free scenario, a fairly high degree of transparency can still be observed in the presence of weak dissipation. However, in the usual scenario for EIT, the system comprises noninteracting atoms, whereas in a waveguide, an effective phase coupling between the emitters persists even at long spatial separations (of the order of a wavelength). Another important feature that surfaces at this choice of phase is the reciprocity in optical transport, i.e., with regard to both the reflected and transmitted amplitudes. More generally, the transport is insensitive to the order in which the atoms are placed in the chain. This is starkly different from the usual scenario, where the phase coupling makes the system strongly nonreciprocal. Reciprocity relations for a large class

*debsosu16@tamu.edu

†girish.agarwal@tamu.edu

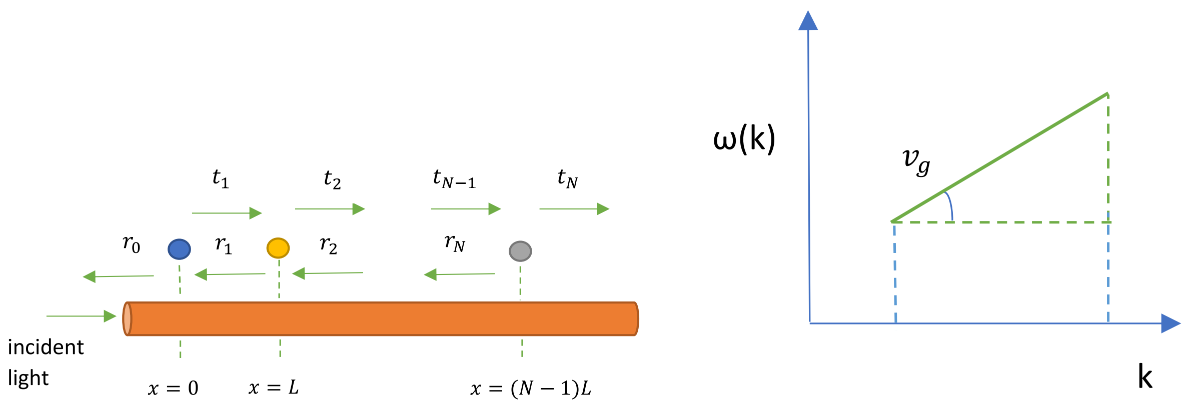

FIG. 1. Left: Atomic array coupled to a 1D waveguide. The atoms have been colored differently to indicate nonidentical transition frequencies. Right: The linear dispersion relation used in the text; the range between the two vertical dashed lines describes the region in which the atomic transition frequencies lie. Thus, $\omega(k) = \alpha + v_g k$ with $k > 0$. Here, α is a constant which is unimportant in the Hamiltonian.

of one-dimensional systems were derived in [73]. A general discussion on the subject of Lorentz reciprocity for lossless systems can be found in [74].

The paper is organized as follows: In Sec. II, we review the transport model for a single photon through a waveguide coupled to a chain of nonidentical emitters. In Sec. III, we demonstrate how one can observe transparent behavior for an atomic chain consisting of two disparately detuned atoms, when kL is chosen to be an integral multiple of π. In Sec. IV, we write down the analytical forms for the reflection and transmission coefficients for an arbitrary chain size and generalize some of the observations to the case of an even chain size. Concurrently, we indicate, for an odd chain size, how one can effect a complete suppression of collective effects so as to recover single-atom behavior. In Sec. V, we argue that the commutativity between the transfer matrices leads to reciprocal transport properties. Finally, in Sec. VI, we probe the effect of including radiative decay into nonwaveguide modes on the spectral characteristics, with particular emphasis on the observability of transparent behavior. Section VII summarizes the key ideas explored.

The ideas in this paper can be mapped onto a variety of concrete experimental models such as many coupled resonators on a transmission line or quantum dots coupled to plasmonic excitations in a nanowire. Recent techniques, such as those used in Refs. [52,53], testify to the experimental feasibility of our optical setup. In both these papers, the authors reported enhanced Bragg reflection off a one-dimensional waveguide system with atoms trapped periodically to satisfy the Bragg condition. In a more recent work [54], the observation of a single collective atomic excitation in the waveguide was reported. Dielectric waveguides, as used in their experiment, are apt for realizing strong atom-photon coupling, which is central to the design of a strong waveguide channel with minimal dissipative couplings.

II. SINGLE-PHOTON TRANSPORT IN A WAVEGUIDE COUPLED TO *N* NONIDENTICAL EMITTERS

We start with a coherent review of photon transport through a waveguide channel based on the transfer matrix formalism in the weak excitation regime. It, therefore, makes sense to restrict our model to the single-photon manifold of the relevant Hilbert space. For a periodic array of N two-level emitters evanescently coupled to a 1D continuum, as depicted in Fig. 1 (which also shows the assumed linearized dispersion of the waveguide), when the atomic transition frequency far exceeds the waveguide cutoff frequency, one can write down the Hamiltonian of the system in real space as

$$\begin{aligned}\mathcal{H}_{\text{eff}} &= i\hbar v_g \int_{-\infty}^{\infty} dx \left[a_L^\dagger(x) \frac{\partial a_L(x)}{\partial x} - a_R^\dagger(x) \frac{\partial a_R(x)}{\partial x} \right] \\ &\quad + \hbar \sum_{n=1}^{N} (\omega_n - i\Gamma_0) |e\rangle_n \langle e| \\ &\quad + \hbar \sum_{n=1}^{N} \mathcal{J}[\{a_L(x_n) + a_R(x_n)\}|e\rangle_n \langle g| + \text{H.c.}], \qquad (1)\end{aligned}$$

where $a_L(x)$ and $a_R(x)$ describe the real-space bosonic operators corresponding to the left and the right propagating fields, ω_n corresponds to the transition frequency of the nth atom and $x_n = (n-1)L$ to its location along the waveguide, v_g is the group velocity of the waveguide modes, and $\mathcal{J}$ denotes the coupling strength between the propagating field and any of the emitters. Γ_0 denotes the rate of spontaneous emission into all modes outside of the waveguide continuum, assumed equal for all the atoms. We disregard the dipole-dipole interaction between the atoms by assuming the interatomic separation to be comparable or larger than the resonance wavelength.We can solve for the transport properties by postulating a scattering eigenstate in the single-photon manifold,

$$\begin{aligned}|\mathcal{E}_k\rangle &= \int_{-\infty}^{\infty} dx[\phi_{kL}(x) a_L^\dagger(x) + \phi_{kR}(x) a_R^\dagger(x)]|\Psi\rangle \\ &\quad + \sum_{n=1}^{N} c_k^{(n)} |0; e_n\rangle, \qquad (2)\end{aligned}$$

where $|\Psi\rangle$ refers to the state with the field in vacuum and all atoms in the ground state, and $|0; e_n\rangle$ to the one where the field is still in vacuum but only the nth atom has been excited. ϕ_{kL} and ϕ_{kR} denote the wave functions corresponding to left- and right-propagating photonic excitations, respectively. The time-independent Schrödinger equation $\mathcal{H}|\mathcal{E}_k\rangle =$

$\hbar v_g k|\mathcal{E}_k\rangle$ leads to a system of ordinary differential equations (ODEs) for the various probability amplitudes:

$$\left(-iv_g\frac{d}{dx}-v_gk\right)\phi_{kR}(x)+\mathcal{J}\sum_{n=1}^{N}c_k^{(n)}\delta(x-x_n)=0,$$
$$\left(iv_g\frac{d}{dx}-v_gk\right)\phi_{kL}(x)+\mathcal{J}\sum_{n=1}^{N}c_k^{(n)}\delta(x-x_n)=0,$$
$$(\Omega_n-v_gk)c_k^{(n)}+\mathcal{J}\phi_{kL}(x_n)+\mathcal{J}\phi_{kR}(x_n)=0, \quad (3)$$

where $\Omega_n=\omega_n-i\Gamma_0$. In these considerations, k lies in the optical domain and is positive. For a wave incident from the left, one can solve the above ODEs subject to the boundary condition that $\phi_{kL/kR}(x_n)=\frac{1}{2}[\phi_{kL/kR}(x_n^+)+\phi_{kL/kR}(x_n^-)]$ and also the discontinuity imposed on the wave functions due to the delta-function source, i.e., $-iv_g[\phi_{kR}(x_n^+)-\phi_{kR}(x_n^-)]+\mathcal{J}c_k^{(n)}=0$ and $iv_g[\phi_{kL}(x_n^+)-\phi_{kL}(x_n^-)]+\mathcal{J}c_k^{(n)}=0$. The solutions take the form

$$\phi_{kL}(x)=\begin{cases}r_1e^{-ikx}, & x<0\\ r_{n+1}e^{-ik(x-nL)}, & (n-1)L<x<nL\\ 0, & x>(N-1)L\end{cases}, \quad (4)$$

$$\phi_{kR}(x)=\begin{cases}e^{ikx}, & x<0\\ t_ne^{ik(x-nL)}, & (n-1)L<x<nL\\ t_Ne^{ik(x-NL)}, & x>(N-1)L\end{cases}, \quad (5)$$

which subsequently lead to a system of simultaneous equations involving the transmission and reflection coefficients and the atomic excitation amplitudes. Eliminating the excitation amplitudes from the system engenders in a recursive matrix relation

$$\begin{bmatrix}r_n\\ t_{n-1}\end{bmatrix}=\mathcal{L}_n\begin{bmatrix}r_{n+1}\\ t_n\end{bmatrix}, \quad (6)$$

where

$$\mathcal{L}_n=\begin{bmatrix}e^{ikL}\left(1-i\delta_{k(n)}^{-1}\right) & -ie^{-ikL}\delta_{k(n)}^{-1}\\ ie^{ikL}\delta_{k(n)}^{-1} & e^{-ikL}\left(1+i\delta_{k(n)}^{-1}\right)\end{bmatrix}, \quad (7)$$

with $\delta_{k(n)}=\frac{\omega_k-\Omega_n}{\Gamma}$ and $\Gamma=\frac{\mathcal{J}^2}{v_g}$, and $\omega_k=v_gk$. Iterative use of this relation, i.e., by applying it repeatedly at the level of each emitter in succession, conjoined with appropriate boundary conditions $t_0=1$, $t_N=te^{iNkL}$, $r_{N+1}=0$, and $r_1=r$, yields the final connective relation between the reflection and transmission coefficients

$$\begin{bmatrix}r\\ 1\end{bmatrix}=\prod_{n=1}^{N}\mathcal{L}_n\begin{bmatrix}0\\ te^{ikNL}\end{bmatrix}. \quad (8)$$

$$r=\frac{\left(\prod_{n=1}^{N}\mathcal{L}_n\right)_{12}}{\left(\prod_{n=1}^{N}\mathcal{L}_n\right)_{22}}, \qquad t=\frac{e^{-ikNL}}{\left(\prod_{n=1}^{N}\mathcal{L}_n\right)_{22}}. \quad (9)$$

The way Γ_0 enters in this description is through Ω_n for each n. Because of the presence of the phase factors $e^{\pm ikL}$ and the differential detunings assigned to the emitters, it is analytically hard to find out compact expressions for the above. The matrix product, however, becomes simple to evaluate for $kL=n\pi$, n being any natural number (and not the dummy index used in the matrix product).

It is to be noted that here we present an exact solution without adiabatically eliminating the waveguide modes and thus, our approach keeps account of the propagation of light from one atom to another in the waveguide, i.e., the retardation effects are included and that is the reason for the appearance of phase factors like $e^{\pm ikL}$.

III. TRANSPARENCY AND COLLECTIVE BEHAVIOR FOR A TWO-ATOM SYSTEM

Let us first consider the simpler scenario of two differentially detuned atoms in a waveguide and define the mean detuning of the incident photon $\overline{\Delta}=\omega_k-\frac{1}{2}(\omega_1+\omega_2)$ and relative atomic detuning $s=\omega_1-\omega_2$. It turns out that while $[\mathcal{L}_1\mathcal{L}_2]_{22}$ is symmetric in s, $[\mathcal{L}_1\mathcal{L}_2]_{12}$ is not, owing to the phase coupling between the emitters mediated by the waveguide. In other words, in view of Eq. (9), even though transmission is perfectly reciprocal, reflection is not. For this system, the reflection and transmission coefficients reduce to

$$r=\frac{-i\Gamma[(e^{i\alpha}+1)(\overline{\Delta}+i\Gamma_0)-(e^{i\alpha}-1)s/2]-\Gamma^2(e^{i\alpha}-1)}{[\overline{\Delta}+i(\Gamma+\Gamma_0)]^2+\Gamma^2e^{i\alpha}-(s/2)^2},$$
$$t=\frac{(\overline{\Delta}+i\Gamma_0+s/2)(\overline{\Delta}+i\Gamma_0-s/2)}{[\overline{\Delta}+i(\Gamma+\Gamma_0)]^2+\Gamma^2e^{i\alpha}-(s/2)^2}, \quad (10)$$

where we have taken $\alpha=2kL$. We note, however, in the special case $kL=n\pi$, that the above expressions turn symmetric in s, thereby leading to reciprocity in the transport properties. In subsequent considerations, we analyze the results pertaining to this special choice of phase.

We also assume an idealized scenario where Γ_0 can be ignored. The effect of Γ_0 on the transmission is studied later in Sec. VI. With Γ_0 set to 0, one observes an emergence of transparent behavior when the two atoms are equally detuned, albeit in opposite directions, with respect to the laser frequency. In other words, t becomes unity when $\overline{\Delta}$ equals zero, or $\omega_k-\omega_1=\omega_2-\omega_k$, i.e., for a pair of antisymmetrically detuned emitters. It follows, from Eq. (10) and the plots in Fig. 2, that there is a transmission peak at $\overline{\Delta}=0$, while there are two roots of the profile at $\overline{\Delta}=\frac{s}{2}$ (or $\omega_k=\omega_1$) and $\overline{\Delta}=-\frac{s}{2}$ (or $\omega_k=\omega_2$) corresponding to perfect reflection. The peak has unit height in the absence of decay, signifiying transparent behavior. The height of this peak is strictly less than unity for any other choice of phase, as can be verified from Eq. (10) (for instance, in the specific scenario, when $kL=\frac{n\pi}{2}$ with odd n, the height of this peak is $[\frac{s^2}{s^2+8\Gamma^2}]^2$; see Fig. 3). For a sufficiently small yet nonzero value of $|\omega_1-\omega_2|$, one finds a very narrow window of size s over which the system is capable of demonstrating both opacity as well as transparency. As $s\to 0$, the two roots come progressively closer. Figure 2 illustrates this scenario for various choices of $|\omega_1-\omega_2|$.

We also note, in passing, that the poles of the transmission and reflection demonstrate features remindful of level attraction. These poles occur at

$$\overline{\Delta}_{\pm}^{(\mathrm{p})}=-i\Gamma\pm\sqrt{\left(\frac{s}{2}\right)^2-\Gamma^2}. \quad (11)$$

Level attraction is typically observed between the normal modes of two coupled oscillators when one of the bare modes has negative energy and the modes have comparable decay rates. When the coupling strength equals or exceeds a critical

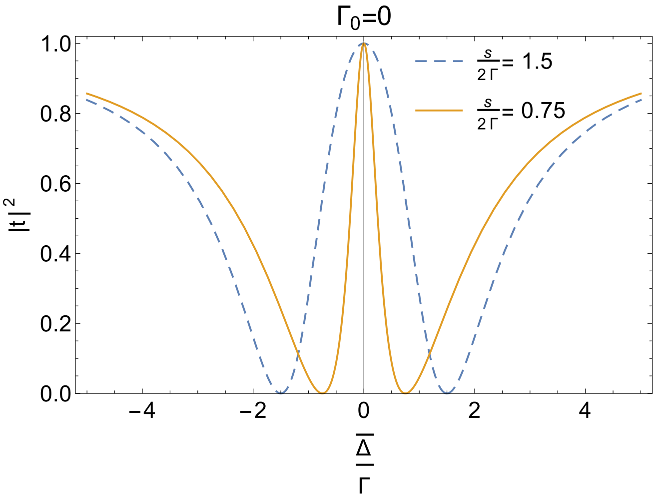

FIG. 2. Transmission for a two-atom system without decay for a couple of values of $s = \omega_1 - \omega_2$ and with $kL = n\pi$. Perfect transparency is observed at $\omega_k = \frac{1}{2}(\omega_1 + \omega_2)$ (zero mean detuning), unless $\omega_1 = \omega_2$, in which case, the system is perfectly reflecting at zero detuning. The two roots of the transmission come closer as the atomic frequencies approach each other.

value, the level separation vanishes. As a direct analogy, we see, in our case, that the real parts of the transmission poles coincide and become 0 in the region $\frac{s}{2} \leqslant \Gamma$, while the imaginary parts expand and shrink, respectively. The point of transition where the coupling equals this critical value is referred to as an exceptional point where the complex eigenfrequencies coincide [75]. Realizing level attraction has been quite a challenge from an experimental perspective and consequently, there is burgeoning interest in level attraction and phenomena occurring in the vicinity of exceptional points. Recently, level attraction has been observed in a variety of systems and topological behavior around an exceptional point has also been explored [76–82].

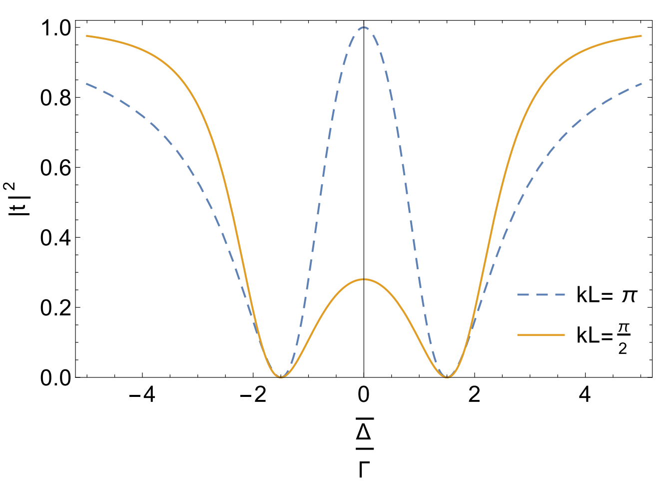

FIG. 3. Comparison of transmission spectra corresponding to $kL = \pi$ and $kL = \frac{\pi}{2}$, with $s = 1.5\Gamma$. The transmission peak attains unit height for $kL = \pi$, whereas it is much shorter than unity for $kL = \frac{\pi}{2}$.

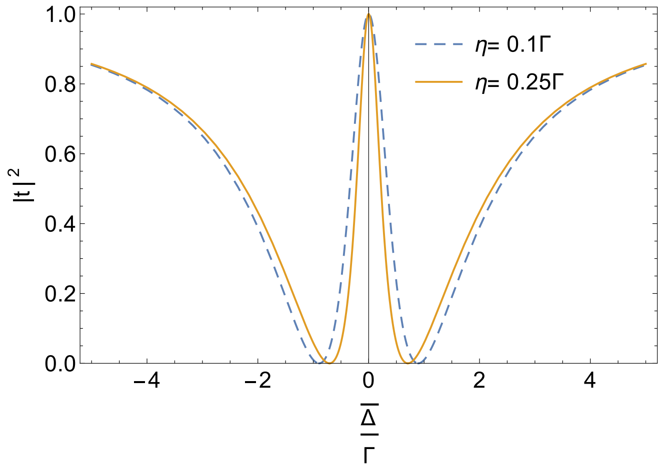

FIG. 4. Transmission at $kL = n\pi$ for $\frac{s}{2} = \Gamma - \eta$ with $\eta = 0.1\Gamma$ and $\eta = 0.25\Gamma$.

In the waveguide case with two atoms, we see that the transmission has zeros at

$$\overline{\Delta}_{\pm}^{(r)} = \pm\frac{s}{2}, \tag{12}$$

which simultaneously determine the peaks of the reflection spectrum. These zeros are close to the real parts of the poles when $\frac{s}{2} \gg \Gamma$. In the complementary regime, when $\frac{s}{2}$ is comparable to Γ, the real parts of the poles become small compared to the respective imaginary parts, as a consequence of which, the resolution between the two levels (or the two poles) becomes difficult. This problem of resolution arises fundamentally because Γ not only appears in the discriminant of the poles, but also acts as a natural broadening term.

The shrinking of the transmission width as $\frac{s}{2}$ goes below Γ is clearly reflected in the transmission plots shown in Fig. 4. Note that for $\frac{s}{2} = \Gamma$, the poles given by Eq. (11) become degenerate. Thus, if we define $\frac{s}{2} = \Gamma - \eta$, then η depicts how far we are off the degeneracy point. For a positive value of $\eta > 0$, the pole $\overline{\Delta}_{+}^{(p)}$ shrinks in width, with the relevant width given by $(1 - \sqrt{\frac{2\eta}{\Gamma}})\Gamma$. Choosing $\frac{\eta}{\Gamma} \sim 10^{-1}$, we find, in Fig. 4, that the transmission window becomes narrower as η becomes larger.

IV. TRANSPARENCY IN A MULTIATOM CHAIN

We now bring out some interesting features of the spectral behavior collectively induced by a chain of multiple emitters with nonidentical detunings, corresponding to a spatial periodicity so chosen that $kL = n\pi$. In Ref. [38], the analytical expressions for the spectral amplitudes were derived for a system of identical emitters for which the transfer matrices were also identical. Through a diagonalization procedure, the matrix product was calculated. However, $kL = n\pi$ is an exceptional point of the transfer matrices, since the eigenvalues coincide and become $(-1)^n$. Hence, diagonalization fails. However, in the special case when kL is an integral multiple of π, one can derive exact analytical expressions for the relevant matrix product in Eq. (9):

$$\prod_{j=1}^{N}\mathcal{L}_j = (-1)^{nN}\begin{bmatrix} 1 - i\Gamma\sum_{j=1}^{N}\delta_{k(j)}^{-1} & -i\Gamma\sum_{j=1}^{N}\delta_{k(j)}^{-1} \\ i\Gamma\sum_{j=1}^{N}\delta_{k(j)}^{-1} & 1 + i\Gamma\sum_{j=1}^{N}\delta_{k(j)}^{-1}\end{bmatrix}. \tag{13}$$

It is easy to check this result for $N = 2$, and the general result for arbitrary N can be verified using the procedure of mathematical induction. In other words, if the expression holds for $N = l - 1$, it is easy to check algebraically, the validity of this expression for $N = l$. One can also verify this result using *Mathematica*. By virtue of this simplification, the following transmission and reflection coefficients entail

$$t = \frac{1}{1 + i\Gamma\sum_{j=1}^{N}(\omega_k - \omega_j + i\Gamma_0)^{-1}},$$
$$r = -\frac{i\Gamma\sum_{j=1}^{N}(\omega_k - \omega_j + i\Gamma_0)^{-1}}{1 + i\Gamma\sum_{j=1}^{N}(\omega_k - \omega_j + i\Gamma_0)^{-1}}. \tag{14}$$

The collective effect due to emission from multiple periodically spaced emitters is clearly embodied in the aforementioned expressions. The spectral dependence on the detunings has a close resemblance with that in the single-emitter scenario. The key factor that modifies the spectrum is $\sum_{j=1}^{N}(\omega_k - \Omega_j)^{-1}$, an additive effect of the inverse detunings pertaining to the individual emitters. The expression is, of course, not as simple for other choices of phase. As a further simplification, let us focus on the case where Γ_0 is small enough to be dropped from consideration. This, in principle, entails the possibility of generating perfect transmission through suitable arrangements of the individual detunings. The condition for transparency ($r = 0$ and $t = 1$) is given by

$$\sum_{j=1}^{N}\frac{1}{\Delta_{k(j)}} = 0, \tag{15}$$

where $\Delta_{k(j)} = \omega_k - \omega_j$ is the laser detuning relative to the transition frequency of the jth atom. For a single emitter, this relation is clearly impossible to satisfy and therefore, a single atom in a waveguide cannot suppress reflection completely. One must have multiple atoms to be able to achieve transparency. In a double-emitter scenario, where $N = 2$, the condition translates to $2\omega_k = \omega_1 + \omega_2$, which implies an exactly antisymmetric assignment of detunings to the two emitters. This is in line with what was highlighted in the previous section dedicated to the study of a two-atom chain (see Fig. 2).

For $N = 3$, the corresponding constraint appears as a quadratic equation

$$3\omega_k^2 - 2(\omega_1 + \omega_2 + \omega_3)\omega_k + \omega_1\omega_2 + \omega_2\omega_3 + \omega_3\omega_1 = 0, \tag{16}$$

with roots given by

$$\tfrac{1}{3}\left[\omega_1 + \omega_2 + \omega_3 \pm \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2 - \omega_1\omega_2 - \omega_2\omega_3 - \omega_3\omega_1}\right].$$

The discriminant can be reexpressed as $\frac{1}{2}[(\omega_1 - \omega_2)^2 + (\omega_2 - \omega_3)^2 + (\omega_3 - \omega_1)^2]$, which, being a sum of squares, is strictly non-negative, and hence, the roots are real.

In general, it is easy to see that for an even number of emitters in the chain, it is always possible to make the system transparent if the atomic transition frequencies can be so adjusted that for any atom detuned by a certain amount, there exists another atom in the chain detuned by the same amount but in the opposite sense (Fig. 5). In other words, for a chain of $2l$ atoms, an assignment of the frequency detunings $+\Delta_1, -\Delta_1, +\Delta_2, -\Delta_2, \ldots, +\Delta_l, -\Delta_l$, in no particular order, would give rise to transparency in the system. Such an asymmetric pairwise assignment of detunings lead to Fano minima in the reflection spectrum, which signifies a destructive interference between the reflected waves emanating from the emitters. Concomitantly, the transmitted waves constructively interfere, leading to perfect transmission. This extreme resonant inhibition of the reflection amplitude relative to the single-atom emission is a new phenomenon that is not observed at $kL = n\pi$ for a system of identically detuned emitters. We have concentrated on the discussion of the transmission and reflection for $kL = n\pi$. It turns out that the corresponding states of the atom could be subradiant or superradiant depending on whether n is even or odd. For example, for the case of two antisymmetrically detuned atoms and for $kL = 2n\pi$, the atom goes over to a subradiant state.

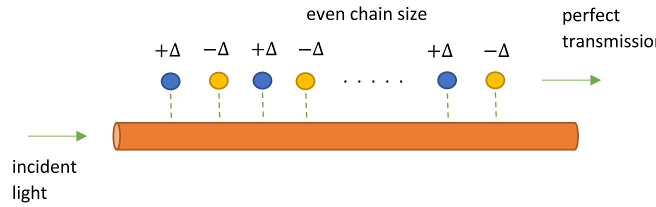

FIG. 5. Even number of emitters with equal and opposite detunings assigned in pairs generates transparency. The order of the atoms is not important, so the arrangement shown here is just one of the possible permutations.

On the other hand, if one has an odd number of emitters in the chain, one can recover the single-atom emission spectra by assigning pairwise asymmetric detunings to any randomly chosen $(N - 1)/2$ emitter pairs, leaving out a single atom. It then follows from Eqs. (14), that the remaining atom completely determines the spectral characteristics. That is, if this particular atom has a transition frequency ω_0, the transmitted spectrum due to the entire atomic chain reduces simply to

$$t = \frac{1}{1 + i\Gamma(\omega_k - \omega_0)^{-1}}, \tag{17}$$

which is identical to the transmission coefficient with just that single atom coupled to the waveguide (Fig. 6). Stated differently, when an even number of atoms with a pairwise asymmetric assignment of frequency detunings are added, in a periodic fashion, to a single atom coupled to a waveguide, no discernible collective effects emerge. The order of this arrangement and therefore, the location of the odd atom are not important. This makes sense from the perspective of Fano interference, since the reflected waves from the appended atoms destructively interfere, while that from the residual atom effectively goes through unperturbed. As a consequence, if the odd atom is in resonance with the laser frequency, the system resembles a perfectly reflecting mirror, regardless of the frequency detunings of the other atoms.

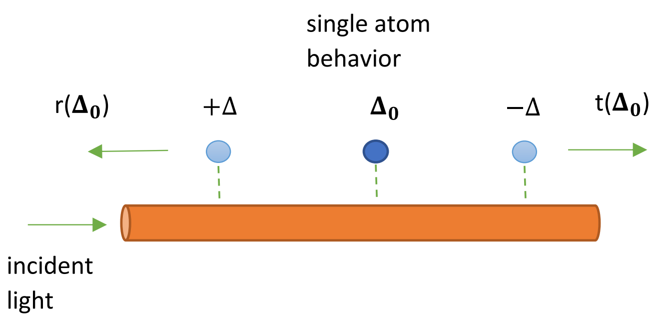

FIG. 6. A system of three atoms, out of which two carry equal and opposite detunings $+\Delta$ and $-\Delta$. The odd one out (the middle one, in this figure) with a detuning of $\boldsymbol{\Delta_0}$ determines the spectral behavior, and no collective effects exist. This behavior transcends to the case of any odd number of emitters in the chain with a commensurate assignment of frequency detunings. When $\boldsymbol{\Delta_0}= 0$, the system behaves as a perfectly reflecting mirror.

One might wonder how robust the transparency effect happens to be against sign-flip error in the detunings. If we take the instance of a simple two-emitter system and ignore dissipative couplings to keep the physics transparent, we know that perfect transparency ensues from assigning detunings that are equal in magnitude but opposite in signature to the individual emitters. Practical setups are not devoid of noise, and perfect flip in signature would be too ideal to achieve. In order to test the robustness against this indispensable error, we can reexamine the case of an atomic array of $2l$ atoms, furnished with pairwise frequency detunings such that the jth pair is assigned a set of detunings $\{+\Delta_j, -\Delta_j + \epsilon_j\}$. The transmission becomes

$$t = \frac{1}{1 - i\Gamma \sum_{j=1}^{N} \frac{\epsilon_j}{\Delta_j^2}} \sim 1 + i \sum_{j=1}^{N} \frac{\Gamma \epsilon_j}{\Delta_j^2} + \cdots . \tag{18}$$

It is useful (and perhaps, obvious) to note that the leading order correction term in the transmitted intensity is quadratic in $\frac{\Gamma\epsilon_j}{\Delta_j}$ for sign-flip error ensuing from the jth pair. This is a testimony to the fact that transparency in our optical setup achievable via this protocol can largely withstand fluctuations in sign flip, as long as the fluctuations are tiny compared to the magnitude of the detunings. An error analysis can also be made in relation to the proclaimed reemergence of single-atom behavior in a chain of odd size, subjected to a similar assignment protocol, which will similarly vindicate its robustness against imperfections in sign flip.

Finally, in the event that all the atoms have identical frequencies, one can observe Dicke-type superradiant behavior due to enhancement of the reflection amplitude. If the atomic frequencies are all set equal to ω_0, the corresponding transmitted spectrum is obtained to be

$$t = \frac{1}{1 + iN\Gamma(\omega_k - \omega_0)^{-1}}, \tag{19}$$

which pertains to a Lorentzian with a half-width of $N\Gamma$.

The superradiant behavior observed for a collection of identical emitters coupled to a 1D continuum, at $kL = n\pi$, was also derived in [38] and also in the context of Bragg reflection [83,84]. However, the possibility of controlling collective effects by tuning the individual atomic frequencies was not explored in that work. Having this added flexibility of adjusting the atomic frequencies brings out different types of interesting radiant behavior that one can observe, in principle. We do not merely encounter the possibility of superradiant reflection, but also come across new points of transparency. In particular, if we have an even number of emitters coupled to the waveguide, a pairwise asymmetric allotment of detunings generates new Fano minima. For an odd chain size, we see how a similar assignment of frequencies to any $N - 1$ of the atoms can lead to a complete disappearance of collective behavior and give rise to a spectrum governed entirely by the transition property of the leftover atom.

V. RECIPROCAL BEHAVIOR FOR $kL = n\pi$

As has been discussed previously, in the context of a two-atom system, reciprocity entails the choice of phase $kL = n\pi$. It follows, quite generally, from the expressions in Eqs. (14), that $kL = n\pi$ ensures perfect optical reciprocity for any arbitrary chain size. In fact, this choice of phase is both necessary and sufficient for reciprocity in both the reflection and the transmission. The fundamental property that brings about this reciprocal character is the commutativity between any two transfer matrices, i.e.,

$$[\mathcal{L}_r, \mathcal{L}_s] = 0, \qquad \forall\ r, s. \tag{20}$$

The commutation relation follows from the form in Eq. (7). As an essential implication of this, one finds that the matrix product $\prod_{j=1}^{N} \mathcal{L}_j$ is insensitive to the order in which the individual matrices are multiplied. Consequently, no matter what the order of the atoms is, one has the same transmission and reflection coefficients. Of course, this result is valid under the assumption that each emitter couples identically to the left- as well as the right-propagating fields, as far as the two coupling strengths are concerned. It is easy to verify that Eq. (20) holds true for any arbitrary assignment of detunings if and only if $kL = n\pi$.

VI. EFFECT OF DISSIPATION INTO NONWAVEGUIDE MODES ON TRANSPARENCY

The simplistic results laid out in the preceding discussions in Secs. III and IV hold only when Γ_0 is small enough to be ignored. However, we can look at more realistic scenarios with dissipation included ($\Gamma_0 \neq 0$) and examine the effect of the same on those observations. For a two-atom chain, we discover that the behavior changes drastically depending on how the relative detuning between the atomic frequencies compares to this decay rate. Figure 7 shows the plots for $|t|^2$ vs $|\frac{\bar{\Delta}}{\Gamma}|$ for a good-quality waveguide with a weak dissipative channel ($\Gamma_0 = 0.1\Gamma$), for varying values of $|\omega_1 - \omega_2|$. It is observed that for sufficiently small values of the latter, the transmission peak almost disappears, whereas for large values, the height of the peak approaches unity. In other words, by adjusting the relative frequency detuning between the emitters, one can achieve either high opacity or high transparency around $\omega_k = \frac{1}{2}(\omega_1 + \omega_2)$. For perfectly matched up atomic frequencies, i.e., $\omega_1 = \omega_2$, one observes a diametrically opposite behavior as the two roots coincide—the central peak is replaced by a

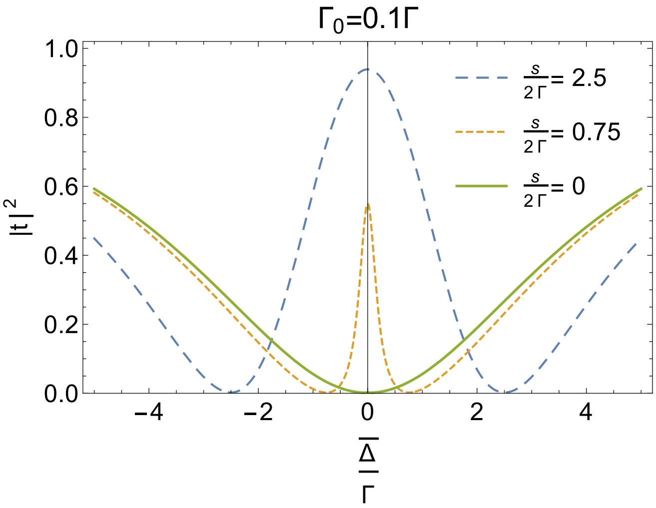

FIG. 7. Effect of dissipation on the transmission of a two-atom system. If the dissipative channel is weak compared to the waveguide channel, the profile closely resembles the dissipation-free spectrum, except when the frequency mismatch between the atoms is smaller than or comparable to the rate of dissipation. The central peak disappears as $s \to 0$ and is replaced by a trough at $s = 0$.

trough. This is a Dicke-type superradiant effect—for negligible decay, the transmission profile is a vertically inverted Lorentzian with a half-width of 2Γ.

One can analytically understand this behavior by considering two specific regimes: (i) $s \ll 2\Gamma_0$ and (ii) $s \gg 2\Gamma_0$. At $\bar{\Delta} = 0$, one obtains

$$t = \frac{\left(\frac{s}{2}\right)^2 + \Gamma_0^2}{\left(\frac{s}{2}\right)^2 + \Gamma_0(\Gamma_0 + 2\Gamma)}. \tag{21}$$

For small relative detuning between the atoms, i.e., $s \ll 2\Gamma_0$, the approximate form is given as $|t|^2 \approx \frac{\Gamma_0^2}{4\Gamma^2}$, which is vanishingly diminutive, as long as the decay rate is much less than Γ. In the opposite scenario when $s \gg 2\Gamma_0$, we get $|t|^2 \approx 1 - O(\frac{8\Gamma\Gamma_0}{\delta^2})$. Thus, a fairly high degree of transparency can be achieved by specifically working with a large relative detuning $|\omega_1 - \omega_2|$.

For a system of even number of emitters, in which half of them have detuning $+\Delta$, whereas the other half have detuning $-\Delta$, the transmission goes as

$$t = \frac{\Delta^2 + \Gamma_0^2}{\Delta^2 + \Gamma_0(\Gamma_0 + N\Gamma)}. \tag{22}$$

For $\Delta \ll \Gamma_0$, $|t|^2 \approx (\frac{\Gamma_0}{N\Gamma})^2$, which testifies to highly reflecting behavior. However, when $\Delta \gg \Gamma_0$, we have $|t|^2 \approx 1 - O(\frac{4N\Gamma\Gamma_0}{\Delta^2})$, implying near transparency. The situation here is reminiscent of EIT where perfect transparency emerges in the absence of dissipative transitions [71,72].

Similarly, when there are odd number of emitters, with $(N-1)/2$ emitters each having a detuning of $+\Delta$ and $(N-1)/2$ other emitters each detuned by $-\Delta$, one has, for the transmission coefficient

$$t = \frac{1}{1 + \frac{(N-1)\Gamma\Gamma_0}{\Delta^2+\Gamma_0^2} + \frac{i\Gamma}{\omega_k-\omega_0+i\Gamma_0}}, \tag{23}$$

where ω_0 is the frequency of the remaining atom. When Δ is large compared to Γ_0, one can discern a reemergence of single-atom behavior.

VII. CONCLUSIONS

To summarize, we have thrown light on new possibilities that emerge in relation to the collective effects of a chain of atoms side-coupled to a waveguide when the interemitter separation is fixed to satisfy $kL = n\pi$, where n is an integer. For a chain of N atoms, we have demonstrated the emergence of new Fano minima (transparency points) in the reflection spectrum for negligible dissipation. When N is even, we have seen how transparency can be generated by assigning equal and opposite detunings to the atoms in pairs, while for odd N, we have highlighted the possibility of reproducing single-atom behavior through a similar assignment, so that the odd one out completely determines the emission spectrum. A system of identically detuned emitters, on the other hand, demonstrates superradiant behavior, similar to the reflection from a system of equidistant quantum wells under Bragg condition. We have also shown that the optical system demonstrates reciprocal behavior with respect to both transmission and reflection. In general, the system turns out to be insensitive to the order in which the atoms are arranged. Finally, it has been demonstrated, both analytically and graphically, that when dissipation into nonwaveguide modes cannot be neglected, one can still produce highly transparent behavior by implementing a considerable disparity in the atomic transition frequencies. For a small mismatch in the frequencies, one, however, observes predominantly opaque behavior in its place.

We also remark, for the sake of completeness, that our results hold, by and large, for symmetrical atom-photon couplings with regard to either direction of photon propagation. The problem of what happens in a chiral setting [13,32,63,65,67,85–87] where the couplings are asymmetrical would, therefore, be an interesting subject of future theoretical investigation.

ACKNOWLEDGMENTS

D.M. is supported by the Herman F. Heep and Minnie Belle Heep Texas A&M University Endowed Fund held and administered by the Texas A&M Foundation. G.S.A. acknowledges the support of Air Force Office of Scientific Research (Award No. FA-9550-18-1-0141).

[1] J. T. Shen and S. Fan, Coherent photon transport from spontaneous emission in one-dimensional waveguides, Opt. Lett. **30**, 2001 (2005).

[2] L. Zhou, Z. R. Gong, Y.-X. Liu, C. P. Sun, and F. Nori, Controllable Scattering of a Single Photon Inside a One-Dimensional Resonator Waveguide, Phys. Rev. Lett. **101**, 100501 (2008).

[3] J.-Q. Liao, J.-F. Huang, Y.-X. Liu, L.-M. Kuang, and C. P. Sun, Quantum switch for single-photon transport in a coupled superconducting transmission-line-resonator array, Phys. Rev. A **80**, 014301 (2009).

[4] D. Witthaut and A. S. Sørensen, Photon scattering by a three-level emitter in a one-dimensional waveguide, New J. Phys. **12**, 043052 (2010).

[5] N. C. Kim, J.-B. Li, Z.-J. Yang, Z.-H. Hao, and Q.-Q. Wang, Switching of a single propagating plasmon by two quantum dots system, Appl. Phys. Lett. **97**, 061110 (2010).

[6] N.-C. Kim, M.-C. Ko, and Q.-Q. Wang, Single plasmon switching with n quantum dots system coupled to one-dimensional waveguide, Plasmonics **10**, 611 (2015).

[7] M.-T. Cheng, X.-S. Ma, M.-T. Ding, Y.-Q. Luo, and G.-X. Zhao, Single-photon transport in one-dimensional coupled-resonator waveguide with local and nonlocal coupling to a nanocavity containing a two-level system, Phys. Rev. A **85**, 053840 (2012).

[8] Z. Liao, X. Zeng, S.-Y. Zhu, and M. S. Zubairy, Single-photon transport through an atomic chain coupled to a one-dimensional nanophotonic waveguide, Phys. Rev. A **92**, 023806 (2015).

[9] W.-B. Yan and H. Fan, Control of single-photon transport in a one-dimensional waveguide by a single photon, Phys. Rev. A **90**, 053807 (2014).

[10] C.-H. Yan and L. F. Wei, Photonic switches with ideal switching contrasts, Phys. Rev. A **94**, 053816 (2016).

[11] T. S. Tsoi and C. K. Law, Quantum interference effects of a single photon interacting with an atomic chain inside a one-dimensional waveguide, Phys. Rev. A **78**, 063832 (2008).

[12] T. S. Tsoi and C. K. Law, Single photon scattering on type three-level atoms in a one-dimensional waveguide, Phys. Rev. A **80**, 033823 (2009).

[13] D. F. Kornovan, M. I. Petrov, and I. V. Iorsh, Transport and collective radiance in a basic quantum chiral optical model, Phys. Rev. B **96**, 115162 (2017).

[14] J. T. Shen and S. Fan, Theory of single-photon transport in a single-mode waveguide. I. Coupling to a cavity containing a two-level atom, Phys. Rev. A **79**, 023837 (2009).

[15] J. T. Shen and S. Fan, Theory of single-photon transport in a single-mode waveguide. II. Coupling to a whispering gallery resonator containing a two-level atom, Phys. Rev. A **79**, 023838 (2009).

[16] D. Roy, Two-Photon Scattering by a Driven Three-Level Emitter in a One-Dimensional Waveguide and Electromagnetically Induced Transparency, Phys. Rev. Lett. **106**, 053601 (2011).

[17] P. Longo, P. Schmitteckert, and K. Busch, Few-Photon Transport in Low-Dimensional Systems: Interaction-Induced Radiation Trapping, Phys. Rev. Lett. **104**, 023602 (2010).

[18] H. Zheng, D. J. Gauthier, and H. U. Baranger, Cavity-Free Photon Blockade Induced by Many-Body Bound States, Phys. Rev. Lett. **107**, 223601 (2011).

[19] Y.-L. L. Fang and H. U. Baranger, Waveguide QED: Power spectra and correlations of two photons scattered off multiple distant qubits and a mirror, Phys. Rev. A **91**, 053845 (2015).

[20] M. Bradford, K. C. Obi, and J.-T. Shen, Efficient Single-Photon Frequency Conversion Using a Sagnac Interferometer, Phys. Rev. Lett. **108**, 103902 (2012).

[21] L. Neumeier, M. Leib, and M. J. Hartmann, Single-Photon Transistor in Circuit Quantum Electrodynamics, Phys. Rev. Lett. **111**, 063601 (2013).

[22] L. Zhou, L.-P. Yang, Y. Li, and C. P. Sun, Quantum Routing of Single Photons with a Cyclic Three-Level System, Phys. Rev. Lett. **111**, 103604 (2013).

[23] X. Li and L. F. Wei, Designable single-photon quantum routings with atomic mirrors, Phys. Rev. A **92**, 063836 (2015).

[24] M.-T. Cheng, X.-S. Ma, J.-Y. Zhang, and B. Wang, Single photon transport in two waveguides chirally coupled by a quantum emitter, Opt. Express **24**, 19988 (2016).

[25] E. Sanchez-Burillo, D. Zueco, J. J. Garcia-Ripoll, and L. Martin-Moreno, Scattering in the Ultrastrong Regime: Nonlinear Optics with One Photon, Phys. Rev. Lett. **113**, 263604 (2014).

[26] S. Derouault and M. A. Bouchene, One-photon wave packet interacting with two separated atoms in a one-dimensional waveguide: Influence of virtual photons, Phys. Rev. A **90**, 023828 (2014).

[27] Y. S. Greenberg and A. A. Shtygashev, Non-Hermitian Hamiltonian approach to the microwave transmission through a one-dimensional qubit chain, Phys. Rev. A **92**, 063835 (2015).

[28] M.-T. Cheng and Y.-Y. Song, Fano resonance analysis in a pair of semiconductor quantum dots coupling to a metal nanowire, Opt. Lett. **37**, 978 (2012).

[29] M.-T. Cheng, J. Xu, and G. S. Agarwal, Waveguide transport mediated by strong coupling with atoms, Phys. Rev. A **95**, 053807 (2017).

[30] W. Konyk and J. Gea-Banacloche, One- and two-photon scattering by two atoms in a waveguide, Phys. Rev. A **96**, 063826 (2017).

[31] W. Konyk and J. Gea-Banacloche, Passive, deterministic photonic conditional-phase gate via two-level systems, Phys. Rev. A **99**, 010301 (2019).

[32] I. M. Mirza and J. C. Schotland, Multiqubit entanglement in bidirectional-chiral-waveguide QED, Phys. Rev. A **94**, 012302 (2016).

[33] Z. Liao, H. Nha, and M. S. Zubairy, Dynamical theory of single photon transport in a one-dimensional waveguide coupled to identical and nonidentical emitters, Phys. Rev. A **94**, 053842 (2016).

[34] F. LeKien and A. Rauschenbeutel, Propagation of nanofiber-guided light through an array of atoms, Phys. Rev. A **90**, 063816 (2014).

[35] S. Das, V. E. Elfving, F. Reiter, and A. S. Sorensen, Photon scattering from a system of multilevel quantum emitters. I. Formalism, Phys. Rev. A **97**, 043837 (2018).

[36] S. Das, V. E. Elfving, F. Reiter, and A. S. Sorensen, Photon scattering from a system of multilevel quantum emitters. II. Application to emitters coupled to a one-dimensional waveguide, Phys. Rev. A **97**, 043838 (2018).

[37] A. Asenjo-Garcia, M. Moreno-Cardoner, A. Albrecht, H. J. Kimble, and D. E. Chang, Exponential Improvement in Photon Storage Fidelities Using Subradiance and "Selective Radiance"in Atomic Arrays, Phys. Rev. X **7**, 031024 (2017).

[38] D. Mukhopadhyay and G. S. Agarwal, Multiple Fano interferences due to waveguide-mediated phase coupling between atoms, Phys. Rev. A **100**, 013812 (2019).

[39] A. V. Akimov, A. Mukherjee, C. L. Yu, D. E. Chang, A. S. Zibrov, P. R. Hemmer, H. Park, and M. D. Lukin, Generation of single optical plasmons in metallic nanowires coupled to quantum dots, Nature (London) **450**, 402 (2007).

[40] H. Wei, D. Ratchford, X. Li, H. Xu, and C.-K. Shih, Propagating surface plasmon induced photon emission from quantum dots, Nano Lett. **9**, 4168 (2009).

[41] A. Huck, S. Kumar, A. Shakoor, and U. L. Andersen, Controlled Coupling of a Single Nitrogen-Vacancy Center to a Silver Nanowire, Phys. Rev. Lett. **106**, 096801 (2011).

[42] T. M. Babinec, B. J. M. Hausmann, M. Khan, Y. Zhang, J. R. Maze, P. R. Hemmer, and M. Loncar, A diamond nanowire single-photon source, Nat. Nanotechnol. **5**, 195 (2010).

[43] J. Claudon, J. Bleuse, N. S. Malik, M. Bazin, P. Jaffrennou, N. Gregersen, C. Sauvan, P. Lalanne, and J.-M. Gérard, A highly efficient single-photon source based on a quantum dot in a photonic nanowire, Nat. Photonics **4**, 174 (2010).

[44] O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov, Jr., Yu. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J. S. Tsai, Resonance fluorescence of a single artificial atom, Science **327**, 840 (2010).

[45] I.-C. Hoi, C. M. Wilson, G. Johansson, T. Palomaki, B. Peropadre, and P. Delsing, Demonstration of a Single-Photon Router in the Microwave Regime, Phys. Rev. Lett. **107**, 073601 (2011).

[46] R. Yalla, M. Sadgrove, K. P. Nayak, and K. Hakuta, Cavity Quantum Electrodynamics on a Nanofiber Using a Composite Photonic Crystal Cavity, Phys. Rev. Lett. **113**, 143601 (2014).

[47] A. Javadi, I. Söllner, M. Arcari, S. Lindskov Hansen, L. Midolo, S. Mahmoodian, G. Kiršanskė, T. Pregnolato, E. H. Lee, J. D. Song, S. Stobbe, and P. Lodahl, Single-photon non-linear optics with a quantum dot in a waveguide, Nat. Commun. **6**, 8655 (2015).

[48] A. Goban, C.-L. Hung, J. D. Hood, S.-P. Yu, J. A. Muniz, O. Painter, and H. J. Kimble, Superradiance for Atoms Trapped Along a Photonic Crystal Waveguide, Phys. Rev. Lett. **115**, 063601 (2015).

[49] A. Sipahigil, R. E. Evans, D. D. Sukachev, M. J. Burek, J. Borregaard, M. K. Bhaskar, C. T. Nguyen, J. L. Pacheco, H. A. Atikian, C. Meuwly, R. M. Camacho, F. Jelezko, E. Bielejec, H. Park, M. Loncar, and M. D. Lukin, An integrated diamond nanophotonics platform for quantum-optical networks, Science **354**, 847 (2016).

[50] D. Roy, C. M. Wilson, and O. Firstenberg, Strongly interacting photons in one-dimensional continuum, Rev. Mod. Phys. **89**, 021001 (2017).

[51] A. Albrecht, T. Caneva, and D. E. Chang, Changing optical band structure with single photons, New J. Phys. **19**, 115002 (2017).

[52] N. V. Corzo, B. Gouraud, A. Chandra, A. Goban, A. S. Sheremet, D. V. Kupriyanov, and J. Laurat, Large Bragg Reflection from One-Dimensional Chains of Trapped Atoms Near a Nanoscale Waveguide, Phys. Rev. Lett. **117**, 133603 (2016).

[53] H. L. Sorensen, J. B. Beguin, K. W. Kluge, I. Iakoupov, A. S. Sorensen, J. H. Muller, E. S. Polzik, and J. Appel, Coherent Backscattering of Light Off One-Dimensional Atomic Strings, Phys. Rev. Lett. **117**, 133604 (2016).

[54] N. V. Corzo, J. Raskop, A. Chandra, A. S. Sheremet, B. Gouraud, and J. Laurat, Waveguide-coupled single collective excitation of atomic arrays, Nature (London) **566**, 359 (2019).

[55] H. Zheng and H. U. Baranger, Persistent Quantum Beats and Long-Distance Entanglement from Waveguide-Mediated Interactions, Phys. Rev. Lett. **110**, 113601 (2013).

[56] G.-Y. Chen, N. Lambert, C.-H. Chou, Y.-N. Chen, and F. Nori, Surface plasmons in a metal nanowire coupled to colloidal quantum dots: Scattering properties and quantum entanglement, Phys. Rev. B **84**, 045310 (2011).

[57] C. Gonzalez-Ballestero, E. Moreno, and F. J. Garcia-Vidal, Generation, manipulation, and detection of two-qubit entanglement in waveguide QED, Phys. Rev. A **89**, 042328 (2014).

[58] P. Facchi, M. S. Kim, S. Pascazio, F. V. Pepe, D. Pomarico, and T. Tufarelli, Bound states and entanglement generation in waveguide quantum electrodynamics, Phys. Rev. A **94**, 043839 (2016).

[59] X. Li and L. F. Wei, Probing a single dipolar interaction between a pair of two-level quantum system by scatterings of single photons in an aside waveguide, Opt. Commun. **366**, 163 (2016).

[60] Fatih Dinc, Å. Ercan, and A. M. Braäczyk, Exact Markovian and non-Markovian time dynamics in waveguide QED: Collective interactions, bound states in continuum, superradiance and subradiance, Quantum **3**, 213 (2019).

[61] Y. S. Greenberg and A. G. Moiseev, Influence of impurity on the rate of single photon superradiance and photon transport in disordered N qubit chain, Phys. E (Amsterdam, Neth.) **108**, 300 (2019).

[62] G.-Z. Song, L.-C. Kwek, F.-G. Deng, and G.-L. Long, Microwave transmission through an artificial atomic chain coupled to a superconducting photonic crystal, Phys. Rev. A **99**, 043830 (2019).

[63] I. M. Mirza and J. C. Schotland, Two-photon entanglement in multi-qubit bi-directional waveguide QED, Phys. Rev. A **94**, 012309 (2016).

[64] Y-L. L. Fang and H. U. Baranger, Multiple emitters in a waveguide: Non-reciprocity and correlated photons at perfect elastic transmission, Phys. Rev. A **96**, 013842 (2017).

[65] I. M. Mirza, J. G. Hoskins, and J. C. Scholtand, Chirality, band structure, and localization in waveguide quantum electrodynamics, Phys. Rev. A **96**, 053804 (2017).

[66] X. H. H. Zhang and H. U. Barnger, Quantum interference and complex photon statistics in waveguide QED, Phys. Rev. A **97**, 023813 (2018).

[67] I. M. Mirza and J. C. Schotland, Influence of disorder on electromagnetically induced transparency in chiral waveguide quantum electrodynamics, J. Opt. Soc. Am. B **35**, 1149-1158 (2018).

[68] V. Paulisch, H. J. Kimble, and A. G.-Tudela, Universal quantum computation in waveguide QED using decoherence free subspaces, New J. Phys. **18**, 043041 (2016).

[69] M. Bello, G. Platero, J. I. Cirac, and A. G. Tudela, Unconventional quantum optics in topological waveguide QED, Sci. Adv. **5**, eaaw0297 (2019).

[70] I. M. Mirza, J. G. Hoskins, and J. C. Schotland, Dimer chains in waveguide quantum electrodynamics, Opt. Commun. **463**, 125427 (2020).

[71] K.-J. Boller, A. Imamoğlu, and S. E. Harris, Observation of electromagnetically induced transparency, Phys. Rev. Lett. **66**, 2593 (1991).

[72] M. Fleischhauer, A. Imamoğlu, and J. P. Marangos, "Electromagnetically induced transparency: Optics in coherent media," Rev. Mod. Phys. **77**, 633 (2005).

[73] G. S. Agarwal and S. D. Gupta, Reciprocity relations for reflected amplitudes, Opt. Lett. **27**, 1205 (2002).

[74] B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics*, Wiley Series in Pure and Applied Optics (Wiley, New York, 2007).

[75] W. D. Heiss, The physics of exceptional points, J. Phys. A **45**, 444016 (2012).

[76] N. R. Bernier, L. D. Tóth, A. K. Feofanov, and T. J. Kippenberg, Level attraction in a microwave optomechanical circuit, Phys. Rev. A **98**, 023841 (2018).

[77] M. Harder, Y. Yang, B. M. Yao, C. H. Yu, J. W. Rao, Y. S. Gui, R. L. Stamps, and C.-M. Hu, Level Attraction Due to Dissipative Magnon-Photon Coupling, Phys. Rev. Lett. **121**, 137203 (2018).

[78] C. M. Bender, M. Gianfreda, Sąhin K. Özdemir, Bo Peng, and Lan Yang, Twofold transition in $\mathcal{PT}$-symmetric coupled oscillators, Phys. Rev. A **88**, 062111 (2013).

[79] M. Brandstetter, M. Liertzer, C. Deutsch, P. Klang, J. Schöberl, H. E. Türeci, G. Strasser, K. Unterrainer, and S. Rotter, Reversing the pump dependence of a laser at an exceptional point, Nat. Commun. **5**, 4034 (2014).

[80] C. Dembowski, H.-D. Gräf, H. L. Harney, A. Heine, W. D. Heiss, H. Rehfeld, and A. Richter, Experimental Observation of the Topological Structure of Exceptional Points, Phys. Rev. Lett. **86**, 787 (2001).

[81] J. Doppler, A. A. Mailybaev, J. Böhm, U. Kuhl, A. Girschik, F. Libisch, T. J. Milburn, P. Rabl, N. Moiseyev, and S. Rotter, Dynamically encircling an exceptional point for asymmetric mode switching, Nature (London) **537**, 76 (2016).

[82] H. Xu, D. Mason, L. Jiang, and J. G. E. Harris, Topological energy transfer in an optomechanical system with exceptional points, Nature (London) **537**, 80 (2016).

[83] E. L. Ivchenko, A. I. Nesvizhskii, and S. Jorda, Bragg reflection of light from quantum well structures, Fiz. Tverd. Tela **36**, 2118 (1994) [Phys. Solid State **36**, 1156 (1994)].

[84] A. Poddubny and E. Ivchenko, Resonant diffraction of electromagnetic waves from solids (a review), Phys. Solid State **55**, 905 (2013).

[85] P. Lodahl, S. Mahmoodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, Chiral quantum optics, Nature (London) **541**, 473 (2017).

[86] S. Hughes and G. S. Agarwal, Anisotropy-Induced Quantum Interference and Population Trapping Between Orthogonal Quantum Dot Exciton States in Semiconductor Cavity Systems, Phys. Rev. Lett. **118**, 063601 (2017).

[87] W. Liu, V. M. Menon, S. Gao, and G. S. Agarwal, Chiral emission of electric dipoles coupled to optical hyperbolic materials, Phys. Rev. B **100**, 245428 (2019).