Quantum correlations of light mediated by gravity

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We propose to explore the quantum nature of gravity using the correlation of light between two optomechanical cavities, and the quantumness of the correlation is witnessed by squeezing. As long as the gravity between the end mirrors of two cavities is quantum in the Newtonian limit, we show that the squeezing is always nonzero and monotonically increases as the mechanical property of the mirrors is improved. The proposed scheme provides a new pathway for testing the quantum nature of gravity systematically with tabletop experiments.

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I. INTRODUCTION

Constructing a consistent and verifiable quantum theory of gravity has been a longstanding challenge of modern physics [1–3], which is partially due to the difficulty of experimentally observing quantum effects of gravity. This, to a certain extent, has motivated some theoretical models that treat gravity as a fundamental classical entity [4–11] or as having emerged from yet unknown underlying microphysics [12–15]. Experimentally probing the quantum nature of gravity is therefore essential for providing hints towards constructing the correct model [16,17]. Recently, two experimental proposals have been made to demonstrate gravity-induced quantum entanglement between two mesoscopic test masses [18,19], motivated by an early suggestion of Feynman [20]. Both involve two matter-wave interferometers located close to each other such that their test masses can be entangled through the gravitational interaction. Whether or not gravity-mediated entanglement in the Newtonian limit establishes the quantumness of gravity has been debated [21–25], because the radiative degrees of freedom—the graviton—are not directly probed in these experiments. Nonetheless, such experiments are important steps towards understanding gravity in the quantum regime [26–31].

The challenge of demonstrating gravity-induced entanglement is achieving a very low thermal decoherence rate and is beyond what can be achieved with state-of-the-art instruments (illustrated in Appendix A). In this paper, we propose a tabletop optomechanical experiment to explore gravity-mediated quantum correlation of light. The strength of the correlation is quantified by squeezing, which is nonclassical according to the Glauber-Sudarshan distribution function [32–34]. The setup is shown schematically in Fig. 1. Two optomechanical cavities are placed close to each other with their end mirrors interacting through gravity. In contrast to the single-photon nonlinear regime studied by Balushi *et al.* [35], we consider the linear regime with the cavity driven by a coherent laser field. The quantum correlation is inferred by squeezing of the outgoing field of cavity A conditional on the homodyne measurement of the outgoing field of B.

If the gravitational interaction between two mirrors is quantum in the Newtonian limit, namely,

$$\hat{H}_{AB} = -\frac{G m_A m_B}{|\hat{q}_A - \hat{q}_B|}, \tag{1}$$

we show that such a conditional squeezing is always nonzero. Observing a sizable squeezing, however, requires the optomechanical cavities to be quantum radiation pressure limited, in which case the squeezing is approximately

$$\mathcal{S} = 10\log_{10}\left[1+\left(\frac{2Q_m G\rho}{\omega_m^2}\right)^2\right] \approx 2\ \text{dB}\left(\frac{0.5\ \text{Hz}}{\omega_m/2\pi}\right)^4\left(\frac{Q_m}{3\times10^6}\right)^2\left(\frac{\rho}{20\ \text{g/cm}^3}\right)^2. \tag{2}$$

It depends only on the gravitational constant G, material density ρ, mechanical frequency ω_m, and quality factor Q_m.

The statistical uncertainty of the measurement will affect the squeezing signal. Fortunately, because the system is at steady state, the signal-to-noise ratio (SNR) increases with the measurement time τ. Achieving an SNR of unity requires

$$\tau \approx 1\ \text{year}\left(\frac{\omega_m/2\pi}{0.5\ \text{Hz}}\right)^3\left(\frac{3\times10^6}{Q_m}\right)\left(\frac{20\ \text{g/cm}^3}{\rho}\right)^2. \tag{3}$$

Both $\mathcal{S}$ and τ scale rapidly with ω_m, and low-frequency mechanical oscillators are therefore preferable. Several optomechanical experiments have achieved the quantum radiation-pressure-limited regime but with high-frequency mechanical oscillators [36–41]; in particular, Ref. [40] reported a

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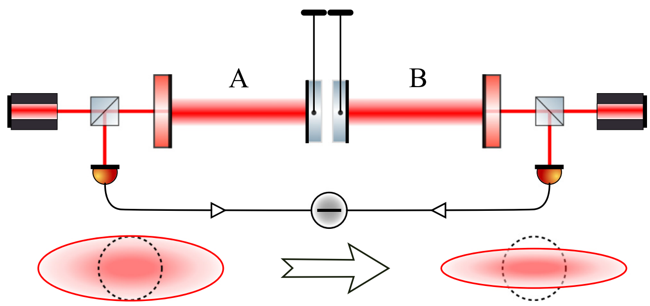

FIG. 1. Schematics showing the proposed experimental setup. Squeezing of the amplitude quadrature of the outgoing field of A conditional on the measurement of the phase quadrature of B manifests the gravity-mediated quantum correlation. The dashed circle denotes the vacuum level.

steady-state entanglement between light mediated by a mechanical oscillator. Extending these experimental techniques towards low frequencies, also an effort in the gravitational-wave community [42–45], is the key to measuring the gravity-mediated quantum correlation.

The outline of this paper goes as follows. In Sec. II, we show the mathematical description of the optomechanical dynamics. In Sec. III, we analyze the quantum correlation and derive the expression for the conditional squeezing. In Sec. IV, we discuss the implications of different outcomes of the proposed measurement and conclude the paper.

II. DYNAMICS

The derivation of Eq. (2) follows the linear-dynamics analysis in quantum optomechanics [46,47]: Solving the linear Heisenberg equations of motion for dynamical variables, which are the mirror position and quadratures of the outgoing optical fields, and representing them in terms of external fields, which are the ingoing optical fields and the thermal bath field.

The total Hamiltonian of the system is $\hat{H}_{\rm tot} = \hat{H}_A + \hat{H}_B + \hat{H}_{AB}$. The individual cavity is quantified by the standard linearized optomechanical Hamiltonian, which describes the radiation-pressure coupling between the optical field and the center-of-mass motion of the mirror (mechanical degree of freedom). The interaction part of $\hat{H}_A$ for cavity A is (similarly for B)

$$\hat{H}_A^{\rm int} = \hbar\, \omega_q \hat{X}_A \hat{Q}_A. \tag{4}$$

We denote $\hat{X}_A$ as the amplitude quadrature of the cavity mode, which is conjugate to the phase quadrature $\hat{Y}_A$, $[\hat{X}_A, \hat{Y}_A] = i$, and $\hat{Q}_A$ as the mirror position $\hat{q}_A$ normalized with respect to its zero-point motion $\sqrt{\hbar/(2m\omega_m)}$. The parameter ω_q describes the optomechanical coupling strength,

$$\omega_q \equiv \sqrt{\frac{2P_{\rm cav}\omega_0}{mcL\omega_m}}, \tag{5}$$

which depends on the intracavity optical power $P_{\rm cav}$, the laser frequency ω_0, the mirror mass m, and the cavity length L.

Up to the second order of the mirror position, the nontrivial interaction part of $\hat{H}_{AB}$ in Eq. (1) is

$$\hat{H}_{AB} = \hbar\frac{\omega_g^2}{\omega_m}\hat{Q}_A\hat{Q}_B. \tag{6}$$

Here we have assumed two mirrors having the same mechanical frequency and mass $m_A = m_B = m$. The characteristic gravitational interaction frequency ω_g is equal to $\sqrt{Gm/d^3}$ when the two mirrors have a mean separation d much larger than their size, which is the case for mesoscopic levitating masses considered in Refs. [18,19,41,48]. For macroscopic test-mass mirrors of the gram or kilogram scale, their separation can be made comparable to their size (yet not affected by, e.g., the Casimir force), and we have

$$\omega_g = \sqrt{\Lambda G\rho}, \tag{7}$$

which does not explicitly depend on the mirror mass. The form factor Λ is determined by the geometry of the two mirrors. It is $\pi/3$ for two spheres with a mean separation equal to twice the radius, and we assume $\Lambda = 2.0$ throughout the paper, which is a good approximation for two closely located disks with the radius 1.5 times the thickness (see Appendix B for details).

Solving the Heisenberg equations of motion results in the following frequency-domain input-output relation for cavity A (similarly for cavity B):

$$\hat{X}_A^{\rm out}(\omega) = \hat{X}_A^{\rm in}(\omega), \tag{8}$$

$$\hat{Y}_A^{\rm out}(\omega) = \hat{Y}_A^{\rm in}(\omega) + \sqrt{2/\gamma}\,\omega_q\hat{Q}_A(\omega), \tag{9}$$

where we have assumed that the cavity bandwidth γ is much larger than the frequency of interest so that the cavity mode can be adiabatically eliminated (cf. Eq. (2.68) in Ref. [46]). The position of mirror A satisfies

$$\hat{Q}_A = \chi_{qq}\big[-\sqrt{8/\gamma}\,\omega_q\hat{X}_A^{\rm in} - \big(\omega_g^2/\omega_m\big)\hat{Q}_B + 2\sqrt{\gamma_m}\hat{Q}_A^{\rm th}\big]. \tag{10}$$

Here $\chi_{qq} \equiv -\omega_m/(\omega^2 - \omega_m^2 + i\gamma_m\omega)$ is the susceptibility with the mechanical damping rate $\gamma_m \equiv \omega_m/Q_m$, $\hat{Q}^{\rm th}$ is the normalized thermal Langevin force according to the fluctuation-dissipation theorem [49,50], and its double-sided spectral density is equal to $\bar{n}_{\rm th} + (1/2)$ with the thermal occupation number $\bar{n}_{\rm th} \equiv k_B T/(\hbar\omega_m)$ in the high-temperature limit.

The final input-output relation involving both cavities is

$$\begin{bmatrix}\hat{X}_A^{\rm out}\\ \hat{Y}_A^{\rm out}\\ \hat{X}_B^{\rm out}\\ \hat{Y}_B^{\rm out}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ \mathcal{K} & 1 & \mathcal{G} & 0\\ 0 & 0 & 1 & 0\\ \mathcal{G} & 0 & \mathcal{K} & 1\end{bmatrix}\begin{bmatrix}\hat{X}_A^{\rm in}\\ \hat{Y}_A^{\rm in}\\ \hat{X}_B^{\rm in}\\ \hat{Y}_B^{\rm in}\end{bmatrix} + \begin{bmatrix}0 & 0\\ \alpha & \beta\\ 0 & 0\\ \beta & \alpha\end{bmatrix}\begin{bmatrix}\hat{Q}_A^{\rm th}\\ \hat{Q}_B^{\rm th}\end{bmatrix}. \tag{11}$$

Here $\mathcal{K} \equiv -4\omega_q^2\chi_{qq}/\gamma$ quantifies the correlation between the amplitude quadrature and the phase quadrature in the individual cavity and is responsible for the optomechanical squeezing [51–55]. The two parameters $\alpha \equiv 2\sqrt{2\gamma_m/\gamma}\,\omega_q\chi_{qq}$ and $\beta \equiv \alpha\chi_{qq}(\omega_g^2/\omega_m)$ quantify the output response to the thermal force noise. As illustrated in Fig. 2, the dimensionless parameter $\mathcal{G}$ quantifies the mutual correlation between two

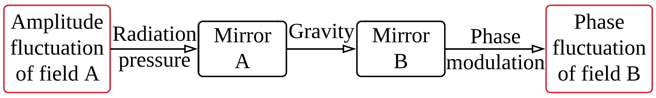

FIG. 2. Flowchart illustrating the physical meaning of $\mathcal{G}$ introduced in the input-output relation, Eq. (11).

cavities and is defined as $\mathcal{G} \equiv 4\omega_q^2\omega_g^2\chi_{qq}^2/(\gamma\omega_m)$. Its magnitude reaches the maximum at the mechanical frequency:

$$|\mathcal{G}(\omega_m)| = 2\,C\,Q_m\left(\frac{\omega_g}{\omega_m}\right)^2. \tag{12}$$

The optomechanical cooperativity, defined as

$$C \equiv \frac{2\omega_q^2}{\gamma\gamma_m}, \tag{13}$$

is proportional to the number of intracavity photons [47]. The fact that $|\mathcal{G}|$ is proportional to C shows that the optomechanical interaction coherently enhances the correlation by amplifying the quantum fluctuation of light.

III. QUANTUM CORRELATION AND CONDITIONAL SQUEEZING

Note that the correlation reaches the maximum around the mechanical frequency within a narrow frequency bandwidth defined by γ_m. We can therefore focus on the quadratures of the outgoing fields around ω_m with a bandwidth $\Delta\omega$ comparable to γ_m (or the measurement time comparable to the damping time $\tau_m \equiv 2\pi Q_m/\omega_m$). The corresponding normalized quadrature operators are defined as

$$\hat{\mathcal{X}} \equiv \sqrt{\Delta\omega/\pi}\,\hat{X}^{\rm out}(\omega_m), \quad \hat{\mathcal{Y}} \equiv \sqrt{\Delta\omega/\pi}\,\hat{Y}^{\rm out}(\omega_m). \tag{14}$$

They satisfy $[\hat{\mathcal{X}}, \hat{\mathcal{Y}}^\dagger] = 2i$, where we have approximated the Dirac delta function $\delta(0)$ as $1/\Delta\omega$. With such a normalization, the uncertainty of $\hat{\mathcal{X}}$ or $\hat{\mathcal{Y}}$ for the vacuum or coherent state is equal to 1.

Due to the quantum correlation, the uncertainty of the amplitude quadrature of A can be reduced after we measure the phase quadrature of B. The conditional uncertainty is obtained by minimizing the residue over the filtering function $\mathcal{F}$,

$$\begin{aligned}\sigma_{XX}^{\rm cond} &= \min_{\mathcal{F}} {\rm Tr}[\hat{\varrho}(\hat{\mathcal{X}}_A - \mathcal{F}\hat{\mathcal{Y}}_B)^2] = \sigma_{XX} - \frac{|\sigma_{XY}|^2}{\sigma_{YY}}\\ &= 1 - \frac{|\mathcal{G}|^2}{1+|\mathcal{K}|^2+|\mathcal{G}|^2+(2\bar{n}_{\rm th}+1)(|\alpha|^2+|\beta|^2)},\end{aligned} \tag{15}$$

where we define the variance $\sigma_{XX} \equiv {\rm Tr}[\hat{\varrho}\,\hat{\mathcal{X}}_A\hat{\mathcal{X}}_A^\dagger]$ (similarly for σ_{YY} of $\hat{\mathcal{Y}}_B$) and the covariance $\sigma_{XY} \equiv {\rm Tr}[\hat{\varrho}\,(\hat{\mathcal{X}}_A\hat{\mathcal{Y}}_B^\dagger + \hat{\mathcal{Y}}_B^\dagger\hat{\mathcal{X}}_A)/2]$, with $\hat{\varrho}$ being the density matrix. In obtaining the above result, we have used the fact that the ingoing optical field is in the vacuum state because the coherent amplitude is absorbed by the coupling rate ω_q [46,47]. The corresponding optimal Wiener filter is given by $\mathcal{F}_{\rm opt} = \sigma_{XY}/\sigma_{YY} = \mathcal{G}/[1+|\mathcal{K}|^2+|\mathcal{G}|^2+(2\bar{n}_{\rm th}+1)(|\alpha|^2+|\beta|^2)]$.

As we can see from Eq. (15), the conditional uncertainty of $\hat{\mathcal{X}}_A$ is always smaller than 1, which implies squeezing.

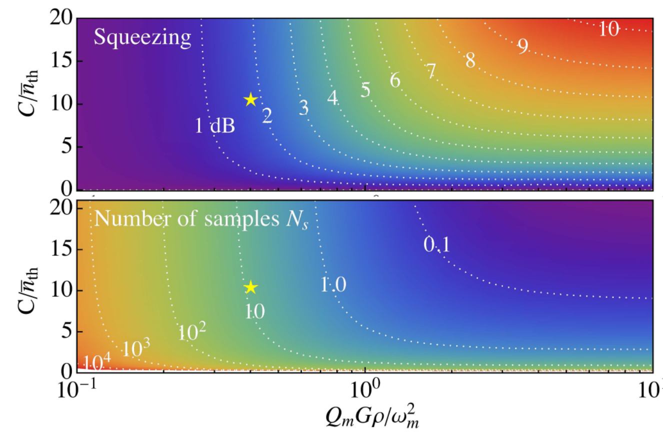

FIG. 3. Top: Squeezing (in dB) as a function of two dimensionless parameters: $C/\bar{n}_{\rm th}$ and $Q_m G\rho/\omega_m^2$. Bottom: Minimum N_s needed to achieve a signal-to-noise ratio of unity ($N_s < 1$ implies that one sample is sufficient). A small N_s does not mean a short measurement time, which is equal to N_s times the mechanical damping time $2\pi Q_m/\omega_m$. The two stars mark the parameters assumed in Eq. (2) and Eq. (3).

To observe such a conditional squeezing experimentally, the estimation error due to a finite number of measurements needs to be smaller than the squeezing level. According to the standard estimation theory, the unbiased estimator for the conditional uncertainty for a known average is

$$\sigma_{XX}^{\rm est} = \frac{1}{N_s}\sum_{k=1}^{N_s}\tilde{\sigma}_{XX}^{\rm cond}(k), \tag{16}$$

where $\tilde{\sigma}_{XX}^{\rm cond}(k)$ is the conditional variance for the kth measurement sample and N_s is the total number of samples. In our case, each sample corresponds to a measurement time of the order of the mechanical damping time τ_m. For a total measurement time of τ, we have

$$N_s \equiv \frac{\tau}{\tau_m} = \frac{\omega_m\tau}{2\pi Q_m}. \tag{17}$$

Since $\sum_{k=1}^{N_s}\tilde{\sigma}_{XX}^{\rm cond}(k)$ follows the chi-squared distribution with N_s degrees of freedom, the estimation error is equal to $\sqrt{2/N_s}\,\sigma_{XX}^{\rm cond}$. It needs to be smaller than the squeezing level to achieve an SNR of unity, which implies

$$\sqrt{\frac{2}{N_s}}\,\sigma_{XX}^{\rm cond} \leqslant \frac{|\mathcal{G}|^2}{1+|\mathcal{K}|^2+|\mathcal{G}|^2+(2\bar{n}_{\rm th}+1)(|\alpha|^2+|\beta|^2)}. \tag{18}$$

The above condition leads to a requirement on the minimum measurement time τ. For experimentally relevant parameters, we have $\bar{n}_{\rm th} \gg 1$ and $|\mathcal{K}| \gg 1$, and we can approximate the denominator of Eq. (15) and Eq. (18) as $|\mathcal{K}|^2 + |\mathcal{G}|^2 + 2\bar{n}_{\rm th}(|\alpha|^2+|\beta|^2)$. The resulting squeezing and also the minimum number of samples are shown in Fig. 3. They depend only on two characteristic dimensionless parameters: $C/\bar{n}_{\rm th}$, the ratio between the optomechanical cooperativity and the thermal occupation number; and $Q_m G\rho/\omega_m^2$, determined solely by the gravity and the mechanical property of the mirror.

To obtain a sizable squeezing, we learn from Fig. 3 that, first, $Q_m G\rho/\omega_m^2$ needs to be large, which implies high-quality-factor, low-frequency test-mass mirrors, and, second, the cooperativity must be much larger than the mean thermal occupation number, namely,

$$C \gg \bar{n}_{\rm th}. \tag{19}$$

This corresponds to the quantum radiation-pressure-limited regime in optomechanics [47]. In this regime, the squeezing and minimum number of samples turn out to become independent of the optical property and depend only on the mechanical property. In particular, we have

$$\sigma_{XX}^{\rm cond} \approx \frac{1}{1+|\mathcal{G}/\mathcal{K}|^2} = \frac{1}{1+\left(2Q_m G\rho/\omega_m^2\right)^2}, \tag{20}$$

which, written in terms of decibels, gives rise to Eq. (2) shown in Sec. I. The minimum number of samples N_s to achieve an SNR of unity can be approximated as

$$N_s \approx 1 + 4|\mathcal{K}/\mathcal{G}|^2 \approx \left(\frac{\omega_m^2}{Q_m G\rho}\right)^2. \tag{21}$$

The second approximation is satisfied for those parameter values assumed in Eq. (3), where we have shown the equivalent minimum measurement time.

IV. DISCUSSION AND CONCLUSION

To summarize, our approach to probing the quantum nature of gravity takes advantage of recent advancements in quantum optomechanical experiments. It is complementary to other approaches based on matter-wave interferometers. In general, achieving a sizable squeezing requires quantum radiation-pressure-limited systems with high-quality-factor, low-frequency mechanical test-mass mirrors. Even though the squeezing signal does not explicitly depend on the size of the test-mass mirror, having a low mechanical frequency usually implies macroscopic test masses. For illustration, we provide a possible set of sample parameters to reach $C/\bar{n}_{\rm th}$ of the order of 10 implicitly assumed in Eq. (2) for $\omega_m/(2\pi) = 0.5$ Hz and $Q_m = 3\times 10^6$,

$$\frac{C}{\bar{n}_{\rm th}} \approx 10\left(\frac{1\ {\rm g}}{m}\right)\left(\frac{P_{\rm cav}}{2\ {\rm kW}}\right)\left(\frac{\rm Finesse}{4000}\right)\left(\frac{300\ {\rm K}}{T}\right), \tag{22}$$

which corresponds to a suspended high-finesse cavity with a gram-scale test-mass mirror at room temperature, close to what has been achieved by the Massachusetts Institute of Technology group [56]. The gravity experiments with milligram test masses [57,58] can be promising if pushed to the low-frequency regime.

Let us consider the consequence of different outcomes of the measurement that we propose. If we do not detect a predicted level of squeezing after a careful calibration of the system, it will imply that the assumption on the gravity sector is invalid [cf. Eq. (1)], as the quantum aspects of the optomechanical interactions have already been established experimentally. One compelling possibility, then, is that gravity is classical, so that it does not appear in the quantum interaction Hamiltonian. If we do observe a nonzero squeezing, we will be able to rule out classical models of gravity, in particular, the Schrödinger-Newton (SN) type of classical gravity model: the gravity is sourced by the expectation value of quantum matters [4–11], which does not lead to quantum correlation. This is because the corresponding signal-noise (SN) two-body interaction for the optomechanical setup would be (cf. Eq. (27) in Ref. [9])

$$\hat{H}_{\rm AB}^{\rm SN} = \hbar\frac{\omega_g^2}{2\omega_m}(\langle\hat{Q}_A\rangle\hat{Q}_B + \hat{Q}_A\langle\hat{Q}_B\rangle). \tag{23}$$

According to Eq. (10), the quantum part of $\langle\hat{Q}_A\rangle$ or $\langle\hat{Q}_B\rangle$ is 0, as the expectation value of the quantum fluctuation $\hat{X}_A^{\rm in}$ is 0. For future study, it would be interesting also to explore the predictions of emergent gravity models [12–15] on the conditional squeezing level in this proposed optomechanical setup.

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APPENDIX A: CONDITION FOR REALIZING GRAVITY-MEDIATED ENTANGLEMENT

Here we try to derive the general condition for achieving gravity-mediated entanglement. We first consider the optomechanical setup proposed and study the entanglement between the outgoing fields of the two cavities. Similarly to the analysis presented in the text [cf. Eq. (14)], we also focus on the optical quadratures at the mechanical frequency ω_m. The entanglement measure can be derived from the total covariance matrix of the quadratures, $\boldsymbol{\sigma} \equiv {\rm Tr}\{\hat{\varrho}\,[\hat{\mathcal{X}}_A\ \hat{\mathcal{Y}}_A\ \hat{\mathcal{X}}_B\ \hat{\mathcal{Y}}_B]^{\rm T}[\hat{\mathcal{X}}_A^\dagger\ \hat{\mathcal{Y}}_A^\dagger\ \hat{\mathcal{X}}_B^\dagger\ \hat{\mathcal{Y}}_B^\dagger]\}_{\rm sym}$, where the superscript "T" indicates transpose and the subscript "sym" means symmetrization—more explicitly, ${\rm Tr}[\hat{\varrho}\,\hat{\mathcal{X}}\hat{\mathcal{Y}}]_{\rm sym} \equiv {\rm Tr}[\hat{\varrho}\,(\hat{\mathcal{X}}\hat{\mathcal{Y}}^\dagger + \hat{\mathcal{Y}}^\dagger\hat{\mathcal{X}})/2]$:

$$\boldsymbol{\sigma} \equiv \begin{bmatrix} \boldsymbol{\sigma}_A & \boldsymbol{\sigma}_{AB} \\ \boldsymbol{\sigma}_{AB}^{\rm T} & \boldsymbol{\sigma}_B \end{bmatrix}. \tag{A1}$$

The diagonal components $\boldsymbol{\sigma}_A = \boldsymbol{\sigma}_B$ are

$$\boldsymbol{\sigma}_A = \begin{bmatrix} 1 & \mathcal{K}^* \\ \mathcal{K} & 1+|\mathcal{K}|^2+|\mathcal{G}|^2+(2\bar{n}_{\rm th}+1)(|\alpha|^2+|\beta|^2) \end{bmatrix}. \tag{A2}$$

The off-diagonal one, describing the cross correlation, is

$$\boldsymbol{\sigma}_{AB} = \begin{bmatrix} 0 & \mathcal{G} \\ \mathcal{G} & 0 \end{bmatrix}. \tag{A3}$$

All the above quantities, $\mathcal{K}$, $\mathcal{G}$, α, and β, refer to their values at ω_m, in particular,

$$\mathcal{K}(\omega_m) = -2iC, \quad \alpha(\omega_m) = 2i\sqrt{C}. \tag{A4}$$

Note that $\mathcal{K}(\omega_m)$ is complex and it leads to complex squeezing, which is unaccessible with the standard homodyne detection [54,55]. This is why the noise ellipse of A illustrated in Fig. 1 in the text shows no correlation between the amplitude quadrature and the phase quadrature of A.

The figure of merit for quantifying such a bipartite Gaussian entanglement is the so-called logarithmic negativity $\mathcal{E}_N$ [59,60], which is defined as

$$\mathcal{E}_N = \max\{-(1/2)\ln[(\Sigma - \sqrt{\Sigma^2 - 4\det\boldsymbol{\sigma}})/2],\ 0\}, \tag{A5}$$

where $\Sigma \equiv \det\boldsymbol{\sigma}_A + \det\boldsymbol{\sigma}_B - 2\det\boldsymbol{\sigma}_{AB}$. A nonzero $\mathcal{E}_N$ implies the existence of entanglement. In our case, the first term is equal to

$$-\ln[\sqrt{1+|\mathcal{G}|^2+(2\bar{n}_{\rm th}+1)(|\alpha|^2+|\beta|^2)} - |\mathcal{G}|]. \tag{A6}$$

Having it larger than 0 requires

$$(2\bar{n}_{\rm th}+1)(|\alpha|^2+|\beta|^2) < 2|\mathcal{G}|. \tag{A7}$$

Using the fact that $|\alpha| \gg |\beta|$ and $\bar{n}_{\rm th} \gg 1$, we arrive at the following condition:

$$\gamma_m k_B T \leqslant \hbar G\rho. \tag{A8}$$

As an order of magnitude, it implies

$$\frac{T}{Q_m} \leqslant 3.0\times10^{-18}\ {\rm K}\left(\frac{0.5\ {\rm Hz}}{\omega_m/2\pi}\right)\left(\frac{\rho}{20\ {\rm g/cm}^3}\right). \tag{A9}$$

This requirement is beyond what we can achieve with the state-of-the-art instruments and needs further experimental efforts. Note that a related analysis of steady-state Gaussian entanglement in the case of two levitating nanobeads has also been presented by Qvarfort *et al.* [48].

The above requirement, Eq. (A8), can be generalized to the free-mass case with the resonant frequency $\omega_m \to 0$ and also not limited to the optomechanical setups, because Eq. (A8) depends neither on ω_m nor on the property of the optical fields. To make the conclusion general, we consider two free test masses coupled through gravity and assume the standard thermal decoherence model. The corresponding master equation for the density matrix $\hat{\varrho}$ of the two test masses takes the diffusive form

$$\dot{\hat{\varrho}}(t) = \frac{i}{\hbar}[\hat{\varrho}(t),\ \hat{H}_{AB}] - \frac{2m\gamma_m k_B T\,\delta x_q^2}{\hbar^2}\sum_{j=A,B}[\hat{Q}_j,\ [\hat{Q}_j,\ \hat{\varrho}(t)]], \tag{A10}$$

where δx_q is the characteristic length scale and is equal to the standard quantum limit [61] for Gaussian states and the size of the quantum superposition for non-Gaussian states. For the quantum entanglement to survive in the presence of the thermal decoherence, we require the interaction rate to be larger than the decoherence rate,

$$\frac{||\hat{H}_{AB}||}{\hbar} \geqslant \frac{2m\gamma_m k_B T\,\delta x_q^2}{\hbar^2}, \tag{A11}$$

where $||\hat{H}_{AB}||$ is the norm that quantifies the magnitude of the gravitational-interaction energy when A and B are at the quantum level.

In the case of δx_q much smaller than the mean separation d, we have, according to Eq. (6) in the text,

$$||\hat{H}_{AB}|| \approx 2\Lambda G m\rho\,\delta x_q^2, \tag{A12}$$

where we have assumed that δx_q is the same for A and B. The condition Eq. (A11) leads to Eq. (A8) for Λ being of the order of 1. Similarly, when δx_q is much larger than the mean separation d, e.g., the non-Gaussian superposition state in the setup using matter-wave interferometers [18,19], the corresponding gravitational interaction energy is simply

$$||\hat{H}_{AB}|| = \frac{Gm^2}{d}. \tag{A13}$$

Equation (A11) results in

$$\gamma_m k_B T \leqslant \frac{\hbar G m}{2d\,\delta x_q^2} < \frac{\hbar G m}{2d^3} \leqslant \hbar G\rho, \tag{A14}$$

where in the last inequality we have used the fact that m/d^3 is at most of the order of the matter density ρ. Therefore, regardless whether the two test masses (being either the free mass or the harmonic oscillator) are prepared in Gaussian states or non-Gaussian states, the same requirement universally applies for achieving gravity-mediated entanglement in the presence of thermal decoherence.

APPENDIX B: DEPENDENCE OF Λ ON THE TEST-MASS GEOMETRY

Depending on the geometry of the two test masses, the form factor in defining ω_g in Eq. (7) is different. The simplest case is having two identical spheres with a uniform density, and $\Lambda = \pi/3$ when their mean separation is equal to twice their radius. Here we consider two test masses that have the shape of a disk, which is usually the geometry for mirrors of optical cavities. Since there is no analytical expression for the Newtonian force between two disks, we perform numerical

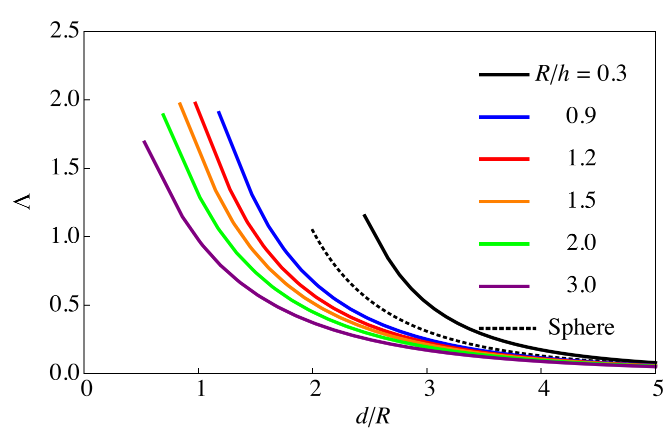

FIG. 4. Form factor Λ as a function of the distance for different ratios between the radius R and the thickness h of the disk. As a reference, we also show the case of two spheres by the dotted line. The lower bounds of the distance for different curves are defined by the one when the two disks touch each other.

integration of the force for disks with different ratios between the radius R and the thickness h. We then take the derivative numerically with respect to their mean separation d along the optical axis to obtain Λ for different mean separations, and the maximum Λ is achieved when their surfaces are close to each other with d approximately equal to h. Figure 4 shows the result, and we can see that the maximum value of Λ for $R/h = 1.5$ is around 2.0, which is what we assumed in the text.

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