Quantum Fredkin gate based on synthetic three-body interactions in superconducting circuits

Wei Feng and Da-wei Wang

Interdisciplinary Center for Quantum Information and State Key Laboratory of Modern Optical Instrumentation, Zhejiang Province Key Laboratory of Quantum Technology and Device and Department of Physics, Zhejiang University, Hangzhou 310027, China

(Received 25 November 2019; revised manuscript received 25 March 2020; accepted 11 May 2020; published 4 June 2020)

We propose the generation of effective three-body interactions in superconducting circuits by coupling qubits or resonators to a bus qubit. Such interactions are characterized by energy exchange between two qubits or resonators depending on the state of the bus qubit. We show that a controlled-iSWAP (-$\sqrt{\text{iSWAP}}$) gate can be naturally implemented based on the three-body interactions and it can be used to construct a quantum Fredkin (controlled-SWAP) gate. A generalized Fredkin gate which controls the swapping of photons between two resonators can be realized in a similar way. It can be used to generate the entangled state of a high number of photons. This proposal is promising to be demonstrated with superconducting circuits previously reported and will stimulate the implementation of multiqubit quantum gates based on many-body interactions.

DOI: 10.1103/PhysRevA.101.062312

I. INTRODUCTION

While fundamental interactions between particles are two-body interactions, effective many-body interactions are responsible for exotic phenomena such as topological order with anyons [1–4], Majorana zero modes [5,6], superfluids [7], and supersolids [8,9]. They also play an important role in the implementation of topological quantum computation [10,11] and memory [12]. However, many-body interactions are usually negligible compared with the dominant two-body interactions in most systems found in nature. Proposals for generating effective three-body interactions have been made with controllable systems such as optical lattices with cold atoms or molecules [13–16], trapped ions [17], and semiconductor quantum dots [18], but related experimental demonstrations are still lacking.

Superconducting circuits with advantages in tunability, flexibility, and scalability are a promising platform for quantum simulation and quantum computation [19–24]. Recently, tremendous progress has been made with superconducting circuits in quantum simulation [25–27] and in the achievement of quantum supremacy [28,29]. In superconducting circuits, it is straightforward to implement a two-body exchange interaction through capacitive or inductive coupling. By periodically modulating superconducting qubits, arbitrary two-body interactions can be engineered, and spin models such as the transverse Ising model and Kitaev honeycomb model can be simulated [30,31]. The generation of many-body interactions in superconducting circuits is of great significance to the quantum simulation of strongly correlated systems, as well as the realization of multiparticle entanglement and topological codes. There are already theoretical proposals for engineering many-body interactions in superconducting circuits [32,33]. However, specific superconducting qubits such as the tunable-coupling transmon and fluxonium are required in these schemes.

In this paper, we show that effective three-body interactions can be generated with readily accessible superconducting circuits. Our idea is inspired by experiments and theories in which superconducting qubits are connected to a bus resonator [34–38]. Mediated by the bus resonator, any two of the qubits can have effective couplings which are independent of the state of the resonator. If we replace the bus resonator with a bus qubit, there will be an interaction between qubits mediated by the bus qubit, and the interaction is dependent on the state of the bus qubit. Such a three-body interaction was previously suggested in optical lattices [15,16]. The design of our superconducting circuits is similar to those released in several recent works [29,39–41], where the intermediary couplers stay in their ground states and the characteristics of the three-body interaction are not demonstrated.

Intuitively, three-body interactions can be used to construct three-qubit gates. The quantum Fredkin gate is a three-qubit gate which swaps the quantum states of two target qubits conditioned on the state of a control qubit. It has important applications in quantum computation and quantum information processing such as quantum routers [42], quantum fingerprinting [43], and error correction [44]. Although the Fredkin gate can be decomposed into a sequence of single- and two-qubit gates [45,46], its direct implementation can simplify complex quantum circuits and optimize large-scale quantum processors with a higher fidelity. The design of the quantum Fredkin gate has flourished in the field of optics [47–52], and two related linear-optical experiments have recently been demonstrated using entangled photons with a low success probability [53,54]. Schemes of the deterministic quantum Fredkin gate have been proposed in hybrid atom-photon [55,56] and ion-phonon [57] systems. Recently, a deterministic controlled-SWAP operation of bosonic modes has been experimental demonstrated in superconducting circuits [58], where the three-body operation is decomposed to three two-body operations.

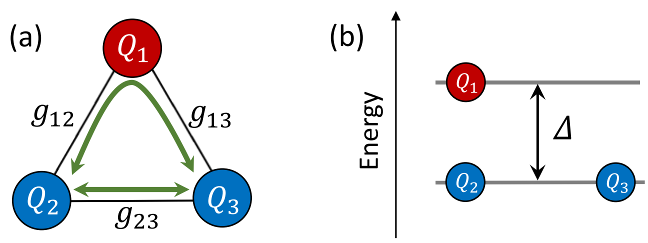

FIG. 1. (a) Schematic of three qubits coupled to each other. The resonant frequencies of the qubits are tunable. (b) Q_2 and Q_3 are set at the same frequency and detuned from Q_1 by Δ. The energy exchange between Q_2 and Q_3 has two different paths, directly and mediated by Q_1. The latter one depends on the state of Q_1.

Here we propose a simple and efficient method for implementing a controlled-iSWAP (-$\sqrt{\text{iSWAP}}$) gate based on the effective three-body interaction. Apart from the indirect interaction mediated by the bus qubit, we engineer a direct interaction for the two target qubits. The overall interaction can be turned on or off by the state of the bus qubit. Therefore, a controlled-iSWAP (-$\sqrt{\text{iSWAP}}$) gate can be naturally implemented and the quantum Fredkin gate can be constructed based on it. If we replace the two target qubits with two microwave resonators, there will still be a three-body interaction between the two resonators and the bus qubit. We can obtain a generalized Fredkin gate for the two resonators, which can be used to generate NOON states.

This paper is organized as follows: In Sec. II, we report the method for generating effective three-body interactions in superconducting circuits. In Sec. III, we present the scheme for implementing a quantum Fredkin gate based on the controlled-$\sqrt{\text{iSWAP}}$ gate which is naturally generated using the effective three-body interaction. In Sec. IV, we extend the method to a system with two resonators and show a generalized Fredkin gate which can be used to generate a NOON state. Finally, we draw a conclusion in Sec. V.

II. EFFECTIVE THREE-BODY INTERACTIONS

The types of interqubit interactions in superconducting circuits can be enriched by engineering different connections between superconducting qubits and resonators. Here we show that effective three-body interactions can be achieved if qubits are connected through a bus qubit. We consider that the system consists of three qubits (Q_j for $j = 1$–3) coupled to each other, as shown in Fig. 1. The Hamiltonian is (assuming $\hbar = 1$)

$$H = H_0 + H_i, \tag{1}$$

with

$$H_0 = -\sum_{j=1}^{3} \frac{\omega_j}{2}\sigma_j^z, \tag{2}$$

$$H_i = \sum_{i\neq j} g_{ij}(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+), \tag{3}$$

where $\sigma_j^z = |g_j\rangle\langle g_j| - |e_j\rangle\langle e_j|$, $\sigma_j^+ = |e_j\rangle\langle g_j|$, and $\sigma_j^- = |g_j\rangle\langle e_j|$ are the Pauli operators, with $|e_j\rangle$ ($|g_j\rangle$) being the excited (ground) states of Q_j, ω_j the resonant frequencies of Q_j, and g_{ij} the coupling strength between Q_i and Q_j. We select Q_1 as the bus qubit and detune it from Q_2 and Q_3 by the same amount Δ, i.e., $\omega_1 - \omega_2 = \omega_1 - \omega_3 = \Delta$. At $\Delta \gg g_{12}, g_{13}$, there is no energy exchange between Q_1 and $Q_{2,3}$. However, there are energy conserving second-order transitions $|g_1e_2g_3\rangle \leftrightarrow |g_1g_2e_3\rangle$ and $|e_1e_2g_3\rangle \leftrightarrow |e_1g_2e_3\rangle$. When the bus qubit Q_1 is in the ground state, the effective coupling strength for the transition $|g_1e_2g_3\rangle \leftrightarrow |g_1g_2e_3\rangle$ mediated by $|e_1g_2g_3\rangle$ is

$$\begin{aligned}\lambda_g &= \frac{\langle g_1g_2e_3|H_i|e_1g_2g_3\rangle\langle e_1g_2g_3|H_i|g_1e_2g_3\rangle}{-\Delta}\\ &= \frac{g_{12}g_{13}}{-\Delta}.\end{aligned} \tag{4}$$

When the bus qubit Q_1 is in the excited state, the effective coupling strength for the transition $|e_1e_2g_3\rangle \leftrightarrow |e_1g_2e_3\rangle$ mediated by $|g_1e_2e_3\rangle$ is

$$\begin{aligned}\lambda_e &= \frac{\langle e_1g_2e_3|H_i|g_1e_2e_3\rangle\langle g_1e_2e_3|H_i|e_1e_2g_3\rangle}{\Delta}\\ &= \frac{g_{12}g_{13}}{\Delta}.\end{aligned} \tag{5}$$

Note that λ_g and λ_e have opposite signs. This is because the intermediate state $|e_1g_2g_3\rangle$ has a higher energy than the initial (final) state $|g_1e_2g_3\rangle$ ($|g_1g_2e_3\rangle$), while the intermediate state $|g_1e_2e_3\rangle$ has a lower energy than the initial (final) state $|e_1e_2g_3\rangle$ ($|e_1g_2e_3\rangle$). The effective Hamiltonian under a second-order perturbation is

$$\begin{aligned}H_e &= H_0 + \lambda_e(2|e_1\rangle\langle e_1| - |e_2\rangle\langle e_2| - |e_3\rangle\langle e_3|)\\ &\quad + g_{23}(\sigma_2^+\sigma_3^- + \sigma_2^-\sigma_3^+) - \lambda_e\sigma_1^z(\sigma_2^+\sigma_3^- + \sigma_2^-\sigma_3^+),\end{aligned} \tag{6}$$

where the second term is the Stark shifts due to the Q_1-$Q_{2,3}$ off-resonant interactions, and the last term is the effective interaction between Q_2 and Q_3 mediated by Q_1 and it can be rewritten as

$$-\lambda_e\sigma_1^z(\sigma_2^+\sigma_3^- + \sigma_2^-\sigma_3^+) = -\frac{\lambda_e}{2}\sigma_1^z(\sigma_2^x\sigma_3^x + \sigma_2^y\sigma_3^y), \tag{7}$$

which is a typical three-body interaction. It indicates that the energy exchange between Q_2 and Q_3 depends on the state of Q_1.

III. IMPLEMENTATION OF A QUANTUM FREDKIN GATE

The XY-type interaction $H_{XY} = (g/2)[\sigma^x\sigma^x + \sigma^y\sigma^y]$ familiar in superconducting circuits naturally leads to a two-qubit iSWAP ($\sqrt{\text{iSWAP}}$) gate with the evolution

$$U_{XY}(t) = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & \cos(gt) & -i\sin(gt) & 0\\ 0 & -i\sin(gt) & \cos(gt) & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \tag{8}$$

during the time $t_{\text{iSWAP}} = \pi/2g$ ($t_{\sqrt{\text{iSWAP}}} = \pi/4g$). With appropriate parameters to meet the condition $g_{23} = \lambda_e$, the last two terms in Hamiltonian (6) cancel each other when Q_1 is in the ground state and sum up when Q_1 is in the excited state. The bus qubit Q_1 serves as a control qubit to switch on and off the XY-type interaction between Q_2 and Q_3. This controlled-XY-type interaction $H_{CXY} = (g/4)[I_1 - \sigma_1^z] \otimes [\sigma_2^x\sigma_3^x + \sigma_2^y\sigma_3^y]$ has

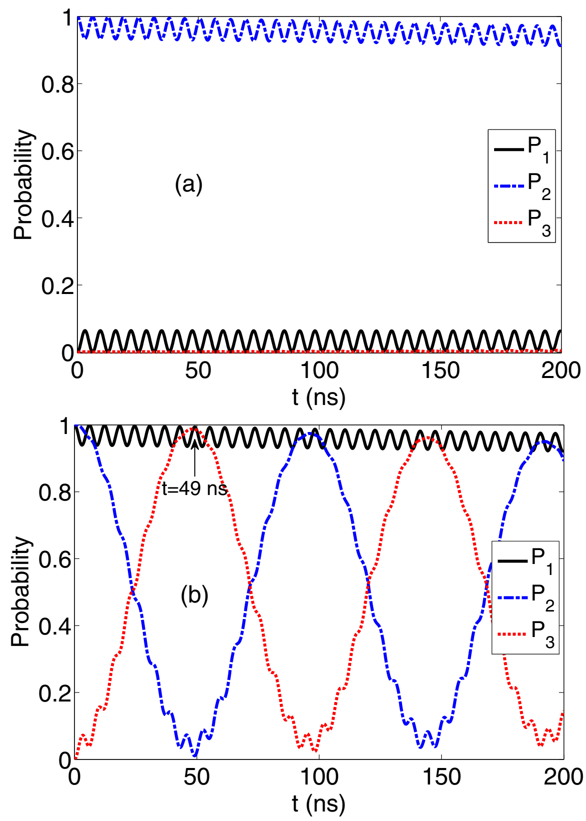

FIG. 2. Controlled-swapping dynamics with effective three-body interactions. Time evolution of populations P_j are simulated using the original Hamiltonian H with initial state (a) $|g_1e_2g_3\rangle$ and (b) $|e_1e_2g_3\rangle$. Relevant parameters are chosen as $g_{12} = g_{13} = 20$ MHz, $\Delta = 150$ MHz, and $g_{23} = g_{12}g_{13}/\Delta = 2.67$ MHz, and the relaxation and pure dephasing times of the superconducting qubits are $T_1 = 10$ μs and $T_2^* = 2$ μs, respectively.

the evolution

$$U_{CXY}(t) = |g_1\rangle\langle g_1| \otimes I_2 \otimes I_3 + |e_1\rangle\langle e_1| \otimes [U_{XY}(t)]_{23}, \quad (9)$$

where I is the identity operator. We simulate the dynamics of the original Hamiltonian H with experimentally feasible parameters of superconducting circuits and the results are shown in Fig. 2. One can see that, despite the fast oscillations due to the off-resonant transitions, Q_2 and Q_3 exchange energy when Q_1 is in the excited state, while they remain almost unchanged when Q_1 is in the ground state.

It is natural to obtain a controlled-iSWAP (-$\sqrt{\text{iSWAP}}$) gate based on a controlled-XY-type interaction. Apart from the term H_{CXY}, Stark shifts and the detuning Δ in Hamiltonian (6) can induce phase errors for a controlled-iSWAP (-$\sqrt{\text{iSWAP}}$) gate. In practice, the phase errors can be corrected by applying single-qubit Z rotations. Another error source is population leakage between the target qubits and the control qubit, as the fast oscillations shown in Fig. 2. The influence of population leakage on the gate fidelity can be reduced with appropriate parameters Δ and g_{ij}, which make the gate time an integral multiple of the period of fast oscillations. Just as the results shown in Fig. 2(b), the population of the control qubit P_1 nearly returns to its initial value at the controlled-iSWAP gate time $t_{\text{CiSWAP}} \approx 49$ ns.

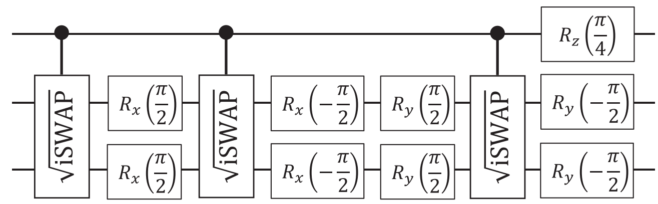

FIG. 3. Construction of a quantum Fredkin gate from three controlled-$\sqrt{\text{iSWAP}}$ gates and single-qubit rotations.

Strictly speaking, the quantum Fredkin gate refers to the controlled-SWAP gate. While the SWAP operation replaces product states with product states for two target qubits, the output of the iSWAP operation is entangled in general. Since a SWAP gate cannot be constructed by applying an H_{XY}-based gate only once [59,60], a quantum Fredkin gate cannot be generated directly based on a controlled-XY-type interaction. Following the construction of a SWAP gate from an iSWAP ($\sqrt{\text{iSWAP}}$) gate [61,62], we can generate a quantum Fredkin gate based on the controlled-$\sqrt{\text{iSWAP}}$ gate and single-qubit rotations (see Fig. 3). We numerically simulate this process and characterize the quantum Fredkin gate with quantum process tomography (see Fig. 4). The simulated quantum process tomography matrix χ_{sim} has a fidelity of 0.953, where realistic errors in single-qubit rotations and measurement are not counted. Qubit decoherence is the main error source to limit the gate fidelity.

Candidates of the circuit for implementing the scheme include frequency-tunable transmons [63–66], capacitively shunted fluxonium [67], and flux qubits [68,69]. If the scheme is implemented using transmons with weak anharmonicity, the influence of higher states should be accounted for. When the bus qubit Q_1 is in the excited state, besides $|g_1e_2e_3\rangle$, there is another intermediate state, $|f_1g_2g_3\rangle$, for the second-order transition $|e_1e_2g_3\rangle \leftrightarrow |e_1g_2e_3\rangle$, where $|f\rangle$ is the second excited state of the transmons. The additional coupling strength is $\lambda_e' = g_{12}'g_{13}'/(\eta - \Delta)$, where $\eta = \omega_{eg} - \omega_{fe}$, with ω_{eg} (ω_{fe}) being the resonant frequency of the transition $|g\rangle \leftrightarrow |e\rangle$ ($|e\rangle \leftrightarrow |f\rangle$), and $g_{ij}' \approx \sqrt{2}g_{ij}$ are the coupling strengths associated with the transition $|e\rangle \leftrightarrow |f\rangle$. To turn off the Q_2-Q_3 coupling when Q_1 is in the ground state, we still set $g_{23} = \lambda_e$. Then the total coupling strength is $2\lambda_e + \lambda_e'$ when Q_1 is in the excited state. The presence of the second excited states increases the total coupling strength with the condition $\Delta < \eta$. Therefore, the controlled-iSWAP (-$\sqrt{\text{iSWAP}}$) gate time can be shortened, which is good for the gate fidelities. However, the $|f\rangle$ energy level also leads to ZZ crosstalk coupling between the transmons. When there is no population in $|f\rangle$ of any of the three qubits during the qubit initialization stage, the total effective Hamiltonian of the system can be restricted to the subspace $\{|g\rangle, |e\rangle\}$ for each qubit, which is

$$\begin{aligned} H_e' = &H_0 + \lambda_e(2|e_1\rangle\langle e_1| - |e_2\rangle\langle e_2| - |e_3\rangle\langle e_3|) \\ &+ \lambda_f(|e_1e_2\rangle\langle e_1e_2| + |e_1e_3\rangle\langle e_1e_3|) + \lambda_f'|e_2e_3\rangle\langle e_2e_3| \\ &+ \left(\lambda_e + \frac{1}{2}\lambda_e'\right)\left(1 - \sigma_1^z\right)\left(\sigma_2^+\sigma_3^- + \sigma_2^-\sigma_3^+\right), \end{aligned} \quad (10)$$

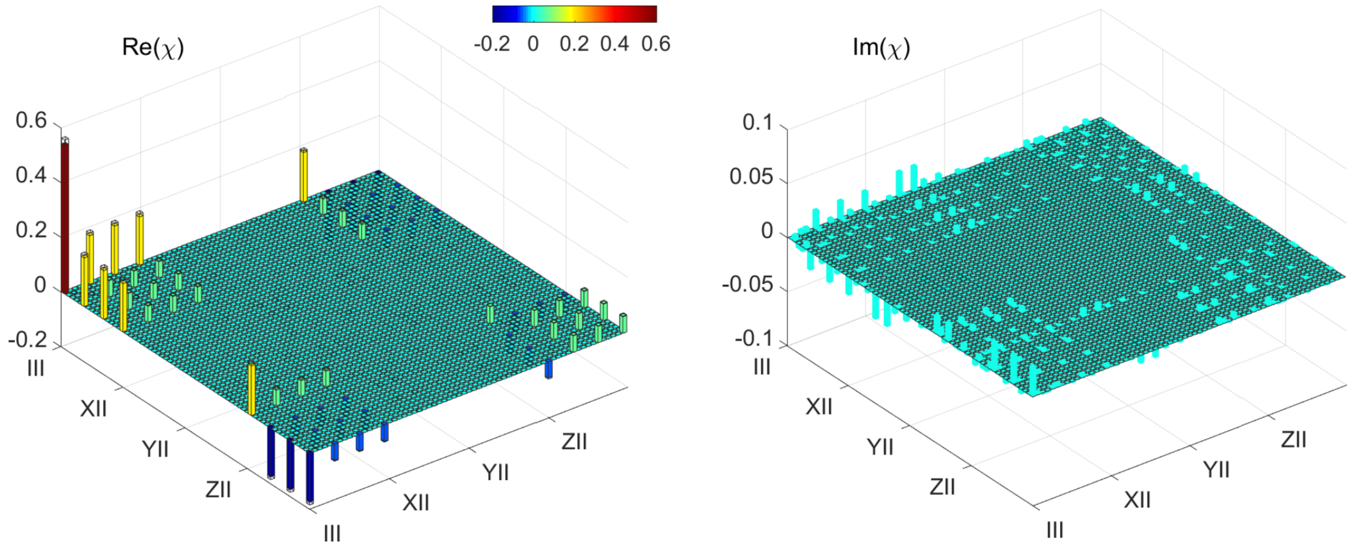

FIG. 4. Quantum process tomography of a quantum Fredkin gate, obtained with the sequence shown in Fig. 3. The ideal $\chi_{\rm ideal}$ (open bars with black outlines) and simulated $\chi_{\rm sim}$ (filled bars) process matrices are shown in the operator basis $\{I\otimes I\otimes I,\ I\otimes I\otimes X, \dots, Z\otimes Z\otimes Z\}$. The underlying controlled-$\sqrt{\rm iSWAP}$ gate is simulated with the same parameters as in Fig. 2, and the gate time is set to $t_{\rm C\sqrt{iSWAP}} = 24.8$ ns. Ignoring the errors of the single-qubit rotations, $\chi_{\rm sim}$ has a fidelity of $\mathrm{Tr}(\chi_{\rm deal}\chi_{\rm sim}) = 0.953$.

where $\lambda_f = g_{12,13}'^2[1/(\eta-\Delta)+1/(\eta+\Delta)]$ and $\lambda_f' = 2g_{23}'^2/\eta$ are the ZZ crosstalk coupling strengths under the second-order perturbation. As mentioned above, the influence of the Stark shifts in the first line of Eq. (10) can be removed easily with single-qubit Z rotations. The additional phase induced by ZZ crosstalk is a common problem for implementing, e.g., iSWAP and $\sqrt{\rm iSWAP}$ gates with transmon-type qubits. Recently, an experiment has shown that ZZ crosstalk coupling can be turned off with an ancillary qubit and proper detuning [41]. In another experiment from Google [70], a U_{XY}-like gate with an additional phase due to ZZ crosstalk is introduced for near-term quantum algorithms, where the additional phase is arbitrarily adjustable by changing the coupling strength and qubit detuning. The same approach can be used to remove the additional phase for the controlled-iSWAP (-$\sqrt{\rm iSWAP}$) gate.

IV. GENERALIZED FREDKIN GATE AND ITS APPLICATION

The core foundation of the controlled-iSWAP (-$\sqrt{\rm iSWAP}$) gate is that the interaction between the two target qubits depends on the state of the bus qubit. If we replace the bus qubit with a resonator, there will be no such kind of state-dependent interaction. However, if we replace the two target qubits with two resonators ($R_{1,2}$), the controlled-swapping dynamics will remain. The Hamiltonian for such a system is

$$H_R = -\frac{\omega}{2}\sigma^z + \sum_{j=1}^{2}[\nu_j a_j^\dagger a_j + g_j(\sigma^+ a_j + \sigma^- a_j^\dagger)] + g_{R12}(a_1^\dagger a_2 + a_1 a_2^\dagger), \tag{11}$$

where σ^z and $\sigma^\pm$ are the Pauli operators of the bus qubit, $a_j^\dagger$ (a_j) is the creation (annihilation) operator of resonator R_j, ω is the resonant frequency of the bus qubit, ν_j are the resonant frequencies of R_j, g_j is the coupling strength between the bus qubit and R_j, and g_{R12} is the direct coupling strength between R_1 and R_2. We set the frequency of the bus qubit higher than the same frequencies of the resonators with detuning δ, i.e., $\omega-\nu_1=\omega-\nu_2=\delta$. Under the condition $\delta\gg\sqrt{N}g_j$, with N being the excitation number of the system, the effective Hamiltonian is

$$H_R' = -\frac{\omega+\kappa}{2}\sigma^z + \sum_{j=1}^{2}(\nu_j-\kappa\sigma^z)a_j^\dagger a_j + (g_{R12}-\kappa\sigma^z)(a_1^\dagger a_2 + a_1 a_2^\dagger), \tag{12}$$

where $\kappa = g_1 g_2/\delta$. A controllable interaction between two resonators can be realized by introducing another ancillary bus qubit and keeping it in the ground state with suitable detuning.

With the condition $g_{R12}=\kappa$, the coupling between the two resonators will be switched off (on) when the bus qubit is in the ground (excited) state, and we obtain a controlled exchange interaction, $H_{RC} = \kappa[I-\sigma^z]\otimes[a_1^\dagger a_2 + a_1 a_2^\dagger]$. Based on this interaction, we can obtain a generalized Fredkin gate which can control the swapping of photons between the two resonators by the state of the bus qubit. With Hamiltonian (12), the evolution of $a_1^\dagger$ is

$$e^{-iH_R't}a_1^\dagger e^{iH_R't} = e^{-i\kappa t}[\cos(2\kappa t)a_1^\dagger - i\sin(2\kappa t)a_2^\dagger]\,|e\rangle\langle e| + e^{i\kappa t}a_1^\dagger|g\rangle\langle g|, \tag{13}$$

where the additional phases $e^{\pm i\kappa t}$ are due to the energy shifts in the first line of Eq. (12). When $2\kappa t=\pi/2$, the photons in R_1 will transfer to R_2 conditioned on the state of the bus qubit. We define the generalized Fredkin gate $U_R = e^{-iH_R't}$ with $t=\pi/4\kappa$.

The generalized Fredkin gate can be used to generate an N-photon number-path-entangled state, which is also known as the NOON state [71]. NOON states have important applications in quantum lithography and metrology [72,73].

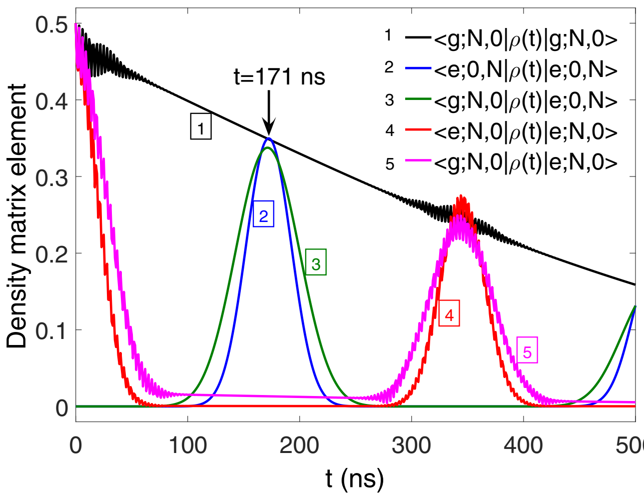

FIG. 5. Evolution of the density matrix components: (1) $\langle g;N,0|\rho(t)|g;N,0\rangle$, black line; (2) $\langle e;0,N|\rho(t)|e;0,N\rangle$, blue line; (3) $\langle g;N,0|\rho(t)|e;0,N\rangle$, green line; (4) $\langle e;N,0|\rho(t)|e;N,0\rangle$, red line; and (5) $\langle g;N,0|\rho(t)|e;N,0\rangle$, magenta line. The initial state is $\frac{1}{\sqrt{2}}(|g\rangle+|e\rangle)|N,0\rangle$, where $N=15$. At $t=171$ ns, $\langle e;0,N|\rho(t)|e;0,N\rangle$ and $\langle g;N,0|\rho(t)|e;0,N\rangle$ reached their maximum. Relevant parameters are chosen as $g_1=g_2=15$ MHz, $\delta=300$ MHz, and $g_{R12}=g_1g_2/\delta=0.75$ MHz. The relaxation and pure dephasing times of the qubits are $T_1=10\ \mu s$ and $T_2^*=2\ \mu s$, respectively, and the relaxation time of the resonator is $T_R=10\ \mu s$.

However, efficient and scalable methods for preparing high-NOON states are still rare and the largest number realized in experiments is only five [74]. In contrast, high-number Fock states containing 15 photons have been prepared in superconducting resonators [75]. Therefore, it is feasible to generate high-NOON states from high-number Fock states by using this generalized Fredkin gate.

To generate NOON states, we first prepare the bus qubit in a superposition state $\frac{1}{\sqrt{2}}(|g\rangle+|e\rangle)$, one resonator in the Fock state $|N\rangle$, and another resonator in the vacuum state $|0\rangle$. Then we apply the generalized Fredkin gate

$$\begin{aligned}&U_R\frac{1}{\sqrt{2}}(|g\rangle+|e\rangle)|N,0\rangle\\&=\frac{1}{\sqrt{2}}\big(e^{iN\frac{\pi}{4}}|g\rangle|N,0\rangle+e^{-iN\frac{3\pi}{4}}|e\rangle|0,N\rangle\big).\end{aligned}\tag{14}$$

The phase shifts $e^{iN\frac{\pi}{4}}$ and $e^{-iN\frac{3\pi}{4}}$ are not defects for the NOON state. Actually, the superresolving power of the NOON state comes from the N-fold relative phase between $|N,0\rangle$ and $|0,N\rangle$ [72,73]. Moreover, we can modulate the relative phase by postprocessing the detuning of the two resonators. Despite the phase shifts, by applying a $\pi/2$ pulse to the qubit, the state in Eq. (14) is transformed to $[|g\rangle(|N,0\rangle+|0,N\rangle)+|e\rangle(|N,0\rangle-|0,N\rangle)]/2$. After a projection measurement on the qubit with basis $\{|g\rangle,|e\rangle\}$, we can obtain the NOON states $(|N,0\rangle+|0,N\rangle)/\sqrt{2}$ and $(|N,0\rangle-|0,N\rangle)/\sqrt{2}$ with the qubit in $|g\rangle$ and $|e\rangle$, respectively. In Fig. 5, the evolution of the density matrix components is plotted with the initial state $\frac{1}{\sqrt{2}}(|g\rangle+|e\rangle)|15,0\rangle$. At time $t=171$ ns, i.e., the operating time of the generalized Fredkin gate, the density matrix has a major overlap with the target state $\frac{1}{\sqrt{2}}(|g\rangle|15,0\rangle+|e\rangle|0,15\rangle)$ and its fidelity is 0.70. The main error sources to limit the fidelity of NOON states are decoherences of the qubit and the resonators. The fidelity heavily depends on the number of photons. If a transmon is employed as the bus qubit, the second excited level will induce an additional coupling similar to the one in the case of three qubits, but there will not be the trouble of ZZ crosstalk. The detuning δ should be set to an appropriate value to avoid photon leakage to the transmon.

V. CONCLUSION

We present an approach to conveniently generate effective three-body interactions in superconducting circuits. By employing a detuned bus qubit as the intermediation, we can obtain interactions between two qubits or resonators which are dependent on the state of the bus qubit. By introducing a direct interaction to match the indirect interaction mediated by the bus qubit, energy swapping between the two qubits or resonators can be switched on and off conditioned on the state of the bus qubit. This controlled-swapping dynamics can be used to implement a quantum Fredkin gate. The working principle of the three-qubit controlled gate is extended to construct a generalized Fredkin gate which can control the swapping of photons between two resonators. We show its application in preparing high-NOON states. Our work is a step towards quantum simulation of strongly correlated systems with many-body interactions and quantum information processing with multiqubit gates.

ACKNOWLEDGMENTS

This research was supported by the National Key Research and Development Program of China (Grant No. 2019YFA0308100), the National Natural Science Foundation of China (Grant No. 11874322), the Fundamental Research Funds for the Central Universities, Zhejiang Province Key Research and Development Program (Grant No. 2020C01019), and the Basic Research Funding of Zhejiang University.

[1] X. G. Wen, *Quantum Field Theory of Many-Body Systems* (Oxford University Press, New York, 2004).

[2] X. G. Wen, F. Wilczek, and A. Zee, Phys. Rev. B **39**, 11413 (1989).

[3] Y. Taguchi, Y. Oohara, H. Yoshizawa, N. Nagaosa, and Y. Tokura, Science **291**, 2573 (2001).

[4] Y.-X. Chen, S.-W. Li, and Z. Yin, Phys. Rev. A **82**, 052320 (2010).

[5] Y. Niu, S. B. Chung, C.-H. Hsu, I. Mandal, S. Raghu, and S. Chakravarty, Phys. Rev. B **85**, 035110 (2012).

[6] W. DeGottardi, M. Thakurathi, S. Vishveshwara, and D. Sen, Phys. Rev. B **88**, 165111 (2013).

[7] B. Capogrosso-Sansone, S. Wessel, H. P. Büchler, P. Zoller, and G. Pupillo, Phys. Rev. B **79**, 020503(R) (2009).
[8] K. P. Schmidt, J. Dorier, and A. M. Läuchli, Phys. Rev. Lett. **101**, 150405 (2008).
[9] X.-F. Zhang, Y.-C. Wen, and Y. Yu, Phys. Rev. B **83**, 184513 (2011).
[10] A. Y. Kitaev, Ann. Phys. **303**, 2 (2003).
[11] V. W. Scarola, K. Park, and S. Das Sarma, Phys. Rev. Lett. **93**, 120503 (2004).
[12] E. Dennis, A. Kitaev, and A. Landahl, and J. Preskill, J. Math. Phys. **43**, 4452 (2002).
[13] H. P. Büchler, A. Micheli, and P. Zoller, Nat. Phys. **3**, 726 (2007).
[14] L. Bonnes, H. Büchler, and S. Wessel, New J. Phys. **12**, 053027 (2010).
[15] J. K. Pachos and M. B. Plenio, Phys. Rev. Lett. **93**, 056402 (2004).
[16] J. K. Pachos and E. Rico, Phys. Rev. A **70**, 053620 (2004).
[17] A. Bermudez, D. Porras, and M. A. Martin-Delgado, Phys. Rev. A **79**, 060303(R) (2009).
[18] P. Xue, Phys. Rev. A **81**, 052331 (2010).
[19] J. Q. You and F. Nori, Nature (London) **474**, 589 (2011).
[20] Y. Makhlin, G. Schon, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[21] J. Clarke and F. K. Wilhelm, Nature (London) **453**, 1031 (2008).
[22] A. A. Houck, H. E. Tureci, and J. Koch, Nat. Phys. **8**, 292 (2012)
[23] I. M. Georgescu, S. Ashhab, and F. Nori, Rev. Mod. Phys. **86**, 153 (2014).
[24] P. Roushan, C. Neill, J. Tangpanitanon, V. M. Bastidas, A. Megrant, R. Barends, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, A. Fowler, B. Foxen, M. Giustina, E. Jeffrey, J. Kelly, E. Lucero, J. Mutus, M. Neeley, C. Quintana, D. Sank, A. Vainsencher, J. Wenner, T. White, H. Neven, D. G. Angelakis, and J. Martinis, Science **358**, 1175 (2017).
[25] P. J. J. O'Malley, R. Babbush, I. D. Kivlichan, J. Romero, J. R. McClean, R. Barends, J. Kelly, P. Roushan, A. Tranter, N. Ding *et al.*, Phys. Rev. X **6**, 031007 (2016).
[26] A. Kandala, A. Mezzacapo, K. Temme, M. Takita, M. Brink, J. M. Chow, and J. M. Gambetta, Nature (London) **549**, 242 (2017).
[27] A. Kandala, K. Temme, A. D. Córcoles, A. Mezzacapo, J. M. Chow, and J. M. Gambetta, Nature (London) **567**, 491 (2017).
[28] C. Neill, P. Roushan, K. Kechedzhi, S. Boixo, S. V. Isakov, V. Smelyanskiy, A. Megrant, B. Chiaro, A. Dunsworth, K. Arya, R. Barends, B. Burkett, Y. Chen, Z. Chen, A. Fowler, B. Foxen, M. Giustina, R. Graff, E. Jeffrey, T. Huang, J. Kelly, P. Klimov, E. Lucero, J. Mutus, M. Neeley, C. Quintana, D. Sank, A. Vainsencher, J. Wenner, T. C. White, H. Neven, and J. M. Martinis, Science **360**, 195 (2018).
[29] F. Arute *et al.*, Nature **574**, 505 (2019).
[30] O. Kyriienko and A. S. Sørensen, Phys. Rev. Appl. **9**, 064029 (2018).
[31] M. Sameti and M. J. Hartmann, Phys. Rev. A **99**, 012333 (2019).
[32] A. Mezzacapo, L. Lamata, S. Filipp, and E. Solano, Phys. Rev. Lett. **113**, 050501 (2014).
[33] M. Hafezi, P. Adhikari, and J. M. Taylor, Phys. Rev. B **90**, 060503(R) (2014).
[34] J. Majer, J. M. Chow, J. M. Gambetta, J. Koch, B. R. Johnson, J. A. Schreier, L. Frunzio, D. I. Schuster, A. A. Houck, A. Wallraff, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **449**, 443 (2007).
[35] S. Filipp, M. Göppl, J. M. Fink, M. Baur, R. Bianchetti, L. Steffen, and A. Wallraff, Phys. Rev. A **83**, 063827 (2011).
[36] C. Song, K. Xu, W. Liu, C.-P. Yang, S.-B. Zheng, H. Deng, Q. Xie, K. Huang, Q. Guo, L. Zhang, P. Zhang, D. Xu, D. Zheng, X. Zhu, H. Wang, Y.-A. Chen, C.-Y. Lu, S. Han, and J.-W. Pan, Phys. Rev. Lett. **119**, 180511 (2017).
[37] K. Xu, J.-J. Chen, Y. Zeng, Y.-R. Zhang, C. Song, W. Liu, Q. Guo, P. Zhang, D. Xu, H. Deng, K. Huang, H. Wang, X. Zhu, D. Zheng, and H. Fan, Phys. Rev. Lett. **120**, 050507 (2018).
[38] C. Song, K. Xu, H. Li, Y. Zhang, X. Zhang, W. Liu, Q. Guo, Z. Wang, W. Ren, J. Hao, H. Feng, H. Fan, D. Zheng, D. Wang, H. Wang, and S. Zhu, Science **365**, 574 (2019).
[39] F. Yan, P. Krantz, Y. Sung, M. Kjaergaard, D. L. Campbell, T. P. Orlando, S. Gustavsson, and W. D. Oliver, Phys. Rev. Appl. **10**, 054062 (2018).
[40] P. Mundada, G. Zhang, T. Hazard, and A. Houck, Phys. Rev. Appl. **12**, 054023 (2019).
[41] X. Li, T. Cai, H. Yan, Z. Wang, X. Pan, Y. Ma, W. Cai, J. Han, Z. Hua, X. Han, Y. Wu, H. Zhang, H. Wang, Y. Song, L. Duan, and L. Sun, arXiv:1912.10721.
[42] B. K. Behera, T. Reza, A. Gupta, and P. K. Panigrahi, Quant. Info. Proc. **18**, 328 (2019).
[43] H. Buhrman, R. Cleve, J. Watrous, and R. de Wolf, Phys. Rev. Lett. **87**, 167902 (2001).
[44] I. L. Chuang and Y. Yamamoto, Phys. Rev. Lett. **76**, 4281 (1996).
[45] H. F. Chau and F. Wilczek, Phys. Rev. Lett. **75**, 748 (1995).
[46] J. A. Smolin and D. P. DiVincenzo, Phys. Rev. A **53**, 2855 (1996).
[47] G. J. Milburn, Phys. Rev. Lett. **62**, 2124 (1989).
[48] J. Fiurášek, Phys. Rev. A **73**, 062313 (2006).
[49] J. Fiurášek, Phys. Rev. A **78**, 032317 (2008).
[50] Y. X. Gong, G. C. Guo, and T. C. Ralph, Phys. Rev. A **78**, 012305 (2008).
[51] Q. Lin and J. Li, Phys. Rev. A **79**, 022301 (2009).
[52] M. S. Everitt and B. M. Garraway, Phys. Rev. A **90**, 012335 (2014).
[53] R. B. Patel, J. Ho, F. Ferreyrol, T. C. Ralph, and G. J. Pryde, Sci. Adv. **2**, e1501531 (2016).
[54] T. Ono, R. Okamoto, M. Tanida, H. F. Hofmann, and S. Takeuchi, Sci. Rep. **7**, 45353 (2017).
[55] B. Wang and L. M. Duan, Phys. Rev. A **75**, 050304(R) (2007).
[56] J. Song, Y. Xia, and H. S. Song, Europhys. Lett. **87**, 50005 (2009).
[57] X. B. Zou, J. Kim, and H. W. Lee, Phys. Rev. A **63**, 065801 (2001).
[58] Y. Y. Gao, B. J. Lester, K. S. Chou *et al.*, Nature (London) **566**, 509 (2019).
[59] G. Burkard, D. Loss, D. P. DiVincenzo, and J. A. Smolin, Phys. Rev. B **60**, 11404 (1999).
[60] Y. Makhlin, Quant. Info. Proc. **1**, 243 (2002).
[61] N. Schuch and J. Siewert, Phys. Rev. A **67**, 032301 (2003).
[62] A. V. Lebedev, G. B. Lesovik, V. M. Vinokur, and G. Blatter, Phys. Rev. B **98**, 214502 (2018).
[63] P. Roushan, C. Neill, A. Megrant *et al.*, Nat. Phys. **13**, 146 (2017).

[64] X. Li, Y. Ma, J. Han, T. Chen, Y. Xu, W. Cai, H. Wang, Y. P. Song, Z.-Y. Xue, Z.-Q. Yin, and L. Sun, Phys. Rev. Appl. **10**, 054009 (2018).

[65] D.-W. Wang, C. Song, W. Feng, H. Cai, D. Xu, H. Deng, H. Li, D. Zheng, X. Zhu, H. Wang, S.-Y. Zhu, and M. O. Scully, Nat. Phys. **15**, 382 (2019).

[66] Y. Wu, L. Yang, M. Gong, Y. Zheng, H. Deng, Z. Yan, Y. Zhao, K. Huang, A. D. Castellano, W. J. Munro, K. Nemoto, D. Zheng, C. P. Sun, Y. X. Liu, X. Zhu, and L. Lu, npj Quantum Info. **4**, 50 (2018).

[67] Y.-H. Lin, L. B. Nguyen, N. Grabon, J. San Miguel, N. Pankratova, and V. E. Manucharyan, Phys. Rev. Lett. **120**, 150503 (2018).

[68] M. Steffen, S. Kumar, D. P. DiVincenzo, J. R. Rozen, G. A. Keefe, M. B. Rothwell, and M. B. Ketchen, Phys. Rev. Lett. **105**, 100502 (2010).

[69] F. Yan, S. Gustavsson, A. Kamal *et al.*, Nat. Commun. **7**, 12964 (2016).

[70] B. Foxen, C. Neill, A. Dunsworth, P. Roushan *et al.*, arXiv:2001.08343.

[71] A. N. Boto, P. Kok, D. S. Abrams, S. L. Braunstein, C. P. Williams, and J. P. Dowling, Phys. Rev. Lett. **85**, 2733 (2000).

[72] J. P. Dowling, Contemp. Phys. **49**, 125 (2008).

[73] W. Feng, K. Jiang, Michelle L.-J. Lollie, M. S. Zubairy, and J. P. Dowling, Phys. Rev. A **89**, 043824 (2014).

[74] I. Afek, O. Ambar, and Y. Silberberg, Science **328**, 879 (2010).

[75] H. Wang, M. Hofheinz, M. Ansmann, R. C. Bialczak, E. Lucero, M. Neeley, A. D. O'Connell, D. Sank, J. Wenner, A. N. Cleland, and J. M. Martinis, Phys. Rev. Lett. **101**, 240401 (2008).