Influence of acceleration on multibody entangled quantum states

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We study the influence of acceleration on the twin-Fock state which is a class of specific multibody entangled quantum state and was already realized experimentally with high precision and sensitivity. We show that the multibody quantum entanglement can be increased with the acceleration, consistent with the "anti-Unruh effect" in reference to the counterintuitive cooling previously pointed out for an accelerated detector coupled to the vacuum. In particular, this kind of entanglement increase can lead to the improvement of the phase sensitivity, which provides a way to test the anti-Unruh effect in the future experiments.

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I. INTRODUCTION

In 1976, Unruh discovered that an observer with uniform acceleration would feel a thermal bath of particles in the Minkowski vacuum of a free quantum field [1], which implicates that the particle content of a quantum field is observer dependent [2]. This effect was put forward soon after that Hawking discovered that a black hole could emit thermal radiation [3] and could help to clarify some conceptual issues [4] raised by black-hole evaporation due to the equivalence. So, the understanding of the Unruh effect is also significant for Hawking radiation and the related problems (i.e., information loss problems). In the past years, the Unruh effect was digested and extended to many different situations (see the review [2] and references therein), but the observation of the Unruh effect has not been realized up to now because of the pretty low Unruh temperature $T = \hbar a/(2\pi c k_B)$ where a is the proper acceleration of the observer, $\hbar$ is the reduced Planck constant, c is the speed of the light, and k_B is the Boltzman constant. The acceleration must be about 10^{20} m/s^2 in order to realize a photon bath at 1 K.

Although it was claimed that the Unruh effect is a direct result of quantum field theory and does not require any experimental confirmation if the quantum field theory is correct [5], there exists still some problems needed to be clarified through experiments or observations, i.e., whether the particles felt by the accelerated observers are real [6], whether the effect is applicable to the extended systems [7], and even some theoretical calculation implies that the possible inversion from Bose to Fermi statistics for many-particle states observed by an accelerated observer [8]. Thus, the experimental quest for the evidence of the Unruh effect is necessary for the final confirmation. As well known, the most observational proposals are related to a model called the Unruh-DeWitt detector [9]. Based on the model, it is found that a quantum system consisting of such a detector uniformly accelerating in Minkowski vacuum sees a thermal field and, thus, cause decoherence due to the coupling with the thermal field. The first attempt is to observe such an effect by the deexcitation of the electron in storage rings by the thermal Unruh radiation [10]. Then some other possible detections related to proton decays [11,12], accelerated charges [13,14], neutrino oscillations [15], and the recent theoretical [16] and observational [17] methods using Larmor radiation were proposed. In particular, an interesting observation for Unruh radiation using quantum simulation in Bose-Einstein condensates was reported, which is significant for the future research of the dynamics of quantum many-body systems in a curved spacetime [18].

On the other hand, the recent found anti-Unruh effect [19] states that a particle detector in uniform acceleration coupled to the vacuum can cool down with increasing acceleration under certain conditions, which is opposite to the celebrated Unruh effect. Since the experiments are always made in the range of finite length and time, it must distinguish the two situations of Unruh and anti-Unruh effects carefully. An interesting way for this is to see the change in quantum entanglement by acceleration. According to the previous results [20–28], the quantum entanglement would be degraded by the Unruh effect, which helps to establish the general conclusion that entanglement is also observer dependent. In particular, a recent calculation showed that the anti-Unruh effect can lead to the increase for the quantum entanglement [29], which might be significant for the task of quantum information in a large spatial or temporal scale. In this paper, we will consider the influence of acceleration on the spin-squeezed states [30,31] and the corresponding experimental feasibility through the change in entanglement. Spin-squeezed states have attracted much attention due to their use in the measurement of the correlation or entanglement among particles and in the improvement of measurement precision in quantum metrology. We will focus on twin-Fock (TF) states [32] which can be seen as a kind of limit for spin-squeezed states and had been realized in a recent experiment with more than 10^4 atoms [33].

This paper is organized as follows. First, in Sec. II, we review the theory about two-level Unruh-DeWitt (UDW) detector in Minkowski spacetime, and the change in

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entanglement between two atoms for the Unruh and anti-Unruh effects. This is followed in Sec. III by the discussions on the influence of acceleration on entanglement for TF states where the spin-squeezing parameter is used to measure the change in entanglement. Then, when the atoms in the TF state are accelerated, how the phase sensitivity is changed under the background of the Ramsey interferometer is investigated in Sec. IV. Finally, we give a conclusion in Sec. V. In this paper, we use units with $c = \hbar = k_B = 1$, except the part of analyzing the experimental feasibility in Sec. IV.

II. THE UNRUH-DEWITT MODEL

We start with the model of the UDW detector in order to investigate the interaction between accelerated atoms and vacuum. The detector, usually considered as a pointlike two-level quantum system or atom (as required in this paper), consists of two quantum states, i.e., the ground $|g\rangle$ and excited $|e\rangle$ states, which are separated by an energy gap Ω whereas experiencing accelerated motion in a vacuum field. But for the accelerated atom, the vacuum appears thermal due to the Unruh effect, which will influence the state of the atom. This could be described according to the following interaction Hamiltonian in a $(1+1)$-dimensional model,

$$H_I = \lambda\chi(\tau/\sigma)\mu(\tau)\phi[x(\tau)], \tag{1}$$

where ϕ is a scalar field related to the vacuum in Minkowski spacetime and interacts with the accelerated atom, λ is the coupling strength, τ is the atom's proper time along its trajectory $x(\tau)$, $\mu(\tau)$ is the atom's monopole momentum, and $\chi(\tau/\sigma)$ is a switching function that is used to control the interaction timescale σ. This can be easily generalized to more complex situations, such as a quantum oscillator [34], as confirmed with Kubo-Martin-Schwinger (KMS) conditions for thermal equilibrium [35]. For an atom accelerating in a vacuum cavity, the evolution of the total quantum state is determined perturbatively by the unitary operator up to first order given by

$$U = I - i\int d\tau\, H_I(\tau) + O(\lambda^2). \tag{2}$$

The atom is accelerated along the trajectory,

$$t(\tau) = a^{-1}\sinh(a\tau),$$
$$x(\tau) = a^{-1}[\cosh(a\tau) - 1], \tag{3}$$

with the proper acceleration a. Note that the extra term a^{-1} in the expression of $x(\tau)$ is related to the initial condition and only for convenience of the calculation below without changing the influence of the acceleration. Thus, within the first-order approximation and in the interaction picture, the evolution of the atom could be described by [19],

$$U|g\rangle|0\rangle = D_0(|g\rangle|0\rangle - i\eta_0|e\rangle|1\rangle),$$
$$U|e\rangle|0\rangle = D_1(|e\rangle|0\rangle + i\eta_1|g\rangle|1\rangle), \tag{4}$$

where k denotes the mode of the $(1+1)$-dimension scalar field with (bosonic) annihilation (creation) operator a_k ($a_k^\dagger$), $a_k|0\rangle = 0$, and $a_k^\dagger|0\rangle = |1_k\rangle$, and $D_{0,1}$ is the state normalization factor. It is noted that the created state $|1_k\rangle$ is dependent on the wave-vector k, so the coupling in Eq. (5) has to be understood by writing $\eta_0|1\rangle = \lambda\int dk\, I_{+,k}|1_k\rangle$ and $\eta_1|1\rangle = \lambda\int dk\, I_{-,k}|1_k\rangle$ where $I_{\pm,k}$ is given as

$$I_{\pm,k} = \frac{1}{\sqrt{4\pi\omega}}\int_{-\infty}^{\infty}\chi(\tau/\sigma)\exp[\pm i\Omega\tau + i\omega t(\tau) - ikx(\tau)]d\tau. \tag{5}$$

The notations $\eta_0|1\rangle$ and $\eta_1|1\rangle$ are inseparable, but, in this paper, we consider $|1\rangle$ is the same for the two cases under the spirit of single mode approximation [36,37], and η_0 and η_1 are related to the excitation and deexcitation probabilities of the atom, i.e., $|\eta_0|^2 = \sum_k|\langle 1_k, e|U^{(1)}|0, g\rangle|^2$ and $|\eta_1|^2 = \sum_k|\langle 1_k, g|U^{(1)}|0, e\rangle|^2$ where $U^{(1)} = -i\int d\tau\, H_I(\tau)$.

It is worth pointing out that the change in the quantum state, i.e., the transition probability, is dependent on the concrete parameters, such as the interaction timescale σ and the energy gap Ω [19]. In particular, under some conditions, for example, when the interaction timescale is far away from the timescale associated to the reciprocal of the detector's energy gap, the probability decreases as the acceleration or the Unruh temperature increases, which makes the atom feel cooler instead of warm up expected by the Unruh effect. This effect was called as anti-Unruh effect. Although the initial discussion for the anti-Unruh effect is made in Ref. [19] for accelerated detectors coupled to a massless scalar field either in a periodic cavity or under a hard-infrared momentum cutoff for the continuum, it has been shown to represent a general stationary mechanism that can exist under a stationary state satisfying the KMS condition [38–40] and is independent on any kind of boundary conditions [19,35]. Thus, like the Unruh effect, the anti-Unruh effect constitutes another new phenomenon for the accelerated observers. Although the physically essential reasons remain to be explored for their difference, some important elements, such as the interaction time, the detector's energy gap, the mass of the quantum field, etc., had been pointed out to distinguish them operationally. Here, we consider massive field with, e.g., $\omega = \sqrt{k^2 + m^2}$ as in Ref. [35] so that the anti-Unruh effect discussed will not be constrained by the finite interaction time and its validity can be extended to situations where the detector is switched on adiabatically over an infinite long time. Without loss of generality, $m = 1$ is used for all numerical calculations.

With that, the change in bipartite entanglement for two atoms was investigated before Ref. [29] in which the initial state is assumed to take the form

$$|\Psi_i\rangle = (\alpha|g\rangle_A|e\rangle_B + \beta|e\rangle_A|g\rangle_B)|0\rangle_A|0\rangle_B, \tag{6}$$

with the complex coefficients satisfying $|\alpha|^2 + |\beta|^2 = 1$. Here, we consider the vacuum as in a product state, and thus, the interaction between either one of two atoms and the scalar field is independent of each other. This could help us to understand the influence of acceleration on the quantum state of the atoms without the disturbance of the complicated vacuum (i.e., it is regarded as an entangled state) [29]. This means that the subscripts A and B in the vacuum state $|\widetilde{0}\rangle \equiv |0\rangle_A|0\rangle_B$ represents the locations related to the atoms A and B. For the case we consider, each atom is independently [41] accelerating in the vacuum and has the same coupling with the scalar field in its respective (spatial) place by the same process presented in Eq. (5). When the two atoms are accelerated simultaneously, the state becomes

$$\begin{aligned}|\Psi_f\rangle = &D_0D_1[(\alpha|g\rangle_A|e\rangle_B + \beta|e\rangle_A|g\rangle_B)|0\rangle_A|0\rangle_B\\ &-i(\alpha\eta_1|g\rangle_A|g\rangle_B + \beta\eta_0|e\rangle_A|e\rangle_B)|0\rangle_A|1\rangle_B\\ &-i(\beta\eta_1|g\rangle_A|g\rangle_B + \alpha\eta_0|e\rangle_A|e\rangle_B)|1\rangle_A|0\rangle_B\\ &+(\alpha\eta_0\eta_1|e\rangle_A|g\rangle_B + \beta\eta_0\eta_1|g\rangle_A|e\rangle_B)|1\rangle_A|1\rangle_B], \quad (7)\end{aligned}$$

where $|\widetilde{1}\rangle \equiv |0\rangle_A|1\rangle_B \equiv |1\rangle_A|0\rangle_B$ represents the single-mode state from a global perspective but $|0\rangle_A|1\rangle_B$ might be different from $|1\rangle_A|0\rangle_B$ locally when the two atoms are separated far apart, and $|\widetilde{2}\rangle \equiv |1\rangle_A|1\rangle_B$ represents the two-mode state from a global perspective. It is necessary to keep the last term in Eq. (7) in order to make the evolution in Eq. (5) intact formally since the so-called single-mode approximation in this paper is performed for the interaction between a single atom and the scalar field. When the two atoms locate nearly at the same place, the forms $|0\rangle_A|1\rangle_B$ and $|1\rangle_A|0\rangle_B$ for the vacuum can be regarded as the same and the two related terms in Eq. (7) can be combined into one, i.e., $i(\alpha+\beta)(\eta_1|g\rangle_A|g\rangle_B + \eta_0|e\rangle_A|e\rangle_B)|0\rangle|1\rangle$. We consider the two atoms (or many atoms considered in the next section) staying nearly in the same place in the whole process of acceleration, but the loss of atoms due to acceleration is not considered in this paper.

The change in entanglement can be quantified by concurrence [42] which is a widely used entanglement measure for a bipartite mixed state. Concurrence is defined by

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (8)$$

where λ_1–λ_4 are the eigenvalues of the Hermitian matrix $\sqrt{\sqrt{\rho}\widetilde{\rho}\sqrt{\rho}}$ with $\widetilde{\rho} = (\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y)$ the spin-flipped state of ρ, σ_y being the y-component Pauli matrix and the eigenvalues listed in decreasing order. For the case of two atoms being accelerated, the change in entanglement can be calculated using concurrence for the reduced density-matrix ρ_{AB} by tracing out the scalar field from the final accelerated quantum state (7), which is shown in the upper plot of Fig. 1 with the initial state $|\Psi_i\rangle$ are taken by $\alpha = \beta = \frac{1}{\sqrt{2}}$. The solid red line represents the case of the anti-Unruh effect as discussed in the paragraph after Eq. (5). A more detailed discussion and other cases for different initial states refer to Ref. [29]. Since the experiment is always performed within a certain timescale and using the certain energy gap, the appearance of the anti-Unruh effect is possible when the experiment is implemented. Therefore, the experimental test must consider this point for which the increase in entanglement would also be the result of acceleration.

III. TWIN-FOCK STATE

The previous section presents the UDW model and the change in entanglement between two atoms in this model. Since it is not easy to implement the corresponding experiment to observe the effect for two atoms, we now attempt to apply it to the case of multibody quantum states for multiple atoms being simultaneously accelerated. Before that, it is noted that the bipartite quantum state for the maximal entangled atoms $|\psi_i\rangle = \frac{1}{\sqrt{2}}(|g\rangle_A|e\rangle_B + |e\rangle_A|g\rangle_B)$ can be regarded as the simplest TF state. The TF state is one kind of Dicke states [43]. For a collection of N identical (pseudo-) spin-

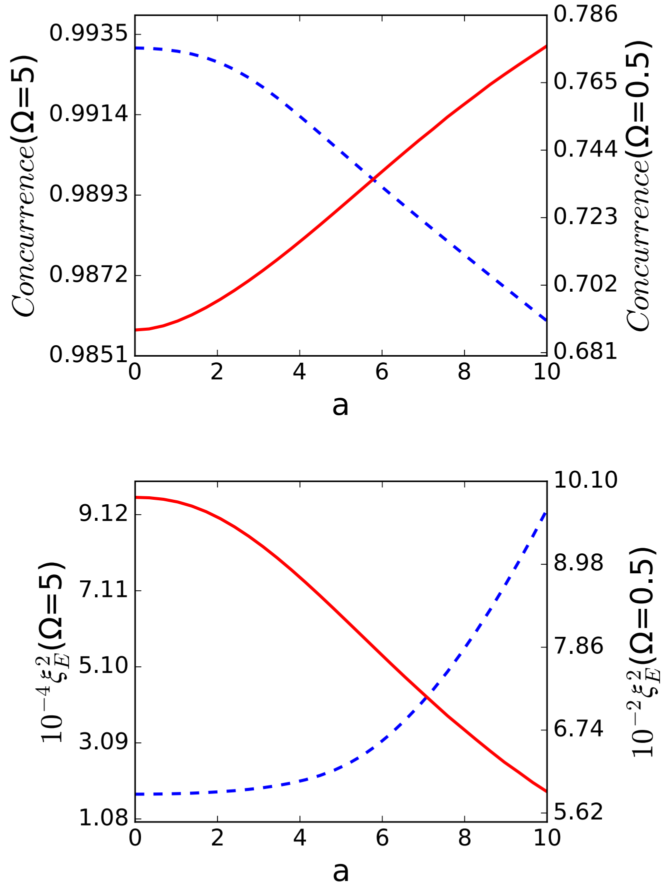

FIG. 1. The concurrence (upper panel) and the spin-squeezing parameter (lower panel) as a function of a when two atoms are accelerated. The model parameters employed are $\lambda = 1$, $\sigma = 0.4$. The solid red line denotes enhanced entanglement with acceleration at $\Omega = 0.5$ (referenced to the right vertical axis), whereas the dashed blue line with respect to the left vertical axis is for the decreased entanglement with acceleration at $\Omega = 5$. We make the total atom number $N = 2$ for the right panel.

1/2 particles, Dicke states can be expressed in Fock space as $|\frac{N}{2}+m\rangle_\uparrow|\frac{N}{2}-m\rangle_\downarrow$ with $(\frac{N}{2}+m)$ particles in spin-up and $(\frac{N}{2}-m)$ particles in spin-down modes for $m = -\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}$. In particular, $m = 0$ represents just the TF state where the number of the particles is the same for each one of the two spin states. On the other hand, Dicke states can be described by the common eigenstate $|j, m\rangle$ of the collective spin operators J^2 and J_z with respective eigenvalues $j(j+1)$ and m. For the system consisting of N two-level atoms, we will consider state $|j = \frac{N}{2}, m\rangle$ indicates that $(j+m)$ atoms are at the excited state $|e\rangle$, $(j-m)$ atoms are at the ground state $|g\rangle$, and it is the TF state when $m = 0$. $J_z = \frac{1}{2}(n_e - n_g)$ represents the difference of the number of atoms between excited (n_e) and ground (n_g) states, and $J^2 = \frac{N}{2}(\frac{N}{2}+1)$ is related to the total number of atoms. With this description, the state of two atoms can be written as $|\psi_i\rangle = |1, 0\rangle$ with which the average of the difference of the number of atoms $\langle J_z\rangle = 0$.

When the two atoms are accelerated, according to the UDW model discussed in last section, state $|\psi_i\rangle$ will become $\rho_f = \mathrm{Tr}_\phi(|\Psi_f\rangle\langle\Psi_f|)$ where $\alpha = \beta = \frac{1}{\sqrt{2}}$ are taken for state

$|\Psi_f\rangle$, and Tr_ϕ indicates the calculation of tracing out the part of the scalar field. Thus, the difference in the number of atoms between excited and ground states is obtained as

$$\langle J_z\rangle = \mathrm{Tr}(\rho_f J_z) = D_0^2 D_1^2(|\eta_0|^2 - |\eta_1|^2), \tag{9}$$

where Tr represents the trace of a matrix. The result means that the atom's number at the excited state is not equal to that at the ground state, different from the requirement of the TF state, unless the probability of transition from the ground state to the excited state equates the probability for the inverse transition.

Now, we extend this to the case of N atoms with the initial TF state $|j,0\rangle$. When all atoms are accelerated simultaneously, the TF state becomes

$$\rho_t = B_0^2|j,0\rangle\langle j,0| + \sum_{m=-N/2}^{N/2}\sum_{m'=-N/2}^{N/2} B_m B_{m'}^*|j,m\rangle\langle j,m'|, \tag{10}$$

up to the normalization factor which is included in our numerical calculation.

$$B_0^2 = \sum_{k=0}^{N/2}\left[\left(C_{N/2}^k\right)^4 (D_0D_1)^N(\eta_0\eta_1)^{2k}\right],$$

$$B_m = \left[\sum_{k=0}^{N/2-|m|} C_{N/2}^k C_{N/2}^{k+|m|}(D_0D_1)^{N/2}(\eta_0\eta_1)^k \times (\theta(m)(-i\eta_0)^m + \theta(-m)(i\eta_1)^{|m|})\right]$$

in which the function $\theta(x)=1$ when $x>0$ and $\theta(x)=0$ otherwise, the star $*$ in $B_{m'}^*$ represents the complex conjugate, and $C_n^r = \frac{n!}{r!(n-r)!}$ denotes the combinatorial factor of choosing r out of n. When all atoms are accelerated, according to Eq. (5), the ground state would change into the form $D_0(|g\rangle|0\rangle - i\eta_0|e\rangle|1\rangle)$, and at the same time, the excited state $|e\rangle$ would change into the form $D_1(|e\rangle|0\rangle + i\eta_1|g\rangle|1\rangle)$. Then, expanding these terms, recombining them and tracing the vacuum state out give the expression (10) of the final state after acceleration. The parameter B_0^2 represents the probability of remaining the original form of the TF state, which includes those cases that if l $(0 \leqslant l \leqslant \frac{N}{2})$ atoms are changed from the ground states to the excited states, there must be other l atoms which are changed from the excited states to the ground states simultaneously. Similarly, the parameter B_m can be worked out by choosing the terms that in every term either there are m more excited states than ground states (that is the case for $m>0$) or there are m more ground states than excited states (that is the case for $m<0$). No crossed terms, such as $|j,0\rangle\langle j,m|$ because we consider the vacuum state including the same number of photons as the same state no matter which atoms emitted these photons. It is not difficult to confirm this for the cases $N=2$ and $N=4$.

In order to quantify the change of entanglement in the process of accelerating atoms that is initially in the TF state, we choose the spin-squeezing parameter [31],

$$\xi_E^2 = \frac{\min_{\vec{n}}\left[(N-1)(\Delta J_{\vec{n}})^2 + \langle J_{\vec{n}}^2\rangle\right]}{\langle J^2\rangle - N/2}, \tag{11}$$

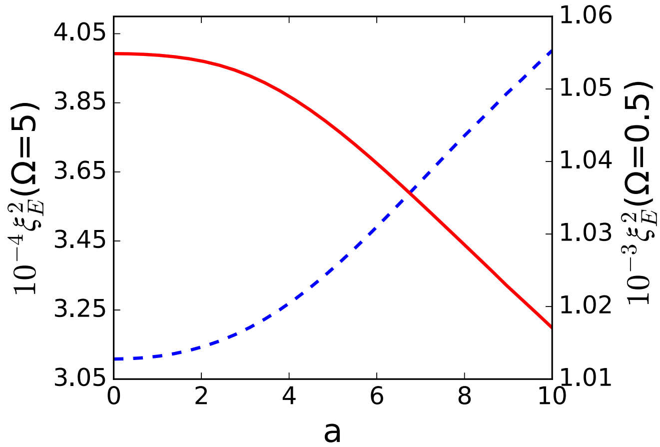

FIG. 2. The spin-squeezing parameter as a function of the acceleration a. We make the total atom number $N=100$, and the other parameters are the same as in Fig. 1.

which is rotationally invariant and related closely to entanglement. The relation between spin squeezing and entanglement could be shown, such as that in Refs. [44,45] in which the inequality,

$$(N-1)(\Delta J_{\vec{n}})^2 + \langle J_{\vec{n}}^2\rangle \geqslant \langle J^2\rangle - N/2 \tag{12}$$

holds for any separable states, and the violation of this inequality indicates entanglement. More related works refer to Ref. [46]. If $\xi_E^2 < 1$, the state is spin squeezed and entangled. In particular, the smaller the value of ξ_E^2, the more the entanglement will be, which can be seen by comparing the upper and lower panels of Fig. 1. Since the mean-spin direction of Dicke states can be set along the z direction, we take the z direction as the direction of $\vec{n}$, and the expression for the spin-squeezing parameter is written as

$$\xi_E^2 = \frac{(N-1)(\Delta J_z)^2 + \langle J_z^2\rangle}{\langle J^2\rangle - N/2}. \tag{13}$$

For TF states, $\xi_E^2 = 0$ due to $\langle J_z^2\rangle = \langle J_z\rangle = 0$, which means that the initial TF state is the most spin-squeezed and entangled state under the measure of ξ_E^2.

After acceleration, the TF state becomes ρ_t described in Eq. (10). With this, we can calculate

$$\langle J_z\rangle = \mathrm{Tr}(\rho_t J_z) = \sum_{m=-N/2}^{N/2} m|B_m|^2, \tag{14}$$

and

$$\langle J_z^2\rangle = \mathrm{Tr}(\rho_t J_z^2) = \sum_{m=-N/2}^{N/2} m^2|B_m|^2. \tag{15}$$

Thus, according to $(\Delta J_z)^2 = \langle J_z^2\rangle - \langle J_z\rangle^2$, ones can calculate ξ_E^2 by substituting these results (14) and (15) into Eq. (13), which is presented in Fig. 2 for different energy gaps (see also the lower panel of Fig. 1 for $N=2$). As seen, the anti-Unruh effect is represented with the solid red line, and it shows that the entanglement increases with the acceleration as expected. It is noted that the entanglement at $a=0$ for the accelerated state (10) is less than that for the initial maximal

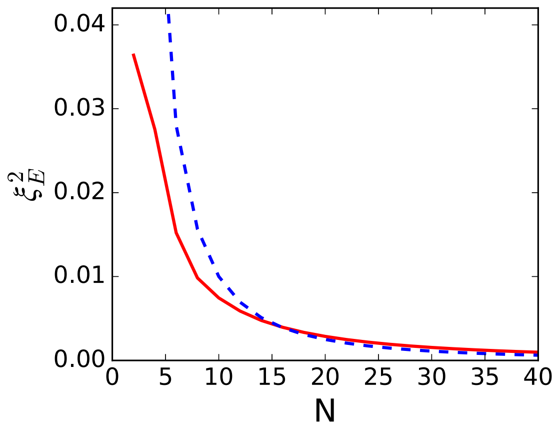

FIG. 3. The spin-squeezing parameter as a function of the total atom number N. The acceleration is taken as $a = 10$, and the other parameters are the same as in Fig. 1. The diagram is drawn by linking the different points that are calculated for every even number from $N = 2$ to $N = 40$.

entangled state due to the presence of switching function. This had been pointed out before [19,34], and its corresponding behavior in entanglement was also presented clearly [29]. Moreover, we calculate the change in ξ_E^2 with regard to the total number N of atoms, which is presented in Fig. 3. It shows that entanglement with regard to the initial entanglement or the change in entanglement increases when the number N increases for a given acceleration, no matter what energy gap is taken. This makes the observation easier experimentally for the influence of acceleration on the quantum state with a larger number. Although the trend of the change is the same both for the Unruh and the anti-Unruh effects, it appears that the change in entanglement with N atoms from the Unruh effect is more violent than that from the anti-Unruh effect. This can be understood by noting that the atoms feel hotter in the case that the Unruh effect works than that the anti-Unruh effect works as seen for $a = 10$ from Fig. 2.

IV. PHASE SENSITIVITY

Since the change in spin squeezing or entanglement influences the phase sensitivity of the measurement, in this section, we will study the influence of acceleration on the phase sensitivity and compare it with the present experiment. In order to do this, we first give the general expressions, and then compare the results for the TF state and its corresponding accelerated state (10).

Consider the Ramsey interferometer [47,48] with the initial input state ρ_i and the output state $\rho_o = U\rho_i U^\dagger$ where $U = \exp(-i\theta J_y)$ is the unitary operator for the evolution, and θ is the phase shift. According to the error propagation formula [31], the phase sensitivity $\Delta\theta$ can be calculated as

$$(\Delta\theta)^2 = \frac{\left(\Delta J_z^2\right)_o^2}{\left|d\left\langle J_z^2\right\rangle_o/d\theta\right|^2}, \tag{16}$$

where the subscript o denotes that the average is taken under the output state. Using $UJ_zU^\dagger = J_z\cos\theta - J_x\sin\theta$, it is easy to calculate $\langle J_z^2\rangle_o = \langle J_z^2\rangle_i\cos^2\theta + \langle J_x^2\rangle_i\sin^2\theta$ where the subscript i denotes the average is taken under the input state. It is seen that the phase shift can be deduced by measuring $\langle J_z^2\rangle_o$ in the experiment. The corresponding fluctuation of J_z^2 is $(\Delta J_z^2)_o^2 = \langle J_z^4\rangle_o - \langle J_z^2\rangle_o^2 = (\Delta J_z^2)_i^2\cos^4\theta + (\Delta J_x^2)_i^2\sin^4\theta + V_{xz}\sin^2\theta\,\cos^2\theta$, where $V_{xz} = \langle (J_xJ_z + J_zJ_x)^2\rangle_i + \langle J_z^2J_x^2 + J_x^2J_z^2\rangle_i - 2\langle J_z^2\rangle_i\langle J_x^2\rangle_i$. Thus, the phase sensitivity becomes

$$(\Delta\theta)^2 = \frac{\left(\Delta J_z^2\right)_i^2\cot^2\theta + \left(\Delta J_x^2\right)_i^2\tan^2\theta + V_{xz}}{4\left(\left\langle J_x^2\right\rangle_i - \left\langle J_z^2\right\rangle_i\right)^2}. \tag{17}$$

When the phase shift satisfies $\tan^2\theta_p = \frac{(\Delta J_z^2)_i}{(\Delta J_x^2)_i}$, the optimal phase sensitivity is obtained as

$$(\Delta\theta)_P^2 = \frac{2\left(\Delta J_z^2\right)_i\left(\Delta J_x^2\right)_i + V_{xz}}{4\left(\left\langle J_x^2\right\rangle_i - \left\langle J_z^2\right\rangle_i\right)^2}, \tag{18}$$

which is our main formula for investigating the change in phase sensitivity due to the influence of acceleration on the spin squeezing or entanglement of TF states.

For Dicke states $|j, m\rangle$, the optimal phase sensitivity occurs at $\theta = 0$ due to $(\Delta J_z^2)_i = 0$. It is calculated easily that $\langle J_x^2\rangle_i = \frac{1}{2}[j(j+1) - m^2]$, $\langle J_z^2\rangle_i = m^2$, $V_{xz} = \frac{1}{2}(4m^2 + 1)[j(j+1) - m^2] - 2m^2$. According to Eq. (18), the optimal phase sensitivity for Dicke states is obtained as

$$(\Delta\theta)_{PD}^2 = \frac{(4m^2+1)[j(j+1) - m^2] - 4m^2}{2[j(j+1) - 3m^2]^2}. \tag{19}$$

When $m = j$, the result is $\frac{1}{2j}$ which is the standard quantum limit and can be reached by the spin coherent state [49]. When $m = 0$, we have

$$(\Delta\theta)_{PD}^2 = \frac{1}{2j(j+1)}, \tag{20}$$

which gives the phase sensitivity with $\sqrt{\frac{2}{N(N+2)}}$ approaching the Heisenberg limit [50].

For the accelerated state in Eq. (10), a direct but tedious calculation within the approximation $m, m' \ll j$ and $m, m' \ll j$ gives

$$\begin{aligned}
\left\langle J_z^2\right\rangle &= \sum_{m=-N/2}^{N/2} m^2|B_m|^2,\\
\Delta J_z^2 &= \sqrt{\sum_{m=-N/2}^{N/2} m^4|B_m|^2 - \left(\sum_{m=-N/2}^{N/2} m^2|B_m|^2\right)^2},\\
\left\langle J_x^2\right\rangle &\simeq \frac{1}{2}j(j+1)B_0^2,\\
\Delta J_x^2 &\simeq \frac{B_0}{2\sqrt{2}}j(j+1),\\
V_{xz} &\simeq \frac{1}{2}j(j+1)\left[1 + \sum_{m=-N/2}^{N/2}|B_m|^2(4m^2+1)\right].
\end{aligned} \tag{21}$$

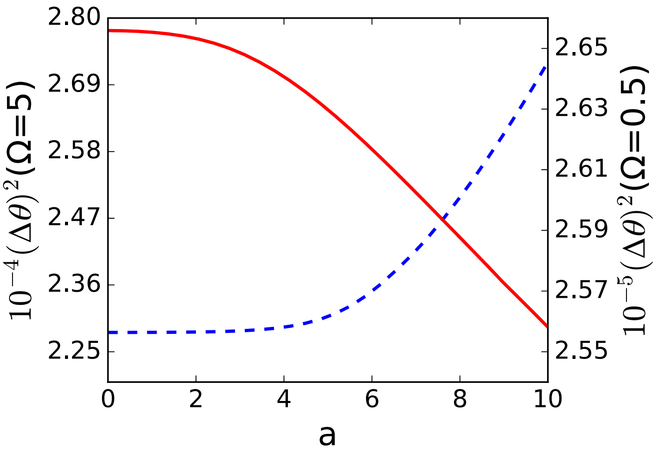

FIG. 4. The phase sensitivity as a function of the acceleration a. We make the total atom number $N = 100$, and the other parameters are the same as in Fig. 1.

Put these results into the Eq. (18), and the phase sensitivity is obtained as

$$(\Delta\theta)^2_{PA} \simeq \frac{1}{2j(j+1)} + \frac{\sqrt{2}B_0\Delta J_z^2}{2j(j+1)} + \frac{\sum_{m=-N/2}^{N/2}|B_m|^2(4m^2+1)}{2j(j+1)}, \tag{22}$$

where the terms related to the summation are ignored in the denominator and we have confirmed this approximation numerically. Figure 4 presents the behavior of the phase sensitivity with regard to the acceleration. It is seen that when the acceleration increases, the anti-Unruh effect can lead to the improvement of the phase sensitivity. This is expected since entanglement will increase (this corresponds to the decrease in the squeezing parameter) when the quantum state is accelerated in the case that the anti-Unruh effect works.

Now, we estimate the feasibility to test the effect through the corresponding experiments. In the recent experiment that generates the TF state [33], the temperature is decreased to the level of 10^{-9} K that is required to form Bose-Einstein condensates. Thus, in order to test the Unruh or anti-Unruh effect, the acceleration has to reach the level of 10^{10} m/s^2, at least, which is smaller than other experimental proposals [11–14,16] to test such an effect in which the acceleration is more than 10^{17} m/s^2. Together with the experimentally allowable parameters $\Omega \sim 2\pi$ Hz, $N \sim 10\,000$, it is obtained that

$$(\Delta\theta)^2_{PA} \sim 10^{-6}, \tag{23}$$

which is consistent with the present sensitivity in the experiment. This does not mean that the Unruh or anti-Unruh effect can be tested in the experiment instantly because the required acceleration is still too large for the practical implementation. However, such a suggestion is promising by reducing the acceleration through decreasing the experimental temperature or increasing the number of atoms under the present sensitivity. It is also feasible to reduce the acceleration by improving the sensitivity of measurement by some means other than changing the temperature or the number of atoms.

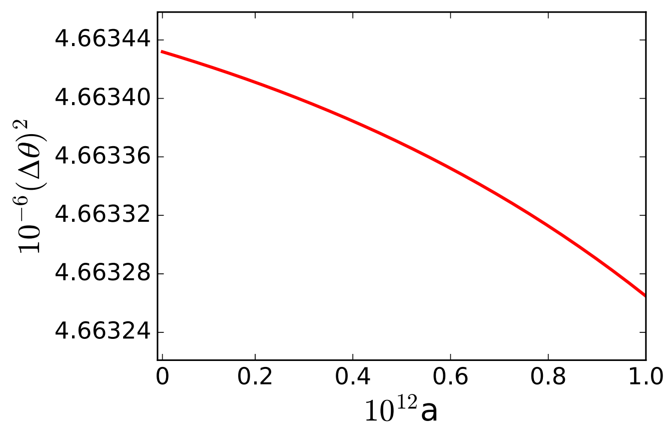

FIG. 5. The phase sensitivity as a function of the acceleration a. We take the parameters according to the experiment performed in Ref. [33] with $\lambda = 1$, $\sigma = 30$, $\Omega = 2\pi$, and $N = 10\,000$.

In particular, for the anti-Unruh effect, it is noted that the influence of acceleration on the TF state could be extracted, even though the temperature generated by the acceleration is lower than the temperature of background due to the fact that the thermal effect from the background cannot lead to the increase in entanglement similar to the analysis performed in Ref. [35]. This is attractive for the future experiment with higher sensitivity. Figure 5 presents the possibility to realize the case of anti-Unruh effect with the accelerating TF state.

V. CONCLUSION

In this paper, we revisit the influence of acceleration on quantum entanglement and the possible test for this effect through accelerating one class of experimentally feasible multibody entangled quantum states. We have calculated the change in the TF state due to acceleration and studied the change of entanglement among atoms using the spin-squeezing parameter as the measurement of entanglement. It is shown that entanglement among atoms not only decreases, but also increases with the acceleration for the certain range of the parameters. We have also compared the measurement of entanglement for two atoms using concurrence and the spin-squeezing parameter, respectively, and found that the same conclusion is obtained. In order to investigate the feasibility of testing the effect from acceleration, we have calculated the phase sensitivity of measurement using the distorted TF state due to acceleration. It is interesting to note that the case for the anti-Unruh effect can appear for such accelerated states, which is favorable for the possible future experiment since this effect is distinctive and different from that coupled to a thermal environment directly by inertial observers [35].

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