# Intermittent chaos in cavity optomechanics

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We theoretically demonstrate intermittent chaos induced by radiation-pressure nonlinearity in a general optomechanical system. In contrast to the periodic and chaotic dynamics, this optomechanical intermittent chaos is characterized by a nearly periodic motion interrupted irregularly by the chaotic motion, and exists in a transitional parameter regime between the normal chaos and periodic windows. The route to intermittent chaos is identified by bifurcation diagrams, and the optimal parameter regime for achieving this intermittent chaos is presented by a phase diagram. This work broadens the realm of optomechanical nonlinear dynamics, and is feasible with currently available optomechanical technology.

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## I. INTRODUCTION

Cavity optomechanics, exploring radiation-pressure interactions between electromagnetic and mechanical systems, has achieved rapid progress in recent years [1]. It is shown that optomechanical nonlinear interactions could induce many interesting phenomena in the field of quantum optics, such as optical bistability, self-induced oscillations, and synchronization phenomena [2–12]. More interestingly, chaotic motion in both the optical and mechanical modes can be induced by this optomechanical nonlinearity, when the strength of the driving laser is increased above a certain threshold, as demonstrated both theoretically [13–20] and experimentally [21–24]. Chaotic dynamics is an aperiodic long-term behavior in a deterministic system that exhibits a sensitive dependence on the initial conditions [25]. It is useful for implementing secret information processing and optical sensing [26–30], and hence the generation of chaos under optomechanics has recently attracted enormous attention.

Intermittent chaos, as an important nonlinear phenomenon different from normal chaos, is characterized by a nearly periodic motion interrupted irregularly by chaotic motion (irregular bursts). Intermittent chaos includes two states, nearly periodic motion and chaotic motion, which appear alternately with time evolution. The time between bursts is statistically distributed, much as a random variable, even though the system is completely deterministic. As the control parameter is varied, the bursts become more frequent until the system is fully chaotic [25]. Intermittent chaos is fundamentally different from periodic motion and chaotic motion, and can provide a new method to generate physical random numbers based on temporal randomness [31]. This behavior was first introduced by Pomeau and Manneville [32] and it has been investigated in different systems, including plasmas [33], optically pumped lasers [34], semiconductor superlattices [35], and gene circuit motifs [36]. A natural question is whether optomechanical nonlinearity could induced intermittent chaos. The study of intermittent chaos in cavity optomechanics remains unexplored, which may enrich the nonlinear dynamics associated with optomechanical interactions.

Here, we propose optomechanical intermittent chaos by investigating the nonlinear dynamics induced by radiation-pressure interactions. This optomechanical intermittent chaos includes nearly periodic motion and chaotic motion, which appear alternately during a single dynamical evolution, and features a ladder evolution for the perturbation of optical and mechanical trajectories. By numerically calculating the bifurcation diagrams, we identify the route to intermittent chaos, and clearly show the transitional parameter regime between normal chaos and periodic windows for obtaining intermittent chaos. We also complete the phase diagram of optomechanical nonlinear dynamics, which shows the regimes of periodic motion, intermittent chaos, and chaos. This intermittent chaos is identified in optomechanics, and it can be implemented experimentally in various optomechanical systems. Our work is fundamentally interesting for completing the optomechanical nonlinear dynamics, and also provides an alternative method to generate physical random numbers by utilizing the randomness of the time interval of nearly periodic motion.

This paper is organized as follows: In Sec. II, we describe the model and present the system Hamiltonian. Then we give the detailed derivation of the analytical expression for the dynamical equation and describe the intermittent chaos. In Sec. III, we follow the route to intermittent chaos dynamics in detail and give a phase diagram of the system as a function of light intensity and the detuning between the driving laser and cavity field. In Sec. IV, we discuss the experimental feasibility for achieving intermittent chaos, and in Sec. V, we finally summarize our results.

## II. MODE AND INTERMITTENT CHAOS

We consider a general optomechanical system, whose Hamiltonian in a frame rotating with $\omega_l$ is

$$H/\hbar = -\Delta\hat{a}^\dagger\hat{a} + \omega_b\hat{b}^\dagger\hat{b} - g\hat{a}^\dagger\hat{a}(\hat{b}^\dagger + \hat{b}) + \Omega(\hat{a}^\dagger + \hat{a}), \quad (1)$$

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where $\hat{a}$ ($\hat{a}^\dagger$) and $\hat{b}$ ($\hat{b}^\dagger$) are the annihilation (creation) operators of the cavity mode and the mechanical mode, respectively. The cavity mode $a$ (with the resonance frequency $\omega_c$) is driven by a driving laser with frequency $\omega_l$ and amplitude $\Omega$. Here, $\Omega$ is related to the driving power $P_l$ and decay rate $\kappa$ by $\Omega = \sqrt{\kappa P_l/(\hbar\omega_l)}$. The detuning between the driving laser and the cavity field $\Delta = \omega_l - \omega_c$. The third term in Eq. (1) describes the radiation-pressure interaction between the cavity and the mechanical oscillator with coupling strength $g$.

Here, we focus on the mean response of the system, and therefore any optical or mechanical operator $\hat{o}$ is reduced to an algebraic number. The photon-phonon quantum correlations can be safely ignored in the semiclassical approximation, which is valid in the concerned weak-coupling regime, i.e., $g \ll \kappa$ [21]. To obtain a general result under the semiclassical approximations in optomechanical systems, we define the dimensionless time $\tau$ by $\tau = \omega_b t$, and the dimensionless driven power parameters $P = 8\Omega^2 g^2/\omega_b^4$. Correspondingly, the mean values of optical and mechanical operators $\hat{a}$ and $\hat{b}$ are rescaled as $\alpha = [\omega_b/(2\Omega)]\langle\hat{a}\rangle$ and $\beta = (g/\omega_b)\langle\hat{b}\rangle$, respectively. In order to discuss the dynamic properties of chaos, we define the mean value of each operator as $o = o_r + io_i$ ($o_r$ and $o_i$ are real numbers). The Heisenberg-Langevin equations read

$$\dot{\alpha}_r = -\frac{\Delta}{\omega_b}\alpha_i - \frac{\kappa}{2\omega_b}\alpha_r + 2\beta_r\alpha_i, \tag{2a}$$

$$\dot{\alpha}_i = \frac{\Delta}{\omega_b}\alpha_r - 2\beta_r\alpha_r - \frac{\kappa}{2\omega_b}\alpha_i - \frac{1}{2}, \tag{2b}$$

$$\dot{\beta}_r = \beta_i - \frac{\gamma}{2\omega_b}\beta_r, \tag{2c}$$

$$\dot{\beta}_i = -\beta_r - \frac{\gamma}{2\omega_b}\beta_i - \frac{P}{2}\left(\alpha_r^2 + \alpha_i^2\right). \tag{2d}$$

Physically. the nonlinearity strength of the system plays an important role in generating intermittent chaos. Generally speaking, for the dissipative system, its equations of motion, when put in a standard autonomous form, must be nonlinear and the variables are greater than three [37]. It is shown from Eq. (2) and the system Hamiltonian that the optomechanical nonlinearity interaction leads to an interrelationship between the intracavity field intensity and the mechanical deformation, which satisfies the above condition and might induce the appearance of intermittent chaos.

Besides the evolution trajectory of the system corresponding to the specified initial condition, we also need to calculate the evolution of the perturbation $\varepsilon_o$ with Eq. (2), where $\varepsilon_o = (\varepsilon_{a_r}, \varepsilon_{a_i}, \varepsilon_{b_r}, \varepsilon_{b_i})$. The equations of the divergence of nearby trajectories read

$$\dot{\varepsilon}_{\alpha_r} = -\frac{\Delta}{\omega_b}\varepsilon_{\alpha_i} - \frac{\kappa}{2\omega_b}\varepsilon_{\alpha_r} + 2(\alpha_i\varepsilon_{\beta_r} + \varepsilon_{\alpha_i}\beta_r), \tag{3a}$$

$$\dot{\varepsilon}_{\alpha_i} = \frac{\Delta}{\omega_b}\varepsilon_{\alpha_r} - \frac{\kappa}{2\omega_b}\varepsilon_{\alpha_i} - 2(\alpha_r\varepsilon_{\beta_r} + \varepsilon_{\alpha_r}\beta_r), \tag{3b}$$

$$\dot{\varepsilon}_{\beta_r} = \varepsilon_{\beta_i} - \frac{\gamma}{2\omega_b}\varepsilon_{\beta_r}, \tag{3c}$$

$$\dot{\varepsilon}_{\beta_i} = -\varepsilon_{\beta_r} - \frac{\gamma}{2\omega_b}\varepsilon_{\beta_i} - \frac{P}{2}(2\alpha_r\varepsilon_{\alpha_r} + 2\alpha_i\varepsilon_{\alpha_i}), \tag{3d}$$

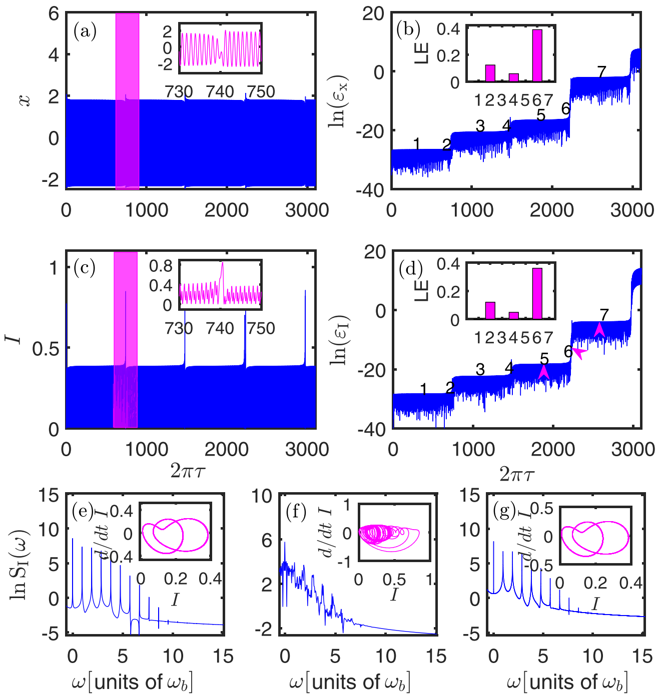


FIG. 1. Evolution of (a) mechanical deformation $x$, and (c) intracavity field intensity $I$ in a general optomechanical system. The evolution corresponding perturbations $\ln(\varepsilon_x), \ln(\varepsilon_I)$ are shown in (b) and (d). Power spectra $\ln S_I(\omega)$ corresponding to time intervals 5, 6, and 7 of (d), indicated by three arrows, are shown in (e)–(g). The insets of (b) and (d) present the LE of different time periods. The optical trajectories in phase space are presented in the insets of (e)–(g). The parameters are dimensionless by setting $\omega_b = 1$, and hence $g = 5 \times 10^{-5}$, $\kappa = 1$, $\gamma = 1.1 \times 10^{-3}$, $\Delta = -1.2$, and $P = 2.6224$.

which can offer the stochastic properties of the system by characterizing the divergence of nearby trajectories in phase space. This is important for demonstrating the appearance of intermittent chaos.

By numerically solving Eqs. (2) and (3), in Figs. 1(a) and 1(c), we present the time evolution of the mechanical deformation $x = 1/\sqrt{2}(\beta + \beta^*)$ and the intracavity field intensity $I = |\alpha|^2$. It clearly shows the intermittent chaotic dynamics both in the mechanical oscillator and the optical mode, i.e., nearly periodic motion interrupted by occasional irregular bursts. This property is demonstrated more clearly in the ladder evolution of the perturbations $\ln(\varepsilon_I)$ and $\ln(\varepsilon_x)$ in Figs. 1(b) and 1(d). The flat part indicates that the trajectories of two systems with infinitesimally different initial conditions will not diverge, and the dynamic evolution of $I$ is periodic. The oblique part indicates that neighboring trajectories separate exponentially fast, and the dynamic evolution of $I$ is chaotic. The power spectra of $I$ corresponding to the flat and oblique parts are shown in Figs. 1(e)–1(g). During a single trajectory, the discrete and continuous spectra appear alternately, which also demonstrates that the intermittent chaotic dynamics includes two states, i.e., periodic and chaotic motions. From the perspective of phase space trajectory [see the insets of Figs. 1(e)–1(g)], the system first evolves along the limit cycles, and then becomes very complicated due to the strong

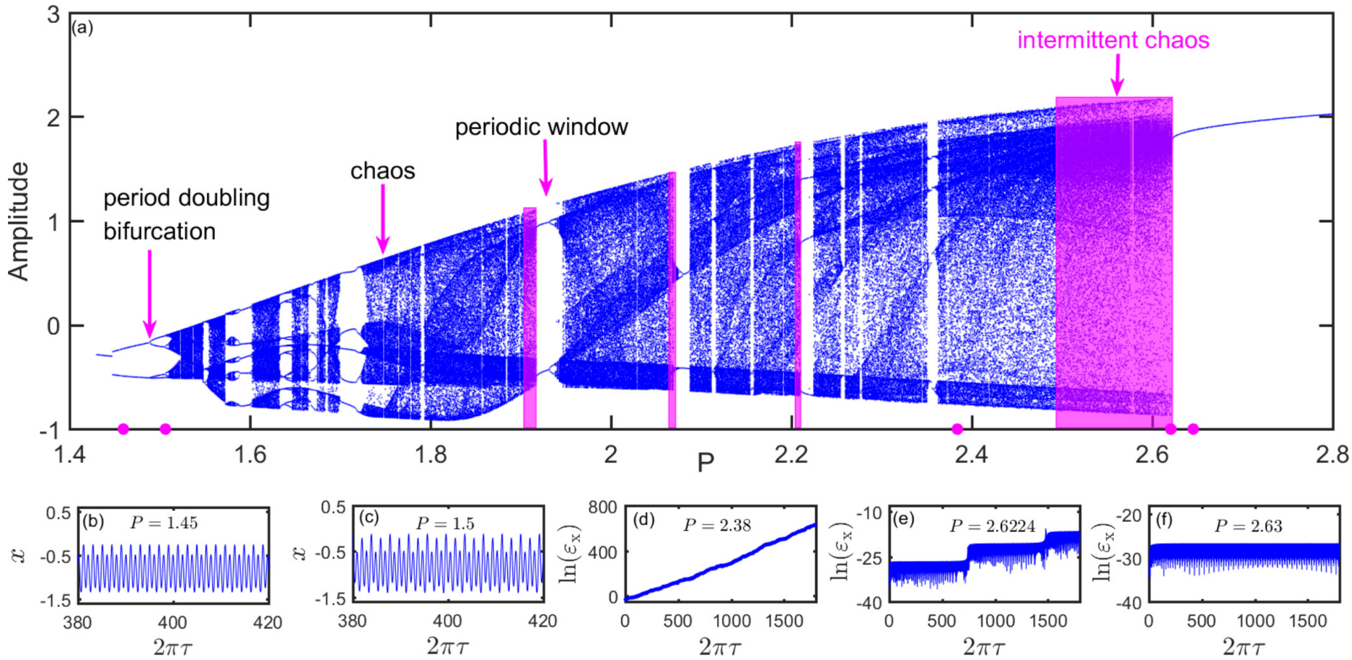


FIG. 2. Bifurcation diagram of the amplitude of the mechanical limit cycle (top), and the corresponding evolutions of the mechanical deformation $x$ and $\ln(\varepsilon_x)$ (bottom) at $\Delta = -1.2$. (b)–(f) correspond to the values of $P$ indicated by the dots in (a), and the parameters are the same as in Fig. 1.

attractor, where nearby points in phase space evolve into completely different states, and lastly the trajectories feature a set of limit cycles again. The above process continues along with time evolution, which is more evidence of intermittent chaos. In the inset of Figs. 1(b) and 1(d), we present the Lyapunov exponent (LE) of different time intervals, which is defined by the logarithmic slope of the perturbation $\varepsilon_I$ versus time $t$. It quantifies the sensitivity of the cavity system to the initial conditions, and features alternately appearing nonzero values for the intermittent chaotic dynamics.

## III. ROUTE TO INTERMITTENT CHAOS

To show the route to intermittent chaos in optomechanics, in Fig. 2(a) we present the bifurcation diagram of the amplitude of mechanical oscillation. As increasing the dimensionless driven power $P$, the system dynamics experiences regular, period-doubling bifurcation, chaos to intermittent chaos (the shaded areas). Specifically, for a weak driving power, the system dynamics experiences the sequence of period-doubling bifurcations, which also can be seen by the evolution of mechanical displacement $x$ in Figs. 2(b) and 2(c). For example, when $P = 1.5$, a period-doubling bifurcation taken place, and a new limit cycle with twice the period of the original simple periodic cycle appears. Increasing the driving field $P$ further, chaotic motion emerges, such as the increasing $\ln(\varepsilon_x)$ that is obtained at $P = 2.4$ shown in Fig. 2(d). Interestingly, intermittent chaotic dynamics is obtained when one increases driven power to the regime before periodic motion appears again. As shown in Fig. 2(e), the ladder evolution of $\ln(\varepsilon_x)$ clearly shows the appearance of intermittent chaos. It also shows that the periodic dynamics appears again when the driven power $P$ is enhanced a little, corresponding to the flat evolution of $\ln(\varepsilon_x)$ in Fig. 2(f). Now, a general result is obtained, that is, that optomechanical intermittent chaos exists only in a transitional regime of driving field power $P$ between the chaos and periodic windows. Note that the dimensionless parameter range of intermittent chaos seems narrow, but, based on a typical whispering-gallery microcavity system [38–41], it corresponds to a relatively broad range ($\sim$10 $\mu$W) even for the narrowest shading area of Fig. 2(a).

Physically, the above intermittent chaos is induced by optomechanical nonlinearity, which is decided by the driving detuning $\Delta$ and power $P$ in a general optomechanical system. To fully show the optimal parameter regime of intermittent chaos, we present the phase diagram versus the detuning $\Delta$ and the driving strength $P$ in Fig. 3, which completely offers the rich dynamics phenomena induced by optomechanical nonlinearity. We distinguish the periodic and multiple periodic trajectories through the amplitude of the mechanical oscillation, and distinguish chaotic and regular trajectories through the LE. The LE separates regular motion with a negative value from chaotic motion with a positive value [16]. The intermittent and chaotic trajectories are distinguished by the evolution of $\ln(\varepsilon_x)$, i.e., the oblique line and ladder form, corresponding to chaos and intermittent chaos, respectively. The various zones are denoted with different colors. The black zone confirms that, in a general optomechanical system, the intermittent chaotic dynamics appears in the transitional parameter regime between chaos and the periodic window. For a fixed driving power, such as $P = 2.5$, the system goes through self-induced oscillations (periodic), intermittent chaos to chaos, along with increasing the modulus of detuning $|\Delta|$ in the blue-detuning regime. For a fixed detuning, such as $\Delta = -1.2$, the system experiences regular, period-doubling bifurcation, chaos, intermittent chaos, to regular as increasing $P$, which is consistent with the above results from the bifurcation diagram.

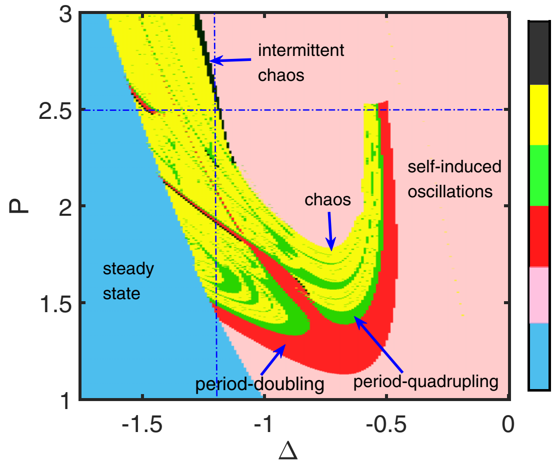


FIG. 3. Phase diagram in terms of the detuning Δ and the driving strength $P$, indicating the intermittent chaos, chaos, period quadrupling, period doubling, self-induced oscillations, and steady-state regimes in a general optomechanical system. The system parameters are the same as in Fig. 1.

## IV. EXPERIMENTAL IMPLEMENTATIONS

Regarding experimental implementations, our work could be realized in a variety of optomechanical platforms. As an example, here we choose the whispering-gallery microcavity system, which has been widely used in a variety of devices on account of its relatively high-quality factor $Q$ and small mode volume $V$ [38–41]. Based on the above theoretical results, the feasible experimental parameters for achieving intermittent chaos are $P = 10.9$ mW, $\omega_b/2\pi = 51.8$ MHz, $m = 20$ ng, $G/2\pi = 28.77$ GHz/nm, $\kappa/2\pi = 51.8$ MHz, $\gamma/2\pi = 56.98$ kHz, and $\Delta = -1.2\omega_b$, respectively. Corresponding to Fig. 3, the frequency range of intermittent chaos is about 0.5 MHz when the driving power $P = 10.9$ mW, which is feasible with current experimental technology.

## V. CONCLUSIONS

In conclusion, we have shown that intermittent chaotic dynamics, characterized by nearly periodic motion interrupted irregularly by chaotic motion, could be realized in a general optomechanical system. By numerically calculating the bifurcation diagram and phase diagram, we identified the route to intermittent chaos and the corresponding optimal parameter regime, i.e., optomechanical intermittent chaos exists in a transitional parameter regime between normal chaos and periodic windows. Our work is general and can be implemented in a variety of optomechanical systems. It enriches the nonlinear dynamics induced by optomechanical interactions, and provides a promising route for generating on-chip intermittent chaos.

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