Erratum: Flexible scheme for the implementation of nonadiabatic geometric quantum computation [Phys. Rev. A 101, 032322 (2020)]

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In Appendix D, there exist errors in the derivations of Eqs. (D9) and (D10) for the estimation of the rank of matrix $\mathcal{M}(t)$. Here, we would like to show the result that the matrix $\mathcal{M}(t)$ is still not full rank but the estimation of rank of $\mathcal{M}(t)$ is different from that given by Eq. (D10). The result only corrects the estimation of the rank of matrix $\mathcal{M}(t)$ given in Eq. (D10) and does not influence other results of the original paper.

In fact, for the dynamic invariant,

$$I(t) = \sum_{\wp<\wp'} \tilde{\xi}_{\wp\wp'}(t)\tilde{G}_{\wp\wp'}, \tag{1}$$

with $\tilde{G}_{\wp\wp'} \in \mathrm{so}(N)$, noting that $I(t)$ is a Hermitian and antisymmetric matrix, the nonzero eigenvalues of $I(t)$ are always pairs of positive and negative real numbers. Therefore, we can rewrite $I(t)$ as

$$I(t) = \sum_n \epsilon_n[|\phi_n^+(t)\rangle\langle\phi_n^+(t)| - |\phi_n^-(t)\rangle\langle\phi_n^-(t)|]. \tag{2}$$

Here, $|\phi_n^\pm(t)\rangle$ is the eigenvector of $I(t)$ with nonzero eigenvalue $\pm\epsilon_n$, satisfying $[|\phi_n^\pm(t)\rangle]^{\mathrm{T}} = \langle\phi_n^\mp(t)|$ because of $I^{\mathrm{T}}(t) = -I(t)$. We assume that the rank of $I(t)$ is 2μ with μ as a positive integer and the dimension of the kernel $\mathcal{K} = \ker[I(t)]$ of $I(t)$ is $N - 2\mu$. When $\mu \geqslant 2$, we can easily find a Hermitian and antisymmetric matrix as

$$X(t) = \epsilon_{n'}[|\phi_n^+(t)\rangle\langle\phi_n^+(t)| - |\phi_n^-(t)\rangle\langle\phi_n^-(t)|] - \epsilon_n[|\phi_{n'}^+(t)\rangle\langle\phi_{n'}^+(t)| - |\phi_{n'}^-(t)\rangle\langle\phi_{n'}^-(t)|], \tag{3}$$

with $n \neq n'$, which satisfies $I(t)X(t) = 0$. Since any Hermitian and antisymmetric matrix can be spanned by the generators of $\mathrm{so}(N)$, we have $X(t) \in \mathrm{so}(N)$, $\mathcal{M}(t)X(t) = -i[I(t), X(t)] = 0$. For the case of $\mu = 1$, we have $N - 2\mu \geqslant 2$ when $N \geqslant 4$, and we can derive the projection operator $P_{\mathcal{K}}$ from subspace $\mathcal{K}$ as

$$P_{\mathcal{K}} = \mathbb{1}_N - P_I, \quad P_I = \sum_n [|\phi_n^+(t)\rangle\langle\phi_n^+(t)| + |\phi_n^-(t)\rangle\langle\phi_n^-(t)|], \tag{4}$$

which is a real symmetric matrix since $P_I^{\mathrm{T}} = P_I = P_I^\dagger$. Therefore, $P_{\mathcal{K}}$ has eigenvectors $\{|\phi_0^n(t)\rangle\}$ with all real components. We select two eigenvectors $\{|\phi_0^1(t)\rangle, |\phi_0^2(t)\rangle\}$ and construct a matrix $X(t)$ as

$$X(t) = |\phi_0^+(t)\rangle\langle\phi_0^+(t)| - |\phi_0^-(t)\rangle\langle\phi_0^-(t)|, \tag{5}$$

with $|\phi_0^\pm(t)\rangle = [|\phi_0^1(t)\rangle \pm i|\phi_0^2(t)\rangle]/\sqrt{2}$. $X(t)$ is a Hermitian and antisymmetric matrix, thus, $X(t) \in \mathrm{so}(N)$. Moreover, we have $I(t)X(t) = 0$ due to $X(t) \in \mathcal{K}$. Consequently, $\mathcal{M}(t)X(t) = -i[I(t), X(t)] = 0$ can be also obtained. Therefore, when $N \geqslant 4$, we can always find two nonzero operators $I(t)$ and $X(t)$ satisfying $\mathcal{M}(t)I(t) = \mathcal{M}(t)X(t) = 0$, and the rank of $\mathcal{M}(t)$ should be less or equal to $[N(N-1)]/2 - 2$. Noting that $\mathcal{M}(t)$ is also an antisymmetric matrix, its rank should be even. When $[N(N-1)]/2 - 2$ is odd, we have $\mathrm{rank}[\mathcal{M}(t)] \leqslant [N(N-1)]/2 - 3$. For example, for $I(t) \in \mathrm{so}(4)$, $\mathrm{rank}[\mathcal{M}(t)] \leqslant 4$, for $I(t) \in \mathrm{so}(5)$, $\mathrm{rank}[\mathcal{M}(t)] \leqslant 8$, and for $I(t) \in \mathrm{so}(6)$, $\mathrm{rank}[\mathcal{M}(t)] \leqslant 12$. The results given above all accord with the numerical results. For $N = 3$, we have $\mu = 1$ and $N - 2\mu = 1$; only $\mathcal{M}(t)I(t) = 0$ can be found, accordingly, $\mathrm{rank}[\mathcal{M}(t)] \leqslant (3\times 2)/2 - 1 = 2$.

