Quantum optics and moving dissipative media: A phenomenological approach

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In this paper, we present a phenomenological quantization of the electromagnetic field in the presence of a moving absorptive and dispersive magnetodielectric slab (MDS) with uniform velocity in the direction parallel to its interface. As our main result, we use this quantization scheme to derive the quantum input-output relations for the case in which quantum states propagate perpendicularly to the moving MDS. We thoroughly investigate the impact of the motion of the moving MDS on quantum properties of the incident states. To illustrate this, we compute the quadrature squeezing and the Mandel parameter for the transmitted state when the incident states from left and right sides are, respectively, the coherent and the quantum vacuum states. We find that the quantum features of the incident state are degraded through transmission in the moving MDS in the low- and moderate-velocity ranges.

I. INTRODUCTION

The electrodynamics of moving media is a fundamental issue dating back to 1908, when Minkowski presented a covariant theory of electrodynamics in moving media by using Einstein's special relativity theory [1–3]. In particular, he introduced the relativistic constitutive equations to explain the electromagnetic phenomena related to the moving media. Since then, the electrodynamics of moving media has been the subject of numerous theoretical and experimental investigations and has been applied in a variety of physical fields such as the optics of moving media [3–8], radiation of fast charged particles in media [9], and astrophysics [10].

Among these studies, the problem of the scattering of electromagnetic waves from moving media is a problem of both fundamental interest and practical importance which has long received much attention. At first, Pauli and Sommerfeld studied the frequency shift of a reflected plane wave by a moving mirror [11,12]. Later on, many authors analyzed in detail this problem for the case of a moving half-space dielectric and moving dielectric slab, and then generalized these calculations to an arbitrary direction of motion [10,13–20].

There has been a revival of interest in the scattering of electromagnetic waves by moving media over the past decade, and it has now become a rather topical area of research. Most of these works deal with fascinating and fundamental issues in the classical theory of electrodynamics such as the analogy between light propagating in moving media and in curved space-times [21,22], the optical analog of the Aharonov-Bohm effect [23], the generation of negative

refraction [24–27], the optically induced magnetoelectric effect [28,29], the occurrence of linear birefringence [30,31], and generation of coherent light by a moving medium [32]. On the other hand, there are some problems such as the emission of light due to fast changes of the geometry [33,34], uniform motion of media [35–40], and rotating objects [41–44] which can only be illuminated in the framework of a full quantum approach.

It was recently shown that quantum optics can pose a less known challenge to effective-medium theories when a stream of photons rather than an electromagnetic wave is used to probe metamaterials [45,46]. It is surprising is that quantum emitters embedded near metamaterials, such as invisibility clocking devices and optical black holes, can provide a challenge to the classical operation of these devices in the few-photon regime [47–49]. One may ask whether such challenges can be applied to media in uniform motion which are closely related to negative refraction and similarly associated with certain metamaterials. In this paper, we will investigate this question.

The scattering of quantum states of light from a dispersive and absorptive medium at rest has been discussed in Refs. [45,46,50–57]. However, to the best of our knowledge, this problem has not been studied before for the case of a moving medium whose velocity may have any value up to the speed of light in free space. An incident light with a nonclassical nature that propagates through a moving absorbing medium will be affected by quantum noise associated with the loss. Definitely, the motion of the medium affects the quantum noise and, in turn, quantum statistical properties of the output light. As our main result, we derive how the flux of noise photons emitted by a moving magnetodielectric slab (MDS) depends on the velocity and the material property of the slab. We explore how quantum states of light can be used to analyze

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properties of the moving media, and investigate the scattering of quantum states of light from moving media in detail.

In order to analyze these problems, a quantum treatment for the propagation of electromagnetic fields in moving media seems to be indispensable. To this end, we first need to quantize the electromagnetic field in the presence of an absorptive and dispersive moving medium. This is a more complicated task than in the case of stationary media. Nevertheless, two phenomenological and canonical quantization schemes have been developed in unbounded moving media [58–63].

The use of a sophisticated canonical approach, although strictly rigorous, may tend to obscure the simple physical concepts. Instead, we follow the phenomenological quantization scheme presented in [59,60] and extend this method to the more general and practical case in which a MDS surrounded by free space is in motion. This makes it possible to investigate the scattering of an incident nonclassical light field by a moving polarizable and magnetizable slab in the laboratory, and we would expect that the optical properties of light propagating through a moving slab are modified differently from light propagating through the same slab at rest. In the quantization process, we found that the motion leads to effective velocity-dependent electric permittivity and magnetic permeability which exhibit an anisotropic character for an observer in the laboratory frame, even if the medium is isotropic in its rest frame. These effective parameters enable us to reduce the phenomenological scheme of the electromagnetic field quantization in a moving slab to that of a stationary slab in a straightforward manner.

The paper is organized as follows: In Sec. II, we present the phenomenological quantization of the electromagnetic field in the presence of a moving MDS. We derive the optical input-output relations for quantized electromagnetic waves normally incident upon a MDS moving with uniform velocity in the direction parallel to its interface, and further analyze the reflection and transmission coefficients of the moving MDS. Given that a coherent state (CS) of light is of considerable practical interest in low-noise optical communication systems and in general a dielectric object is not at rest, we study the effects of transmission of the CS through a moving slab on some statistical properties, such as quadrature squeezing and photon counting statistics, in Sec. III. A summary and interesting conclusions are deduced in Sec. IV. We provide further details of our calculations for the square root of the imaginary part of effective tensors, boundary conditions, the elements of transformation, and absorbing matrices in Appendices A, B, C and D, respectively.

II. QUANTIZATION OF THE ELECTROMAGNETIC FIELD IN THE PRESENCE OF A MOVING MDS

A. Basic equations

Consider a homogeneous isotropic MDS moving uniformly at velocity **v** with respect to the laboratory frame. In the rest frame of the medium, the electromagnetic response of the slab is characterized by the electric permittivity ε and magnetic permeability μ that satisfy the Kramers-Kronig relations. We assume that the moving MDS is surrounded by vacuum, i.e., the permittivity and permeability in the regions

outside the moving MDS are unity. The propagation of the classical electromagnetic waves in this slab can be described by the macroscopic Maxwell equations together with suitable constitutive relations. From the point of view of a reference system comoving with the medium, the macroscopic Maxwell equations have the same mathematical form as that of an observer in the laboratory frame [3]. However, due to the uniform motion of the medium, the relativistic relations between the macroscopic electromagnetic fields **E**, **D**, **B**, and **H** in the laboratory frame are given by the well known Minkowski constitutive relations. Taking the Fourier transform of these fields with respect to time, we can write the Minkowski constitutive relations for the positive frequency part of the fields as [64]

$$\mathbf{D}^+(\mathbf{r}, \omega) = \varepsilon_0 \varepsilon \bar{\bar{\boldsymbol{\alpha}}} \cdot \mathbf{E}^+(\mathbf{r}, \omega) + \frac{1}{c}\bar{\bar{\mathbf{m}}} \cdot \mathbf{H}^+(\mathbf{r}, \omega), \quad (1a)$$

$$\mathbf{B}^+(\mathbf{r}, \omega) = -\frac{1}{c}\bar{\bar{\mathbf{m}}} \cdot \mathbf{E}^+(\mathbf{r}, \omega) + \mu_0 \mu \bar{\bar{\boldsymbol{\alpha}}} \cdot \mathbf{H}^+(\mathbf{r}, \omega), \quad (1b)$$

where boldface symbols and boldface symbols along with double overlines are used to identify vector and second-rank tensor quantities, respectively. Here, $\bar{\bar{\mathbf{I}}}$ is the unit tensor, $\bar{\bar{\mathbf{m}}}$ is an antisymmetric tensor defined as $\mathbf{m} \times \bar{\bar{\mathbf{I}}}$, and the symmetric tensor $\bar{\bar{\boldsymbol{\alpha}}}$ and the vector **m** are defined as

$$\bar{\bar{\boldsymbol{\alpha}}} = \bar{\bar{\mathbf{I}}}\alpha + (1 - \alpha)\hat{v}\hat{v}, \quad (2)$$

$$\mathbf{m} = m\hat{v}, \quad (3)$$

where $m = \beta(n^2 - 1)/(1 - n^2\beta^2)$, $\beta = v/c$, and $\alpha = (1 - \beta^2)/(1 - n^2\beta^2)$ with $n = \sqrt{\varepsilon\mu}$ being the refractive index of the medium in the rest frame of the medium, $\hat{v}$ the unit vector of the velocity of the medium, and c the velocity of light in free space.

As mentioned in Introduction, the phenomenological quantization of the electromagnetic fields in unbounded moving media was developed previously [59,60]. Here, we briefly summarized the main results needed for an understanding of the present paper, and then extend this approach to the practical case of a moving MDS.

Unfortunately, the above form of the constitutive relations are not convenient for the phenomenological approach. Unlike the electromagnetic fields, the electric permittivity and the magnetic permeability are parameters observed in the rest frame of the medium. Using the Maxwell equations in reciprocal space, these constitutive relations can be cast into a more convenient form as [59,60]

$$\mathbf{D}^+(\mathbf{k}, \omega) = \varepsilon_0 \bar{\bar{\boldsymbol{\varepsilon}}}_{\text{eff}} \cdot \mathbf{E}^+(\mathbf{k}, \omega) + \mathbf{P}_N^+(\mathbf{k}, \omega), \quad (4a)$$

$$\mathbf{H}^+(\mathbf{k}, \omega) = \mu_0^{-1} \bar{\bar{\boldsymbol{\mu}}}_{\text{eff}}^{-1} \cdot \mathbf{B}^+(\mathbf{k}, \omega) - \mathbf{M}_N^+(\mathbf{k}, \omega), \quad (4b)$$

where $\boldsymbol{\varepsilon}_{\text{eff}}$ and $\boldsymbol{\mu}_{\text{eff}}$ are the effective electric permittivity and the magnetic permeability tensors which show the material parameters of the moving medium as seen by the observer in the laboratory frame. The explicit forms of these nonsymmetric tensors are given by [59,60]

$$\bar{\bar{\boldsymbol{\varepsilon}}}_{\text{eff}} = \varepsilon\left(\bar{\bar{\boldsymbol{\alpha}}} + \frac{(\bar{\bar{\boldsymbol{\alpha}}} \cdot \mathbf{P})\mathbf{m} - (\mathbf{m} \cdot \mathbf{P})\bar{\bar{\boldsymbol{\alpha}}}}{\alpha^2 n^2}\right), \quad (5a)$$

$$\bar{\bar{\mu}}_{\text{eff}} = \mu \left(\bar{\bar{\alpha}} + \frac{(\bar{\bar{\alpha}} \cdot \mathbf{P}) \mathbf{m} - (\mathbf{m} \cdot \mathbf{P}) \bar{\bar{\alpha}}}{\alpha^2 n^2} \right), \tag{5b}$$

where $\mathbf{P} = q^{-1} \mathbf{k} + \mathbf{m}$ in which $q = \omega/c$ and $\mathbf{k}$ is the wave vector. From above equations, one observes that the field $\mathbf{D}$ ($\mathbf{B}$) is not in the direction of $\mathbf{E}$ ($\mathbf{H}$); hence a moving medium is anisotropic even thought it is isotropic in the rest frame of the the medium. This is the case because the isotropy and anisotropy properties are not invariant under the Lorentz transformation [65]. Furthermore, it is seen that the noise polarization $\mathbf{P}_N$ and the noise magnetization $\mathbf{M}_N$ are added to the Minkowski constitutive relations (1) due to material absorption associated with the electric and magnetic losses. These noise operators are unavoidably required in order to preserve the well-known canonical field commutation relations, and therefore lead to the correct Heisenberg equations of motion for the fields [59,66,67]. In this manner, the quantization of electromagnetic fields in presence of a moving slab can be carried out similarly to the method accomplished for a stationary anisotropic slab in Refs. [68,69].

By making use of the Weyl gauge, which has the advantage that the scalar potential is zero, and by combining the Maxwell equation and Minkowski constitutive relations, the wave equation for the the positive frequency part of the vector potential operator is obtained as

$$(\bar{\bar{\mathbf{k}}} \cdot \bar{\bar{\mu}}_{\text{eff}}^{-1} \cdot \bar{\bar{\mathbf{k}}} + q^2 \bar{\bar{\varepsilon}}_{\text{eff}}) \cdot \hat{\mathbf{A}}^+(\mathbf{k}, \omega) = -\mu_0 \hat{\mathbf{J}}_N^+(\mathbf{k}, \omega), \tag{6}$$

where $\bar{\bar{\mathbf{k}}} = \mathbf{k} \times \bar{\bar{\mathbf{I}}}$ is an antisymmetric tensor and $\hat{\mathbf{J}}_N^+(\mathbf{k}, \omega) = -i\omega \mathbf{P}_N^+(\mathbf{k}, \omega) + i\bar{\bar{\mathbf{k}}} \cdot \mathbf{M}_N^+(\mathbf{k}, \omega)$ is the Fourier-transformed current operator. Neglect of this noise operator leads to a spatially damped vector potential, and therefore the canonical field commutation relations are not preserved. This noise operator operator can be described in term of the fundamental variables of the system as follows [59]:

$$\hat{\mathbf{J}}_N^+(\mathbf{k}, \omega) = \omega \sqrt{\frac{2\hbar \varepsilon_0}{S} \bar{\bar{\varepsilon}}_{\text{eff}}^I} \cdot \hat{\mathbf{f}}_e(\mathbf{k}, \omega)$$
$$+ i\bar{\bar{\mathbf{k}}} \cdot \sqrt{\frac{-2\hbar}{S\mu_0} \bar{\bar{\mu}}_{\text{eff}}^{-1 I}} \cdot \hat{\mathbf{f}}_m(\mathbf{k}, \omega), \tag{7}$$

where the superscript I stands for the imaginary part of a function, S is the transverse normalized area, and $\hat{\mathbf{f}}_e(\mathbf{k}, \omega)$ and $\hat{\mathbf{f}}_m(\mathbf{k}, \omega)$ are the Fourier transforms of the bosonic field operators $\hat{\mathbf{f}}_e(\mathbf{r}, \omega)$ and $\hat{\mathbf{f}}_m(\mathbf{r}, \omega)$ for the electric and magnetic excitations of the system. The components of these bosonic operators satisfy the commutation relations:

$$[\hat{f}_{\lambda,i}(\mathbf{k}, \omega), \hat{f}_{\lambda',j}^\dagger(\mathbf{k}', \omega')] = \delta_{\lambda\lambda'} \delta_{ij} \delta(\mathbf{k} - \mathbf{k}') \delta(\omega - \omega'), \tag{8a}$$

$$[\hat{f}_{\lambda,i}(\mathbf{k}, \omega), \hat{f}_{\lambda',j}^\dagger(\mathbf{k}', \omega')] = 0, \tag{8b}$$

where $\lambda, \lambda' = e, m$ and $i, j = x, y, z$. Moreover, in writing Eq. (7), the square roots of the imaginary part of the effective optical tensors are defined as [59]

$$\sqrt{\bar{\bar{\varepsilon}}_{\text{eff}}^I} \cdot \sqrt{\bar{\bar{\varepsilon}}_{\text{eff}}^I}^\dagger = \frac{i}{2} (\bar{\bar{\varepsilon}}_{\text{eff}} - \bar{\bar{\varepsilon}}_{\text{eff}}^\dagger)^*, \tag{9a}$$

$$\sqrt{\bar{\bar{\mu}}_{\text{eff}}^{-1 I}} \cdot \sqrt{\bar{\bar{\mu}}_{\text{eff}}^{-1 I}}^\dagger = \frac{i}{2} (\bar{\bar{\mu}}_{\text{eff}}^{-1} - \bar{\bar{\mu}}_{\text{eff}}^{-1 \dagger})^*. \tag{9b}$$
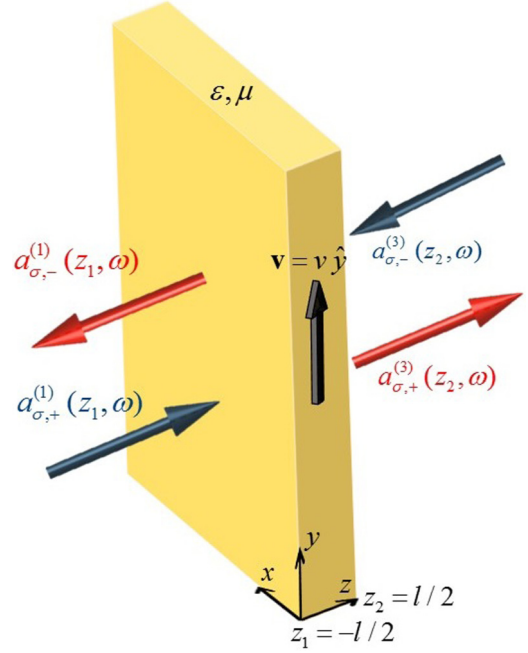

FIG. 1. The geometry representation of the system for the fields impinging leftwards and rightwards on the MDS which is moving perpendicular to the incident field and parallel to its outer surface. The arrows together with the bosonic operators show the input and the output modes defined in the input-output relations (26).

With the help of Eq. (6), and by using the inverse Fourier transforms, the explicit form of the vector potential operator in the real space can be obtained as

$$\hat{\mathbf{A}}(\mathbf{r}, \omega) = -\frac{\mu_0}{4\pi^2} \int_0^\infty d\omega \int d^3\mathbf{k}$$
$$\times (\bar{\bar{\mathbf{G}}}(\mathbf{k}, \omega) \cdot \mathbf{J}_N(\mathbf{k}, \omega) e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} + \text{H.c.}), \tag{10}$$

where $\bar{\bar{\mathbf{G}}}$ is the electromagnetic Green's tensor (GT) of the system as uniquely defined by the Helmholtz equation, $(\bar{\bar{\mathbf{k}}} \cdot \bar{\bar{\mu}}_{\text{eff}}^{-1} \cdot \bar{\bar{\mathbf{k}}} + q^2 \bar{\bar{\varepsilon}}_{\text{eff}}) \cdot \bar{\bar{\mathbf{G}}}(\mathbf{k}, \omega) = \bar{\bar{\mathbf{I}}}$, subject to the appropriate boundary condition. It is known that the solution of this Helmholtz equation for an infinite moving medium is given by [59]

$$\bar{\bar{\mathbf{G}}}(\mathbf{k}, \omega) = \frac{1}{q^2 \varepsilon} \frac{(\mathbf{k} + q\mathbf{m})(\mathbf{k} + q\mathbf{m}) - n^2 q^2 \alpha^2 \bar{\bar{\alpha}}^{-1}}{(\mathbf{k} + q\mathbf{m}) \cdot \bar{\bar{\alpha}} \cdot (\mathbf{k} + q\mathbf{m}) - n^2 q^2 \alpha^2}. \tag{11}$$

Let us consider the z axis of the laboratory Cartesian frame that is normal to the interface of the slab moving with a constant velocity in the direction parallel to its interface. Interfaces are in the xy plane, and the coordinate origin is taken at the center of the slab. Without loss of generality, we assume that the slab is moving with a constant speed along the y axis, i.e., $\mathbf{v} = v\hat{y}$, as illustrated schematically in Fig. 1.

By recalling Eq. (5), the effective tensors $\bar{\bar{\boldsymbol{\varepsilon}}}_{\text{eff}}$ and $\bar{\bar{\boldsymbol{\mu}}}_{\text{eff}}$ of the moving MDS are simplified as

$$\bar{\bar{\boldsymbol{\varepsilon}}}_{\text{eff}} = \varepsilon \begin{pmatrix} \frac{\alpha^2 n^2 - m^2}{\alpha n^2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{ck_z m}{\omega \alpha n^2} & \frac{\alpha^2 n^2 - m^2}{\alpha n^2} \end{pmatrix}, \quad (12\text{a})$$

$$\bar{\bar{\boldsymbol{\mu}}}_{\text{eff}} = \mu \begin{pmatrix} \frac{\alpha^2 n^2 - m^2}{\alpha n^2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{ck_z m}{\omega \alpha n^2} & \frac{\alpha^2 n^2 - m^2}{\alpha n^2} \end{pmatrix}. \quad (12\text{b})$$

Here, for the sake of convenience, the effective tensors are written in matrix form. Accordingly, after lengthy calculations, the square roots of the imaginary part of the effective tensors $\sqrt{\bar{\bar{\boldsymbol{\varepsilon}}}_{\text{eff}}^I(\omega)}$ and $\sqrt{\bar{\bar{\boldsymbol{\mu}}}_{\text{eff}}^{-1\,I}(\omega)}$ can be obtained as

$$\sqrt{\bar{\bar{\boldsymbol{\varepsilon}}}_{\text{eff}}^I(\omega)} = \begin{pmatrix} e_{11} & 0 & 0 \\ 0 & e_{22} & e_{23} \\ 0 & e_{32} & e_{33} \end{pmatrix}, \quad (13\text{a})$$

$$\sqrt{\bar{\bar{\boldsymbol{\mu}}}_{\text{eff}}^{-1\,I}(\omega)} = \begin{pmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{pmatrix}, \quad (13\text{b})$$

where the explicit form of these elements has been presented in Appendix A. From here on, we restrict our attention to the case in which a quantized linearly polarized wave is normally incident along the z axis toward the moving slab. Using Eq. (11) and the fact that the slab is translationally invariant in the plane of the surface, the GT of system in the reciprocal space is obtained as

$$\bar{\bar{\mathbf{G}}}(\mathbf{k}, \omega) = \frac{1}{q^2 \varepsilon (k_z^2 \alpha + q^2 m^2 - n^2 q^2 \alpha^2)}$$
$$\times \begin{pmatrix} -n^2 q^2 \alpha & 0 & 0 \\ 0 & q^2 m^2 - n^2 q^2 \alpha^2 & qm k_z \\ 0 & qm k_z & k_z^2 - n^2 q^2 \alpha^2 \end{pmatrix}. \quad (14)$$

By using the inverse Fourier transformation and taking the contour integration over k_z, we straightforwardly arrive at the electromagnetic GT in the coordinate space as follows:

$$\bar{\bar{\mathbf{G}}}(z, z', \omega) = e^{ik|z-z'|} \begin{pmatrix} \frac{-i\mu}{2k} & 0 & 0 \\ 0 & \frac{-i\mu_{\text{eff,xx}}}{2k} & \frac{im}{2\alpha\varepsilon\omega/c} \\ 0 & \frac{im}{2\alpha\varepsilon\omega/c} & \frac{i(k^2 c^2/\omega^2 - n^2\alpha^2)}{2\alpha\varepsilon k} \end{pmatrix}. \quad (15)$$

Here, $k = n_{\text{eff}}\omega/c$, where $n_{\text{eff}} = \sqrt{(\alpha^2 n^2 - m^2)/\alpha} = \gamma\sqrt{n^2 - \beta^2}$ is the refraction index in the laboratory frame, in which $\gamma = (1 - \beta^2)^{-1/2}$ is the usual Lorentz factor. This refraction index is in agreement with the classical results derived earlier in a different way for the dispersion relation of moving media [64].

By combining Eqs. (15) and (10), the vector potentials for different polarized waves can be represented in a convenient form as

$$\hat{A}_x(z, \omega) = \sqrt{\frac{\hbar\xi(\omega)}{32\pi^4 S\varepsilon_0 c\,\omega}} \frac{\mu}{n_{\text{eff}}} [e^{i\eta\omega z/c}\hat{a}_{x+}(z, \omega)$$
$$+ e^{-i\eta\omega z/c}\hat{a}_{x-}(z, \omega) + \text{H.c.}], \quad (16\text{a})$$

$$\hat{A}_y(z, \omega) = \sqrt{\frac{\hbar\xi'(\omega)}{32\pi^4 S\varepsilon_0 c\,\omega}} \frac{\mu_{\text{eff,xx}}}{n_{\text{eff}}} [e^{i\eta\omega z/c}\hat{a}_{y+}(z, \omega)$$
$$+ e^{-i\eta\omega z/c}\hat{a}_{y-}(z, \omega) + \text{H.c.}], \quad (16\text{b})$$

where the operators

$$\hat{a}_{x\pm}(z, \omega) = i\sqrt{2\kappa(\omega)\omega/c}\, e^{\mp\kappa(\omega)z\omega/c}$$
$$\times \int_{-\infty}^{\pm z} dz' e^{-in_{\text{eff}}(\omega)z'\omega/c} \left[\frac{\sqrt{\bar{\bar{\varepsilon}}_{\text{eff,xx}}^I}\hat{f}_{e,x}(\pm z', \omega)}{\sqrt{\bar{\bar{\varepsilon}}_{\text{eff,xx}}^I + |n_{\text{eff}}(\omega)|^2 E_m}} \right.$$
$$\left. \pm \frac{n_{\text{eff}}(\omega)\sqrt{E_m}\hat{f}_{m,\perp}(\pm z', \omega)}{\sqrt{\bar{\bar{\varepsilon}}_{\text{eff,xx}}^I + |n_{\text{eff}}(\omega)|^2 E_m}} \right], \quad (17\text{a})$$

$$\hat{a}_{y\pm}(z, \omega) = i\sqrt{2\kappa(\omega)\omega/c}\, e^{\mp\kappa(\omega)z\omega/c}$$
$$\times \int_{-\infty}^{\pm z} dz' e^{-in_{\text{eff}}(\omega)z'\omega/c} \left[\frac{\sqrt{E_e}\,\hat{f}_{e,\perp}(\pm z', \omega)}{\sqrt{E_e - |n_{\text{eff}}(\omega)|^2 \bar{\bar{\mu}}_{\text{eff,xx}}^{-1\,I}}} \right.$$
$$\left. \pm \frac{in_{\text{eff}}(\omega)\sqrt{-\bar{\bar{\mu}}_{\text{eff,xx}}^{-1\,I}}\,\hat{f}_{m,x}(\pm z', \omega)}{\sqrt{E_e - |n_{\text{eff}}(\omega)|^2 \bar{\bar{\mu}}_{\text{eff,xx}}^{-1\,I}}} \right] \quad (17\text{b})$$

are associated with the amplitudes of the x and y polarized waves propagating to the right ($+$) and left ($-$). Here, η and κ are, respectively, the real and imaginary parts of the refractive index n_{eff}, and the parameters $\xi(\omega)$ and $\xi'(\omega)$ are, respectively, defined as $(\bar{\bar{\varepsilon}}_{\text{eff,xx}}^I + |n_{\text{eff}}|^2 E_m)/2\kappa$ and $(E_e - |n_{\text{eff}}|^2\bar{\bar{\mu}}_{\text{eff,xx}}^{-1\,I})/2\kappa$, in which $E_e = |e_{22} - (mn_{\text{eff}}/\varepsilon\alpha\mu_{\text{eff,xx}})e_{32}|^2 + |e_{23} - mn_{\text{eff}}e_{33}/\varepsilon\alpha\mu_{\text{eff,xx}}|^2$ and $E_m = |m_{22}|^2 + |m_{23}|^2$. Moreover, the new bosonic field operators $\hat{f}_{e,\perp}(z, \omega)$ and $\hat{f}_{m,\perp}(z, \omega)$ are defined as

$$\hat{f}_{e,\perp}(z, \omega) = \frac{\left(e_{22} - \frac{mn_{\text{eff}}}{\varepsilon\alpha\mu_{\text{eff,xx}}}e_{32}\right)\hat{f}_{e,y}(z, \omega)}{\sqrt{E_e}}$$
$$+ \frac{\left(e_{23} - \frac{mn_{\text{eff}}}{\varepsilon\alpha\mu_{\text{eff,xx}}}e_{33}\right)\hat{f}_{e,z}(z, \omega)}{\sqrt{E_e}}, \quad (18\text{a})$$

$$\hat{f}_{m,\perp}(z, \omega) = \frac{m_{22}\hat{f}_{m,y}(z, \omega) + m_{23}\hat{f}_{m,z}(z, \omega)}{\sqrt{E_m}}, \quad (18\text{b})$$

which satisfy the following bosonic commutation relations:

$$[\hat{f}_{\nu,\perp}(z, \omega), \hat{f}_{\nu',\perp}^\dagger(z', \omega')] = \delta_{\nu\nu'}\delta(z - z')\delta(\omega - \omega'), \quad (19\text{a})$$

$$[\hat{f}_{\nu,\perp}(z, \omega), \hat{f}_{\nu',\perp}(z', \omega')] = 0, \quad (19\text{b})$$

where $\nu = e, m$. With these relations in mind, the amplitude operators $\hat{a}_{x\pm}(z, \omega)$ and $\hat{a}_{y\pm}(z, \omega)$ are found to satisfy the commutation relations as follows:

$$[\hat{a}_{\sigma\pm}(z, \omega), \hat{a}_{\sigma'\pm}^{\dagger}(z', \omega')] = \delta_{\sigma\sigma'}\delta(\omega - \omega'), \quad (20a)$$

$$[\hat{a}_{\sigma\pm}(z, \omega), \hat{a}_{\sigma'\pm}(z', \omega')] = 0, \quad (20b)$$

where $\sigma, \sigma' = x, y$. From Eq. (17), it can be easily shown that these operators satisfy the following quantum Langevin equations:

$$\frac{\partial}{\partial z}\hat{a}_{x\pm}(z, \omega) = \mp \kappa\omega/c\,\hat{a}_{x\pm}(z, \omega) \pm \hat{D}_{x\pm}(z, \omega), \quad (21a)$$

$$\frac{\partial}{\partial z}\hat{a}_{y\pm}(z, \omega) = \mp \kappa\omega/c\,\hat{a}_{y\pm}(z, \omega) \pm \hat{D}_{y\pm}(z, \omega), \quad (21b)$$

in which the Langevin noise operators $\hat{D}_{\sigma\pm}(z, \omega)$ are defined as

$$\hat{D}_{x\pm}(z, \omega) = \pm i\sqrt{2\kappa(\omega)\omega/c}\,e^{\mp i\eta(\omega)\omega z/c}$$
$$\times \frac{\sqrt{\varepsilon_{\text{eff,xx}}^{I}}\hat{f}_{e,x}(\pm z', \omega) \pm n_{\text{eff}}(\omega)\sqrt{E_m}\hat{f}_{m,\perp}(\pm z', \omega)}{\sqrt{\varepsilon_{\text{eff,xx}}^{I} + |n_{\text{eff}}(\omega)|^2 E_m}},$$
$$(22a)$$

$$\hat{D}_{y\pm}(z, \omega) = \pm i\sqrt{2\kappa(\omega)\omega/c}\,e^{\mp i\eta(\omega)\omega z/c}$$
$$\times \frac{\sqrt{E_e}\hat{f}_{e,\perp}(\pm z', \omega) \pm in_{\text{eff}}(\omega)\sqrt{-\mu_{\text{eff,xx}}^{-1I}}\hat{f}_{m,x}(\pm z', \omega)}{\sqrt{E_e - |n_{\text{eff}}(\omega)|^2 \mu_{\text{eff,xx}}^{-1I}}}.$$
$$(22b)$$

Equations (21) and (22) will make it possible to calculate output fields in terms of input fields at any position outside the moving slab, without explicitly using the Green function (15). We do this job in the next subsection.

B. Quantum optical input-output relations for a moving MDS

Let us consider the quantization method developed in the previous section for a magnetodielectric slab moving uniformly with velocity $\mathbf{v} = v\hat{y}$ parallel to its outer surface. From the perspective of an observer in the laboratory frame and based on Eq. (5), the optical properties of the slab can be described by the effective permittivity and permeability tensors (12a) and (12b). As illustrated schematically in Fig. 1, the magnetodielectric slab is taken to have a thickness l with boundaries at $z = \pm l/2$. We introduce the annihilation operators of the incoming and outgoing radiations on the left and right sides of the slab by $\hat{a}_{\sigma\pm}^{(1)}(z, \omega)$ and $\hat{a}_{\sigma\pm}^{(3)}(z, \omega)$, respectively.

Let us now derive the input-output relations for the moving MDS, without explicitly applying the GT (15). To accomplish this goal, we proceed in three steps: First, by using Eqs. (17a) and (17b), we relate the amplitude operators $\hat{a}_{\sigma\pm}^{(2)}(z, \omega)$ within the slab at the positions $z = \pm l/2$ to each other as

$$\begin{pmatrix} \hat{a}_{\sigma+}^{(2)}(l/2, \omega) \\ \hat{a}_{\sigma-}^{(2)}(l/2, \omega) \end{pmatrix} = \mathbb{R}_\sigma \begin{pmatrix} \hat{a}_{\sigma+}^{(2)}(-l/2, \omega) \\ \hat{a}_{\sigma-}^{(2)}(-l/2, \omega) \end{pmatrix} + \begin{pmatrix} \hat{d}_{\sigma+} \\ \hat{d}_{\sigma-} \end{pmatrix}, \quad (23)$$

where $\sigma = x, y$ denotes the polarization of different modes, and $\mathbb{R}_\sigma$ is a diagonal 2×2 matrix with $R_{\sigma,11} = 1/R_{\sigma,22} = e^{-\kappa\omega l/c}$. The quantum noise operators in the matrix equation (23) are given by

$$\hat{d}_{\sigma\pm} = e^{\mp\kappa\omega l/2c} \int_{-l/2}^{l/2} dz'\hat{D}_{\sigma\pm}(z', \omega)e^{\pm\kappa\omega z'/c}, \quad (24)$$

where the Langevin noise operators $\hat{D}_{\sigma\pm}(z, \omega)$ were previously defined in Eqs. (22a) and (22b). In the second step, we relate the operators $\hat{a}_{\sigma\pm}^{(j+1)}(z_j, \omega)$ and $\hat{a}_{\sigma\pm}^{(j)}(z_j, \omega)$ in neighboring layers across the interface to each other by imposing the boundary conditions at $z = z_j$ ($j = 1, 2$), in which the tangential components of the electric and magnetic field operators must be continuous (see Appendix B). Therefore, after some manipulations, we arrive at

$$\begin{pmatrix} \hat{a}_{\sigma+}^{(j+1)}(z_j, \omega) \\ \hat{a}_{\sigma-}^{(j+1)}(z_j, \omega) \end{pmatrix} = \mathbb{S}_\sigma^{(j)} \begin{pmatrix} \hat{a}_{\sigma+}^{(j)}(z_j, \omega) \\ \hat{a}_{\sigma-}^{(j)}(z_j, \omega) \end{pmatrix}. \quad (25)$$

Here, $z_{1(2)} = -(+)l/2$ and the elements of the transformation matrix $\mathbb{S}_\sigma^j$ are given in Appendix C. In the last step, by applying Eq. (23) and twice Eq. (25), as our main result we get the quantum-optical input-output relations

$$\begin{pmatrix} \hat{a}_{\sigma-}^{(1)}(-l/2, \omega) \\ \hat{a}_{\sigma+}^{(3)}(l/2, \omega) \end{pmatrix} = \begin{pmatrix} R_\sigma & T_\sigma \\ T_\sigma & R_\sigma \end{pmatrix} \begin{pmatrix} \hat{a}_{\sigma+}^{(1)}(-l/2, \omega) \\ \hat{a}_{\sigma-}^{(3)}(l/2, \omega) \end{pmatrix}$$
$$+ \mathbb{A}_\sigma \begin{pmatrix} \hat{g}_{\sigma+}(\omega) \\ \hat{g}_{\sigma-}(\omega) \end{pmatrix}, \quad (26)$$

where the reflection and transmission coefficients of the moving MDS, R_σ and T_σ, are given by the classical expressions

$$R_\sigma = \frac{(e^{2in_{\text{eff}}(\omega)\omega l/c} - 1)[n_{\text{eff}}^2(\omega) - \zeta_\sigma^2]e^{-i\omega l/c}}{[\zeta_\sigma + n_{\text{eff}}(\omega)]^2 - [\zeta_\sigma - n_{\text{eff}}(\omega)]^2 e^{2in_{\text{eff}}(\omega)\omega l/c}}, \quad (27a)$$

$$T_\sigma = \frac{4n_{\text{eff}}(\omega)\zeta_\sigma e^{-i\omega l/c}e^{in_{\text{eff}}(\omega)\omega l/c}}{[\zeta_\sigma + n_{\text{eff}}(\omega)]^2 - [\zeta_\sigma - n_{\text{eff}}(\omega)]^2 e^{2in_{\text{eff}}(\omega)\omega l/c}}. \quad (27b)$$

Here, the parameter ζ_σ ($\sigma = x, y$) is equal to $\mu_{\text{eff,yy}}$ and $\mu_{\text{eff,xx}}$ for the polarizations along x and y directions, respectively. In Eq. (26), $\mathbb{A}_\sigma$ is the absorption matrix which arises from dissipative nature of the slab. The elements of this 2×2 matrix are given in Appendix D. Also, the quantum noise operators $\hat{g}_{\sigma\pm}(\omega)$ in Eq. (26) are given by

$$\hat{g}_{\sigma\pm}(\omega) = [2c_{\sigma\pm}(l, \omega)]^{-1/2}[\hat{g}'_{\sigma-}(\omega) \pm \hat{g}'_{\sigma+}(\omega)], \quad (28)$$

where

$$g'_{x\pm}(\omega) = i\sqrt{\frac{\omega}{c}}e^{in_{\text{eff}}(\omega)\omega l/2c} \int_{-l/2}^{l/2} dz'\, e^{\mp in_{\text{eff}}(\omega)z'\omega/c}$$
$$\times \left[\frac{\sqrt{\varepsilon_{\text{eff xx}}^{I}}\hat{f}_{e,x}(z', \omega)}{\sqrt{\varepsilon_{\text{eff,xx}}^{I} + E_m|n_{\text{eff}}(\omega)|^2}} \right.$$
$$\left. \pm \frac{n_{\text{eff}}(\omega)\sqrt{E_m}\hat{f}_{m,\perp}(\pm z', \omega)}{\sqrt{\varepsilon_{\text{eff,xx}}^{I} + E_m|n_{\text{eff}}(\omega)|^2}} \right], \quad (29a)$$

$$g'_{y\pm}(\omega) = i\sqrt{\frac{\omega}{c}} e^{in_{\text{eff}}(\omega)\omega l/2c} \int_{-l/2}^{l/2} dz' \, e^{\mp in_{\text{eff}}(\omega)z'\omega/c}$$

$$\times \left[\frac{\sqrt{E_e}\hat{f}_{e,\perp}(z',\omega)}{\sqrt{E_e - \mu_{\text{eff,xx}}^{-1I}|n_{\text{eff}}(\omega)|^2}} \right.$$

$$\left. \pm \frac{in_{\text{eff}}(\omega)\sqrt{-\mu_{\text{eff,xx}}^{-1I}}\hat{f}_{m,x}(z',\omega)}{\sqrt{E_e - \mu_{\text{eff,xx}}^{-1I}|n_{\text{eff}}(\omega)|^2}} \right], \quad (29b)$$

and

$$c_{x\pm}(l,\omega) = e^{-\kappa\omega l/c} \left(\frac{\sinh(\kappa\omega l/c)}{\kappa} \right.$$

$$\left. \pm \frac{\varepsilon_{\text{eff,xx}}^I - E_m|n_{\text{eff}}(\omega)|^2}{\varepsilon_{\text{eff,xx}}^I + E_m|n_{\text{eff}}(\omega)|^2} \frac{\sin(\eta\omega l/c)}{\eta} \right), \quad (30a)$$

$$c_{y\pm}(l,\omega) = e^{-\kappa\omega l/c} \left(\frac{\sinh(\kappa\omega l/c)}{\kappa} \right.$$

$$\left. \pm \frac{E_e + \mu_{\text{eff,xx}}^{-1I}|n_{\text{eff}}(\omega)|^2}{E_e - \mu_{\text{eff,xx}}^{-1I}|n_{\text{eff}}(\omega)|^2} \frac{\sin(\eta\omega l/c)}{\eta} \right).$$

$$(30b)$$

By applying Eqs. (19) and making use of Eqs. (28)–(30), the quantum noise operators $\hat{g}_{\sigma\pm}(\omega)$ are found to satisfy the following commutation relations:

$$[\hat{g}_{\sigma\pm}(\omega), \hat{g}_{\sigma'\pm}^{\dagger}(\omega')] = \delta_{\sigma\sigma'}\delta(\omega - \omega'), \quad (31a)$$

$$[\hat{g}_{\sigma\pm}(\omega), \hat{g}_{\sigma'\mp}^{\dagger}(\omega')] = 0. \quad (31b)$$

Equations (26)–(31) make it possible to calculate the quantum statistical properties of the output fields at any position outside the slab, from the properties of the input fields and the noise operators. Interestingly, the input-output relations (26) are completely consistent with those reported in [68,69] for the input-output relation of a stationary anisotropic slab with $\varepsilon_{yz} = \mu_{yz} = 0$, because, as stated earlier, an isotropic moving MDS behaves somewhat like an anisotropic MDS.

To compare the input-output relations of the moving MDS (26) with the corresponding relations for a dielectric slab at rest [45,46,50,52,54,69], we define new noise operators as $\begin{pmatrix} \hat{F}_{\sigma+}(\omega) \\ \hat{F}_{\sigma-}(\omega) \end{pmatrix} = \mathbb{A}_\sigma \begin{pmatrix} \hat{g}_{\sigma+}(\omega) \\ \hat{g}_{\sigma-}(\omega) \end{pmatrix}$. They represent the quantum noises associated with loss inside the moving MDS, and have the following expectation values at the finite temperature Θ:

$$\langle F|\hat{F}_{\sigma\pm}^{\dagger}(\omega)|F\rangle = \langle F|\hat{F}_{\sigma\pm}|F\rangle = 0,$$

$$\langle F|\hat{F}_{\sigma\pm}^{\dagger}(\omega)\hat{F}_{\sigma\pm}(\omega')|F\rangle = N(\gamma\omega, \Theta) \quad (32)$$

$$\times (1 - |R_\sigma|^2 - |T_\sigma|^2)\delta(\omega - \omega'),$$

where $|F\rangle$ represents the noise state of the MDS, and $N(\omega, \Theta) = [\exp(\hbar\omega/k_B\Theta) - 1]^{-1}$ is the mean number of thermal photons at frequency ω and temperature Θ, in which $\hbar$ is the Planck constant per 2π and k_B is the Boltzmann constant. Since the input fields in the free space cannot sense the presence of the moving MDS before arriving at it, the optical

input operators must satisfy the usual bosonic commutation relations as

$$\left[\hat{a}_{\sigma+}^{(1)}(z,\omega), \hat{a}_{\sigma'+}^{(1)\dagger}(z',\omega')\right] = \left[\hat{a}_{\sigma-}^{(3)}(z,\omega), \hat{a}_{\sigma'-}^{(3)\dagger}(z',\omega')\right]$$

$$= \delta_{\sigma\sigma'}\delta(\omega - \omega'). \quad (33)$$

By using the above commutation relations and the input-output relations (26), the bosonic commutation relations for the outgoing operators are obtained as follows:

$$\left[\hat{a}_{\sigma-}^{(1)}(z,\omega), \hat{a}_{\sigma'-}^{(1)\dagger}(z',\omega')\right] = \left[\hat{a}_{\sigma+}^{(3)}(z,\omega), \hat{a}_{\sigma'+}^{(3)\dagger}(z',\omega')\right]$$

$$= \delta_{\sigma\sigma'}\delta(\omega - \omega'), \quad (34a)$$

$$\left[\hat{a}_{\sigma-}^{(1)}(z,\omega), \hat{a}_{\sigma'+}^{(3)\dagger}(z',\omega')\right] = \left[\hat{a}_{\sigma+}^{(3)}(z,\omega), \hat{a}_{\sigma'-}^{(1)\dagger}(z',\omega')\right]$$

$$= 0. \quad (34b)$$

In the limiting case that the moving MDS is at rest, that is $v = 0$, the effective tensors $\bar{\bar{\varepsilon}}_{\text{eff}}$ and $\bar{\bar{\mu}}_{\text{eff}}$ reduce to the stationary parameters ε and μ. Consequently, the input-output relations (26) for $\mu = 1$ tend to those derived in [52–54] for a stationary dielectric slab.

III. NUMERICAL CALCULATIONS AND ANALYSIS

A. Transmission, reflection, and absorption coefficients

Due to the complexity of Eq. (27), it is difficult to predict the results analytically. We start our numerical analysis by examining the motion effects of the moving MDS on the transmission and reflection properties. Consider a single-resonance MDS of Lorentz type whose complex permittivity and permeability functions in the rest frame of the MDS are given by [45,46,56]

$$\varepsilon(\omega) = \varepsilon_\infty \left(1 + \frac{\omega_{pe}^2}{\omega_0^2 - \omega^2 - i\gamma_e\omega} \right), \quad (35a)$$

$$\mu(\omega) = \mu_\infty \left(1 + \frac{\omega_{pm}^2}{\omega_0^2 - \omega^2 - i\gamma_m\omega} \right), \quad (35b)$$

where ε_∞ and μ_∞ are, respectively, the high-frequency limit of ε and μ, ω_p and ω_0 are, respectively, the plasma and the resonance frequency, and γ is the absorbtion coefficient of the MDS.

In Figs. 2 and 3, the squared moduli of the transmission and reflection coefficients, $|T_x|^2$ and $|R_x|^2$, and the absorption coefficient, $1 - |R_x|^2 - |T_x|^2$, for x-polarized light are plotted as functions of dimensionless parameters ω/ω_0 and β. Here, positive (negative) value of β represents that the MDS is moving in the positive (negative) y direction. It is seen that all plots are symmetric with respect to $\beta = 0$, because these optical coefficients depend on even powers of β. Furthermore, as β varies from 0 to 1, $|R_x|^2$ increases very slowly with β and then shows an oscillatory behavior followed by a fast enhancement to 1, in the limit of $\beta \to 1$. In contrast, $|T_x|^2$ decreases very slowly and then shows an oscillatory behavior followed by a fast decrease to zero, as β approaches to 1. Similar behaviors are seen in Figs. 3(a) and 3(b) for the y-polarized light incident on the moving MDS.
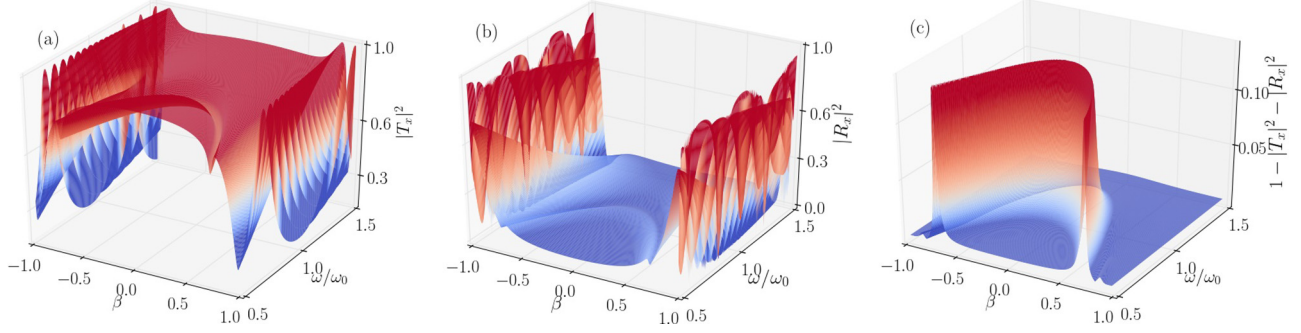
FIG. 2. (a) The square modulus of the transmission coefficient $|T_x|^2$, (b) the reflection coefficient $|R_x|^2$, and (c) the absorption coefficient $1 - |R_x|^2 - |T_x|^2$ for x-polarized light incident on the moving MDS with thickness $\omega_0 l / c = 1$. The permittivity and permeability functions of the MDS in its rest frame are described by the Lorentz model (35), with parameters $\varepsilon_\infty = 2$, $\mu_\infty = 1$, $\gamma_e / \omega_0 = 0.1$, $\gamma_m / \omega_0 = 0.2$, $\omega_{pe} / \omega_0 = 0.1$, and $\omega_{pm} / \omega_0 = 0.05$.

In the low velocity limit $v \ll c$ ($\beta \ll 1$), the absorption coefficient reaches the maximum value of 0.14 near the resonance frequency. This maximum value shifts to frequencies below the resonant frequency of the MDS with increasing β, as seen in Figs. 2(c) and 3(c).

In the ultrarelativistic limit $v \simeq c$ ($\beta \simeq 1$), the relation $|R_x|^2 + |T_x|^2 \approx 1$ holds, and the moving MDS acts like a lossless slab, with $|T_x|^2 \approx 0$ and $|R_x|^2 \approx 1$; i.e., the moving MDS behaves as a perfectly conducting slab to the incident quantum light. This is in confirmation of the results obtained in [20].

B. Quadrature squeezing

In this subsection, we shall proceed to study the significant impacts of the motion of the moving MDS on the noise properties of the transmitted states. To do so, let us start with the definition of the quadrature operators of the output field in the region $z > l/2$ as follows:

$$\hat{X}_\sigma^{(3)} = \frac{1}{2} \left(\hat{a}_{\sigma+}^{(3)} + \hat{a}_{\sigma+}^{(3)\dagger} \right), \tag{36a}$$

$$\hat{Y}_\sigma^{(3)} = \frac{i}{2} \left(\hat{a}_{\sigma+}^{(3)\dagger} - \hat{a}_{\sigma+}^{(3)} \right). \tag{36b}$$

These quadratures are the analogues of the dimensionless position and momentum operators and are subject to a similar uncertainty relation $\langle (\Delta \hat{X}_\sigma^{(3)})^2 (\Delta \hat{Y}_\sigma^{(3)})^2 \rangle > 1/16$, where the variance of the arbitrary operator $\hat{O}$ is defined as $\langle \Delta \hat{O}^2 \rangle = \langle \hat{O}^2 \rangle - \langle \hat{O} \rangle^2$. We can now quantify the quantum fluctuations of the transmitted light through the moving MDS by evaluat-ing the variance of these field quadratures. Consider that the incident fields from the free space to the left and the right sides of the MDS are, respectively, the single mode CS, $|\alpha_\sigma\rangle_R$, and the conventional quantum vacuum state, $|0\rangle_L$, where the subscript indices R and L denote the direction of propagation. Therefore, the general state of the system is written as $|\psi\rangle = |\alpha_\sigma\rangle_R |0\rangle_L |F\rangle$. By using Eqs. (26) and evaluating the variance of $\hat{X}_\sigma^{(3)}$ with respect to the state $|\psi\rangle$, after some manipulations we obtain

$$\langle (\Delta \hat{X}_\sigma^{(3)})^2 \rangle = \frac{1}{4} (1 + 2\langle \hat{F}_{\sigma+}^\dagger \hat{F}_{\sigma+} \rangle), \tag{37a}$$

$$\langle (\Delta \hat{Y}_\sigma^{(3)})^2 \rangle = \frac{1}{4} (1 + 2\langle \hat{F}_{\sigma+}^\dagger \hat{F}_{\sigma+} \rangle). \tag{37b}$$

It is clearly seen that the transmitted CS through the moving MDS is not a CS due to the presence of the noise flux $\langle \hat{F}_{\sigma+}^\dagger \hat{F}_{\sigma+} \rangle$ at the right-hand side of Eq. (37). Of course, this noise flux vanishes at zero temperature, or at frequencies far from the resonant frequency of the MDS, or at extreme velocities where the moving MDS acts like a lossless slab. In these limiting cases, the output state stays in a minimum uncertainty state.

In what follows, for simplicity, we use the squeezing parameter, $S_{X\sigma}^{(3)} = 4\Delta \hat{X}_\sigma^{(3)2} - 1$, to investigate the quadrature squeezing of the output light. This parameter is zero for the quantum vacuum and the coherent states and the existence of the quadrature squeezing (squeezing in the form of reduced quantum noise with respect to the standard limit) is manifested in a negative-valued variance.
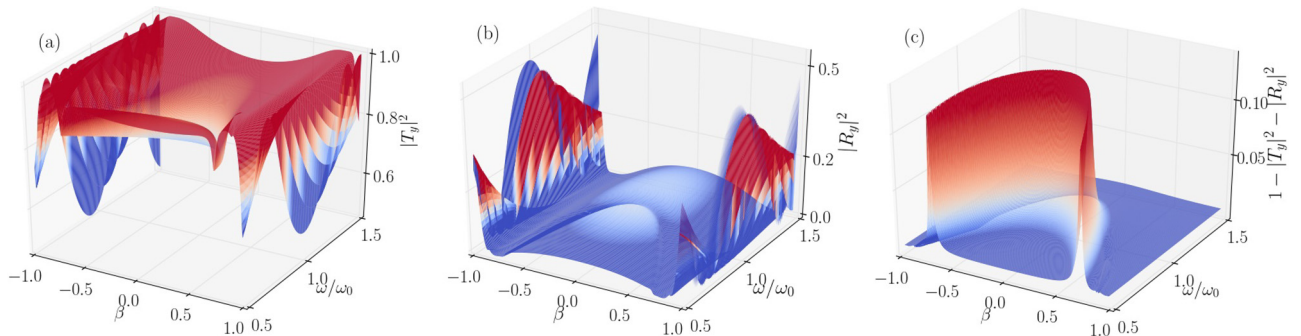

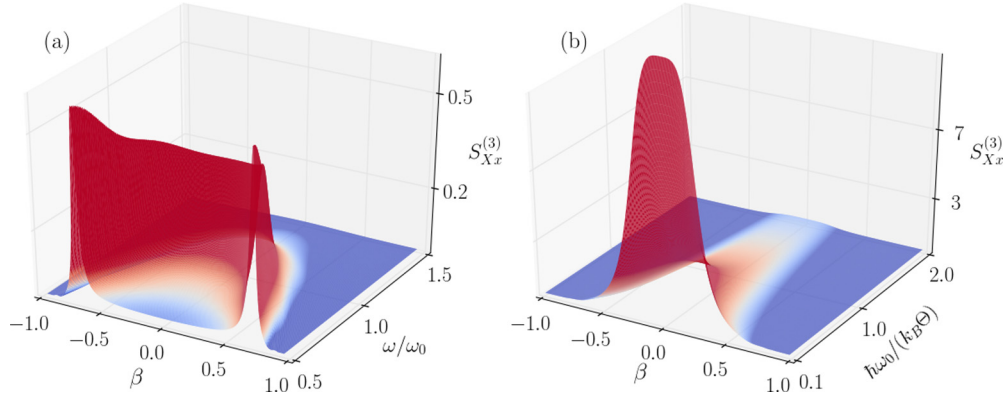FIG. 3. Same as Fig. 2 but now for y-polarized light incident on the moving MDS.

FIG. 4. (a) The squeezing parameter $S_{Xx}^{(3)}$ as a function of dimensionless parameters ω/ω_0 and β for the transmitted CS through the moving MDS at temperature $\hbar\omega_0/k_B\Theta = 10/6$. (b) The squeezing variance $S_{Xx}^{(3)}$ as a function of dimensionless parameters $\hbar\omega_0/k_B\Theta$ and β for the transmitted CS through the moving MDS at fixed frequency $\omega/\omega_0 = 1$. The material properties of the moving MDS slab in its rest frame are described by the Lorentz model (35) with parameters are identical to those used in Fig. 2. Here, the mean number of photons in the coherent state $|\alpha_x\rangle_R$ is 16.

Figure 4 shows the squeezing parameter $S_{X\sigma}^{(3)}$ for the transmitted CS with x polarization as functions of the dimensionless parameters β and ω/ω_0. Here, we adopt the Lorentz model (35) to characterize the dissipative and dispersive effects of the MDS in its rest frame. Since the fluctuations in other quadrature operator $S_{Y\sigma}^{(3)}$ follow a similar behavior, we confine our attention to the squeezing parameter $S_{X\sigma}^{(3)}$.

At the extreme velocity $v \simeq c$, as mentioned above, the moving MDS acts as a perfectly conducting slab to the input states. Therefore, it is expected that the output field is prepared in a state close to the quantum vacuum state, as seen in Fig. 4(a). In contrast, at frequency ranges where the relation $|R_x|^2 + |T_x|^2 \approx 1$ holds except at extreme velocities, the transmitted field is prepared in a state close to a single mode CS. Therefore, in both above situations, the output state is prepared in a minimum uncertainty state, and consequently the squeezing parameter $S_{X\sigma}^{(3)}$ becomes zero.

To compare the thermal effects of the moving MDS with its motion effects, the squeezing parameter $S_{X\sigma}^{(3)}$ at the resonance frequency ω_0 is shown in Fig. 4(b) with respect to the dimensionless parameters β and $\hbar\omega_0/k_BT$. At elevated temperatures, we observe that the squeezing parameter reaches the maximum value around $\beta = 0$, where the noise flux is very significant at the resonance frequency, and then decreases to zero with increasing β, because the absorption becomes very small in the limit of $\beta \to 1$ [see Fig. 2(c)]. Therefore, at low velocities and also at the resonance frequency, the thermal effects have the overall effect of degrading the quantum features of the input state.

C. Mandel parameter

In order to study the photon-counting statistics of the transmitted CSs through the MDS at the finite temperature, we analyze the Mandel parameter [70]

$$Q_{\sigma+}^{(3)} = \frac{\langle \hat{a}_{\sigma+}^{(3)\dagger} \hat{a}_{\sigma+}^{(3)} \hat{a}_{\sigma+}^{(3)\dagger} \hat{a}_{\sigma+}^{(3)} \rangle - \langle \hat{a}_{\sigma+}^{(3)\dagger} \hat{a}_{\sigma+}^{(3)} \rangle^2 - \langle \hat{a}_{\sigma+}^{(3)\dagger} \hat{a}_{\sigma+}^{(3)} \rangle}{\langle \hat{a}_{\sigma+}^{(3)\dagger} \hat{a}_{\sigma+}^{(3)} \rangle}, \quad (38)$$

where the positive, zero, and negative values of this parameter represent super-Poissonian, Poissonian, and sub-Poissonian distributions, respectively [70]. By using Eq. (26), after some

algebra, the Mandel parameter (38) for the output state in the region $z > l/2$ can be obtained as

$$Q_\sigma^{(3)} = \frac{2|T_\sigma \alpha_\sigma|^2 N(\gamma\omega, \Theta)(1 - |T_\sigma|^2 - |R_\sigma|^2)}{|T_\sigma \alpha_\sigma|^2 + N(\gamma\omega, \Theta)(1 - |T_\sigma|^2 - |R_\sigma|^2)}$$
$$- \frac{(N(\gamma\omega, \Theta)(1 - |T_\sigma|^2 - |R_\sigma|^2))^2}{|T_\sigma \alpha_\sigma|^2 + N(\gamma\omega, \Theta)(1 - |T_\sigma|^2 - |R_\sigma|^2)}. \quad (39)$$

With the help of the Lorentz model (35), the motion effect of the moving MDS on the photon counting statistics of the transmitted x-polarized CS is shown in Fig. 5.

In Fig. 5(a), in confirmation of our recent findings in previous subsections, we observe that the Mandel parameter is positive only around regions where the absorption is significant, i.e., the transmitted CS exhibits super-Poisson distribution. Furthermore, at the frequencies where the relation $|R_x|^2 + |T_x|^2 \approx 1$ holds except at extreme velocities, the Mandel parameter becomes zero. Therefore, as far as it is related to the photon-counting statistics, the transmitted CS through the moving MDS is the same CS.

In Fig. 5(b), we have plotted the Mandel parameter $Q_x^{(3)}$ at the resonant frequency as a function of the dimensionless parameters β and $\hbar\omega_0/k_BT$. As can be seen, the thermal effects are predominant only in the low velocity $v < 0.2c$, where the absorption is significant. In the high velocities, the thermal effects are minimal and subsequently the Mandel parameter becomes zero, because the moving MDS with high velocity behaves like a lossless slab at the resonant frequency [see Fig. 2(c)].

IV. CONCLUSIONS

We have developed a phenomenological scheme for the quantization of the electromagnetic field propagating perpendicularly to a moving MDS with uniform velocity in the direction parallel to its interface. We have derived quantum-optical input-output relations for this MDS and described the action of a moving lossy slab on an arbitrary quantum state of light with either s or p polarization. We have investigated the impact of the motion of the moving MDS on quantum
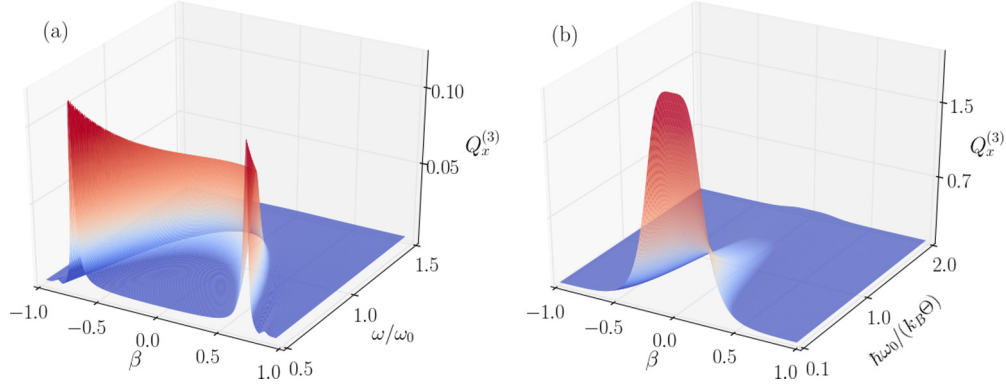
FIG. 5. Mandel parameter $Q_x^{(3)}$ as a function of dimensionless parameters ω/ω_0 and β for the transmitted CS through the moving MDS at temperature $\hbar\omega_0/k_B\Theta = 10/6$. (b) Mandel parameter $Q_x^{(3)}$ as a functions of dimensionless parameters $\hbar\omega_0/k_B\Theta$ and β for the transmitted CS through the moving MDS at fixed frequency $\omega/\omega_0 = 1$. The material parameters are identical to those used in Fig. 2.

properties of the incident states. To this end, the input-output relations are used to investigate the impact of the motion of the moving MDS on the quantum properties of the incident states.

By modeling the dispersive and dissipative effects of the moving MDS by the Lorentz model in its rest frame, we have analyzed the reflection and transmission coefficients of the moving MDS, which is a fundamental problem in classical electrodynamics, and the results may be applied to some aspects of optics and astrophysics. We then evaluated the quadrature squeezing and the Mandel parameter for the transmitted CS through the moving MDS. As a subsidiary result, we found that the moving MDS at extreme velocity acts as a perfectly conducting slab to the input states. Therefore, the output field is prepared in a state close to the quantum vacuum state. At the low and moderate velocities, $v < 0.8c$, and also at the frequencies regions where the absorbtion is weak, the transmitted field is prepared in a state close to a single mode CS. It is shown that the thermal effects have a significant role in degrading the quantum features of the input state only in the low- and moderate-velocity ranges.

APPENDIX A: SQUARE ROOT OF TENSORS

The elements of the square root of the imaginary part of the effective tensors $\sqrt{\bar{\bar{\varepsilon}}_{\text{eff}}^I(\omega)}$ and $\sqrt{\bar{\bar{\mu}}_{\text{eff}}^{-1\,I}(\omega)}$ can be expressed as

$$e_{11} = \sqrt{\bar{\bar{\varepsilon}}_{\text{eff,xx}}^I}, \tag{A1a}$$

$$e_{22} = \frac{\left(-\bar{\bar{\varepsilon}}_{\text{eff,yy}}^I + \bar{\bar{\varepsilon}}_{\text{eff,zz}}^I\right)(\mathcal{M}_e - \mathcal{N}_e)}{2\mathcal{D}_e} + \frac{\mathcal{D}_e(\mathcal{M}_e + \mathcal{N}_e)}{2\mathcal{D}_e}, \tag{A1b}$$

$$e_{23} = \frac{\left(\left(\bar{\bar{\varepsilon}}_{\text{eff,yy}}^I - \bar{\bar{\varepsilon}}_{\text{eff,zz}}^I\right)^2 - \mathcal{D}_e^2\right)(\mathcal{M}_e - \mathcal{N}_e)}{2i\bar{\bar{\varepsilon}}_{\text{eff,zy}}^{*}\mathcal{D}_e}, \tag{A1c}$$

$$e_{32} = \frac{-i\bar{\bar{\varepsilon}}_{\text{eff,zy}}^{*}(\mathcal{M}_e - \mathcal{N}_e)}{2\mathcal{D}_e}, \tag{A1d}$$

$$e_{33} = \frac{\left(\bar{\bar{\varepsilon}}_{\text{eff,yy}}^I - \bar{\bar{\varepsilon}}_{\text{eff,zz}}^I\right)(\mathcal{M}_e - \mathcal{N}_e)}{2\mathcal{D}_e} + \frac{\mathcal{D}_e(\mathcal{M}_e + \mathcal{N}_e)}{2\mathcal{D}_e}, \tag{A1e}$$

$$m_{11} = \sqrt{\bar{\bar{\mu}}_{\text{eff,xx}}^{-1\,I}}, \tag{A1f}$$

$$m_{22} = \frac{\left(-\bar{\bar{\mu}}_{\text{eff,yy}}^{-1\,I} + \bar{\bar{\mu}}_{\text{eff,zz}}^{-1\,I}\right)(\mathcal{M}_m - \mathcal{N}_m)}{2\mathcal{D}_m} + \frac{\mathcal{D}_m(\mathcal{M}_m + \mathcal{N}_m)}{2\mathcal{D}_m}, \tag{A1g}$$

$$m_{23} = \frac{\left(\left(\bar{\bar{\mu}}_{\text{eff,yy}}^{-1\,I} - \bar{\bar{\mu}}_{\text{eff,zz}}^{-1\,I}\right)^2 - \mathcal{D}_m^2\right)(\mathcal{M}_m - \mathcal{N}_m)}{2i\bar{\bar{\mu}}_{\text{eff,zy}}^{-1\,*}\mathcal{D}_m}, \tag{A1h}$$

$$m_{32} = \frac{-i\bar{\bar{\mu}}_{\text{eff,zy}}^{-1\,*}(\mathcal{M}_m - \mathcal{N}_m)}{2\mathcal{D}_m}, \tag{A1i}$$

$$m_{33} = \frac{\left(\bar{\bar{\mu}}_{\text{eff,yy}}^{-1\,I} - \bar{\bar{\mu}}_{\text{eff,zz}}^{-1\,I}\right)(\mathcal{M}_m - \mathcal{N}_m)}{2\mathcal{D}_m} + \frac{\mathcal{D}_m(\mathcal{M}_m + \mathcal{N}_m)}{2\mathcal{D}_m}, \tag{A1j}$$

where $\mathcal{D}_e = \sqrt{(\bar{\bar{\varepsilon}}_{\text{eff,yy}}^I - \bar{\bar{\varepsilon}}_{\text{eff,zz}}^I)^2 + |\bar{\bar{\varepsilon}}_{\text{eff,zy}}^I|^2}$, $\mathcal{M}_e = \sqrt{(\bar{\bar{\varepsilon}}_{\text{eff,yy}}^I + \bar{\bar{\varepsilon}}_{\text{eff,zz}}^I - \mathcal{D}_e)/2}$, and $\mathcal{N}_e = \sqrt{(\bar{\bar{\varepsilon}}_{\text{eff,yy}}^I + \bar{\bar{\varepsilon}}_{\text{eff,zz}}^I + \mathcal{D}_e)/2}$. Furthermore, the parameters $\mathcal{D}_m$, $\mathcal{M}_m$, and $\mathcal{N}_m$ are obtained from the previous relations by replacing $\bar{\bar{\varepsilon}}_{\text{eff}}^I$ with $\bar{\bar{\mu}}_{\text{eff}}^{-1\,I}$.

APPENDIX B: BOUNDARY CONDITIONS

Considering the continuity of the tangential electric and magnetic fields across the boundaries of the MDS, and taking into account that the vector potential with components (16a) and (16b) is continuously differentiable at the interface between the MDS and the vacuum, the boundary conditions at $z = z_j$ ($j = 1, 2$) can be written as

$$\sqrt{\xi_{j+1}} \frac{\mu_{\text{eff}\,j+1,yy}}{n_{j+1}} \left[e^{i\eta_{j+1}\omega z_j/c} \hat{a}_{x+}^{(j+1)}(z_j, \omega) \right.$$
$$\left. + e^{-i\eta_{j+1}\omega z_j/c} \hat{a}_{x-}^{(j+1)}(z_j, \omega) \right]$$
$$= \sqrt{\xi_j} \frac{\mu_{\text{eff}\,j,yy}}{n_j} \left[e^{i\eta_j\omega z_j/c} \hat{a}_{x+}^{(j)}(z_j, \omega) \right.$$
$$\left. + e^{-i\eta_j\omega z_j/c} \hat{a}_{x-}^{(j)}(z_j, \omega) \right], \tag{B1a}$$

$$\sqrt{\xi'_{j+1}}\frac{\mu_{\text{eff xx},j+1}}{n_{j+1}}\Big[e^{i\eta_{j+1}\omega z_j/c}\hat{a}_{y+}^{(j+1)}(z_j,\omega)$$

$$+e^{-i\eta_{j+1}\omega z_j/c}\hat{a}_{y-}^{(j+1)}(z_j,\omega)\Big]$$

$$=\sqrt{\xi'_j}\frac{\mu_{\text{eff xx},j}}{n_j}\Big[e^{i\eta_j\omega z_j/c}\hat{a}_{y+}^{(j)}(z_j,\omega)$$

$$+e^{-i\eta_j\omega z_j/c}\hat{a}_{y-}^{(j)}(z_j,\omega)\Big], \qquad \text{(B1b)}$$

and

$$\sqrt{\xi_{j+1}}\Big[e^{i\eta_{j+1}\omega z_j/c}\hat{a}_{x+}^{(j+1)}(z_j,\omega)$$

$$-e^{-i\eta_{j+1}\omega z_j/c}\hat{a}_{x-}^{(j+1)}(z_j,\omega)\Big]$$

$$=\sqrt{\xi_j}\Big[e^{i\eta_j\omega z_j/c}\hat{a}_{x+}^{(j)}(z_j,\omega)-e^{-i\eta_j\omega z/c}\hat{a}_{x-}^{(j)}(z_j,\omega)\Big], \qquad \text{(B2a)}$$

$$\sqrt{\xi'_{j+1}}\Big[e^{i\eta_{j+1}\omega z_j/c}\hat{a}_{y+}^{(j+1)}(z_j,\omega)$$

$$-e^{-i\eta_{j+1}\omega z_j/c}\hat{a}_{y-}^{(j+1)}(z_j,\omega)\Big]$$

$$=\sqrt{\xi'_j}\Big[e^{i\eta_j\omega z_j/c}\hat{a}_{y+}^{(j)}(z_j,\omega)-e^{-i\eta_j\omega z_j/c}\hat{a}_{y-}^{(j)}(z_j,\omega)\Big], \qquad \text{(B2b)}$$

where η_j and κ_j are, respectively, the real and imaginary parts of the refractive index n_j. Here, $n_2 = n_{\text{eff}}$ is the refractive index of the moving MDS in the laboratory frame. Notice that because of the moving MDS is surrounded by vacuum, we have $\varepsilon_1 = \varepsilon_3 = \mu_1 = \mu_3 = 1$. Therefore, we have $\eta_1 = \eta_3 = 1$, $\kappa_1 = \kappa_3 = 0$, $\mu_{\text{eff }1,\text{xx}} = \mu_{\text{eff }1,\text{yy}} = \mu_{\text{eff }3,\text{xx}} = \mu_{\text{eff }3,\text{yy}} = 1$, and $\varepsilon_{\text{eff }1,\text{xx}} = \varepsilon_{\text{eff }1,\text{yy}} = \varepsilon_{\text{eff }3,\text{xx}} = \varepsilon_{\text{eff }3,\text{yy}} = 1$.

APPENDIX C: ELEMENTS OF THE TRANSFORMATION MATRIX

We can now relate the operators $\hat{a}_{\sigma\pm}^{(j+1)}(z_j,\omega)$ and $\hat{a}_{\sigma\pm}^{(j)}(z_j,\omega)$ to each other using the boundary conditions (B1) and (B2). After straightforward calculations, we arrive at Eq. (25), where the elements of the transformation matrix $\mathbb{S}^{(j)}$ read as

$$\mathbb{S}_{11,x}^{(j)} = \frac{\sqrt{\xi_j}}{\sqrt{\xi_{j+1}}}\frac{n_{j+1}\mu_{\text{eff }j,\text{yy}}+n_j\mu_{\text{eff }j+1,\text{yy}}}{2n_j\mu_{\text{eff }j+1,\text{yy}}}$$
$$\times e^{-i(\eta_{j+1}-\eta_j)z_j\omega/c}$$
$$=\mathbb{S}_{22,x}^{(j)}e^{-i2(\eta_{j+1}-\eta_j)z_j\omega/c}, \qquad \text{(C1a)}$$

$$\mathbb{S}_{12,x}^{(j)} = \frac{\sqrt{\xi_j}}{\sqrt{\xi_{j+1}}}\frac{n_{j+1}\mu_{\text{eff }j,\text{yy}}-n_j\mu_{\text{eff }j+1,\text{yy}}}{2n_j\mu_{\text{eff }j+1,\text{yy}}}$$
$$\times e^{-i(\eta_{j+1}+\eta_j)z_j\omega/c}$$
$$=\mathbb{S}_{21,x}^{(j)}e^{-i2(\eta_{j+1}+\beta_j)z_j\omega/c}, \qquad \text{(C1b)}$$

$$\mathbb{S}_{11,y}^{(j)} = \frac{\sqrt{\xi'_j}}{\sqrt{\xi'_{j+1}}}\frac{n_{j+1}\mu_{\text{eff }j,\text{xx}}+n_j\mu_{\text{eff }j+1,\text{xx}}}{2n_j\mu_{\text{eff }j+1,\text{xx}}}$$
$$\times e^{-i(\eta_{j+1}-\eta_j)z_j\omega/c}$$
$$=\mathbb{S}_{22,y}^{(j)}e^{-i2(\eta_{j+1}-\eta_j)z_j\omega/c}, \qquad \text{(C1c)}$$

$$\mathbb{S}_{12,y} = \frac{\sqrt{\xi'_j}}{\sqrt{\xi'_{j+1}}}\frac{n_{j+1}\mu_{\text{eff }j,\text{xx}}-n_j\mu_{\text{eff }j+1,\text{xx}}}{2n_{e,j}\mu_{\text{eff }j+1,\text{xx}}}$$
$$\times e^{-i(\eta_{j+1}+\eta_j)z_j\omega/c}$$
$$=\mathbb{S}_{21,y}e^{-i2(\eta_{j+1}+\eta_j)z_j\omega/c}. \qquad \text{(C1d)}$$

APPENDIX D: ELEMENTS OF THE ABSORPTION MATRIX

The elements of the characteristic absorption matrix $\mathbb{A}_\sigma$ are expressed as follows:

$$\mathbb{A}_{11,x} = \sqrt{\kappa\xi c_{x+}}t_{12,x}\vartheta e^{-i\omega l/2c}(1+e^{in_{\text{eff}}\omega l/c}r_{23,x}), \qquad \text{(D1a)}$$

$$\mathbb{A}_{12,x} = \sqrt{\kappa\xi c_{x-}}t_{12,x}\vartheta e^{-i\omega l/2c}(1-e^{in_{\text{eff}}\omega l/c}r_{23,x}), \qquad \text{(D1b)}$$

$$\mathbb{A}_{21,x} = \sqrt{\kappa\xi c_{x+}}t_{32,x}\vartheta e^{-i\omega l/2c}(1+e^{in_{\text{eff}}\omega l/c}r_{21,x}), \qquad \text{(D1c)}$$

$$\mathbb{A}_{22,x} = \sqrt{\kappa\xi c_{x-}}t_{32,x}\vartheta e^{-i\omega l/2c}(e^{in_{\text{eff}}\omega l/c}r_{21,x}-1), \qquad \text{(D1d)}$$

$$\mathbb{A}_{11,y} = \sqrt{\kappa\xi' c_{y+}}t_{12,y}\vartheta' e^{-i\omega l/2c}(1+e^{in_{\text{eff}}\omega l/c}r_{23,y}), \qquad \text{(D1e)}$$

$$\mathbb{A}_{12,y} = \sqrt{\kappa\xi' c_{y-}}t_{12,y}\vartheta' e^{-i\omega l/2c}(1-e^{in_{\text{eff}}\omega l/c}r_{23,y}), \qquad \text{(D1f)}$$

$$\mathbb{A}_{21,y} = \sqrt{\kappa\xi' c_{y+}}t_{32,y}\vartheta' e^{-i\omega l/2c}(1+e^{in_{\text{eff}}\omega l/c}r_{21,y}), \qquad \text{(D1g)}$$

$$\mathbb{A}_{22,y} = \sqrt{\kappa\xi' c_{y-}}t_{32,y}\vartheta' e^{-i\omega l/2c}(e^{in_{\text{eff}}\omega l/c}r_{21,y}-1), \qquad \text{(D1h)}$$

where

$$\vartheta' = \big[1-r_{21,y}^2 e^{2in_{\text{eff}}\omega l/c}\big]^{-1}, \qquad \text{(D2a)}$$

$$\vartheta = \big[1-r_{21,x}^2 e^{2in_{\text{eff}}\omega l/c}\big]^{-1}, \qquad \text{(D2b)}$$

and in which

$$r_{12,x} = r_{32,x} = -r_{21,x} = -r_{23,x} = \frac{\mu_{\text{eff,yy}}-n_{\text{eff}}}{\mu_{\text{eff,yy}}+n_{\text{eff}}}, \qquad \text{(D3a)}$$

$$t_{21,x} = t_{23,x} = \frac{2n_{\text{eff}}}{\mu_{\text{eff,yy}}+n_{\text{eff}}}, \qquad \text{(D3b)}$$

$$r_{12,y} = r_{32,y} = -r_{21,y} = -r_{23,y} = \frac{\mu_{\text{eff,xx}}-n_{\text{eff}}}{\mu_{\text{eff,xx}}+n_{\text{eff}}}, \qquad \text{(D3c)}$$

$$t_{12,y} = t_{32,y} = \frac{2\mu_{\text{eff,xx}}}{\mu_{\text{eff,xx}}+n_{\text{eff}}}, \qquad \text{(D3d)}$$

$$t_{21,y} = t_{23,y} = \frac{2n_{\text{eff}}}{\mu_{\text{eff,xx}}+n_{\text{eff}}}, \qquad \text{(D3e)}$$

$$t_{12,x} = t_{32,x} = \frac{2\mu_{\text{eff,yy}}}{\mu_{\text{eff,yy}}+n_{\text{eff}}}. \qquad \text{(D3f)}$$

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