Quantum energy transfer between a nonlinearly coupled bosonic bath and a fermionic chain: An exactly solvable model

Zhao-Ming Wang,[1,2] Da-Wei Luo,[3] Baowen Li,[4] and Lian-Ao Wu[2,5,*]
[1] *Department of Physics, Ocean University of China, Qingdao 266100, China*
[2] *Department of Theoretical Physics and History of Science, The Basque Country University (EHU/UPV), 48008 Bilbao, Spain*
[3] *Center for Quantum Science and Engineering, and Department of Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030, USA*
[4] *Department of Mechanical Engineering, University of Colorado, Boulder, Colorado 80309, USA*
[5] *IKERBASQUE, Basque Foundation for Science, 48011 Bilbao, Spain*

(Received 22 August 2019; accepted 6 April 2020; published 30 April 2020)

The evolution of a quantum system towards thermal equilibrium is usually studied by approximate methods, which have their limits of validity and should be checked against analytically solvable models. In this paper, we propose an analytically solvable model to investigate the energy transfer between a bosonic bath and a fermionic chain which are nonlinearly coupled to each other. The bosonic bath consists of an infinite collection of noninteracting bosonic modes, while the fermionic chain is represented by a chain of interacting fermions with nearest-neighbor interactions. We compare the behaviors of the temperature-dependent energy current J_T and temperature-independent energy current J_{TI} for different bath configurations. With respect to the bath spectrum, J_T decays exponentially for a Lorentz-Drude-type bath, which is the same as the conventional approximations. On the other hand, the decay rate is $1/t^3$ for the Ohmic type and $1/t$ for white noise, which does not have conventional counterparts. For the temperature-independent current J_{TI}, the decay rate is divergent for the Lorentz-Drude-type bath, $1/t^4$ for the Ohmic bath, and $1/t$ for the white noise. When further considering the dynamics of the fermionic chain, the current will be modulated based on the envelope from the bath. As an example, for a bosonic bath with an Ohmic spectrum, when the fermionic chain is uniformly coupled, we have $J_T \propto 1/t^6$ and $J_{TI} \propto 1/t^3$. Moreover, it is interesting that J_T is proportional to $(N-1)^{1/2}$ at certain times for perfect state transfer couplings under a Lorentz-Drude or Ohmic bath.

DOI: 10.1103/PhysRevA.101.042130

I. INTRODUCTION

Dissipative phenomena in open systems [1,2] can give rise to a variety of interesting physical scenarios and have been under extensive study across many different fields, such as quantum optics, many-body physics, and quantum information sciences. Open systems are notoriously difficult to deal with exactly due to the complexity of the quantum reservoir, whose Hilbert space can be prohibitively large. Such systems are usually tackled with a system-plus-reservoir approach where one treats the composite system as a whole, and later traces out the reservoir degrees of freedom to study the reduced dynamical behaviors of the system under consideration. One widely used way to model the quantum reservoir is to treat it as noninteracting harmonic oscillators [3,4]. The dynamics of two Brownian particles in a common reservoir has been studied [5] as well as its thermal equilibrium properties [6]. Using a quantum Langevin description, a system-reservoir model is proposed [7] and a quantum current is observed which is dependent on various parameters of external noise. A spin-boson model also provides a clear physical picture for exploring quantum dissipation effects. This model includes an impurity two-level system (TLS) coupled to a thermal reservoir and displays a rich phase diagram in the equilibrium regime [1,8]. A generalized nonequilibrium polaron-transformed Redfield equation with an auxiliary counting field was developed recently to study the full counting statistics of quantum heat transfer in a driven nonequilibrium spin-boson model [9]. For a subsystem which consists of two interacting spins, this situation effectively corresponds to a subsystem unharmonically coupled to a bosonic bath, allowing one to introduce nonlinear effects [10]. Thermal rectification within a spin-boson nanojunction model is analyzed and analytic solutions are obtained for a separable model and a nonseparable model [11]. The exact dynamics of interacting TLS immersed in separate thermal reservoirs or within a common bath has also been studied [12]. For the device design, such as in molecular devices, one often needs to consider the scaling of heat current with system size and time in order to prevent the devices from disintegrating [13,14], because excess heat buildup during operation may cause device disintegrating.

However, most theoretical investigations of how a quantum system reaches thermal equilibrium use approximation methods, such as quantum master equations [15–17], Born-Oppenheimer methods [18], etc. Typically, such methods only provide numerical results, hindering a direct picture of the microscopic processes involved. Responding to this challenge, we have recently developed an analytic method for describing the energy transfer in a hybrid quantum system.

*Corresponding author: lianao.wu@ehu.es

The hybrid quantum system consists of a bosonic bath and a fermionic chain, which are nonlinearly coupled by using a dressing transformation. Physically, this prototype quantum model could be realized in different systems. For example, the bosonic bath and the fermionic chain correspond to harmonic solids and metal (or spin). respectively. In Ref. [19], thermal rectification appears in two different reservoirs connected by molecular vibration. Our results show that the energy current can disappear at some times for this nonlinearly coupled hybrid bath, whereas for two linearly coupled bosonic baths, a steady current will be obtained [20].

II. MODEL

Consider two baths (H_{LB} and H_{RB}) connected by a central system H_S. The central system S consists of two interacting fermions. The left bath LB is modeled as a collection of noninteracting bosonic modes [3,4], maintained at a fixed temperature $T = \beta^{-1}$, with $k_B = 1$. The right bath RB is modeled as a one-dimensional fermionic chain with nearest-neighbor interactions. The total Hamiltonian is given by

$$H = H_S + H_{LB} + H_{RB} + V_L + V_R, \tag{1}$$

where V_L (V_R) is the interaction between the left (right) bath and the central system. Treating the central system and the fermionic right bath as a whole, we denote $H_{\rm ch} = H_S + H_{RB} + V_R$ and rearranged the indices so that the interacting fermions in the central system are labeled 1 and 2 and the sites in the fermionic chain are labeled 3 through N,

$$H_{\rm ch} = H_S + H_{RB} + V_R = -\sum_{i=1}^{N-1} \tau_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i), \tag{2}$$

where $H_S = -\tau_1(c_1^\dagger c_2 + c_2^\dagger c_1)$, $H_{RB} = -\sum_{i=3}^{N-1} \tau_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i)$, $V_R = -\tau_2(c_2^\dagger c_3 + c_3^\dagger c_2)$, $c_i^\dagger$ is a standard fermionic creation operator, and τ_i is the coupling constant between the nearest-neighbor sites. Additionally, we take $\tau_i > 0$ which corresponds to ferromagnetic couplings throughout. $H_{\rm ch}$ may describe a chain of interacting spinless fermions, which can be mapped to a one-dimensional XY chain under a Jordan-Wigner transformation. The mapping of the fermion number $n_i = c_i^\dagger c_i$ to the Pauli matrix σ_z is $2n_i = \sigma_i^z + 1$. This is true for an open-ended chain or a periodic chain [21–24]. Here, we consider an open-ended chain. For simplicity, the total Hamiltonian H can be written as (see Fig. 1)

$$H = H_{\rm ch} + H_B + V, \tag{3}$$

where $H_B = H_{LB} = \sum_\alpha \omega_\alpha b_\alpha^\dagger b_\alpha$ is the bosonic bath's Hamiltonian (or phonons if zero-point energy is added). $b_i^\dagger$ is the bosonic creation operator, and ω_α is the frequency of the αth mode. Note that the total number of excitations $M = \sum_i c_i^\dagger c_i$ in the chain remains constant, and thus the z component of the total spin is a conserved quantity. We can discuss problems in a fixed subspace for certain excitations. For simplicity, we only consider the $M = 1$ case. We will study the energy transfer between the bath and the chain next. The energy current will be zero after some time t for a finite length chain, at which point thermal equilibrium is reached.

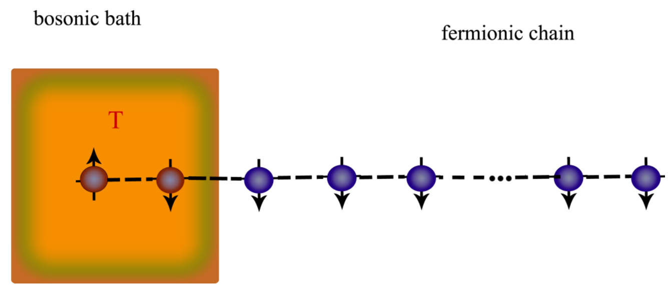

FIG. 1. A schematic representation of the model. The bosonic bath and the fermionic chain are connected by two fermions. The fermionic chain can be mapped to a one-dimensional spin chain and the bosonic bath is at a finite temperature T while the fermionic chain is at zero temperature.

A specific interaction $V = V_L$ can be introduced by the so-called dressing transformation $W^\dagger(H_{\rm ch} + H_B)W$ with a displacement operator $W = \exp[\sum_\alpha (\Gamma_\alpha b_\alpha^\dagger - \Gamma_\alpha^* b_\alpha)n_1]$, where the fermion number $n_1 = \frac{1}{2}(\sigma_i^1 + 1)$. Clearly, it is possible to introduce different types of interactions using different types of dressing transformations [15]. The exact form of the interaction operator is now given by

$$V = \sum_\alpha \omega_\alpha \big(\Gamma_\alpha^* b_\alpha + \Gamma_\alpha b_\alpha^\dagger + |\Gamma_\alpha|^2\big) n_1 + \tau_1 (c_1^\dagger c_2 e^{\Gamma_\alpha^* b_\alpha - \Gamma_\alpha b_\alpha^\dagger} - 1 + \text{H.c.}). \tag{4}$$

Γ_α parametrizes the system-bath coupling strength, and is assumed to be a complex number.

III. CALCULATION OF ENERGY EXPECTATION VALUE AND ENERGY CURRENT

In this section, we will give a general analysis of the energy current between the bosonic bath and the fermionic chain. The energy current operator can be defined as [15]

$$J = i[V, H_B]. \tag{5}$$

The expectation value of the energy current is given by

$$J(t) = \text{Tr}[\rho_0 \widehat{J}], \tag{6}$$

where $\rho_0 = \rho_B \otimes \rho_{\rm ch}$ is the initial density operator of the whole system and $\widehat{J} = e^{iHt}\widehat{J}(0)e^{-iHt}$. Equivalently, the energy current may be rewritten as

$$J(t) = \frac{\partial \langle H_B(t)\rangle}{\partial t}, \tag{7}$$

where $\langle H_B(t)\rangle = \text{Tr}[\rho_0 \widehat{H_B}(t)]$ is the energy expectation value of the bath, where $\widehat{H_B} = e^{iHt}\widehat{H}_B(0)e^{-iHt}$. Note that a positive value represents an energy current from the bath to the chain and vice versa. The bosonic bath at temperature T is modeled as a canonical ensemble with distribution $\rho_B = \exp(-\beta H_B)/\text{Tr}[\exp(-\beta H_B)]$.

It can be readily shown that

$$U_0^\dagger b_\alpha^\dagger U_0 = b_\alpha^\dagger e^{i\omega_\alpha t}, \tag{8}$$

and

$$U_0^\dagger c_1^\dagger U_0 = c_1^\dagger(t) = \sum_l f_{1,l}^* c_l^\dagger, \tag{9}$$

where $U_0 = e^{-i(H_B+H_{\rm ch})t}$ and $f_{1,l}$ is the transition amplitude of an excitation (the $|1\rangle$ state) from site 1 to site l in the chain. We restrict the fermion chain to have only one excitation. Denoting $|\bar{l}\rangle$ as the state where the one fermion excitation is at site l, we write the initial state of the fermion chain as $|\Psi(0)\rangle = \frac{1}{\sqrt{a}}\sum_{l=1}^{a}|\bar{l}\rangle$, where the 1 excitation is restricted to the first a sites. Then, $\langle H_B(t)\rangle$ can be expressed as (neglecting the time-independent part)

$$\langle H_B(t)\rangle = \langle H_B(t)\rangle_T + \langle H_B(t)\rangle_{TI}, \tag{10}$$

where $\langle H_B(t)\rangle_T$ is the temperature-dependent part,

$$\langle H_B(t)\rangle_T = \frac{1}{a}\sum_\alpha \omega_\alpha|\Gamma_\alpha|^2\langle D(\Gamma)\rangle_{\rm eq}\bigg\{2\coth\frac{\beta\omega_\alpha}{2}\sin\omega_\alpha t \times \ {\rm Im}[F(t)] + 2(1-\cos\omega_\alpha t){\rm Re}[F(t)]\bigg\}, \tag{11}$$

and $\langle H_B(t)\rangle_{TI}$ is the temperature-independent part,

$$\langle H_B(t)\rangle_{TI} = \frac{1}{a}\sum_\alpha \omega_\alpha|\Gamma_\alpha|^2[(1-2\cos\omega_\alpha t)|f_{11}|^2 + G(t)], \tag{12}$$

where $\langle D(\Gamma)\rangle_{\rm eq} = \exp(-\frac{1}{2}\sum_\alpha|\Gamma_\alpha|^2\coth\frac{\beta\omega_\alpha}{2})$ is the expectation value of the displacement operator in the thermal equilibrium state, and $F(t), G(t)$ indicate the dynamics of the chain and depend on the initial state of the chain. The explicit definitions of $F(t)$ and $G(t)$ are given for three different initial states in the following sections. Next, we will discuss the behavior of the energy current for different initial states, different spectra for the bosonic bath, and different coupling configurations in the fermionic chain.

IV. ENERGY CURRENT FOR DIFFERENT INITIAL STATES OF THE CHAIN

Now we discuss the energy current for some different initial states. The energy current in the contact can also be expressed as two parts,

$$J = J_T + J_{TI}, \tag{13}$$

where

$$J_T = \frac{2}{a}\sum_\alpha \omega_\alpha|\Gamma_\alpha|^2\langle D(\Gamma_\alpha)\rangle_{\rm eq} \times\bigg\{\coth\frac{\beta\omega_\alpha}{2}[\omega_\alpha\cos\omega_\alpha t\,{\rm Im}[F(t)] + \sin\omega_\alpha t\, d\,{\rm Im}[F(t)]/dt] + \omega_\alpha\sin\omega_\alpha t\,{\rm Re}[F(t)] + (1-\cos\omega_\alpha t)d\,{\rm Re}[F(t)]/dt\bigg\} \tag{14}$$

is the temperature-dependent current, and

$$J_{TI} = \frac{1}{a}\sum_\alpha \omega_\alpha|\Gamma_\alpha|^2[2\omega_\alpha\sin\omega_\alpha t|f_{11}|^2 + (1-\cos\omega_\alpha t)d(|f_{11}|^2)/dt + dG(t)/dt] \tag{15}$$

is the temperature-independent current.

(i) $|\Psi(0)\rangle = |\bar{1}\rangle$. In this case, $a=1$, $F(t)=0$, $G(t)=0$, and $J_T=0$. Therefore, the current is independent of the temperature and represents a pure quantum current. We also note that if the initial state is prepared as $\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle$, i.e., the first site is an arbitrary pure state $\alpha|0\rangle+\beta|1\rangle$ and all other sites at state $|0\rangle$, the state on site one cannot be transferred to other sites under the chain dynamics.

(ii) $|\Psi(0)\rangle = \frac{1}{\sqrt{N}}(|\bar{1}\rangle + |\bar{2}\rangle + \cdots + |\bar{N}\rangle)$, now $a=N$, $F(t) = f_{1,1}^*\sum_{l=2}^N f_{1,l}$, $G(t) = \sum_{l,m=2} f_{1,l}^* f_{1,m}$.

(iii) $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|\bar{1}\rangle + |\bar{2}\rangle)$, $a=2$, $F(t) = f_{1,1}^* f_{12}$, $G(t) = |f_{1,2}|^2$.

Note that the analytical expression of the energy current above is obtained without approximations. In the next section, we will discuss case (iii) as an illustrative example.

V. EFFECTS OF THE SPECTRUM DISTRIBUTION FOR THE BOSONIC BATH

When the chain-bath couplings are weak ($\Gamma_\alpha/\tau_\alpha \to 0$), which corresponds to the Markovian limit, the displacement operator expectation value in the thermal equilibrium can be approximated as its first-order term $\exp(-\frac{1}{2}\sum_\alpha|\Gamma_\alpha|^2\coth\frac{\beta\omega_\alpha}{2}) \approx 1$. Additionally, in the high-temperature T, or low-frequency limit $\omega_\alpha \to 0$, the hyperbolic cotangent $\coth\frac{\omega_\alpha}{2T} \to \infty$, so the dominant term will be the first two terms in Eq. (14). Using the Taylor expansion for the hyperbolic cotangent $\coth x = \sum_{n=1}^{\infty}\frac{2^{2n}B_{2n}x^{2n-1}}{(2n)!}$ (where $0<|x|<\pi$ and B_n is the nth Bernoulli number) and taking the first order for $x\to 0$, the current in Eq. (14) can be further reduced to

$$J_T = 2T\sum_\alpha|\Gamma_\alpha|^2\bigg[\sin\omega_\alpha t\frac{d\,{\rm Im}[F(t)]}{dt} + \omega_\alpha\cos\omega_\alpha t\,{\rm Im}[F(t)]\bigg]. \tag{16}$$

Denoting Γ as an overall coupling strength factor, the energy current J_T, J_{TI} for different spectrum distributions of the bath can then be written in this simplified form.

(i) Lorentz-Drude-type bath [25], whose the spectrum density $\rho(\omega) = \frac{\omega}{\omega_d^2+\omega^2}$. The temperature-dependent energy current is given by

$$J_T = \pi T|\Gamma|^2 e^{-\omega_d t}\bigg[\frac{d\,{\rm Im}[F(t)]}{dt} - \omega_d\,{\rm Im}[F(t)]\bigg]. \tag{17}$$

In this case, the current decays exponentially with time t, modulated by $F(t)$, which comes from the chain's dynamics and finally it reaches zero. Then the thermal equilibrium state is obtained. The envelope is an exponentially decreasing line. For J_{TI}, it is divergent in the Lorentz-Drude bath.

(ii) Ohmic bath, with a spectrum density $\rho(\omega) = \frac{\pi}{2}\omega e^{-\omega/\omega_c}$, where ω_c is the cutoff frequency. For a long-time limit $(\omega_c t) \gg 1$,

$$J_T \approx \frac{2\pi T|\Gamma|^2}{\omega_c t^4}\bigg[t\frac{d\,{\rm Im}[F(t)]}{dt} - 3\,{\rm Im}[F(t)]\bigg], \tag{18}$$

$$J_{TI} \approx \frac{\pi|\Gamma|^2}{2}\bigg[\frac{3}{t^4}|f_{1,1}(t)|^2 + \frac{6}{\omega_c t^4}d(|f_{1,1}(t)|^2)/dt + \omega_c^3 d(|f_{1,1}(t)|^2 + |f_{1,2}(t)|^2)/dt\bigg]. \tag{19}$$

Therefore, for J_T the envelope becomes $1/t^3$ while for J_{TI} it is $1/t^4$ for Ohmic baths. The quantum current decreased more quickly than the classical current for the Ohmic bath. Note that for J_{TI}, in the long-time limit, the current J_{TI} will only depend on the chain's dynamics.

(iii) A "white-noise" spectrum where the frequency distributes uniformly with a cutoff frequency,

$$\rho(\omega) = \begin{cases} 1 & (0 < \omega \leqslant \Omega), \\ 0 & (\text{otherwise}), \end{cases}$$

then the energy current in the long-time limit ($t \gg 1$) is given by

$$J_T \approx 2T|\Gamma|^2\left\{\left[\frac{\Omega}{t}\sin\Omega t\right]\text{Im}[F(t)]+\frac{(1-\cos\Omega t)}{t}\frac{d\,\text{Im}\,F(t)}{dt}\right\}, \tag{20}$$

$$J_{TI} \approx |\Gamma|^2\left\{\left[\frac{\Omega^2\sin\Omega t}{t}\right]|f_{1,1}(t)|^2 - \frac{\Omega\sin\Omega t}{t}d(|f_{1,1}(t)|^2)/dt + \frac{\Omega^2}{4}d(|f_{1,1}(t)|^2+|f_{1,2}(t)|^2)/dt\right\}. \tag{21}$$

The envelope for the currents is $1/t$. Note that for $J_{TI}(t)$ the current depends on the chain's dynamics only in the long-time limit.

VI. EFFECTS OF THE FERMIONIC CHAIN CONFIGURATION

Now we reveal how different configurations of the fermionic chain can affect the energy current. The transition amplitude $f_{m,m'}$ depends on the types of couplings in the chain. First, we discuss the perfect state transfer (PST) couplings $\tau_k = 2\tau\sqrt{k(N-k)/N^2}$. The transition amplitude reads

$$f_{m,m'}(t) = \exp\left[i\frac{\pi}{2}(m-m')\right]d^l_{m',m}(2\tau t), \tag{22}$$

where $d^l_{m',m}(2\tau t)$ is the Wigner d matrix [26]. The indices of the site number of a one-dimensional chain can be mapped onto the magnetic quantum numbers m of the total angular momentum l, such that $l = \frac{N-1}{2}$, $m = -\frac{N-1}{2} + k - 1$, where $k = 1, 2, \ldots, N$ [27]. Using this relation, we obtain $f_{1,1}(t) = [\cos\tau t]^{N-1}$, $f_{1,2}(t) = i\sqrt{N-1}\sin\tau t[\cos\tau t]^{N-2}$, $f_{1,N}(t) = (-1)^N\exp[i\frac{\pi}{2}(N-1)][\sin\tau t]^{N-1}$. Note that the transition amplitudes $f_{1,n}(t)$ $(n = 1, 2, N)$ are periodic functions. When transferring a quantum state from one end to the other end of the chain, the transmission fidelity is a function of the transition amplitude and it can periodically reach 1. This is the so-called perfect state transfer (PST) [27].

For the Lorentz-Drude spectrum, from Eq. (17), the current is given by

$$J_T = \sqrt{N-1}\pi T|\Gamma|^2e^{-\omega_d t}\{\tau[\cos(\tau t)]^{2(N-1)} - \tau(2N-3)\sin^2\tau t[\cos(\tau t)]^{2(N-2)} - \omega_d\sin(\tau t)[\cos(\tau t)^{2N-3}]\}. \tag{23}$$

It is interesting to note that when $t = 2n\pi/\tau$, $n = 1, 2, \ldots$, $\cos(\tau t) = 1$, $\sin(\tau t) = 0$, so we have

$$J_T = \sqrt{N-1}\tau\pi T|\Gamma|^2e^{-2\omega_d n\pi/\tau}, \tag{24}$$

i.e., $J_T \propto (N-1)^{\frac{1}{2}}$. Note that $[\cos(\tau t)]^N$ becomes a δ function for big N. The current will appear suddenly when $t = 2n\pi/\tau$, and disappear at other t, and this behavior will be modulated by the exponential decreasing which comes from the thermal bath. A larger bath size will absorb more energy for the PST couplings.

For an Ohmic spectrum at $t = 2n\pi/\tau$, from Eq. (18), $J_T \propto (N-1)^{\frac{1}{2}}$. For J_{TI} at a long-time limit, it is proportional to

$$\frac{\tau}{4}(N-1)[\cos 2\tau t(\cos\tau t)^{2N-6} - (2N-6)\sin\tau t\sin 2\tau t \times (\cos\tau t)^{2N-8}] - 2\tau(N-1)\sin\tau t[\cos\tau t]^{2(N-1)-1}.$$

Then there exists an oscillating quantum current J_{TI} even for the long-time limit in the PST couplings with an Ohmic bath, whose time average vanishes. The underlying physics could be a quantum effect similar to a persistent alternating electric current or the superfluid current in Ref. [28]. It might also be associated with pure dephasing models where the system-bath coupling commutes with the system Hamiltonian. For example, in the pure dephasing model [29] where a spin is coupled to a bosonic bath the same as that in our model, it is easy to check that there could be a temperature-independent oscillating quantum current even for the long-time limit. We mention that the system-bath couplings both in the pure dephasing and in our model are introduced in terms of essentially the same dressing transformation W. Since the system-bath couplings in the two models share the same origin, it seems to suggest that the temperature-independent currents from the two models have the same physical mechanisms. However, even so, special caution is still necessary in the interpretation of the long-time limit, because, different from the pure dephasing model, the system-bath coupling in our model does not commute with the system Hamiltonian.

For uniform couplings $\tau_i = \tau/2$, the transition amplitudes $f_{j,l}$ from site j to l are

$$f_{j,l} = \frac{2}{N+1}\sum_{m=1}^{N}\sin(q_m j)\sin(q_m l)e^{iE_m t}, \tag{25}$$

where $q_m = \pi m/(N+1)$, $E_m = -\tau\cos q_m$.

Note that the transition probability $f_{1,l}(t)$ is real when l is odd, and imaginary when l is even. That is a typical odd-even effect and it is a universal property for finite systems [30]. The more evident effects will be displayed for smaller N. Clearly, when $N = 2$, $\text{Re}(f^*_{1,1}f_{1,2}) = 0$.

When N is infinite, the transition amplitude can be calculated as $f_{1,1} = \frac{1}{2}[J_0(\tau t) + J_2(\tau t)]$, where $J_n(t)$ is the Bessel function of the first kind. When $l > 1$ and $n = 0, 1, 2, \ldots$,

$$f_{1,l} = \begin{cases} \frac{1}{\tau t}lJ_l(\tau t), & l = 4n+1, \\ -\frac{i}{\tau t}lJ_l(\tau t), & l = 4n+2, \\ -\frac{1}{\tau t}lJ_l(\tau t), & l = 4n+3, \\ \frac{i}{\tau t}lJ_l(\tau t), & l = 4n+4. \end{cases} \tag{26}$$

From the expression of $f_{1,1}$ and $f_{1,2}$, we can see that for both PST couplings and uniform couplings, $f_{1,1}$ is real and $f_{1,2}$ is imaginary. Then $\text{Re}[F(t)] \equiv 0$ $[F(t) = f^*_{1,1}f_{1,2}]$ in Eq. (14). Thus even if we do not consider the high temperature or low frequency, the last two terms can be neglected for PST or uniform couplings. Using an Ohmic spectrum for the bosonic bath as an example,

$$J_T \approx \frac{2\pi T|\Gamma|^2}{\omega_c}\left\{\frac{2J_1J_3 - J_2[J_0 - J_2]}{4\tau^2 t^5} + \frac{6J_1J_2}{\tau^2 t^6}\right\}, \tag{27}$$

$$J_{TI} \approx \frac{\pi|\Gamma|^2}{2}\left[\frac{3}{t^4}\left|f_{1,1}(t)\right|^2 + \frac{6}{\omega_c t^4} d(\left|f_{1,1}(t)\right|^2)/dt + \omega_c^3 d(\left|f_{1,1}(t)\right|^2 + \left|f_{1,2}(t)\right|^2)/dt\right]. \tag{28}$$

When $t \to \infty$, $J_n(t) \approx \sqrt{\frac{2}{\pi t}}\cos(t - \frac{n\pi}{2} - \frac{\pi}{4})$, so $J_T \propto 1/t^6 = (1/t^3)^2$, where the bath spectrum and chain's dynamics both contribute a factor of $1/t^3$. On the other hand, $J_{TI} \propto 1/t^3$ only, which corresponds to the dissipation in the uniform chain. Here, we would like to point out that when $N \to \infty$ and $t \to \infty$, it is shown that the asymptotic behaviors of J_T might be unclear due to complexities of the asymptotic processes, for instance, the order of taking limits $N \to \infty$ and $t \to \infty$. A simple analytical asymptotic expression of the PST couplings sheds light on the issue: $J_T \propto |\ \Gamma\ |^2 \sqrt{N}/t^3$ is allowed to be a finite value if $N \to \infty$ and $t^3 \to \infty$ share the same asymptotic speeds. The underlying physics for the possibility of the existence of this finite current and the relation between the asymptotic behavior and the Markovian assumption need further research. The ratio of $J_T(t)/J_{TI}(t) \approx \frac{T\tan(\tau t - \frac{\pi}{4})}{2\tau\omega_c^4 t^3}$ is proportional to temperature T, coupling intensity $1/\tau$, cutoff frequency $1/\omega_c^4$, and time $1/t^3$ modulated by a tan function.

VII. CONCLUSIONS

We analytically calculate the energy current between a bosonic bath and a fermionic chain. For our system, only an initial entanglement state in the chain can induce a temperature-dependent energy current. The energy current $J(t)$ depends on both the bosonic bath spectrum and the coupling mechanisms within the fermion chain. With respect to the effects of the bath spectrum, J_T will decrease to zero, with an exponential decay for the Lorentz-Drude type which is in accordance with the conventional Markovian approximation. On the other hand, it is proportional to $1/t^3$ for an Ohmic spectrum, and $1/t$ for white noise. For J_{TI}, the effect of the bath spectrum becomes divergent, $1/t^4$ and $1/t$, respectively. When different coupling configurations in the fermion chain are introduced, the envelope of the energy current will be modulated. For PST couplings, the oscillation is governed by a periodical function and for uniform couplings it is governed by the Bessel function of the first kind. Additionally, for PST couplings, with a Lorentz-Drude or Ohmic bosonic bath, J_T is found to be proportional to $(N-1)^{1/2}$ at certain times. This behavior can be interpreted as larger baths have the capacity to absorb more energy. Our work gives an exactly analytical expression for the energy current in hybrid nonlinear quantum structures.

ACKNOWLEDGMENTS

This material is based upon work supported by the National Natural Science Foundation of China (Grants No. 11475160, No. 61575180, No. 11575071), the Natural Science Foundation of Shandong Province (No. ZR2014AM023, No. ZR2014AQ026), the Basque Country Government (Grant No. IT986-16), and PGC2018-101355-B-I00 (MCIU/AEI/FEDER,UE).

[1] A. J. Leggett, S. Chakravarty, A. T. Dorsey, Matthew P. A. Fisher, Anupam Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
[2] A. O. Caldeira and A. J. Leggett, Ann. Phys. (NY) **149**, 374 (1983).
[3] P. Hedegard and A. O. Caldeira, Phys. Scr. **35**, 609 (1987).
[4] A. H. Castro Neto and A. O. Caldeira, Phys. Rev. E **48**, 4037 (1993).
[5] O. S. Duarte and A. O. Caldeira, Phys. Rev. Lett. **97**, 250601 (2006).
[6] D. M. Valente and A. O. Caldeira, Phys. Rev. A **81**, 012117 (2010).
[7] S. Bhattacharya, P. Chaudhury, S. Chattopadhyay, and J. R. Chaudhuri, Phys. Rev. E **78**, 021123 (2008).
[8] K. Le Hur, Ann. Phys. **323**, 2208 (2008).
[9] C. Wang, J. Ren, and J. Cao, Phys. Rev. A **95**, 023610 (2017).
[10] C. Vierheilig, J. Hausinger, and M. Grifoni, Phys. Rev. A **80**, 052331 (2009).
[11] D. Segal and A. Nitzan, Phys. Rev. Lett. **94**, 034301 (2005).
[12] L.-A. Wu, C. X. Yu, and D. Segal, New J. Phys. **15**, 023044 (2013).
[13] G. Schulze *et al.*, Phys. Rev. Lett. **100**, 136801 (2008).
[14] E. Pop, Nano Res. **3**, 147 (2010).
[15] L.-A. Wu and D. Segal, J. Phys. A **42**, 025302 (2009).
[16] J. Liu, C.-Y. Hsieh, and J. Cao, J. Chem. Phys. **148**, 234104 (2018).
[17] M. Ramezani, M. Golshani, and A. T. Rezakhani, Phys. Rev. E **97**, 042101 (2018).
[18] L.-A. Wu and D. Segal, Phys. Rev. E **83**, 051114 (2011).
[19] L.-A. Wu and D. Segal, Phys. Rev. Lett. **102**, 095503 (2009).
[20] F. Bonetto, J. Lebowitz, and L. Rey-Bellet, *Mathematical Physics 2000* (World Scientific, Singapore, 2000).
[21] E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. **16**, 407 (1961).
[22] L. Amico and A. Osterloh, J. Phys. A **37**, 291 (2004).
[23] L. Amico, A. Osterloh, F. Plastina, R. Fazio, and G. M. Palma, Phys. Rev. A **69**, 022304 (2004).
[24] L.-A. Wu, M. Byrd, Z. D. Wang, and B. Shao, Phys. Rev. A **82**, 052339 (2010).
[25] J. Thingna and J.-S. Wang, Europhys. Lett. **104**, 37006 (2013).
[26] L. C. Biedenharn and J. D. Louck, *Angular Momentum in Quantum Physics* (Addison-Wesley, Reading, MA, 1981).
[27] M. Christandl, N. Datta, A. Ekert, and A. J. Landahl, Phys. Rev. Lett. **92**, 187902 (2004).
[28] L. Kohn, P. Silvi, M. Gerster, M. Keck, R. Fazio, G. E. Santoro, and S. Montangero, Phys. Rev. A **101**, 023617 (2020).
[29] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, UK, 2002).
[30] L.-A. Wu and H. Toki, Phys. Lett. B **407**, 207 (1997).