Operational causality in spacetime

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The no-signaling principle preventing superluminal communication is a limiting paradigm for physical theories. Within the information-theoretic framework it is commonly understood in terms of admissible correlations in composite systems. Here we unveil its complementary incarnation—the "dynamical no-signaling principle"—which forbids superluminal signaling via measurements on simple physical objects (e.g., particles) evolving in time. We show that it imposes strong constraints on admissible models of dynamics. The posited principle is universal; it can be applied to any theory (classical, quantum, or postquantum) with well-defined rules for calculating detection statistics in spacetime. As an immediate application we show how one could exploit the Schrödinger equation to establish a fully operational superluminal protocol in the Minkowski spacetime. This example illustrates how the principle can be used to identify the limits of applicability of a given model of quantum or postquantum dynamics.

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I. INTRODUCTION

The problem of causality in quantum theory, ignited by the famous Einstein–Bohr debate [1,2], has long been a controversial topic. It took a few decades to realize that although quantum correlations are stronger than those available classically, they do not allow for superluminal transfer of any information [3,4]. The latter demand, known as the *no-signaling principle* is now recognized as an essential feature of any physical theory. It prevents the logical inconsistencies that might emerge from the incompatibility of correlations between spacelike-separated events with the causal structure of spacetime. While met in both classical and quantum physics, it turned out to leave room for postquantum theories [5–12].

Yet, there exists a second face of no-signaling connected with the inherent dynamics of simple physical systems, such as the time evolution of a single particle. In the domain of classical physics, the existence of tachyons (when allowed to interact with ordinary matter) would readily imply the possibility of superluminal communication. However, quantum particles do not have a definite position in space when propagating per se, so the 'classical' protocols of information transfer do not automatically apply. Furthermore, the quantum measurement effectuates a dramatic change in the particle's dynamics. Consequently, although several formal results [13–18] have suggested that quantum wave packets can propagate superluminally, it is unclear whether this implies operational faster-than-light communication (cf., for instance, [16] vs [19] or a more recent work [20]).

In this article we formulate the *dynamical no-signaling principle*, which says that one must not be able to exploit the inherent dynamics of any physical phenomenon for superluminal signaling. From this standpoint we identify the operational constraints on both dynamics and measurement schemes. The adopted formalism of measures on spacetime is very general and can be applied in any theory: classical, quantum, or postquantum. We show that, surprisingly, any conceivable dynamics must abide by a strong classical constraint related to the evolution of a pointlike particle's statistics (see Fig. 1). As a concrete application, we resolve the controversy around the purported (a)causality of wave-packet

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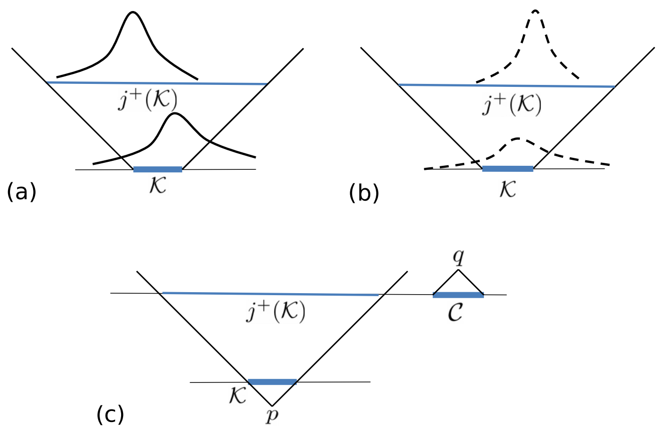

FIG. 1. Summary of the main result. (a) In classical physics, a causal propagation of a probability measure (modeling a spatially distributed physical quantity) must satisfy an optimal-transport theoretic condition, (3), which encodes the demand that infinitesimal portions of probability cannot move faster than light or—equivalently—that the probability 'mass' initially contained within a compact set $\mathcal{K}$ must stay in its causal future $j^+(\mathcal{K})$. (b) The propagation of a quantum wave packet gives rise to a certain "potential detection statistics," which does not exist objectively until an actual measurement. Nevertheless, the present result says—surprisingly—that the causal structure of spacetime coerces the same optimal-transport theoretic condition, (3), on the statistics of *unperformed* [sic] experiments. (c) The violation of condition (3) leads to an operational protocol of faster-than-light signaling. Namely, the free decision "to perform or not to perform" a measurement in the set $\mathcal{K}$ taken at a spacetime point p results in a change in the potential detection statistics in some other set $\mathcal{C}$ outside of the future of both set $\mathcal{K}$ and event p. Since this statistics can be collected at a later event q, this leads to statistical signaling between the spacelike-separated events p and q.

evolution. This is done, first, by demonstrating that the Schrödinger equation together with a local detection can lead to a logical paradox and, second, by recognizing the pertinence of the physical time and length scales of causality violation. The latter provide ultimate bounds on the domain of applicability of a given model of dynamics.

II. MEASUREMENTS AND COMMUNICATION IN SPACETIME

Our starting point is a spacetime $\mathcal{M}$ consisting of events. The latter are associated with classical (i.e., objective) information and form the basic elements of any information processing protocol (cf. [21]). The spacetime has an inbuilt causal structure, that is, a partial order relation $\preceq$ specifying which events might influence one another. If such an influence between two events is precluded, we say that they are *spacelike separated*. Furthermore, a spacetime must admit (at least locally) a splitting $\mathcal{M} \cong I \times \mathcal{S}$ into time range I and space $\mathcal{S}$. This splitting need not be unique and typically depends on the choice of the reference frame.

It is standard to accept that the causal structure of spacetime is defined by the constant speed of light in vacuum, but other options are also conceivable (cf., for instance, [22]). In any case, we say that a dynamical law is *acausal* or *superluminal* if it implies influences between spacelike-separated events. The incompatibility of dynamics with the assumed casual structure typically leads to logical paradoxes, which spoil the model's consistency (see Fig. 4 for an explicit example).

The simplest two-party communication protocol entails two definite events: signal sending p and signal reception q. The sender (Alice) is active; she can freely choose the bit she desires to communicate. On the other hand, the receiver (Bob) passively gathers the incoming information. The communication is effective if Alice's free choice changes Bob's detection statistics registered during the event q. Clearly, if the change in Bob's statistics is tiny, then Alice should send multiple copies of her signal. The communication is called superluminal if q is not in the causal future of p.

A probability measure μ supported on $\mathcal{S}_t := \{t\} \times \mathcal{S}$ is a natural mathematical object tailored to model the statistics of a basic binary measurement at a given time $t \in I$. Indeed, if a detector or an array of detectors is located at time t in a compact region of space $\mathcal{K} \subset \mathcal{S}_t$, μ yields a concrete number, $\mu(\mathcal{K}) \in [0, 1]$. The compactness of $\mathcal{K}$ reflects the demand of the locality of the measuring device (cf. [23,24]).

Let us stress that the "detection" is purely operational and signifies a detector click—which constitutes an objective bit of information—registered in region $\mathcal{K}$. The interpretation is secondary and depends on the theory. For instance, if the signal had been carried by a classical-like particle, the click would mean that the detection was an actual event located somewhere in $\mathcal{K}$. Had it been a quantum particle instead, the only admissible conclusion would be that the particle was not outside of $\mathcal{K}$ at moment t with certainty.

The time evolution of a probability measure on a spacetime $\mathcal{M}$ is defined [18,25] as a map $t \mapsto \mu_t$ such that $\operatorname{supp} \mu_t \subset \mathcal{S}_t$. It models the time evolution of *purely potential* detection statistics associated with the dynamics of a simple physical system, such as a propagating quantum particle. The number $\mu_t(\mathcal{K})$ answers the question, What is the probability of signal detection *if* a detector covering a region $\mathcal{K}$ is switched on at a moment in time t? The basic signaling task requires two local operations, so only the initial and final measures—$\mu := \mu_s$ and $\nu := \mu_t$, respectively—are relevant.

Consider now an actual measurement checking whether the signal at an initial time s is within the compact set $\mathcal{K}$. The impact of the measurement on the final measure at some later time t is taken into account with the help of conditional measures

$$\nu(\,\cdot\,|m_{\mathcal{K}}), \quad m_{\mathcal{K}} \in \{0, 1\},$$

where $m_{\mathcal{K}} = 1$ ($m_{\mathcal{K}} = 0$) corresponds to the situation where the measurement has been (has not been) performed at time s. The statistics of the measurement $\{P(r|m_{\mathcal{K}})\}$ with the possible results $r \in \{+, -, \emptyset\}$ (corresponding to "signal detected," "signal not detected," and "not applicable," respectively) satisfies the rules:

$$\begin{aligned} &P(+|1) = \mu(\mathcal{K}), && P(-|1) = 1 - \mu(\mathcal{K}), && P(\emptyset|1) = 0, \\ &P(+|0) = 0, && P(-|0) = 0, && P(\emptyset|0) = 1. \end{aligned} \tag{1}$$

Another consistency condition,

$$\nu(\cdot|0) = \nu, \tag{2}$$

must hold, because the absence of the measurement does not disturb the dynamics.

III. CAUSAL EVOLUTION OF STATISTICS

A classical particle is bound to travel along a future-directed causal curve. In other words, its propagation marks a one-parameter family of events p_t for $t \in I$, with $p_s \preceq p_t$ for all $s \leqslant t$. Because classical particles carry objective information, any "tachyon" violating the causal order $\preceq$ transfers information to a forbidden region of spacetime, eventually resulting in the logical inconsistency of the model.

This classical picture has been formally extended in [18,26] to the measure-theoretic setting [see Fig. 1(a)].

Causal evolution (CE) condition. The inequality

$$\mu_s(\mathcal{K}) \leqslant \mu_t(j^+(\mathcal{K})), \tag{3}$$

with $j^+(\mathcal{K}) := J^+(\mathcal{K}) \cap \mathcal{S}_t$, must hold for all compact $\mathcal{K} \subset \operatorname{supp} \mu_s$ and for all $s \leqslant t$.

The CE condition is a relativistic invariant, i.e., it does not depend on the adopted splitting $\mathcal{M} \cong I \times \mathcal{S}$ [25]. It encodes, via the theory of optimal transport (cf. [25,26]), the following classical intuition:

Each infinitesimal part of the probability distribution must travel along a future-directed causal curve.

This demand has a clear justification when μ models the statistical distribution of an ensemble of classical particles, e.g., dust or fluid. In this context, condition (3) grasps the demand that none of the elements constituting the considered physical medium can propagate superluminally.

However, it seems unlikely that CE might be the proper incarnation of the no-signaling principle beyond the classical realm. First, the mere fact of the existence (and of dynamical emergence) of statistical dependencies between spacelike-separated parts of a physical system does not necessarily imply the possibility of superluminal information transfer, either in quantum mechanics or in a "postquantum" theory with stronger correlations [5]. Second, CE treats the detection statistics *as if they objectively existed*, whereas in quantum mechanics what evolves is purely potential—"nonexisting"—statistics. Furthermore, CE makes no reference to the actual detection process, which, in quantum theory, does change the signal's dynamics. Finally, there is no straightforward connection between the violation of CE for some set $\mathcal{K}$ and a physical signaling process, which involves two definite events, i.e., points of $\mathcal{M}$ rather than sets.

Curiously enough, it turns out that the violation of CE, when complemented by a minimalistic detection scheme, *always* leads to operational superluminal communication. This surprising conclusion holds independently of whether or not the detection statistics actually exists before the measurement.

IV. DYNAMICAL NO-SIGNALING

Many of the physical detection events involve the demolition of the signal carrier—most notably, the photon absorption in a silicon detector. After such an event, the recorded information "click" or "no click" is objective, hence the following demand is indispensable.

Axiom 1 (A1). If the signal has been detected ($r = +$) at time s in region $\mathcal{K}$ ($m_{\mathcal{K}} = 1$), then it must be present with certainty in that region's future $j^+(\mathcal{K})$ for any later time t:

$$\nu(j^+(\mathcal{K})|+, 1) = 1. \tag{4}$$

Let us stress that the adopted detection scheme includes the von Neumann measurement but is not limited to it. It can be applied equally well in a nondemolition scenario (cf., for instance, [27]), in which case after the detection the particle (or whatever signal carrier one considers) might undergo an entirely different dynamics than before.

The intuition that the detection process in a region $\mathcal{K}$ must not affect the (potential) statistics outside of $J^+(\mathcal{K})$ is formalized with the help of conditional measures, as follows.

Dynamical no-signaling condition (NS). For any compact $\mathcal{C} \subset \mathcal{S}_t \setminus j^+(\mathcal{K})$,

$$\nu(\mathcal{C}\,|\,1) = \nu(\mathcal{C}\,|\,0). \tag{5}$$

Strikingly, it turns out that the intuitive Axiom 1 and condition NS jointly imply that the dynamics of measures must obey the strong classical-like constraint CE. Equivalently,

Proposition 1. Under the assumption of A1, the violation of CE entails the violation of NS.

The somewhat technical proofs of Proposition 1, and of Theorem 2 below, are included in Appendix A.

Condition NS, while intuitive, entails information flow from *region* $\mathcal{K}$ *to region* $\mathcal{C}$, whereas operational signaling requires definite events. Nevertheless, it turns out that the violation of NS can always be exploited for a superluminal communication between two strictly local agents.

Theorem 2. If NS is violated, the set $\mathcal{C}$ for which (5) does not hold can always be chosen so that there exist spacetime points $q, p_1, \ldots, p_k$ such that

$$\mathcal{K} \subset \bigcup_{i=1}^{k} J^+(p_i),\; \mathcal{C} \subset J^-(q),\; \text{and}\; p_i \npreceq q,\; i = 1, \ldots, k.$$

The essence of Theorem 2 is illustrated in Fig. 1(c). In order to send a superluminal signal we first fill both region $\mathcal{K}$ and region $\mathcal{C}$ with detectors. The violation of NS implies that the measurement effectuated by devices in $\mathcal{K}$ changes the detection probability in $\mathcal{C}$. However, to actually execute the (statistical) superluminal signaling we would need, first, to orchestrate the measurement in $\mathcal{K}$ and, second, to gather the statistical information from $\mathcal{C}$. This amounts to the existence of sending event(s) $p_1, \ldots, p_k$ and a readout event q, such that $p_i \npreceq q$ for all i.

In the simplest scenario ($k = 1$) there is a single sending event and so statistical signaling from p_1 to the spacelike-separated q is straightforward. Theorem 2 says that the set $\mathcal{C}$ can always be chosen in such a way that the readout is performed at a single event q. This is exactly the situation depicted in Fig. 1(c).

On the other hand, set $\mathcal{K}$ might have a more complicated shape (see Appendix B for an example), in which case $k > 1$ sending events are needed. For concreteness let us assume that $k = 2$, as in Fig. 2. Observe first that a measurement

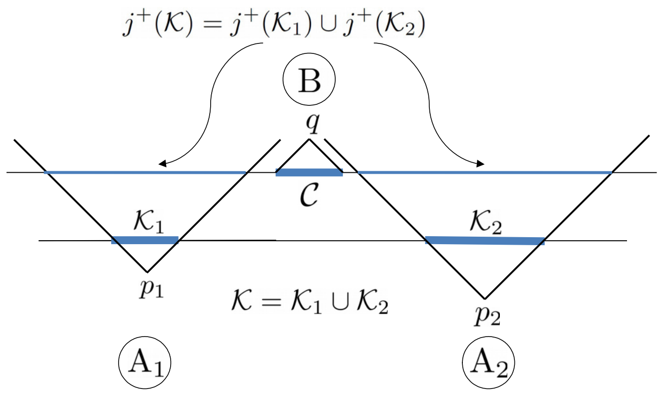

FIG. 2. Diagram illustrating an instance of Theorem 2 with two senders: a passive one, Alice 1 (A_1), and an active one, Alice 2 (A_2).

performed in just one of the regions, $\mathcal{K}_1$ or $\mathcal{K}_2$, does not lead to the violation of NS, for if it were to do so, a single sending event would suffice. Consequently, we can assume that one of the senders, say Alice 1, is passive and has an always-on detector ($m_{\mathcal{K}_1} = 1$). The active sender, Alice 2, decides whether or not to perform a measurement ($m_{\mathcal{K}_2} = 1$ or 0), which fixes the value of the communicated bit because $m_{\mathcal{K}} = m_{\mathcal{K}_1} \cdot m_{\mathcal{K}_2} = m_{\mathcal{K}_2}$. The value of this bit can be statistically inferred by Bob from the difference between $\nu(\mathcal{C} \,|\, m_{\mathcal{K}} = 1)$ and $\nu(\mathcal{C} \,|\, m_{\mathcal{K}} = 0)$. The generalization to $k > 2$ senders is straightforward.

Conclusion. A violation of NS always has operational consequences; it enables a protocol suitable for operational superluminal communication.

We have presented the constraint on free evolution of potential detection statistics and the admissible change of measure on the positive result of the detection by Alice. For completeness, let us now discuss the effect of a negative result of the detection.

V. COMPLEMENTARY AXIOM AND THE TRIAD OF INTERRELATED CONDITIONS

Just as Axiom 1 puts constraints on the possible evolution of the measure $\mu(\,\cdot\,|+,1)$, i.e., the measure conditioned on the *positive* result of the detection measurement in $\mathcal{K}$, the following condition deals with the measure $\mu(\,\cdot\,|-,1)$, i.e., conditioned on the *negative* result.

Axiom 2 (A2). If the signal has *not* been detected at time s within $\mathcal{K}$, then outside of $J^+(\mathcal{K})$ the evolution of $\mu(\,\cdot\,|-,1)$ proceeds with no modification other than renormalization,

$$\nu(\mathcal{C}\,|-,1) = \frac{\nu(\mathcal{C})}{1-\mu(\mathcal{K})}, \tag{6}$$

for any compact $\mathcal{C} \subset \mathcal{S}_t \setminus j^+(\mathcal{K})$.

This axiom can be intuitively justified as follows: Immediately after the measurement, the result of which was positive, we must have $\mu(\mathcal{K}|+,1) = 1$, and thus $\mu(\,\cdot\,|+,1)$ is zero outside of $\mathcal{K}$. Moreover, irrespective of the measurement's result, the statistics of the potential measurements outside of $\mathcal{K}$ must remain unchanged so as not to allow for instantaneous signaling, i.e., $\mu(\mathcal{K}'|1) = \mu(\mathcal{K}'\,|\,0)$ for every $\mathcal{K}' \subset \mathcal{S}_s \setminus \mathcal{K}$.

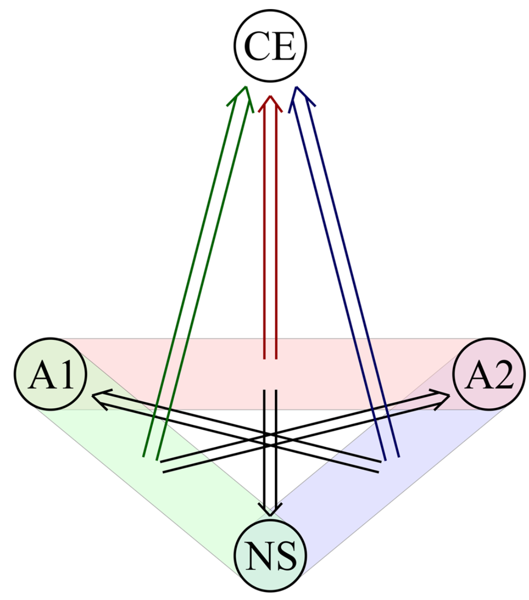

FIG. 3. Diagram illustrating Theorem 3.

Altogether, using subsequently (2), (5), and (1), one gets

$$\begin{aligned}\mu(\mathcal{K}') &= \mu(\mathcal{K}'|0) = \mu(\mathcal{K}'|1)\\ &= \underbrace{\mu(\mathcal{K}'|+,1)}_{=0} P(+|1) + \mu(\mathcal{K}'|-,1)P(-|1)\\ &= \mu(\mathcal{K}'|-,1)(1-\mu(\mathcal{K})).\end{aligned}$$

We see that $\mu(\mathcal{K}'|-,1) = \mu(\mathcal{K}')/(1-\mu(\mathcal{K}))$ for any $\mathcal{K}'$ disjoint with $\mathcal{K}$. Since the evolution outside of $J^+(\mathcal{K})$ should not be altered, this formula 'time-evolves' into (6).

The three conditions A1, A2, and NS, together with the overarching constraint CE, turn out to enjoy an intimate logical interplay captured by the following result (see Fig. 3).

Theorem 3. If any two conditions from the set {NS, A1, A2} hold true, then the third one and CE hold true as well.

In fact, Theorem 3 exhausts the logical dependencies between the four conditions considered. More rigorously speaking: Any combination of the true-false values assigned to the conditions NS, A1, A2, and CE which is not deemed impossible by Theorem 3 can be realized with suitably defined measures μ, ν, and $\nu(\cdot|\pm,1)$. For more details, as well as for the proof of Theorem 3, the reader is invited to consult Appendix A.

VI. AN ILLUSTRATION: LOGICAL PARADOXES FROM QUANTUM WAVE DYNAMICS

Consider two observers, active Alice (A) and passive Bob (B), in the Minkowski spacetime. They have at their disposal a quantum wave packet following a unitary evolution driven by a Hamiltonian $\hat{H}$. Bob has a binary (always-on) detector covering a region $\mathcal{C}$. Now, consider two situations.

(i) Suppose first that Alice can emit (prepare) a wave packet ψ_0 compactly supported within region $\mathcal{K}$, spacelike separated with $\mathcal{C}$. She decides (event p) to emit it ($m_{\mathcal{K}} = 1$) or not to emit it ($m_{\mathcal{K}} = 0$). According to Hegerfeldt's theorem [13,14] if the Hamiltonian $\hat{H}$ is bounded from below, the

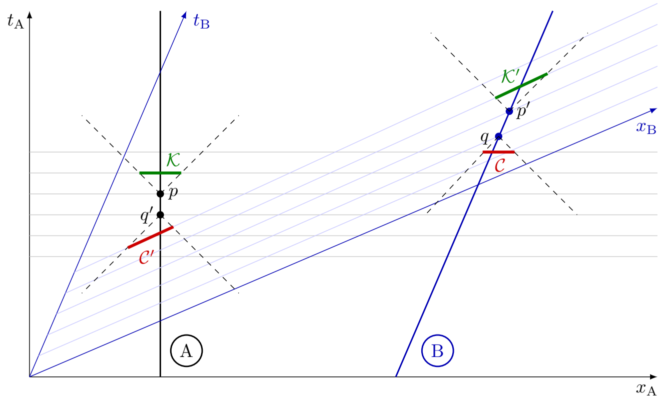

FIG. 4. Logical paradoxes from superluminal wave-packet spreading in the Minkowski spacetime.

initial wave packet ψ_0 immediately becomes spread over the entire space. Consequently, A1 is violated. Moreover, CE is violated, because $\mu(\mathcal{K}) = 1$ and $\nu(j^+(\mathcal{K})) < 1$. Finally, NS is also violated because $\nu(\mathcal{C}\,|\,0) = 0$ (for the wave packet has not been emitted in the first place), but $\nu(\mathcal{C}\,|\,1) > 0$.

(ii) Suppose now that Alice cannot prepare localized states, but she can collapse (event $m_{\mathcal{K}} = 1$) a preexisting quantum wave packet ψ_0 supported over the entire space $\mathcal{S}_s$. Alice registers a click ($m_{\mathcal{K}} = 1,\ r = +$) with probability $\mu(\mathcal{K}) < 1$, in which case $\nu(\mathcal{C}\,|+,\ 1) = 0$, because the particle is no more. In this situation, Axiom 1 is readily verified. However, if Alice's detector fails to click, although it is on ($m_{\mathcal{K}} = 1,\ r = -$), Bob has a nonvanishing probability of detecting it, $\nu(\mathcal{C}\,|-,\ 1) > 0$. If the dynamics is such that $\nu(\mathcal{C}\,|\,0) = (1-\mu(\mathcal{K}))\nu(\mathcal{C}\,|-,\ 1) = \nu(\mathcal{C}\,|\,1)$ (which is exactly Axiom 2), then there is no superluminal signaling, because Bob cannot statistically distinguish the situations ($m_{\mathcal{K}} = 1$) and ($m_{\mathcal{K}} = 0$). However, if the dynamics violates CE, then—by Proposition 1—NS is also violated. Consequently, Axiom 2 fails as well.

In either case, if Alice can statistically signal to Bob, she might fall into a logical paradox. Indeed, suppose that Alice is free-falling and Bob travels away from Alice at a constant relativistic velocity (see Fig. 4). Alice triggers the superluminal protocol (event p, $m_{\mathcal{K}} = 1$). If the bit $m_{\mathcal{K}}$ is successfully communicated to region $\mathcal{C}$ spacelike separated with $\mathcal{K}$, then Bob (automatically) triggers his protocol (event p', $m_{\mathcal{K}'} = 1$). As a consequence, the bit $m_{\mathcal{K}'}$ is in turn statistically communicated to region $\mathcal{C}'$ spacelike separated with $\mathcal{K}'$. If the signaling succeeds, then Alice receives the bit $m_{\mathcal{K}'}$ (event q'). But because $q' \preceq p$, such an event might, for instance, destroy Alice's laboratory, thus extorting $m_{\mathcal{K}} = 0$.

Note that this scenario is covariant in the sense that if we interchange the roles of Alice and Bob, Bob can (statistically) effectuate a causal loop $p' \rightsquigarrow q$ with the passive help of Alice.

The problem(s) with situation (i) are well known and are usually bypassed by a QFT theorem on the nonexistence of strictly localized states [28]. On the other hand, situation (ii) (analyzed in [15] and [26], but without the key role of active measurements) harmonizes with the view that whereas quantum states are inherently nonlocal, the operations are local [23,24]. In this context, it is rather surprising to learn [26] that the unitary evolution under the standard free Hamiltonian $\hat{H} = \hat{p}^2/2m$ *does violate* the CE condition. The fact that the *relativistic* Hamiltonian $\sqrt{\hat{p}^2 + m^2}$ also implies the violation of CE [26] (cf. also [15]) is even more abstruse. In view of the generality of Hegerfeldt's result, one might expect that the violation of CE by quantum wave dynamics is generic. The notable exceptions are, e.g., Dirac and photon wave function (Maxwell) equations, which guarantee CE for any (even compactly supported) initial state [26]. Does this mean that all other quantum wave dynamics, including the standard Schrödinger equation, should be discarded as entailing logical paradoxes?

The appeasement comes from the consideration of the *characteristic scales* of causality violation. Indeed, the explicit account of the spacetime aspect naturally establishes the time scale $t - s$ of the signaling time lapse, the size vol $\mathcal{K}$ of signaling devices, and the "capacity of the superluminal channel" $\max\{0, \nu(\mathcal{C}|0) - \nu(\mathcal{C}|1)\}$.

For example (see [18]), one can safely use the relativistic Hamiltonian $\sqrt{\hat{p}^2 + m^2}$ for modeling the dynamics of a single quantum particle. Indeed, the causality violation effects in this model are transient and restricted to a region of the size of the particle's Compton wavelength—a regime in which the quantum nature of the vacuum can no longer be neglected. On the other hand, the superluminal spreading of a Gaussian wave packet driven by the nonrelativistic Hamiltonian $\hat{p}^2/(2m)$ induces immediate and persistent causality violation effects for $\mathcal{K} = [-\ell, \ell]^3$, with $\ell > \frac{cm\lambda^2}{t\hbar^2}(m\lambda^2 + \sqrt{m^2\lambda^4 + t^2\hbar^2})$, where λ is the width of the initial wave packet. Nevertheless, we observe that the minimal scale of causality violation in this model increases with m and λ, even in the limit of an infinite time lapse. Consequently, for instance, at the scales characteristic for a Bose–Einstein condensate ($m \sim 10^{-26}$ kg, $\lambda \sim 1\,\mu$m), the superluminal spreading would manifest itself only for $\ell \gtrsim 30$ km.

VII. DISCUSSION

We have shown that the embedding of information processing protocols within a spacetime unravels the "dynamical no-signaling principle," which is complementary to the one exploiting correlations. When a minimalistic assumption about the measurement scheme is adopted, the principle coerces a strong constraint on the dynamics of detection statistics. Although the latter embodies the concept of transport along causal curves taken from classical physics, it applies to *any* model within quantum theory or even beyond.

When applied to quantum wave dynamics the unveiled principle leads to an arresting consequence: *The Schrödinger equation facilitates operational superluminal signaling via local detection.* Our finding reinforces and extends the earlier claims around Hegerfeldt's theorem (cf., for instance, [29]) with the help of an explicit communication protocol involving a logical paradox. On the other hand, the adopted formalism justifies the critique of the applicability of Hegerfeldt's theorem raised from a QFT standpoint [19]. It does so by drawing attention to the characteristic scales of causality violation.

In conclusion, we put forward a way to assess the credibility of physical theories: One needs to be able to compute the local detection statistics in an (effective) spacetime, but no details on the dynamics are prerequisite. On the theoretical side, it presents a challenge to understanding the characteristic physical scales of information-theoretic-inspired postquantum theories, such as [5–9,11,12]. On the practical side, it offers a powerful method for exploring the limitations of competing models of quantum wave dynamics, possibly nonunitary and/or nonlinear.

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APPENDIX A: PROOFS

1. Proof of Proposition 1

Any Borel probability measure η living in a Polish space $\mathcal{M}$ (that is a separable completely metrizable topological space) is *tight*, by which we mean that

$$\eta(\mathcal{B}) := \sup\{\eta(\mathcal{C})|\ \mathcal{C} \subset \mathcal{B}, \mathcal{C} - \text{compact}\}$$

for any measurable $\mathcal{B} \subset \mathcal{M}$ (for details, see [30, chap. 12]). The tightness property entails the following lemma, which is instrumental in proving Proposition 1.

Lemma 4. Suppose that two measures η_1 and η_2 in $\mathcal{M}$ satisfy

$$\eta_1(\mathcal{B}) < \eta_2(\mathcal{B})$$

for some measurable set $\mathcal{B} \subset \mathcal{M}$. Then there exists a *compact* subset $\mathcal{C} \subset \mathcal{B}$ for which the above inequality is still valid, i.e.,

$$\eta_1(\mathcal{C}) < \eta_2(\mathcal{C}).$$

Observe that $\mathcal{C}$ must be nonempty.

Proof. Suppose, on the contrary, that $\eta_1(\mathcal{C}) \geqslant \eta_2(\mathcal{C})$ for all compact subsets $\mathcal{C} \subset \mathcal{B}$. Then, of course, the same concerns the suprema: $\sup\{\eta_1(\mathcal{C})|\ \mathcal{C} \subset \mathcal{B}, \mathcal{C} - \text{compact}\} \geqslant \sup\{\eta_2(\mathcal{C})|\ \mathcal{C} \subset \mathcal{B}, \mathcal{C} - \text{compact}\}$. But on the strength of the tightness property, this yields $\eta_1(\mathcal{B}) \geqslant \eta_2(\mathcal{B})$. ■

Moving now to the proof of Proposition 1, suppose that the CE condition, (3), is violated, i.e., that there exist time instants s and t with $s \leqslant t$ and a compact set $\mathcal{K} \subset \operatorname{supp} \mu_s$ such that

$$\mu_s(\mathcal{K}) > \mu_t(j^+(\mathcal{K}))$$

or, switching to the more convenient notation $\mu := \mu_s$ and $\nu := \mu_t$, that

$$\mu(\mathcal{K}) > \nu(j^+(\mathcal{K})). \tag{A1}$$

Our aim is to find a compact $\mathcal{C} \subset \mathcal{S}_t \setminus j^+(\mathcal{K})$ violating the NS condition, (5). To this end, observe first that

$$\begin{aligned} \nu(j^+(\mathcal{K})|1) &= \sum_{r \in \{+,-,\emptyset\}} \nu(j^+(\mathcal{K})|r, 1)P(r|1) \\ &= 1 \cdot \mu(\mathcal{K}) + \nu(j^+(\mathcal{K})|-, 1)(1 - \mu(\mathcal{K})) \geqslant \mu(\mathcal{K}) \\ &> \nu(j^+(\mathcal{K})) = \nu(j^+(\mathcal{K})|0), \end{aligned}$$

where we have employed the law of total probability, consistency conditions (1), Axiom 1, (A1), and, finally, consistency condition (2). Passing to the complement, we thus have

$$\nu(\mathcal{S}_t \setminus j^+(\mathcal{K})|1) < \nu(\mathcal{S}_t \setminus j^+(\mathcal{K})|0).$$

By Lemma 4, there exists a compact subset $\mathcal{C} \subset \mathcal{S}_t \setminus j^+(\mathcal{K})$ for which the above inequality remains valid, i.e.,

$$\nu(\mathcal{C}|1) < \nu(\mathcal{C}|0). \tag{A2}$$

2. Proof of Theorem 2

The proof is somewhat technical and relies on certain notions from Lorentzian causality theory, which we briefly recall for the reader's convenience. Namely, for any event $p \in \mathcal{M}$ the following sets are considered: $J^+(p)$, $J^-(p)$, $I^+(p)$, and $I^-(p)$, called the causal future, the causal past, the chronological future, and the chronological past of p, respectively. Although the precise definition of these sets is not important here, the crucial property for the following proof is that the sets $I^\pm(p)$ are always open (topologically) and contained in $J^\pm(p)$. Moreover, we need to assume that the causal future of any compact set is topologically closed. This is true, in particular, if $\mathcal{M}$ is a globally hyperbolic manifold. For an excellent exposition of causality theory, the reader is referred to [31].

Moving to the actual proof, let $\mathcal{C} \subset \mathcal{S}_t \setminus j^+(\mathcal{K})$ be the (nonempty) set satisfying (A2) and, thus, violating the NS condition, (5). In the first three steps of the proof we construct a compact subset $\mathcal{C}' \subset \mathcal{C}$ still violating NS, together with $q \in \mathcal{M} \setminus J^+(\mathcal{K})$ such that $\mathcal{C}' \subset J^-(p)$. Then, in the last step, we show how to find $p_1, \ldots, p_k \in \mathcal{M} \setminus J^-(q)$ (for some $k \in \mathbb{N}$) such that $\mathcal{K} \subset \bigcup_{i=1}^k J^+(p_i)$.

a. Step 1

Consider the family $\{I^-(q)\}_{q \in \mathcal{M} \setminus J^+(\mathcal{K})}$ of open subsets of $\mathcal{M}$. We claim that it covers $\mathcal{C}$, i.e., that

$$\forall p \in \mathcal{C}\ \exists q \in \mathcal{M} \setminus J^+(\mathcal{K}), \quad p \in I^-(q).$$

Indeed, assuming the contrary, we would have that

$$\exists p \in \mathcal{C}\ \forall q \in \mathcal{M} \setminus J^+(\mathcal{K}), \quad p \notin I^-(q),$$

which in fact can be equivalently written as

$$\exists p \in \mathcal{C}, \quad I^+(p) \subset J^+(\mathcal{K}).$$

But since p lies in the closure of $I^+(p)$ and set $J^+(\mathcal{K})$ is closed [31], we obtain that $\mathcal{C} \cap J^+(\mathcal{K}) \neq \emptyset$, in contradiction to the inclusion $\mathcal{C} \subset \mathcal{S}_t \setminus j^+(\mathcal{K}) = \mathcal{S}_t \setminus J^+(\mathcal{K})$.

b. Step 2

Since $\mathcal{C}$ is compact, there exists a finite subcover $\{I^-(q_i)\}_{i \in F}$, where F is a finite set of indices. However, for a technical reason to become clear soon, we need a pairwise disjoint refinement of this subcover. To this end, one might construct the family $\mathcal{U} := \{U_S\}$, where the index S runs over all nonempty *subsets* of F, by defining

$$\begin{aligned} U_S &:= \bigcap_{i \in S} I^-(q_i) \setminus \bigcup_{j \in F \setminus S} I^-(q_j) \\ &= \{p \in \mathcal{M} \mid \forall i \in \{1, \ldots, l\}, \quad p \in I^-(q_i) \Leftrightarrow i \in S\}. \end{aligned}$$

Observe that every U_S is measurable. Clearly, $\mathcal{U}$ thus defined is a pairwise disjoint family of sets which covers $\mathcal{C}$. That it is also a refinement of the cover $\{I^-(q_i)\}_{i\in F}$ stems from the fact that every S is nonempty, and hence every U_S is contained in at least one of the $I^-(q_i)$'s. We now claim that

$$\exists\, \emptyset \neq S^* \subset F, \quad \nu(\mathcal{C}\cap U_{S^*}|1) < \nu(\mathcal{C}\cap U_{S^*}|0).$$

Indeed, assuming that $\nu(\mathcal{C}\cap U_S|1) \geqslant \nu(\mathcal{C}\cap U_S|0)$ for all nonempty $S\subset F$, one would get

$$\nu(\mathcal{C}|1) = \sum_{\emptyset\neq S\subset F} \nu(\mathcal{C}\cap U_S|1) \geqslant \sum_{\emptyset\neq S\subset F} \nu(\mathcal{C}\cap U_S|0) = \nu(\mathcal{C}|0),$$

in contradiction to inequality (A2). It is at this step that we need the cover to be pairwise disjoint; otherwise, we would not be able to use the measures' additivity property.

c. Step 3

Invoking the above lemma, we obtain the existence of a compact $\mathcal{C}'\subset\mathcal{C}\cap U_{S^*}$ such that

$$\nu(\mathcal{C}'|1) < \nu(\mathcal{C}'|0).$$

Moreover, picking any $i\in S^*$, we get $\mathcal{C}'\subset U_{S^*}\subset I^-(q_i)$ with $q_i\in\mathcal{M}\setminus J^+(\mathcal{K})$, because only such events were involved in the construction of the original open cover in Step 2. Of course, we now set $q =: q_i$. Since $I^-(q)$ is contained in $J^-(q)$, the first part of the proof is complete.

d. Step 4

Consider now the family $\{I^+(p)\}_{p\in\mathcal{M}\setminus J^-(q)}$. We claim that it is an open cover of $\mathcal{K}$. Indeed, assuming the contrary and proceeding analogously as in Step 1, one would obtain that $\mathcal{K}\cap J^-(q)\neq\emptyset$, in contradiction with how q was defined. By the compactness of $\mathcal{K}$, one can now take the finite subcover $\{I^+(p_1),\dots,I^+(p_k)\}$. Observe that the spacetime points p_i satisfy $p_i \npreceq q$, for $i=1,\dots,k$, as desired. Moreover, we have that $\mathcal{K}\subset\bigcup_{i=1}^k I^+(p_i)\subset\bigcup_{i=1}^k J^+(p_i)$, which completes the entire proof. ■

3. Proof of Theorem 3

In what follows, $\mathcal{C}$ is always understood as being bound by the quantifier "for any compact $\mathcal{C}\subset\mathcal{S}_t\setminus j^+(\mathcal{K})$." On the strength of consistency condition (2), the NS condition, (5), can be written as

$$\nu(\mathcal{C}\,|\,1) = \nu(\mathcal{C}). \tag{A3}$$

What is more, Axiom 1 can be reexpressed as

$$\nu(\mathcal{C}\,|+,1) = 0. \tag{A4}$$

Finally, with the help of one of the consistency conditions (1), we also rewrite Axiom 2 as

$$\nu(\mathcal{C}\,|-,1)P(-|1) = \nu(\mathcal{C}). \tag{A5}$$

Invoking the law of total probability $\nu(\mathcal{C}\,|\,1) = \nu(\mathcal{C}\,|+,1)P(+|1)+\nu(\mathcal{C}\,|-,1)P(-|1)$, one can now easily observe that any two equalities from (A3)–(A5) imply the third one, which completes the first part of the proof.

TABLE I. Combinations of truth values assigned to conditions NS, A1, A2, and CE not precluded by Theorem 3, together with sample values of parameters A, B, and C realizing them.

NS	A1	A2	CE	Sample (A,B,C)
T	T	T	T	(1,1,1)
F	F	T	T	(1,0,1)
F	T	F	T	(1,1,0)
T	F	F	T	$(\frac{2}{3},\frac{1}{3},0)$
F	F	F	T	(1,0,0)
F	T	F	F	(0, 1, 0), (0, 1, 1)
T	F	F	F	(0,0,0)
F	F	F	F	(0,0,1)

It now suffices to show, e.g., that Axiom 2 implies CE. Indeed, plugging $\mathcal{C} := \mathcal{S}_t\setminus j^+(\mathcal{K})$ into (6) we obtain that

$$\frac{\nu(\mathcal{S}_t\setminus j^+(\mathcal{K}))}{1-\mu(\mathcal{K})} = \nu(\mathcal{S}_t\setminus j^+(\mathcal{K})\,|\,-,1)\leqslant 1$$

and hence

$$1-\nu(j^+(\mathcal{K}))\leqslant 1-\mu(\mathcal{K}),$$

which is equivalent to inequality (3). This completes the proof of Theorem 3. ■

Finally, let us demonstrate that Theorem 3 completely describes the logical relations between the four considered conditions. Namely, we find concrete realizations of all the logical situations not excluded by Theorem 3.

To this end, let us fix $p,q\in\mathcal{S}_s$ and $p',q'\in\mathcal{S}_t$ such that $p\preceq p'$, $q\preceq p'$, $q\preceq q'$, and $p\npreceq q'$. Let us consider the family of discrete (one- or two-point) measures

$$\begin{aligned}\mu &:= \frac{1}{2}\delta_p + \frac{1}{2}\delta_q,\\ \nu &:= A\delta_{p'} + (1-A)\delta_{q'},\\ \nu(\cdot\,|+,1) &:= B\delta_{p'} + (1-B)\delta_{q'},\\ \nu(\cdot\,|-,1) &:= C\delta_{p'} + (1-C)\delta_{q'},\end{aligned}$$

where $A,B,C\in[0,1]$ are parameters. Without much effort one can convince oneself that, for such defined measures, the four conditions considered here amount to

$$\text{NS:}\quad 2A = B + C,\qquad \text{A1:}\ \ B = 1,$$
$$\text{CE:}\quad 2A\geqslant 1,\qquad\ \ \text{A2:}\ \ 2A = 1 + C.$$

Now, it is not difficult to find sample values of the parameter triple (A,B,C) providing a realization of each combination of truth values assigned to the four conditions that is not excluded by Theorem 3. Table I sums this up, with T and F denoting "true" and "false," respectively.

APPENDIX B: EXAMPLE OF A MULTISENDER SIGNALING SCENARIO

As explained in the text, the shape of set $\mathcal{K}$, for which the CE condition is violated, might require a coordinated measurement of multiple senders. Figure 5 illustrates an example where such a situation arises.

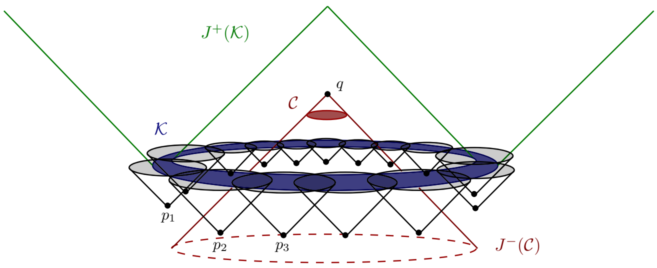

FIG. 5. A signaling scenario in $(2+1)$–dimensional spacetime with set $\mathcal{K}$ being an annulus. To execute a superluminal communication protocol, $k = 15$ sending events are needed. For the sake of readability, only the first three events are captioned.

The spacetime configuration is such that there exists no single event p such that $\mathcal{K} \subset J^+(p)$ and $\mathcal{C} \not\subset J^+(p)$. Consequently, although there is an information flow from $\mathcal{K}$ to the spacelike-separated region $\mathcal{C}$, the communication from any p with $\mathcal{K} \subset J^+(p)$ to any q such that $\mathcal{C} \subset J^-(q)$ is actually subluminal.

Nevertheless, there exist events $\{p_i\}_{i=1}^k$, such that
(i) $p_i \not\preceq p_j$ for all $i \neq j$;
(ii) $p_i \not\preceq q$ for all i; and
(iii) $\mathcal{K}_i := \mathcal{K} \cap J^+(p_i)$ are such that $\mathcal{K} = \bigcup_{i=1}^n \mathcal{K}_i$.

In this case an orchestrated action of k senders is required to trigger a superluminal signal towards the spacelike-separated detector in $\mathcal{C}$ and hence to the receiver located at q. Namely, we can assume that $k-1$ "Alices" are passive and have always-on detectors, and it is the decision of the remaining (active) sender whether or not to perform the measurement, which fixes the value of the communicated bit, in full analogy with the $k = 2$ case discussed in the text.

[1] A. Einstein, B. Podolsky, and N. Rosen, Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **47**, 777 (1935).

[2] N. Bohr, Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **48**, 696 (1935).

[3] D. Dieks, Communication by EPR devices, Phys. Lett. A **92**, 271 (1982).

[4] W. K. Wootters and W. H. Żurek, A single quantum cannot be cloned, Nature **299**, 802 (1982).

[5] S. Popescu, Nonlocality beyond quantum mechanics, Nat. Phys. **10**, 264 (2014).

[6] Č. Brukner, Quantum causality, Nat. Phys. **10**, 259 (2014).

[7] P. Horodecki and R. Ramanathan, The relativistic causality versus no-signaling paradigm for multi-party correlations, Nat. Commun. **10**, 1701 (2019).

[8] A. Kissinger, M. Hoban, and B. Coecke, Equivalence of relativistic causal structure and process terminally, arXiv:1708.04118.

[9] G. Chiribella, G. M. D'Ariano, and P. Perinotti, Informational derivation of quantum theory, Phys. Rev. A **84**, 012311 (2011).

[10] M. Pawłowski, T. Paterek, D. Kaszlikowski, V. Scarani, A. Winter, and M. Żukowski, Information causality as a physical principle, Nature **461**, 1101 (2009).

[11] E. Castro-Ruiz, F. Giacomini, and Č. Brukner, Dynamics of Quantum Causal Structures, Phys. Rev. X **8**, 011047 (2018).

[12] L. Hardy, Probability theories with dynamic causal structure: A new framework for quantum gravity, arXiv:gr-qc/0509120.

[13] G. C. Hegerfeldt, Remark on causality and particle localization, Phys. Rev. D **10**, 3320 (1974).

[14] G. C. Hegerfeldt and S. N. M. Ruijsenaars, Remarks on causality, localization, and spreading of wave packets, Phys. Rev. D **22**, 377 (1980).

[15] G. C. Hegerfeldt, Violation of Causality in Relativistic Quantum Theory? Phys. Rev. Lett. **54**, 2395 (1985).

[16] G. C. Hegerfeldt, Causality Problems for Fermi's Two-Atom System, Phys. Rev. Lett. **72**, 596 (1994).

[17] R. E. Wagner, B. T. Shields, M. R. Ware, Q. Su, and R. Grobe, Causality and relativistic localization in one-dimensional Hamiltonians, Phys. Rev. A **83**, 062106 (2011).

[18] M. Eckstein and T. Miller, Causal evolution of wave packets, Phys. Rev. A **95**, 032106 (2017).

[19] D. Buchholz and J. Yngvason, There are No Causality Problems for Fermi's Two-Atom System, Phys. Rev. Lett. **73**, 613 (1994).

[20] M. Pavšič, Localized states in quantum field theory, Adv. Appl. Cliff. Alg. **28**, 89 (2018).

[21] M. Eckstein and P. Horodecki, The experiment paradox in physics, arXiv:1904.04117.

[22] J. Bhattacharyya, M. Colombo, and T. P. Sotiriou, Causality and black holes in spacetimes with a preferred foliation, Class. Quantum Grav. **33**, 235003 (2016).

[23] R. Haag, Local quantum physics: Fields, particles, algebras, in *Theoretical and Mathematical Physics* (Springer, Berlin, 1996).

[24] A. Peres and D. R. Terno, Quantum information and relativity theory, Rev. Mod. Phys. **76**, 93 (2004).

[25] T. Miller, Polish spaces of causal curves, J. Geom. Phys. **116**, 295 (2017).

[26] M. Eckstein and T. Miller, Causality for nonlocal phenomena, Ann. Henri Poincaré **18**, 3049 (2017).

[27] C. Guerlin, J. Bernu, S. Deleglise, C. Sayrin, S. Gleyzes, S. Kuhr, M. Brune, J.-M. Raimond, and S. Haroche, Progressive field-state collapse and quantum non-demolition photon counting, Nature **448**, 889 (2007).

[28] H. Reeh and S. Schlieder, Bemerkungen zur Unitäräquivalenz von Lorentzinvarianten Felder, Nuovo Cimento **22**, 1051 (1961).

[29] G. C. Hegerfeldt, Instantaneous spreading and Einstein causality in quantum theory, Ann. Phys. **7**, 716 (1998).

[30] C. Aliprantis and K. Border, *Infinite Dimensional Analysis: A Hitchhiker's Guide* (Springer, Berlin, 2006).

[31] E. Minguzzi and M. Sánchez, The causal hierarchy of spacetimes, in *Recent Developments in Pseudo-Riemannian Geometry. ESI Lectures in Mathematics and Physics*, edited by D. V. Alekseevsky and H. Baum (European Mathematical Society, Zurich, 2008), pp. 299–358.