

Decoherence framework for Wigner's-friend experiments

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The decoherence interpretation of quantum measurements is applied to Wigner's-friend experiments. A framework in which all the experimental outcomes arise from unitary evolutions is proposed. Within it, a measurement is not completed until an uncontrolled environment monitorizes the state composed of the system, the apparatus, and the observer. The (apparent) wave-function collapse and the corresponding randomness result from tracing out this environment; it is thus the ultimate responsible party for the emergence of definite outcomes. Two main effects arise from this fact. First, external interference measurements, trademark of Wigner's-friend experiments, modify the memory records of the internal observers; this framework provides a univocal protocol to calculate all these changes. Second, it can be used to build a consistent scenario for the recently proposed extended versions of the Wigner's-friend experiment. In regard to the work of Frauchiger and Renner [*Nat. Commun.* **9**, 3711 (2018)], this framework shows that the agents' claims become consistent if the changes in their memories are properly taken into account. Furthermore, the particular setup discussed by Brukner [*Entropy* **20**, 350 (2018)] cannot be tested against the decoherence framework, because it does not give rise to well-defined outcomes according to this formalism. A variation of this setup, devised to fill this gap, makes it possible to assign joint truth values to the observations made by all the agents. This framework also narrows the requisites for such experiments, making them virtually impossible to apply to conscious (human) beings. Notwithstanding, it also opens the door to future realizations on quantum machines.

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In 1961, Eugene Wigner proposed a thought experiment to show that a conscious being must have a different role in quantum mechanics than that of an inanimate device [1]. This experiment consists of two observers playing different roles. The first one, Wigner's friend, performs a measurement on a particular quantum system in a closed laboratory; as a consequence of it, she observes one of the possible outcomes of her experiment. The second one, Wigner himself, measures the whole laboratory from outside. If quantum theory properly accounts for what happens inside the laboratory, Wigner observes that both his friend and the measured system are in an entangled superposition state. Hence, the conclusions of both observers are incompatible. For Wigner's friend, the reality consists in a definite state equal to one of the possible outcomes of her experiment; for Wigner, it consists in a superposition of all these possible outcomes.

Since then, a large number of discussions, interpretations, and extensions have been done. Among them, this work focus on a recent extended version of this experiment, from which two different no-go theorems have been formulated. The first one shows that different agents, measuring on and reasoning over the same quantum system, are bound to get contradictory conclusions [2]. The second one establishes that it is impossible to assign joint truth values to the observations made by all the agents [3]. This extended version of the Wigner's-friend

experiment consists of two closed laboratories, each one with an observer inside, and two outside observers dealing with a different laboratory. All the measurements are performed on a pair of entangled quantum systems, each one being measured in a different laboratory. An experiment to prove the second no-go theorem recently has been done [4].

The key point of the original and the extended versions of the Wigner's-friend experiment is the quantum treatment of the measurements performed inside the closed laboratories. It is assumed that Wigner's friend observes a definite outcome from her experiment, but the wave function of the whole laboratory in which she lives remains in an entangled superposition state. This is somehow in contradiction with the spirit of the Copenhagen interpretation, since the measurement does not entail a nonunitary collapse. Its main shortcoming is not providing a specific procedure to determine whether a proper measurement has been performed. It is not clear at all whether an agent has observed a definite outcome, or just a simple quantum correlation, implying no definite outcomes, has been crafted. But, at the same time, it can be useful in the era of quantum technologies, because it can describe the evolution of a quantum machine able to perform experiments, infer conclusions from the outcomes, and act as a consequence of them.

The aim of this work is to provide a framework which keeps the quantum character of all the measurements, while supplying a mechanism for the (apparent) wave-function collapse that the agents perceive. This is done by means of the decoherence interpretation of quantum measurements [5], whose origin dates back to almost 40 years ago [6]. The

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key element of this interpretation is that a third party, besides the measured system and the measuring apparatus, is required to complete a quantum measurement. It consists in an uncontrolled environment, which cannot be the object of present or future experiments, and which is the ultimate responsible of the emergence of definite outcomes, and the (apparent) wave-function collapse. Hence, the laboratory in which Wigner's friend lives must include three different objects: the measured system, the measuring apparatus, and the uncontrolled environment—a quantum machine performing such an experiment must contain a set of qubits making up the measuring apparatus and the computer memory, and a second set of qubits forming the environment; the Hamiltonian of the complete machine, including all these qubits, is supposed to be known. The decoherence formalism establishes that this environment, not present in standard Wigner's-friend setups [1–4], determines to which states the memory of Wigner's friend collapses, and therefore which outcomes are recorded by her. At the same time, it guarantees the unitary evolution of the whole laboratory, making it possible for Wigner to observe the system as an entangled superposition of his friend, the measuring apparatus, and the environment. Notwithstanding, our aim is not to support this framework against other possibilities, like wave-function collapse theories, for which the collapse is real and due to slight modifications in the quantum theory that become important only for large systems [7] or recently proposed modifications of the Born rule [8]. We intend just to show the following: (i) this framework provides a univocal protocol to calculate the state of the memories of all the agents involved in the experiment at any time, (ii) it rules out all the inconsistencies arising from the standard interpretations of Wigner's-friend experiments, and (iii) it narrows the circumstances under which such experiments can be properly performed. We can trust in future experiments involving quantum *intelligent* machines to determine which is the correct alternative—if any of these.

Our first step is to build a simple model for the interaction between the measuring apparatus and the environment. This model allows us to determine the properties of the interaction and the size of the environment required to give rise to a proper measurement, as discussed in Ref. [5]. Therefore, it can be used to build a quantum machine to perform Wigner's-friend experiments. Then we profit from it to discuss the original Wigner's-friend experiment [1] and the no-go theorems devised in Refs. [2,3]. We obtain the following conclusions. First, the external interference measurement that Wigner performs on his friend changes her memory record. This change can be calculated, and its consequences on further measurements can be exactly predicted. Second, the decoherence framework rules out all the inconsistencies arising from the usual interpretations of these experiments. Finally, it also establishes restrictive requirements for such experiments.

To avoid all the difficulties that conscious (human) beings entail, all the observers are considered quantum machines, that is, devices operating in the quantum domain, and programmed with algorithms allowing them to reach conclusions from their own observations. This choice facilitates the challenge of the experimental verification (or refutation) of the results that the decoherence framework provides, against, for example, predictions of wave-function collapse models [7] or

modifications of the Born rule [8]. Within this spirit, all the Hamiltonians discussed throughout this paper must be understood as fundamental parts of quantum machines dealing with Wigner's-friend experiments; the Hamiltonians modeling the algorithms used by any particular setup are far beyond the scope of this paper.

The paper is organized as follows. Section II is devoted to the decoherence interpretation of quantum measurements. A simple numerical model is proposed to guide all the discussions. In Sec. III the original Wigner's-friend experiment is studied in terms of the decoherence framework. A numerical simulation is used to illustrate its most significant consequences. In Sec. IV the consistency of the quantum theory is discussed, following the argument devised in Ref. [2]. Section V refers to the possibility of assigning joint truth values to all the measurements in an extended Wigner's-friend experiments, following the point of view given in Ref. [3]. Finally, conclusions are gathered in Sec. VI.

II. DECOHERENCE FRAMEWORK

The first aim of this section is to review the decoherence formalism. We have chosen the examples and adapted the notation to facilitate its application to Wigner's-friend experiments. After this part is completed, we propose a Hamiltonian model giving rise to the definite outcomes observed by any agents involved in any quantum experiment, and we explore its consequences by means of numerical simulations.

A. Decoherence interpretation of quantum measurements

In all the versions of Wigner's-friend experiments, the protocol starts with a measurement performed by a certain agent, I . Let us consider that a single photon is the object of such measurement, and let us suppose that the experiment starts from the following initial state:

$$|\Psi\rangle = \sqrt{\frac{1}{2}}(|h\rangle + |v\rangle), \quad (1)$$

where $|h\rangle$ denotes that it is horizontally polarized, and $|v\rangle$, vertically polarized.

The usual way to model a quantum measurement consists in a unitary evolution, given by the Hamiltonian that encodes the dynamics of the system and the measuring apparatus. It transforms the initial state, in which system and apparatus are uncorrelated, onto a final state in which the system and the apparatus are perfectly correlated:

$$\frac{1}{\sqrt{2}}(|h\rangle + |v\rangle) \otimes |A_0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|h\rangle \otimes |A_h\rangle + |v\rangle \otimes |A_v\rangle), \quad (2)$$

where $|A_0\rangle$ represents the state of the apparatus before the measurement, $\langle A_h|A_v\rangle = 0$, and $\langle A_h|A_h\rangle = \langle A_v|A_v\rangle = 1$. In Ref. [4] an ancillary photon plays the role of the apparatus. In general, such a measurement can be performed by means of a C-NOT gate. As the choice of $|A_0\rangle$ is arbitrary, we can consider that $|A_0\rangle \equiv |A_h\rangle$, and thus the corresponding Hamiltonian is given by

$$H = \frac{g}{2} |v\rangle\langle v| \otimes [|A_h\rangle\langle A_h| + |A_v\rangle\langle A_v| - |A_v\rangle\langle A_h| - |A_h\rangle\langle A_v|], \quad (3)$$

where g is a coupling constant. This Hamiltonian performs Eq. (2), if it is applied during an interaction time given by $g\tau = \pi/2$ [5]. The resulting state, which we denote

$$|\Psi\rangle = \sqrt{\frac{1}{2}}(|h\rangle|A_h\rangle + |v\rangle|A_v\rangle) \quad (4)$$

for simplicity, entails that if the photon has horizontal polarization, then the apparatus is in state $|A_h\rangle$, and if the photon has vertical polarization, then the apparatus is in state $|A_v\rangle$. That is, it is enough to observe the apparatus to know the state of the photon.

As we have just pointed out, this is the usual description of a quantum measurement. Once the state (4) is fixed, the measurement is completed, and the only remaining task is to interpret the results. This is precisely what is done in the experimental facility discussed in Ref. [4]. In both the original and the extended versions of Wigner’s-friend experiments, the interpretation is the following. The observer inside the laboratory, I , sees that the outcome of the experiment is either h or v , with probability $1/2$, following the standard Born rule; it sees the reality as consisting in a definite state corresponding to either $|h\rangle$ or $|v\rangle$, according to the information it has gathered. Even more, it can write that its observation has been completed, making possible for an external observer, E , to know that I is seeing a definite outcome,

$$|\Psi_1'\rangle = \left[\frac{1}{\sqrt{2}}(|h\rangle|A_h\rangle + |v\rangle|A_v\rangle) \right] \otimes |\text{Observation}\rangle. \quad (5)$$

This implies that I has observed a definite outcome, whereas the whole laboratory in which it lives remains in a superposition state that can be observed by E , despite knowing that I sees the photon either in horizontal or vertical polarization, and not in such a superposition state.

This conclusion is the basis of all the versions of the Wigner’s friend experiment. Notwithstanding, it suffers from two important shortcomings. The first one is that the complete laboratory consists just in the measured system and the measuring apparatus. Hence, there is no place for a quantum device able to act as a consequence of its measurement—the reasonings to infer contradictory conclusions, as discussed in Ref. [2], require a complex machine, not just a qbit signaling whether the measured photon is vertically or horizontally polarized. Therefore, as is pointed out in Ref. [4], the consideration of Eq. (4) as a proper measurement is questionable. However, this shortcoming is solvable—at least from a theoretical point of view—just by considering that the apparatus represents, not only the measuring machine, but also the memory of the observer. We will rely on this interpretation throughout the rest of the paper; technical considerations are far beyond its scope. Therefore, from now on, the state of any measuring apparatus will represent the memory record of any quantum machine playing the observer role. After the complete protocol is finished, all these records are supposed to be available as the outputs of the quantum computation.

The second one is the basis ambiguity problem [5]. The very same state in Eq. (4), $|\Psi_1\rangle$, can be written in different basis,

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle|A_\alpha\rangle + |\beta\rangle|A_\beta\rangle), \quad (6)$$

where

$$|\alpha\rangle = \sin\theta|h\rangle + \cos\theta|v\rangle, \quad (7a)$$

$$|\beta\rangle = -\cos\theta|h\rangle + \sin\theta|v\rangle, \quad (7b)$$

$$|A_\alpha\rangle = \sin\theta|A_h\rangle + \cos\theta|A_v\rangle, \quad (7c)$$

$$|A_\beta\rangle = -\cos\theta|A_h\rangle + \sin\theta|A_v\rangle. \quad (7d)$$

That is, the final state of the very same measuring protocol, starting from the very same initial condition, can also be written as the superposition given in Eq. (6) for arbitrary values of θ . This problem blurs the usual interpretation of all the versions of the Wigner’s-friend experiment. *As Eq. (6) is a correct representation of agent I ’s memory, we have no grounds to conclude that outcome of its measurement is either h or v , instead of α or β .* The unitary evolution giving rise to the measurement, Eq. (2), does not determine a preferred basis for the corresponding definite outcome. Hence, a physical mechanism for the emergence of such an outcome must be provided, in order to not get stuck on a fuzzy interpretation issue. The main trademark of the decoherence formalism is providing a plausible mechanism.

There are several ways to solve this problem. One of them consists in modifying the Schrödinger equation to model the wave function collapse and to choose the corresponding preferred basis. These theories are based on the fact that superpositions have been experimentally observed in systems up to 10^{-21} g, whereas the lower bound for a classical apparatus is around 10^{-6} g [7]. This means that the Schrödinger equation is just an approximation, which works pretty well for small systems, but fails for systems as large as measurement devices. Such a real collapse would change all the dynamics of Wigner’s-friend experiments, presumably ruling out all their inconsistencies.

Another possibility, the one which is the object of this work, is that Eq. (2) is *not* a complete measurement, but just a premeasurement—a previous step required for any observation [5,9]. Following this interpretation, the observation is not completed until a third party, an environment which is *not* the object of the measurement, becomes correlated with the measured system and the measuring apparatus. This correlation is given again by a Hamiltonian and therefore consists in a unitary evolution. If such an environment is continuously monitoring the system [10], the state of the whole system becomes

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}[|h\rangle|A_h\rangle|\varepsilon_1(t)\rangle + |v\rangle|A_v\rangle|\varepsilon_2(t)\rangle], \quad (8)$$

where the states of the environment $|\varepsilon_1(t)\rangle$ and $|\varepsilon_2(t)\rangle$ change over time, because the apparatus is continuously interacting with it, and $\langle\varepsilon_1(t)|\varepsilon_1(t)\rangle = \langle\varepsilon_2(t)|\varepsilon_2(t)\rangle = 1$. Note that Eq. (8) entails that the correlations between the system and the apparatus remain untouched despite the continuous monitoring by the environment. Hence, the states $|A_h\rangle$ and $|A_v\rangle$ are called *pointer states*, because they represent the stable states of the apparatus [5,6] and the stable records in the memory of the observers. Furthermore, if such an apparatus-environment interaction implies $\langle\varepsilon_1(t)|\varepsilon_2(t)\rangle = 0, \forall t > \tau$, where τ can be understood as the time required to complete the measurement, the following affirmations hold:

(i) There is no other triorthogonal basis to write the state given by Eq. (8) [11]. That is, the basis ambiguity problem is fixed by the action of the uncontrolled environment.

(ii) As the observer I cannot measure the environment, its memory record and all the further experiments it can perform on the system and the apparatus are compatible with the following mixed state:

$$\rho = \frac{1}{2}(|h\rangle|A_h\rangle\langle h|A_h| + |v\rangle|A_v\rangle\langle v|A_v|), \quad (9)$$

independently of the particular shapes of both $|\varepsilon_1(t)\rangle$ and $|\varepsilon_2(t)\rangle$. That is, the observer I sees the system as if it were randomly collapsed either to $|h\rangle|A_h\rangle$ or to $|v\rangle|A_v\rangle$, even though the real evolution of the complete system—including *itself!*—is deterministic and given by Eq. (8). Relying on the decoherence framework, such an observer can only deduce that the real state of the system, the apparatus and itself must be something like Eq. (8) [12]. Randomness arises through this lack of knowledge.

At this point, it is worth remarking that the decoherent environment must be understood as a fundamental part of the measuring device, not a practical difficulty under realistic conditions—the difficulty of keeping the system aside from external perturbations. If the decoherence framework is applied, any quantum machine must include such an environment as an inseparable part of it. The trademark of this framework is postulating that definite—classical—outcomes arise as a consequence of the continuous environmental monitoring; if such an environment does not exist, no definite outcomes are observed. In other words, the observation is completed when the state given by Eq. (8) is reached: *if the observer sees a collapsed state is because an uncontrolled environment is monitoring the system (including itself!), and thus the complete wave function is given by Eq. (8)*. The decoherence interpretation of quantum measurements also provides a framework to derive the Born rule from fundamental postulates [9]. Notwithstanding, all this work is based just on the previous facts (i) and (ii), and therefore the possible issues in this derivation of the Born rule are not relevant.

Before ending this section, it is interesting to delve into the differences between the standard interpretation of quantum measurements and the one supplied by the decoherence formalism. Under normal circumstances, both interpretations provide indistinguishable results. For example, the standard interpretation establishes that, once an agent has observed a definite outcome in a polarization experiment, say, h , then any further measurements performed in the same basis are bounded to give the same outcome, h . This important fact is exactly reproduced by the decoherence framework. A second measurement with an identical apparatus, denoted A' , performed on Eq. (8) will give

$$|\Psi_3\rangle = \sqrt{\frac{1}{2}}[|h\rangle|A_h\rangle|A'_h\rangle|\varepsilon_1(t)\rangle + |v\rangle|A_v\rangle|A'_v\rangle|\varepsilon_2(t)\rangle], \quad (10)$$

if we logically assume that the Hamiltonian modeling the interaction between the apparatus and the environment is identical for two identical apparatuses. Therefore, the perception of the observer is given by

$$\rho = \frac{1}{2}[|h\rangle|A_h\rangle|A'_h\rangle\langle h|A_h|A'_h| + |v\rangle|A_v\rangle|A'_v\rangle\langle v|A_v|A'_v|]. \quad (11)$$

That is, its internal memory says that if it has observed h in the first measurement, then it has also observed h in the second.

In the next sections we will show that the standard interpretation and the decoherence formalism do show important differences when the observers are the object of external interference experiments. The key point lies, again, in the role played by the environment. To perform a proper interference experiment, the external observer must act coherently on the system, the apparatus (that is, the memory of the internal agent), *and* the environment. As a consequence of this action, the state of the environment will eventually change in a perfectly predictable way. And, as it is the ultimate responsible of the definite outcome observed by the internal agent, its internal memory will also change accordingly. We will discuss below how these changes release quantum theory from inconsistencies.

B. A simple model for the laboratories

The laboratories in which agents I perform their measurements are quantum machines evolving unitarily. Their Hamiltonians must consist of (i) a system-apparatus interaction, performing the premeasurements, and (ii) an apparatus-environment interaction, following the decoherence formalism. For (i) we consider the logical C-NOT gate given in Eq. (3). Following [5], for (ii) we propose a model

$$H = |A_h\rangle\langle A_h| \sum_{n,m} V_{nm}^h |\varepsilon_n\rangle\langle \varepsilon_m| + |A_v\rangle\langle A_v| \sum_{n,m} V_{nm}^v |\varepsilon_n\rangle\langle \varepsilon_m|, \quad (12)$$

where V^h and V^v are the coupling matrices giving rise to the interaction. The only condition for them is to be Hermitian matrices; independently of their particular shapes, the Hamiltonian given by Eq. (12) guarantees that the correlations $|h\rangle|A_h\rangle$ and $|v\rangle|A_v\rangle$ remain unperturbed, that is, $|A_h\rangle$ and $|A_v\rangle$ are the pointer states resulting from this interaction, and the state given by Eq. (8) holds for any time.

To build a simple model, we consider that both V^h and V^v are real symmetric random matrices of the Gaussian orthogonal ensemble (GOE), which is the paradigmatic model for quantum chaos [13]. They are symmetric square matrices of size N , with independent Gaussian random elements with mean $\mu(V_{nm}) = 0$, $\forall n, m = 1, \dots, N$, and standard deviation $\sigma(V_{nn}) = 1$, $\forall n = 1, \dots, N$ (diagonal elements) and $\sigma(V_{nm}) = 1/\sqrt{2}$, $\forall n \neq m = 1, \dots, N$ (nondiagonal elements).

In Fig. 1(a) we show how the overlap between the two states of the environment, $|\varepsilon_1(t)\rangle$ and $|\varepsilon_2(t)\rangle$, evolves with time, and in Fig. 1(b) how it evolves with the environment size. To perform the calculations, we have considered that the environment consists in N qbits, and hence the dimension of its Hilbert space is $d = 2^N$. In all the cases, the initial state is a tensor product

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}[|h\rangle|A_h\rangle + |v\rangle|A_v\rangle] \otimes |\varepsilon_0\rangle, \quad (13)$$

where $|\varepsilon_0\rangle$ is the first element of the environmental basis (as the interaction is a GOE random matrix, the particular shape of the basis is irrelevant [13]). All the results are averaged over 50 different realizations. We have considered $\hbar = 1$.

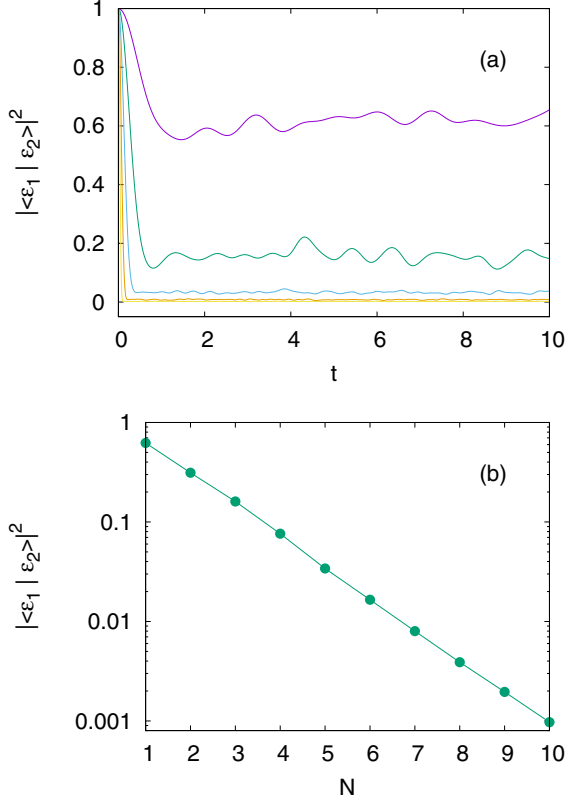


FIG. 1. (a) Value of $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2$ as a function of time, for environments composed of different number of qubits. The solid curves show, from the upper one to the lower one, $N = 1$, $N = 3$, $N = 5$, $N = 7$, and $N = 9$. (b) Finite-size scaling for the long-time average of $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2$, as a function of the number of qubits composing the environment, N .

Figure 1(a) shows the results of $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2$ for $N = 1$ ($d = 2$), $N = 3$ ($d = 8$), $N = 5$ ($d = 32$), $N = 7$ ($d = 128$), and $N = 9$ ($d = 512$). We clearly see that, the larger the number of environmental qubits, the smaller the value of $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2$ at large times, and the smaller the characteristic time τ required to complete the measurement process. Therefore, the condition $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2 \sim 0$ is quickly reached if the number of the environmental qubits is $N \sim 10$. The results plotted in Fig. 1(b) confirm this conclusion. We show there the long-time average of $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2$, calculated for $2 \leq t \leq 10$, as a function of the number of environmental qubits. It is clearly seen that the overlap between these states decreases fast with this number. As a consequence, we can safely conclude that an agent I operating within a laboratory described by Eq. (12) will observe a state given by Eq. (9).

These results imply that the laboratories in which all the agents perform their measurements must have the structure summarized in Table I. It is worth noting that this structure is independent from any further evolution of the measured system, after the premeasurement is completed. For example, let us imagine that the measured system has its own Hamiltonian, and therefore the time evolution for the whole system is governed by

$$H = H_S \otimes I_{A\varepsilon} + I_S \otimes H_{A\varepsilon}, \quad (14)$$

TABLE I. Parts of laboratories in which the agents I perform their measurements in a Wigner's-friend experiment, following the decoherence framework.

L1	The measured system.
L2	The measuring apparatus.
L3	An internal environment, with a chaotic interaction like the one given by Eq. (12), and large enough to guarantee $ \langle \varepsilon_1(t) \varepsilon_2(t) \rangle ^2 \sim 0$.

where H_S is the Hamiltonian for the measured system, $H_{A\varepsilon}$ represents the environment-apparatus interaction, given by Eq. (12), and I_S ($I_{A\varepsilon}$) is the identity operator for the system (environment-apparatus). As the two terms in this Hamiltonian commute pairwise, the time evolution of the whole system is

$$|\Psi(t)\rangle = \sqrt{\frac{1}{2}}|\eta(t)\rangle|A_h\rangle|\varepsilon_1(t)\rangle + \sqrt{\frac{1}{2}}|\nu(t)\rangle|A_v\rangle|\varepsilon_2(t)\rangle, \quad (15)$$

where the notation $\eta(t)$ and $\nu(t)$ has been chosen to denote that $\eta(t)$ is the state which evolves from an initial condition consisting in an horizontally polarized photon, $|\eta(t)\rangle = \exp(-iH_S t)|h\rangle$, and $\nu(t)$ the state which evolves from a vertically polarized photon, $|\nu(t)\rangle = \exp(-iH_S t)|v\rangle$. Therefore all further measurements of the same agents are well described by

$$\rho(t) = \frac{1}{2}|\eta(t)\rangle|A_h\rangle\langle\eta(t)|\langle A_h| + |\nu(t)\rangle|A_v\rangle\langle\nu(t)|\langle A_v|. \quad (16)$$

That is, all the possible experiments that agent I can perform in the future are compatible with the system collapsing onto either $|h\rangle$ or $|v\rangle$ after the measurement, and unitarily evolving from the corresponding initial condition. In other words, and as we have already pointed out, this framework is fully compatible with the Copenhagen interpretation ... *but the wave-function collapse being just a consequence of ignoring the environmental degrees of freedom*. It is worth to remark that this is not a subjective interpretation, but the result of a unitary time evolution including a number of degrees of freedom that cannot be measured by the same observer. Equation (16) establishes that a further reading of the agent memory record would reveal that the photon has collapsed either to h or v , and then it has evolved from the corresponding initial condition.

As the key point in Wigner's-friend experiments consists in further interference measurements on the whole laboratory, a study of the complexity of the state resulting from the time evolution summarized in Fig. 1 is necessary. Such a study can be made by means of a correlation function $C(\tau) = |\langle \varepsilon_1(t) | \varepsilon_1(t + \tau) \rangle|^2$. If $C(\tau) \sim 1$, then the time evolution of the environmental state $|\varepsilon_1(t)\rangle$ is quite simple; its only possible change is an irrelevant global phase. Such a simple evolution would facilitate further interference experiments. On the contrary, if $C(\tau)$ quickly decays to zero, the same evolution is highly involved, implying that the state of the whole laboratory is complex enough to hinder further interference experiments.

Results are summarized in Fig. 2. Figure 2(a) shows $C(\tau)$ for the same environments displayed in the same panel of Fig. 1. It has been obtained after a double average: over

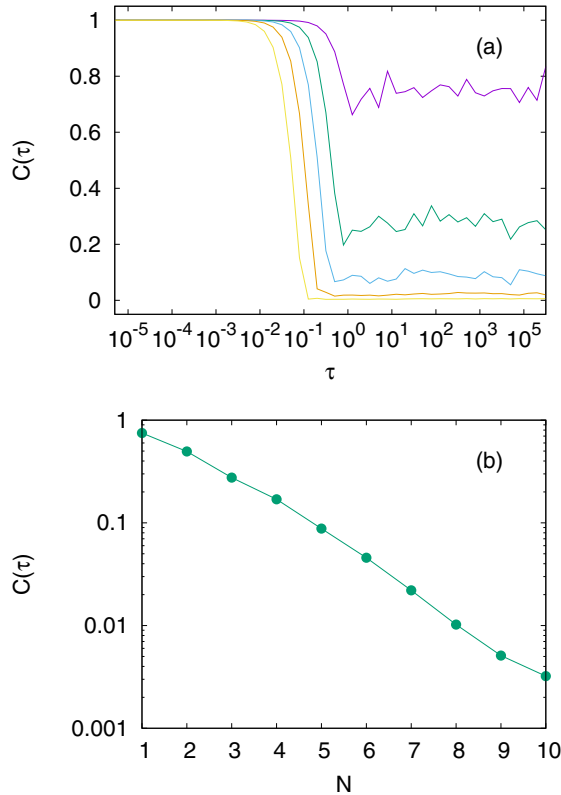


FIG. 2. (a) Value of $C(\tau)$ as a function of τ , for environments composed of different number of qubits. The solid curves show, from the upper one to the lower one, $N = 1$, $N = 3$, $N = 5$, $N = 7$, and $N = 9$. (b) Finite-size scaling for the long-time average of $C(\tau)$, as a function of the number of qubits composing the environment, N .

50 different realizations, and over 10^4 different values of the time t . Figure 2(b) displays a finite-size scaling of $C(\tau)$ for large values of time versus the number of environmental qubits, calculated averaging over $\tau \geq 10$. It is clearly seen that the results shown in this figure are correlated with the ones displayed in Fig. 1. That is, if the environment is large enough to give rise to $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2 \sim 0$, then the environmental states fulfill $C(\tau) \sim 0$; the smaller the overlap between $|\varepsilon_1(t)\rangle$ and $|\varepsilon_2(t)\rangle$, the smaller the value of the correlation function $C(\tau)$. It is also worth noting that $C(\tau)$ decays very fast to zero; for $N = 9$, $C(\tau) \sim 0$ for $\tau \simeq 10^{-1}$. This means that the state of the environment is changing fast, and therefore the state of the whole laboratory, including the measured system, the measuring apparatus, and the environment, is very complex.

As we have pointed out above, the key point of all the versions of Wigner's-friend experiments consist in further interference measurements performed by an external agent, for which the whole laboratory evolves unitarily following Eq. (8). Both in its original [1] and its extended versions, discussed in Refs. [2–4], the external agents perform interference experiments involving only two states, $|A_h\rangle|h\rangle$ and $|A_v\rangle|v\rangle$. The Hilbert spaces of the simplified versions of the laboratories discussed in these papers are spanned by $\{|A_h\rangle|h\rangle, |A_v\rangle|v\rangle, |A_h\rangle|v\rangle, |A_v\rangle|h\rangle\}$. Notwithstanding, the last two states are never occupied, and hence such two-state interference experiments are feasible [4]. The situation arising

TABLE II. Requisites for an extended Wigner's-friend experiment in which the external agent, E , performs an interference experiment involving only two states.

R1	A perfect knowledge of the interaction between the system and the apparatus, H , given by Eq. (12).
R2	A perfect knowledge of the environmental initial state, $ \varepsilon_0\rangle$.
R3	A perfect knowledge of the time at which agent I performs its measurement, t_I .
R4	A perfect choice of the time at which agent E performs its interference experiment, t_E .

from the decoherence framework is far more complex. The dimension of the whole laboratory, composed of the measured system, the measuring apparatus, and an environment with N qubits, is $d = 2^{N+2}$. From the results summarized in Fig. 2, we conjecture that all the 2^N states of the environment are populated, and therefore 2^{N+1} states of the whole laboratory become relevant for further interference experiments. Hence, the first consequence of the results discussed in this section is that experiments like the ones in Refs. [1–4] become extremely difficult. However, as $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2 \sim 0$, it is true that only two states, $|h\rangle|A_h\rangle|\varepsilon_1(t)\rangle$ and $|v\rangle|A_v\rangle|\varepsilon_2(t)\rangle$, are populated at each time t ; the rest of the Hilbert space is irrelevant at that particular value of the time t . Unfortunately, these states change very fast with time, and in a very complex way. Therefore, an interference experiment involving only two states, $|h\rangle|A_h\rangle|\varepsilon_1(t)\rangle$ and $|v\rangle|A_v\rangle|\varepsilon_2(t)\rangle$, would require a very restrictive protocol, whose main requisites are summarized in Table II. Only if such requisites are fulfilled, can the external agent E rely on a simplified basis, composed of $|h(\tau)\rangle \equiv |h\rangle|A_h\rangle|\varepsilon_1(\tau)\rangle$ and $|v(\tau)\rangle \equiv |v\rangle|A_v\rangle|\varepsilon_2(\tau)\rangle$, where $\tau = t_E - t_I$, t_I the time at which agent I performs its measurement, and t_E the same for agent E . A small error in points R1–R4 would imply that the real state of the laboratory, $|\Psi(t)\rangle$, had negligible overlaps with both $|h(\tau)\rangle$ and $|v(\tau)\rangle$, and therefore any interference experiments involving just these two states would give no significative outcomes. Notwithstanding, given the promising state of the art in quantum computing [14], we can trust for future quantum machines to be able to work with enough precision.

Before applying these conclusions to the original and the extended versions of the Wigner's-friend experiments, it makes sense to test if these conclusions depend on the particular model we have chosen for the apparatus-environment interaction. To tackle this task, we consider more general random matrices V^h and V^v in Eq. (12), in which $\mu(V_{nm}) = 0$, $\forall n, m = 1, \dots, N$, $\sigma(V_{nn}) = 1$, $\forall n = 1, \dots, N$ (diagonal elements), and $\sigma(V_{nm}) = 1/(\sqrt{2}|n-m|^\alpha)$ $\forall n \neq m = 1, \dots, N$ (nondiagonal elements). If the parameter α is large, then only a very few nondiagonal elements are relevant, and hence the interaction becomes approximately integrable. On the contrary, if $\alpha = 0$, GOE (chaotic) results are recovered.

We fix our attention in the degree of chaos of the resulting Hamiltonian. To do so, we study the ratio of consecutive level spacings distribution, $P(r)$, where $r_n = s_{n+1}/s_n$ and $s_n = E_{n+1} - E_n$, $\{E_n\}$ being the energy spectrum of the system. It has been shown [15] that the distribution for standard integrable systems is $P(r) = 1/(1+r)^2$, whereas it is $P(r) =$

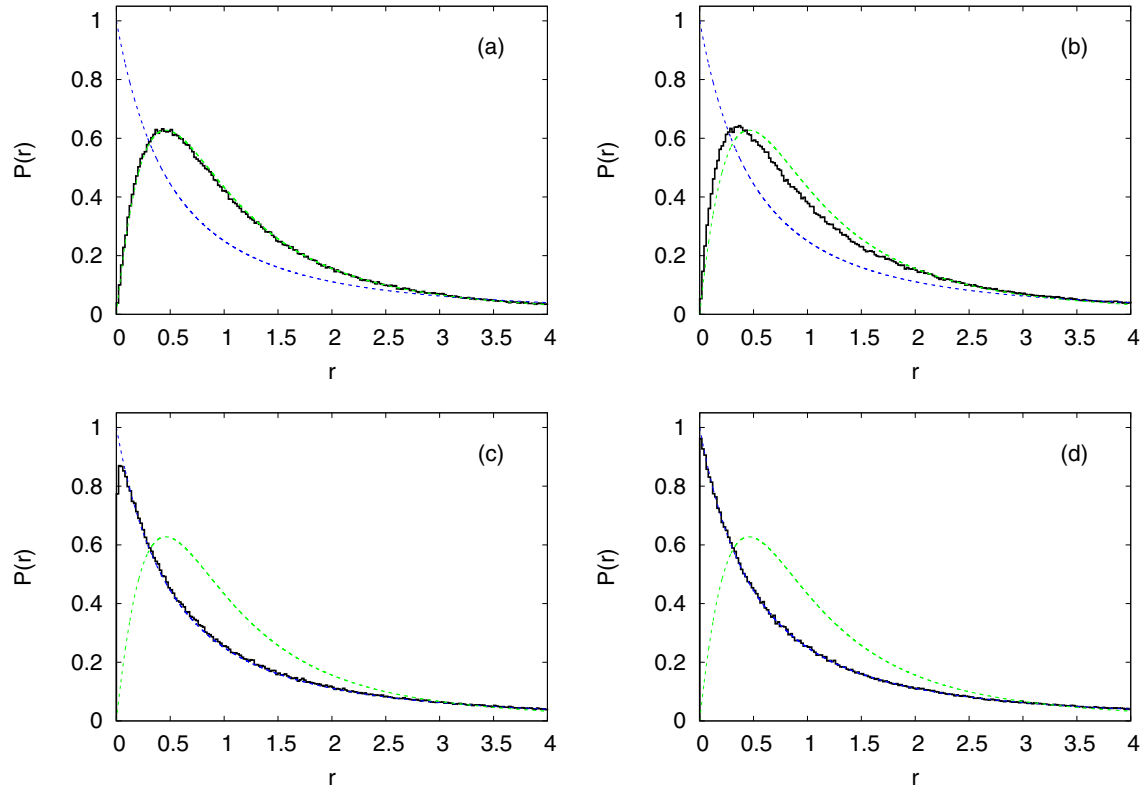


FIG. 3. Ratio of consecutive level spacings distribution, $P(r)$, for $\alpha = 0.5$ (a), $\alpha = 1$ (b), $\alpha = 2$ (c), and $\alpha = 4$ (d). Solid histograms show the numerical results for 2000 matrices with dimension $d = 512$; green dashed line, the result for a GOE system, $P(r) = 27(r + r^2)/(8(1 + r + r^2)^{5/2})$, and the blue dashed line, the result for an integrable system, $P(r) = 1/(1 + r)^2$.

$27(r + r^2)/[8(1 + r + r^2)^{5/2}]$ for GOE systems; a generic interpolating distribution has been recently proposed [16].

In Fig. 3 we show the results for four different values of α , $\alpha = 0.5$, $\alpha = 1$, $\alpha = 2$, $\alpha = 4$. They consist in the average over 2000 realizations of matrices of dimension $d = 512$. The case with $\alpha = 0$ (not shown) exactly recovers the GOE result, as expected. The case with $\alpha = 0.5$ [Fig. 3(a)] is also fully chaotic; its ratio of consecutive level spacings distribution, $P(r)$, is identical to the GOE result. Things become different for larger values of α . The case $\alpha = 1$ [Fig. 3(b)] is yet different from the GOE result, although its behavior is still highly chaotic. The cases $\alpha = 2$ [Fig. 3(c)] and $\alpha = 4$ [Fig. 3(d)] are very close to the integrable result.

In Fig. 4 we show how the long-time average of $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2$, calculated for $2 \leq t \leq 50$, scales with the number of environmental qubits, N , for five different values of $\alpha = 0, 0.5, 1, 2$, and 4 . The results are averaged over 50 different realizations. It is clearly seen that the two fully chaotic cases, $\alpha = 0$ (circles) and $\alpha = 0.5$ (squares), behave in the same way; the overlap $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2$ decreases with the number of environmental qubits, and therefore we can expect $|\varepsilon_1(t)\rangle$ and $|\varepsilon_2(t)\rangle$ to become orthogonal if the environment is large enough. The behavior of the case with $\alpha = 1$ (upper triangles) is different. First, the overlap $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2$ decreases with N , but it seems to reach an asymptotic value for $N \gtrsim 7$. This fact suggests that a fully chaotic apparatus-environment interaction is required for the scenario described by the decoherence framework. This conclusion is reinforced with the results for $\alpha = 2$ (lower triangles) and $\alpha = 4$

(diamonds). These two cases correspond to (almost) integrable Hamiltonians, and their overlaps $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2$ remain large independently of the number of environmental qubits.

C. Summary of results

The results discussed in the previous section narrow the circumstances under which Wigner's-friend experiments are feasible, if we take into account the decoherence interpretation of quantum measurements. First, laboratories in which all the

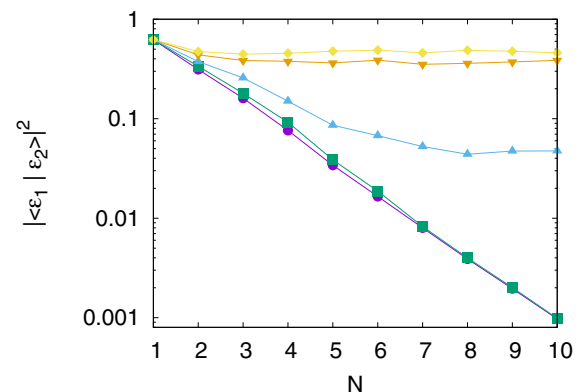


FIG. 4. Finite-size scaling for the long-time average of $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle|^2$, as a function of the number of qubits composing the environment, N . Solid circles represent the case $\alpha = 0$; solid squares, $\alpha = 0.5$; solid upper triangles, $\alpha = 1$; solid lower triangles, $\alpha = 2$, and solid diamonds, $\alpha = 4$.

TABLE III. Summary of the facts consequence of the decoherence interpretation of quantum measurements, for Wigner's-friend experiments.

F1	After the measurement performed by agent I is completed, the real state of the measured system, the measuring apparatus, and the surrounding environment (which includes the agent itself) is given by Eq. (8), with $ \langle \varepsilon_1(t) \varepsilon_2(t) \rangle ^2 \sim 0$.
F2	All the results obtained by the agent I are compatible with the mixed state given by Eq. (9). That is, it sees the system as if it were collapsed onto one of the possible outcomes of its experiment, despite fact F1.

agents work must have the structure given in Table I. Second, if external agents want to perform interference experiments relying on just two basis states, the requirements listed in Table II are mandatory. And, third, if such circumstances hold, then the facts F1 and F2 listed in Table III characterize such experiments. Fact F1 establishes that an observer cannot get a conclusion about the exact state of the whole system (including itself!) just from the outcome of as many experiments as it can perform. On the contrary, the very fact of observing a definite outcome entails that the observer is a part of a larger, entangled superposition state, including an environment from which the observer cannot get information. (Note that the decoherence framework establishes that this happens in any quantum measurement, independently of the existence of an external observer.) Fact F2 refers to the practical consequences of F1. It entails that all the agents involved in an experiment are limited to discuss about the outcomes they obtain, outcomes that depend on both their measuring apparatus *and the environmental degrees of freedom which have been traced out*. If either the apparatus or the environmental degrees of freedom are different, then the whole experiment is also different, and thus different outcomes can be expected.

III. STANDARD WIGNER'S-FRIEND EXPERIMENTS AND THE DECOHERENCE FRAMEWORK

In this section we discuss the consequences of the decoherence framework in the standard Wigner's-friend experiment [1]. This discussion sets the grounds to analyze the extended versions of the experiments [2–4].

Let us consider that an internal agent I has performed a measurement on an initial state given by Eq. (1). As we have explained above, independently of the outcome it observes, the resulting state is given by Eq. (8), which is the result of the unitary evolution due to Hamiltonians (3) and (12). To simplify the notation, we consider the whole state of the laboratory as follows:

$$|h(t)\rangle = |h\rangle|A_h\rangle|\varepsilon_1(t)\rangle, \quad (17a)$$

$$|v(t)\rangle = |v\rangle|A_v\rangle|\varepsilon_2(t)\rangle, \quad (17b)$$

where both $|h(t)\rangle$ and $|v(t)\rangle$ may in general change with time. Thus, the state after the measurement by agent I is

$$|\Psi_1(t)\rangle = \sqrt{\frac{1}{2}}[|h(t)\rangle + |v(t)\rangle]. \quad (18)$$

Following the protocol proposed by Wigner [1], an external agent, E , performs a measurement on $|\Psi_1(\tau)\rangle$, at a particular instant of time τ . Let us consider that the four requisites, R1–R4, of Table II are fulfilled, and therefore an interference experiment can be performed with a two-state basis, $\{|\alpha(\tau)\rangle, |\beta(\tau)\rangle\}$, given by

$$|\alpha(\tau)\rangle = \sin\theta|h(\tau)\rangle + \cos\theta|v(\tau)\rangle, \quad (19a)$$

$$|\beta(\tau)\rangle = -\cos\theta|h(\tau)\rangle + \sin\theta|v(\tau)\rangle, \quad (19b)$$

for an arbitrary value of the angle θ . In this basis, the state $|\Psi_1(\tau)\rangle$ reads

$$|\Psi_1(\tau)\rangle = \sqrt{\frac{1}{2}}(\sin\theta + \cos\theta)|\alpha(\tau)\rangle + \sqrt{\frac{1}{2}}(\sin\theta - \cos\theta)|\beta(\tau)\rangle. \quad (20)$$

Therefore, following the decoherence formalism, and as a consequence of the same kind of unitary evolution than before, the state resulting from agent E measurement is

$$|\Psi_2(\tau)\rangle = \sqrt{\frac{1}{2}}(\sin\theta + \cos\theta)|\alpha(\tau)\rangle|A'_\alpha\rangle|\varepsilon'_1(\tau)\rangle + \sqrt{\frac{1}{2}}(\sin\theta - \cos\theta)|\beta(\tau)\rangle|A'_\beta\rangle|\varepsilon'_2(\tau)\rangle, \quad (21)$$

where A' represents its apparatus, and ε' the environment required by the decoherence framework.

Up to now, we have considered that both the measurement and the correlation between the apparatus A' and the environment ε' happen at time τ . But this consideration is not relevant. Taking into account that both the internal, A , and the external, A' , apparatuses are continuously monitored by their respective environments, the former state unitarily evolves with a Hamiltonian $H = H_I \otimes I_E + I_I \otimes H_E$, where I_I (I_E) represents the identity operator for the internal (external) laboratory. Therefore, in any moment after the measurement the resulting state is

$$|\Psi_2(t)\rangle = \sqrt{\frac{1}{2}}(\sin\theta + \cos\theta)|\alpha(t)\rangle|A'_\alpha\rangle|\varepsilon'_1(t)\rangle + \sqrt{\frac{1}{2}}(\sin\theta - \cos\theta)|\beta(t)\rangle|A'_\beta\rangle|\varepsilon'_2(t)\rangle, \quad (22)$$

with $|\langle \varepsilon'_1(t) | \varepsilon'_2(t) \rangle|^2 \sim 0$. And hence, any further experiment performed by agent E , in which the external environment is not measured, is compatible with the state

$$\rho_E = \frac{1}{2}(\sin\theta + \cos\theta)^2|\alpha(t)\rangle|A'_\alpha\rangle\langle A'_\alpha|\langle\alpha(t)| + \frac{1}{2}(\sin\theta - \cos\theta)^2|\beta(t)\rangle|A'_\beta\rangle\langle A'_\beta|\langle\beta(t)|. \quad (23)$$

Two remarks are useful at this point. First, as we have pointed out above, the real state of the system is given by Eq. (22); the mixed state given by Eq. (23) is only a description of what agent E sees, that is, of what agent E can infer from any further measurements performed by itself, and what it is recorded in its memory. Second, the interpretation of Eq. (23) is independent of the precise forms of $|\alpha(t)\rangle$ and $|\beta(t)\rangle$. The fact that the internal laboratory changes with time, as a consequence of the monitorization by its environment,

due to the Hamiltonian (12), has no influence on agent E conclusions because its apparatus remains pointing at either α or β .

As we have explained in the previous section, the main difference between the decoherence framework and the standard interpretation of quantum measurements, consisting just in a correlation between the system and the apparatus, is that definite outcomes arise as a consequence of the environmental monitorization, given by the Hamiltonian (12), and therefore can be exactly tracked at any instant of time. This fact releases us from the need of choosing a particular perspective to interpret the results without inconsistencies, as is proposed in Ref. [17]; within the decoherence framework, we just need to calculate the state of the agents' memories. So, as the action of an external observer includes an interaction with the internal environment, one may wonder about the consequences of such an action. The measurement performed by agent E has changed the state of the system from

$$|\Psi_1(t)\rangle = \sqrt{\frac{1}{2}}[|h(t)\rangle + |v(t)\rangle] \otimes |A'_0\rangle|\varepsilon'_0\rangle, \quad (24)$$

where $|A'_0\rangle$ and $|\varepsilon'_0\rangle$ are the (irrelevant) initial states of agent E apparatus and the external environment, to Eq. (22). The decoherence framework establishes that agent I sees the system as if it were collapsed either onto $|h\rangle$ or $|v\rangle$ (both with probability $p_h = p_v = 1/2$) as a consequence of tracing out the degrees of freedom of ε , A' , and ε' from Eq. (24). But, as the global state has changed onto Eq. (22) as a consequence of agent E 's measurement, a change of how agent I perceives the reality is possible. To answer this question, we can rewrite Eq. (22) using the basis $\{|h\rangle, |v\rangle\}$. The resulting state is

$$\begin{aligned} |\Psi_2(t)\rangle = & \frac{\sin\theta}{\sqrt{2}}(\sin\theta + \cos\theta)|h\rangle|A_h\rangle|\varepsilon_1(t)\rangle|A'_\alpha\rangle|\varepsilon'_1(t)\rangle \\ & + \frac{\cos\theta}{\sqrt{2}}(\sin\theta + \cos\theta)|v\rangle|A_v\rangle|\varepsilon_2(t)\rangle|A'_\alpha\rangle|\varepsilon'_1(t)\rangle \\ & + \frac{\cos\theta}{\sqrt{2}}(\cos\theta - \sin\theta)|h\rangle|A_h\rangle|\varepsilon_1(t)\rangle|A'_\beta\rangle|\varepsilon'_2(t)\rangle \\ & + \frac{\sin\theta}{\sqrt{2}}(\sin\theta - \cos\theta)|v\rangle|A_v\rangle|\varepsilon_2(t)\rangle|A'_\beta\rangle|\varepsilon'_2(t)\rangle. \end{aligned} \quad (25)$$

As any further measurements performed by agent I will involve neither its environment, ε , nor agent E 's apparatus, A' , nor agent E 's environment, ε' , the resulting outcomes can be calculated tracing out all these three degrees of freedom. The result is

$$\begin{aligned} \rho_I = & \frac{1}{4}(2 - \sin 4\theta)|h\rangle|A_h\rangle\langle h|\langle A_h| \\ & + \frac{1}{4}(2 + \sin 4\theta)|v\rangle|A_v\rangle\langle v|\langle A_v|. \end{aligned} \quad (26)$$

This is the first remarkable consequence of the decoherence framework and shows that one has to be very cautious when testing claims made at different stages of an external interference experiment. Let us imagine that the protocol discussed in this section, with $\theta = \pi/8$, has been performed a large number, N , of times. Then let us suppose that we have access to the memory record of agent I —encoded in

the pointer states of the apparatus, $|A_h\rangle$ and $|A_v\rangle$ —before the external interference measurement takes place, in every realization of the experiment. This reading would reveal that agent I has observed h roughly $N/2$ times, and v roughly the same amount of times. Now, let us imagine that an identical protocol is being performed by a colleague, but with a slight difference: in every realization, she reads the internal memory of agent I after the external interference measurement has been completed. Astonishingly, our colleague's reading would reveal that agent I has observed h roughly $N/4$ times, and v roughly $3N/4$ times. At first sight, this conclusion seems preposterous. Our colleague and we are reading an identical internal memory of an identical quantum machine performing an identical ensemble of experiments, modeled by identical Hamiltonians, Eqs. (3) and (12); but we claim that the machine has observed h $N/2$ times, and our colleague claims that this outcome has occurred only $N/4$ times. This absurd contradiction is easily ruled out if we take into account that the external measurement modifies the state of the internal environment, which is the ultimate responsible of the definite outcomes recorded on the memory of the internal agent, and therefore it also modifies these records. Furthermore, the decoherence framework provides an exact procedure to calculate these changes, as we have pointed out above.

This significant result can be summarized by means of the following statement: *if the internal agent I observes a definite outcome, then the external interference measurement performed by agent E changes its memory record; if this change does not occur it is because agent I has not observed a definite outcome.*

The main conclusion we can gather from this analysis is that a contradiction between two claims, one made before an external interference measurement, and the other made afterwards, can be the logical consequence of this interference measurement. Hence, the arguments given in Ref. [2], which are based on the same kind of contradictions, must be studied with care, taking into account all the changes due to all the measurements performed throughout all the protocol. This is the aim of the next section.

To illustrate this analysis, we perform now a numerical simulation covering all the protocol. We study the case with $\theta = \pi/8$, and we consider that both environments are composed of six qubits—the total size of the Hilbert space is $2^{15} = 32\,768$. We start from the state resulting from agent I premeasurement

$$|\Psi_0\rangle = \sqrt{\frac{1}{2}}(|h\rangle|A_h\rangle + |v\rangle|A_v\rangle)|\varepsilon_1\rangle|A'_\alpha\rangle|\varepsilon'_1\rangle, \quad (27)$$

where ε_1 and ε'_1 represent the first states of the basis used to model the internal and the external environments, respectively. Note that we have considered the state $|A'_\alpha\rangle$ as the zero state of the apparatus, but the results do not depend on this particular choice. From this state, the system passes through three stages:

Stage 1. From $t = 0$ to $t = \tau_1$, the internal environment interacts with apparatus A to complete the measurement. Even though the external agent E has not performed any measurement yet, we also consider a similar interaction for

the external environment—in such a case, the external agent E would see a definite outcome pointing to zero, that in

this case corresponds to the outcome α . The corresponding Hamiltonian is

$$H_1 = \left(|A_h\rangle\langle A_h| \sum_{n,m} V_{nm}^h |\varepsilon_n\rangle\langle \varepsilon_m| + |A_v\rangle\langle A_v| \sum_{n,m} V_{nm}^v |\varepsilon_n\rangle\langle \varepsilon_m| \right) \otimes I_E \\ + \left(|A'_\alpha\rangle\langle A'_\alpha| \sum_{n,m} V_{nm}^\alpha |\varepsilon'_n\rangle\langle \varepsilon'_m| + |A'_\beta\rangle\langle A'_\beta| \sum_{n,m} V_{nm}^\beta |\varepsilon'_n\rangle\langle \varepsilon'_m| \right) \otimes I_I, \quad (28)$$

where I_I represents the identity operator over the laboratory in which agent I lives, and I_E the identity operator over the degrees of freedom corresponding to A' and ε' .

Stage 2. From $t = \tau_1$ to $t = \tau_2$, agent E performs its premeasurement. We consider that the interaction with the external environment is switched off, to model that this part of the measurement is purely quantum [18]. However, the interaction between the internal apparatus and the internal environment still exists, because the monitorization is always present after a measurement is completed. The corresponding Hamiltonian is

$$H_2 = \left(|A_h\rangle\langle A_h| \sum_{n,m} V_{nm}^h |\varepsilon_n\rangle\langle \varepsilon_m| + |A_v\rangle\langle A_v| \sum_{n,m} V_{nm}^v |\varepsilon_n\rangle\langle \varepsilon_m| \right) \otimes I_E \\ + g |\beta(\tau_1)\rangle\langle \beta(\tau_1)| [|A'_\alpha\rangle\langle A'_\alpha| + |A'_\beta\rangle\langle A'_\beta| - |A'_\alpha\rangle\langle A'_\beta| - |A'_\beta\rangle\langle A'_\alpha|] \otimes I_I. \quad (29)$$

It is worth remarking that the requirements R1–R4 of Table II have been explicitly taken into account. The interaction leading to agent E premeasurement is based on $|\beta(\tau_1)\rangle$, which is the exact state of the internal laboratory at time $t = \tau_1$. The duration of this stage is exactly $\tau_2 - \tau_1 = \pi/(2g)$.

Stage 3. From $t = \tau_2$ on, the external environment gets correlated with apparatus A' , to complete the measurement performed by agent E . Hence, the Hamiltonian is again given by Eq. (28).

In summary, the system evolves from the initial state given by Eq. (27), $|\Psi_0\rangle$, by means of H_1 , given by Eq. (28), from $t = 0$ to $t = \tau_1$; by means of H_2 , given by Eq. (29), from $t = \tau_1$ to $t = \tau_2$; and by means of H_1 again, from $t = \tau_2$ on. Agent I 's point of view is directly obtained from the real state of the whole system, $|\Psi(t)\rangle$, by tracing out the degrees of freedom corresponding to ε , A' , and ε' . The resulting state can be written

$$\rho_I(t) = C_{hh}(t) |h\rangle\langle h| |A_h\rangle\langle h| \langle A_h| + C_{hv}(t) |h\rangle\langle h| |A_h\rangle\langle v| \langle A_v| \\ + C_{vh}(t) |v\rangle\langle v| |A_v\rangle\langle h| \langle A_h| + C_{vv}(t) |v\rangle\langle v| |A_v\rangle\langle v| \langle A_v|. \quad (30)$$

If $C_{hv} \sim 0$ and $C_{vh} \sim 0$, agent I sees the system as if it were collapsed onto either $|h\rangle\langle h|$, with probability C_{hh} , or $|v\rangle\langle v|$, with probability C_{vv} [19].

Following the same line of reasoning, agent E 's point of view is obtained from $|\Psi(t)\rangle$ by tracing out the external environment, ε' . The resulting state can be written

$$\rho_E(t) = C_{\alpha\alpha}(t) |\alpha(t)\rangle\langle \alpha(t)| |A_\alpha\rangle\langle \alpha(t)| \langle A_\alpha| \\ + C_{\alpha\beta}(t) |\alpha(t)\rangle\langle \beta(t)| |A_\alpha\rangle\langle \beta(t)| \langle A_\beta| \\ + C_{\beta\alpha}(t) |\beta(t)\rangle\langle \alpha(t)| |A_\beta\rangle\langle \alpha(t)| \langle A_\alpha| \\ + C_{\beta\beta}(t) |\beta(t)\rangle\langle \beta(t)| |A_\beta\rangle\langle \beta(t)| \langle A_\beta|. \quad (31)$$

The interpretation is the same as before. If $C_{\alpha\beta} \sim 0$ and $C_{\beta\alpha} \sim 0$, agent E sees the reality as if it were collapsed onto either $|\alpha(t)\rangle\langle \alpha(t)|$, with probability $C_{\alpha\alpha}$, or $|\beta(t)\rangle\langle \beta(t)|$, with probability $C_{\beta\beta}$. It is worth to note that the states of the

internal laboratory $|\alpha(t)\rangle$ and $|\beta(t)\rangle$, change with time, but this is not relevant for agent E 's point of view.

In Fig. 5(a) we show the results from agent I 's point of view. The coupling constant is set $g = 100$; $\tau_1 = 10$, and $\tau_2 - \tau_1 = \pi/200$. The nondiagonal element, $C_{nd} = \sqrt{|C_{hv}|^2 + |C_{vh}|^2}$ (dotted blue line), is significantly large only at the beginning of the simulation; from results in Fig. 1, we expect that larger environments give rise to smaller values for C_{nd} (see Fig. 6 for a deeper discussion). Hence, our first conclusion is that agent I 's point of view is compatible with the photon collapsing either to horizontal or to vertical polarizations. The measurement performed by agent E , which starts at $\tau_1 = 10$, does not alter this fact. However, as we clearly see in the inset of the same panel, this measurement does change elements C_{hh} (violet line) and C_{vv} (green line). In the main part of the panel, we display the expected values, given in Eq. (26), $C_{hh} = 1/4$, $C_{vv} = 3/4$, as black dashed-dotted lines; we can see that these values are quickly reached. Furthermore, we can also see in the inset that this is a smooth change, due to the physical interaction between the laboratory and the apparatus A' . Therefore, *agent I 's point of view continuously changes during this small period of time*. As we have already pointed out, the dependence of the Hamiltonian (29) on the internal environmental states alter the definite outcomes observed by agent I , and therefore the records of its internal memory. Thus, this simulation illustrates how the apparent contradiction discussed above is solved.

Figure 5(b) represents agent E 's point of view. Before performing the measurement, its apparatus points α because this is chosen as zero. Then, at $t = \tau_1$ this point of view starts to change. $C_{\alpha\alpha}$ (solid violet line) changes to $C_{\alpha\alpha} = 0.854$, the expected value from Eq. (23), and equally $C_{\beta\beta}$ (solid green line) changes to $C_{\beta\beta} = 0.146$. During the first instants of time after the premeasurement, the nondiagonal element $C_{nd} = \sqrt{|C_{\alpha\beta}|^2 + |C_{\beta\alpha}|^2}$ (blue dotted line) is significantly different from zero; but, after the external environment has

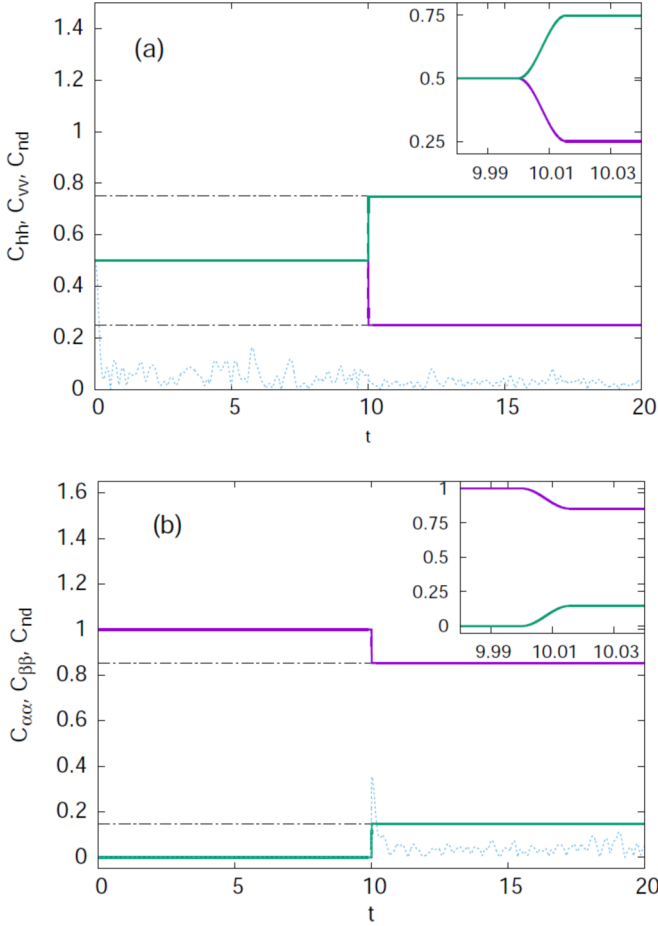


FIG. 5. (a) Matrix elements C_{hh} (solid, violet line), C_{vv} (solid green line), and $C_{nd} = \sqrt{|C_{hv}|^2 + |C_{vh}|^2}$ (dashed blue line), from Eq. (30). Dotted-dashed lines show the expected values at stage 3. The inset show C_{hh} and C_{vv} around stage 2. (b) Matrix elements $C_{\alpha\alpha}$ (solid, violet line), $C_{\beta\beta}$ (solid green line), and $C_{nd} = \sqrt{|C_{\alpha\beta}|^2 + |C_{\beta\alpha}|^2}$ (dashed blue line), from Eq. (31). Dotted-dashed lines show the expected values at stage 3. The inset show $C_{\alpha\alpha}$ and $C_{\beta\beta}$ around stage 2. The number of qubits of both environment is $N = 6$, $g = 10^2$, $\tau_1 = 10$, and $\tau_2 - \tau_1 = \pi/200$.

played its role, agent E 's point of view becomes compatible with the laboratory collapsed either to α (with probability $p = 0.854$) or to β (with probability $p = 0.146$) as expected.

A finite-size scaling analysis of the nondiagonal element of ρ_I is given in Fig. 6. Due to the huge size of the whole Hilbert space, it is not possible to reach large environmental sizes. However, we clearly see in the inset how the size of this nondiagonal element, C_{nd} , averaged from $t = 3$ to $t = 100$, decays with the number of environmental qubits. Furthermore, a visual comparison between the cases with $N = 3$ (green line) and $N = 6$ (red line), given in the main panel of the same figure, corroborates this impression. Therefore, we can conjecture that both agents I and E see their measured systems as if they were collapsed, provided that their corresponding environments are large enough.

Finally, we study how the results depend on the coupling constant between the external apparatus, A' , and the laboratory whose state is measured by agent E . In Fig. 7 we show

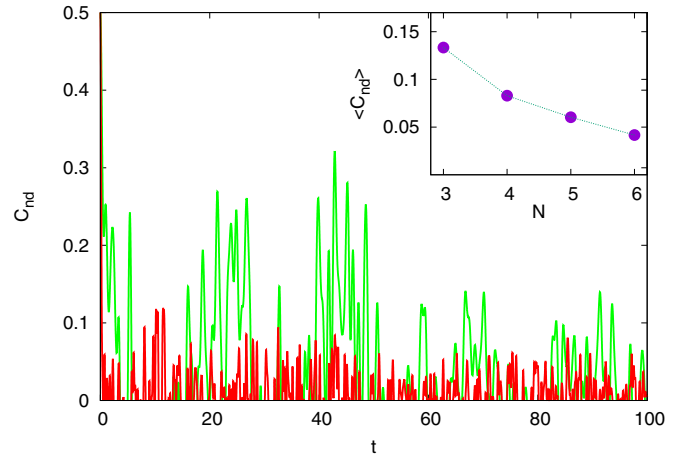


FIG. 6. $C_{nd} = \sqrt{|C_{hv}|^2 + |C_{vh}|^2}$, from Eq. (30), for an environment with $N = 3$ qubits (light green line) (dashed blue line), and for an environment with $N = 6$ qubits (dark red line). In the inset, scaling analysis for the time average of C_{nd} obtained from $t = 3$ to $t = 100$.

C_{hh} for $N = 6$ and $g = 1$ (blue line), $g = 10$ (green line), and $g = 100$ (violet line), together with the expected value, $C_{hh} = 1/4$ (dotted-dashed black line). We conclude that this expected value is reached only if g is large enough. The explanation is quite simple. If g is small, the time required for the external apparatus A' to complete the premeasurement is large compared with the characteristic correlation time of the laboratory, given in Fig. 2. Therefore, the state $\beta(\tau_1)$, used in Eq. (29), ceases to be the real state of the laboratory while the external apparatus, A' , is still performing the premeasurement. As a consequence, the resulting measurement is not correct, and neither agent E nor agent I reaches the expected results. This is an important fact that makes a bit more difficult the external interference measurements trademark of Wigner's-friend experiments. Besides the requirements R1–R4 of Table II, it is also mandatory that the external premeasurement is shorter than the characteristic time of the internal dynamics of the measured laboratory. As is shown in Fig. 2, the larger

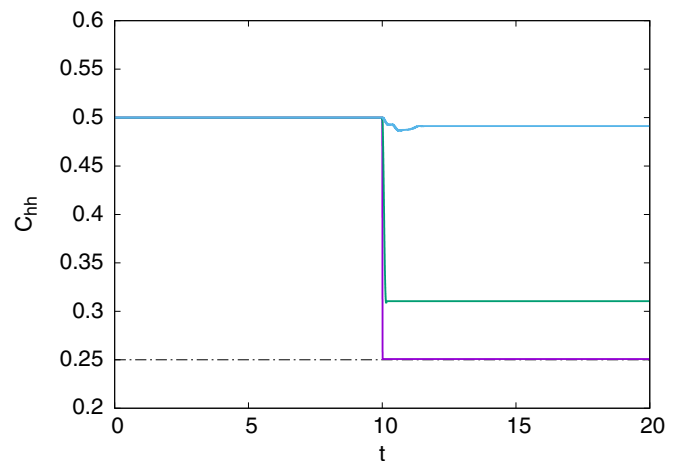


FIG. 7. C_{hh} from Eq. (30) for different coupling constants g in Eq. (29): $g = 1$ (blue line), $g = 10$ (green line), $g = 100$ (violet line). Dotted-dashed line shows the expected value for stage 3.

the internal environment, the shorter this time. Hence, if agent I is a conscious (human) being, composed of a huge number of molecules, the external interference premeasurement must be completed in a tiny amount of time.

The first conclusion we can gather from all these results is that, according to the decoherence framework, *the memory records of all the agents involved in a Wigner's-friend experiment will generically change after the actions of any other agents*, and therefore we must take these changes into account when comparing claims made at different stages of the experiment. As we will see in next sections, this is the clue to interpreting the extended versions of the experiment.

Notwithstanding, agent I still sees the reality as if the measured photon were either horizontally or vertically polarized—not in a superposition of both states. Even more, states ρ_E , given by Eq. (23), and ρ_I , given by Eq. (26), seem incompatible at a first sight. But this is just a consequence of the differences between the experiments performed by these two agents. Agent I sees the universe as if it were in state ρ_I , because it ignores ε , A' , and ε' . On the other hand, agent E sees the universe as if it were in state ρ_E , because it just ignores ε' , and therefore has relevant information about A' and ε . And, even more important, both agents agree that their perceptions about the reality are linked to the limitations of their experiments, and that the real state of the universe is a complex, entangled, and superposition state involving the measured photon, both their apparatus, both the environments that surround them, and themselves—neither ρ_I , nor ρ_E . Nevertheless, one of the most remarkable consequences of the decoherence framework is that this fact does not prevent any of the agents from making correct claims about the outcomes observed by the others. We will see in the next sections that the four agents involved in the extended version of the Wigner's-friend experiment discussed in Refs. [2–4] can make correct—not contradictory—claims about the other agents' outcomes just considering (i) the results of their own experiments, that is, the records of their own memories, and (ii) how external interference measurements change these records. No other ingredients, like the point of view change proposed in Ref. [17], are required.

IV. CONSISTENCY OF THE QUANTUM THEORY

The aim of this section is to discuss the thought experiment proposed in Ref. [2] within the framework presented above. A number of comments and criticisms have been already published, including Ref. [3] and some others [20–23]. This work deals with the original proposal in Ref. [2].

A. No-go theorem and original interpretation

Both no-go theorems discussed in Refs. [2,3] share a similar scheme: (a) A pair of entangled quantum systems is generated. In Ref. [2] it consists in a *quantum coin*, with an orthogonal basis given by $\{|head\rangle_R, |tail\rangle_R\}$, and a 1/2-spin, spanned by $\{|\downarrow\rangle_S, |\uparrow\rangle_S\}$. The initial entangled state is

$$|\Psi\rangle = \sqrt{\frac{1}{3}}|head\rangle_R|\downarrow\rangle_S + \sqrt{\frac{2}{3}}|tail\rangle_R|\rightarrow\rangle_S, \quad (32)$$

where $|\rightarrow\rangle_S = \sqrt{\frac{1}{2}}(|\downarrow\rangle_S + |\uparrow\rangle_S)$.

To simplify the notation and make it compatible with Refs. [3,4], the following changes are made: (i) instead of the quantum coin and the spin in Eq. (32), two polarized photons are used, (ii) the first photon is denoted by the subindex a , and the second one, by the subindex b , and (iii) the superpositions of vertical and horizontal polarization are denoted $|+\rangle = \sqrt{\frac{1}{2}}(|h\rangle + |v\rangle)$ and $|-\rangle = \sqrt{\frac{1}{2}}(|h\rangle - |v\rangle)$, respectively. With this notation, the initial state in Ref. [2] reads

$$|\Psi\rangle = \sqrt{\frac{1}{3}}|h\rangle_a|v\rangle_b + \sqrt{\frac{2}{3}}|v\rangle_a|+\rangle_b. \quad (33)$$

(b) Photon a is sent to a closed laboratory A , and photon b , to a closed laboratory B .

(c) An observer I_A , inside laboratory A , measures the state of photon a ; and an observer I_B , inside laboratory B , measures the state of photon b .

(d) An external observer E_A measures the state of the whole laboratory A , and an external observer E_B measures the state of the whole laboratory B .

Both no-go theorems [2,3] deal with the observations made by I_A , I_B , E_A , and E_B . The one formulated in Ref. [2] is based upon the following assumptions:

Assumption Q. Let us consider that a quantum system is in the state $|\Psi\rangle$. Then let us suppose that an experiment has been performed on a complete basis $\{|x_1\rangle, \dots, |x_n\rangle\}$, giving an unknown outcome x . Then, if $\langle\Psi|\pi_m|\Psi\rangle = 1$, where $\pi_m = |x_m\rangle\langle x_m|$, for a particular state of the former basis, $|x_m\rangle$, then I am certain that the outcome is $x = x_m$.

Assumption C. If I am certain that some agent, upon reasoning within the same theory I am using, knows that a particular outcome x is $x = x_m$, then I am also certain that $x = x_m$.

Assumption S. If I am certain that a particular outcome is $x = x_m$, I can safely reject that $x \neq x_m$.

The theorem says that there exist circumstances under which any quantum theory satisfying these three assumptions is bound to yield contradictory conclusions. The extended version of the Wigner's-friend experiment discussed in Ref. [2] constitutes one paradigmatic example of such circumstances.

Before continuing with the analysis, it is worth remarking that the theorem focuses on particular outcomes that happen for certain—with probability $p = 1$. It refers neither to the real state of the corresponding system nor to a subjective interpretation made by any of the agents. Hence, its most remarkable feature is that contradictions arise as consequences of simple observations.

Let us review now all the steps of the experiment from the four agents' points of view. As it is explained in Ref. [2], to infer their conclusions they need (i) the knowledge of the initial state of the whole system, (ii) their outcomes, and (iii) the details of the experimental protocol, in order to predict future outcomes, or track back past ones, relying on the unitary evolutions of the corresponding (pre)measurements. We do not go into details about the assumptions required to reach each conclusion; we refer the reader to the original paper [2] for that purpose. Moreover, we do not consider now the decoherence framework; all the measurements are

understood as correlations between the measured (part of the) system and the measuring apparatus.

Step 1. Agent I_A measures the initial state, given by Eq. (33), in the basis $\{|h\rangle_a, |v\rangle_a\}$.

Fact 1: Given the shape of the initial state, agent I_A concludes that, if it obtains that photon a is vertically polarized (outcome v_a), then a further measurement of the laboratory B in the basis $\{|+\rangle_B, |-\rangle_B\}$ will lead to the outcome $+_B$.

The resulting state of agent I_A 's measurement is

$$|\Psi\rangle_1 = \sqrt{\frac{1}{3}}|h\rangle_a|v\rangle_b|A_h\rangle_a + \sqrt{\frac{2}{3}}|v\rangle_a|+\rangle_b|A_v\rangle_a. \quad (34)$$

This expression can be simplified considering the whole state of the laboratory A , which consists in the photon a and the measuring apparatus A_a . Hence, let us denote

$$|h\rangle_A \equiv |h\rangle_a|A_h\rangle_a, \quad (35a)$$

$$|v\rangle_A \equiv |v\rangle_a|A_v\rangle_a. \quad (35b)$$

And, therefore, the state after this measurement is

$$|\Psi\rangle_1 = \sqrt{\frac{1}{3}}|h\rangle_A|v\rangle_b + \sqrt{\frac{2}{3}}|v\rangle_A|+\rangle_b. \quad (36)$$

Fact 1 *seems* compatible with this state. There is a perfect correlation between state $|v\rangle_A$, which represents the case in which agent I_A has observed that the photon a is vertically polarized, and state $|+\rangle_b$. Thus, agent I_A can deduce that the laboratory B will evolve from $|+\rangle_b$ to $|+\rangle_B$, as a consequence of agent I_B 's measurement. And hence, considering irrelevant the further action of agent E_A , because it does not deal with laboratory B [2], a further measurement on laboratory B will yield $+_B$, subjected to the outcome v_a . We will see in Sec. IV C that considering irrelevant the action of agent E_A is not important if the decoherence framework is *not* taken into account—if the measurements consist just on correlations between the systems and the apparatuses. In Sec. IV B we will discuss how the decoherence framework alter this fact.

Step 2. Agent I_B measures photon b in the basis $\{|h\rangle_b, |v\rangle_b\}$.

Fact 2: If agent I_B observes that the photon is horizontally polarized, then the outcome of agent I_A cannot correspond to a horizontally polarized photon.

Using the same notation as before (applied to laboratory B), the state after agent I_B completes its measurement is

$$|\Psi\rangle_2 = \sqrt{\frac{1}{3}}|v\rangle_A|h\rangle_B + \sqrt{\frac{1}{3}}|v\rangle_A|v\rangle_B + \sqrt{\frac{1}{3}}|h\rangle_A|v\rangle_B. \quad (37)$$

Therefore, there is a perfect correlation between $|h\rangle_B$ and $|v\rangle_A$; the probability of observing h_b and h_a in the same realization of the experiment is zero. Hence, all the previous conclusions are well supported.

Step 3. Agent E_A measures laboratory A in the basis $\{|+\rangle_A, |-\rangle_A\}$, where

$$|+\rangle_A = \sqrt{\frac{1}{2}}(|h\rangle_A + |v\rangle_A), \quad (38a)$$

$$|-\rangle_A = \sqrt{\frac{1}{2}}(|h\rangle_A - |v\rangle_A). \quad (38b)$$

Then the state after this measurement is

$$|\Psi\rangle_3 = \sqrt{\frac{2}{3}}|+\rangle_A|A'_+\rangle_A|v\rangle_B + \sqrt{\frac{1}{6}}|+\rangle_A|A'_+\rangle_A|h\rangle_B - \sqrt{\frac{1}{6}}|-\rangle_A|A'_-\rangle_A|h\rangle_B, \quad (39)$$

where A' is the measuring apparatus used by agent E_A . From this state, we obtain the following:

Fact 3a: If the outcome obtained by agent E_A is $-_A$, then agent I_B has obtained an horizontally polarized photon, h_b , in its measurement.

Fact 3b: Given facts 3a and 2, the outcome $-_A$, obtained by agent E_A determines that agent I_A could not obtain an horizontally polarized photon.

Fact 3c: Given the facts 3b and 1, the outcome $-_A$ determines that a further measurement on laboratory B , in the basis $\{|+\rangle_B, |-\rangle_B\}$ will necessary yield $+_B$.

The main conclusion we can infer from these sequential reasonings is that, if agent E_A observes $-_A$, then E_B is bounded to observe $+_B$. Therefore, it is not possible that outcomes $-_A$ and $-_B$ occur in the same realization of the experiment. Furthermore, as is discussed in detail in Ref. [2], relying on assumptions Q, S, and C, it is straightforward to show that the four agents agree with that.

The contradiction that (presumably) establishes that quantum theory cannot consistently describe the use of itself consists in that the probability of obtaining $-_A$ and $-_B$ in the same realization of the experiments is $1/12$, *even though all the agents, relying on assumptions Q, C, and S, agree that such probability must be zero*. This can be easily inferred from the final state of the system after measurements performed by all the agents (including E_B) are completed,

$$|\Psi\rangle = \sqrt{\frac{3}{4}}|+\rangle_A|A'_+\rangle_A|+\rangle_B|A'_+\rangle_B - \sqrt{\frac{1}{12}}|+\rangle_A|A'_+\rangle_A|-\rangle_B|A'_-\rangle_B - \sqrt{\frac{1}{12}}|-\rangle_A|A'_-\rangle_A|+\rangle_B|A'_+\rangle_B - \sqrt{\frac{1}{12}}|-\rangle_A|A'_-\rangle_A|-\rangle_B|A'_-\rangle_B. \quad (40)$$

B. The role of the decoherence framework

The first element that the decoherence framework introduces is that every premeasurement has to be fixed by the action of the corresponding environment. Notwithstanding, this fact does not change too much the equations discussed in the previous section. The states of all the laboratories change with time, due to the continuous monitorization by their environments, and all the measurements must be completed at their exact times, following the results in Table II, but the structure of all the resulting equations is pretty much the same. For example, the state of laboratory A must be written

$$|h(t)\rangle_A \equiv |h\rangle_a|A_h\rangle_a|\varepsilon_1(t)\rangle_a, \quad (41a)$$

$$|v(t)\rangle_A \equiv |v\rangle_a|A_v\rangle_a|\varepsilon_2(t)\rangle_a, \quad (41b)$$

including the time-dependent environment, and the final state of the whole setup becomes

$$\begin{aligned}
|\Psi\rangle = & \sqrt{\frac{3}{4}}|+(\tau)\rangle_A|A'_+(\tau)\rangle_A|\varepsilon'_1(\tau)\rangle_A|+(\tau)\rangle_B|A'_+(\tau)\rangle_B|\varepsilon'_1(\tau)\rangle_B - \sqrt{\frac{1}{12}}|+(\tau)\rangle_A|A'_+(\tau)\rangle_A|\varepsilon'_1(\tau)\rangle_A|-(\tau)\rangle_B|A'_-(\tau)\rangle_B|\varepsilon'_2(\tau)\rangle_B \\
& - \sqrt{\frac{1}{12}}|-(\tau)\rangle_A|A'_-(\tau)\rangle_A|\varepsilon'_2(\tau)\rangle_A|+(\tau)\rangle_B|A'_+(\tau)\rangle_B|\varepsilon'_1(\tau)\rangle_B - \sqrt{\frac{1}{12}}|-(\tau)\rangle_A|A'_-(\tau)\rangle_A|\varepsilon'_2(\tau)\rangle_A|-(\tau)\rangle_B|A'_-(\tau)\rangle_B|\varepsilon'_2(\tau)\rangle_B,
\end{aligned} \tag{42}$$

instead of much simpler Eq. (40).

Another important point is that requisites R1–R4 from Table II, together with the fast-enough realization of the external interference experiments, are mandatory to reach the previous conclusion. Hence, assumptions Q, S, and C might lead to contradictory conclusions only if the experiment is performed under very specific circumstances. Results in Fig. 2 suggest that, the larger the laboratories A and B are, the more specific the circumstances of the experiment must be. Thus, if agents are not small quantum machines, composed of just a few qubits, but human beings, composed of a huge number of particles, the probability that such a contradiction might arise is virtually zero. Notwithstanding, this conclusion affects only human beings acting as agents. Quantum machines acting coherently, like the 53-qubit quantum computer recently developed [14], are free from this limitation. Therefore, and despite the huge complexity of such experiments, we can trust that they will be feasible in the future.

Now, let us imagine that quantum technologies are sufficiently developed, and let us go ahead with the experiment. That is, let us wonder if quantum theory can consistently describe the use of itself, relying on the decoherence formalism. For this purpose, we follow the same guidelines of the setup in Ref. [2]: we assume that all agents are aware of the whole experimental procedure, and use the decoherence framework to determine the outcomes that the other agents have obtained or will obtain, conditioned to their own outcomes.

We start our analysis with fact 1. Considering the environmental monitorization, the state after I_A has performed its measurement is

$$|\Psi_1\rangle = \sqrt{\frac{1}{3}}|h(t)\rangle_A|v\rangle_b + \sqrt{\frac{2}{3}}|v(t)\rangle_A|+\rangle_b, \tag{43}$$

with $|h(t)\rangle_A$ and $|v(t)\rangle_A$ given by Eqs. (41a) and (41b). Let us study the conclusions that agent I_A can reach from this state, relying on assumptions Q, C, and S, and the decoherence framework. After tracing out the environmental degrees of freedom, Eq. (43) gives rise to

$$\begin{aligned}
\rho_1 = & \frac{1}{3}|h\rangle_a\langle A_h|_a|v\rangle_b\langle h|_a\langle A_h|_a|v\rangle_b \\
& + \frac{2}{3}|v\rangle_a\langle A_v|_a|+\rangle_b\langle v|_a\langle A_v|_a|+\rangle_b,
\end{aligned} \tag{44}$$

that is, it establishes a correlation between the outcome v_a , obtained by agent I_A , and the state $|+\rangle_b$. Hence, the first conclusion that agent I_A can reach is the following:

(i) If agent I_B measures photon b in the basis $\{|+\rangle_b, |-\rangle_b\}$, it will obtain the outcome $+_b$, if I have obtained v_a .

However, as this measurement is not actually performed, this statement is useless; agent I_A needs further reasonings and calculations to reach a valid conclusion. Thus, it jumps

to the next step in the experimental protocol and takes into account the consequences of agent I_B 's measurement. Relying on the decoherence framework and considering the action of the corresponding unitary operators, agent I_A can calculate that the resulting state is

$$\begin{aligned}
|\Psi_2\rangle = & \sqrt{\frac{1}{3}}|v\rangle_a|A_v\rangle_a|\varepsilon_2(t)\rangle_a|h\rangle_b|A_h\rangle_b|\varepsilon_1(t)\rangle_b \\
& + \sqrt{\frac{1}{3}}|v\rangle_a|A_v\rangle_a|\varepsilon_2(t)\rangle_a|v\rangle_b|A_v\rangle_b|\varepsilon_2(t)\rangle_b \\
& + \sqrt{\frac{1}{3}}|h\rangle_a|A_h\rangle_a|\varepsilon_1(t)\rangle_a|v\rangle_b|A_v\rangle_b|\varepsilon_2(t)\rangle_b,
\end{aligned} \tag{45}$$

which can be written

$$\begin{aligned}
|\Psi_2\rangle = & \sqrt{\frac{2}{3}}|v\rangle_a|A_v\rangle_a|\varepsilon_2(t)\rangle_a|+(\tau)\rangle_B \\
& + \sqrt{\frac{1}{6}}|h\rangle_a|A_h\rangle_a|\varepsilon_1(t)\rangle_a|+(\tau)\rangle_B \\
& - \sqrt{\frac{1}{6}}|h\rangle_a|A_h\rangle_a|\varepsilon_1(t)\rangle_a|-(\tau)\rangle_B.
\end{aligned} \tag{46}$$

And, after tracing out the corresponding environmental degrees of freedom, agent I_A 's perception can be written as

$$\begin{aligned}
\rho_2 = & \frac{2}{3}|v\rangle_a\langle A_v|_a|+(\tau)\rangle_B\langle v|_a\langle A_v|_a|+(\tau)\rangle_B \\
& + \frac{1}{6}|h\rangle_a\langle A_h|_a|+(\tau)\rangle_B\langle h|_a\langle A_h|_a|+(\tau)\rangle_B \\
& + \frac{1}{6}|h\rangle_a\langle A_h|_a|-(\tau)\rangle_B\langle h|_a\langle A_h|_a|-(\tau)\rangle_B \\
& - \frac{1}{6}|h\rangle_a\langle A_h|_a|+(\tau)\rangle_B\langle h|_a\langle A_h|_a|-(\tau)\rangle_B \\
& - \frac{1}{6}|h\rangle_a\langle A_h|_a|-(\tau)\rangle_B\langle h|_a\langle A_h|_a|+(\tau)\rangle_B.
\end{aligned} \tag{47}$$

Therefore, agent I_A can make the following statement, which *seems similar* to fact 1:

(ii) If agent I_B measures photon b in the basis $\{|h\rangle_b, |v\rangle_b\}$, and subsequently, without any other measurement in between, agent E_B measures laboratory B in the basis $\{|+(\tau)\rangle_B, |-(\tau)\rangle_B\}$, the last one will obtain $|+\rangle_B$, if I have obtained $|v\rangle_a$.

However, this statement does *not* represent the thought experiment discussed in Ref. [2]. The experimental protocol establishes that agent I_B measures photon b in the basis $\{|h\rangle_b, |v\rangle_b\}$, then agent E_A measures the whole laboratory A in the basis $\{|+(\tau)\rangle_A, |-(\tau)\rangle_B\}$, and finally agent E_B measures laboratory B in the basis $\{|+(\tau)\rangle_B, |-(\tau)\rangle_B\}$. This is the point at which the differences between the decoherence framework and the standard interpretations—measurements as correlations between systems and apparatuses—emerge. As we have discussed in Sec. III, an external interference

measurement, like the one performed by agent E_A , generally implies changes in the memory record of the measured agent. Notwithstanding, as these changes can be exactly calculated, agent I_A can still rely on the decoherence framework to predict

the correlations between its outcome, v_a , and the one that agent E_B will obtain when measuring laboratory B in the basis $\{|+(\tau)\rangle_B, |-(\tau)\rangle_B\}$. Just before the final measurement by agent E_B , the state of the whole system can be written

$$\begin{aligned} |\Psi_3(\tau)\rangle = & \sqrt{\frac{3}{8}}|h\rangle_a|A_h\rangle_a|\varepsilon_1(\tau)\rangle_a|A'_+\rangle_A|\varepsilon'_1(\tau)\rangle_A|+\rangle_B + \sqrt{\frac{3}{8}}|v\rangle_a|A_v\rangle_a|\varepsilon_2(\tau)\rangle_a|A'_+\rangle_A|\varepsilon'_1(\tau)\rangle_A|+\rangle_B \\ & - \sqrt{\frac{1}{24}}|h\rangle_a|A_h\rangle_a|\varepsilon_1(\tau)\rangle_a|A'_+\rangle_A|\varepsilon'_1(\tau)\rangle_A|-\rangle_B - \sqrt{\frac{1}{24}}|v\rangle_a|A_v\rangle_a|\varepsilon_2(\tau)\rangle_a|A'_+\rangle_A|\varepsilon'_1(\tau)\rangle_A|-\rangle_B \\ & - \sqrt{\frac{1}{24}}|h\rangle_a|A_h\rangle_a|\varepsilon_1(\tau)\rangle_a|A'_-\rangle_A|\varepsilon'_2(\tau)\rangle_A|+\rangle_B + \sqrt{\frac{1}{24}}|v\rangle_a|A_v\rangle_a|\varepsilon_2(\tau)\rangle_a|A'_-\rangle_A|\varepsilon'_2(\tau)\rangle_A|+\rangle_B \\ & - \sqrt{\frac{1}{24}}|h\rangle_a|A_h\rangle_a|\varepsilon_1(\tau)\rangle_a|A'_-\rangle_A|\varepsilon'_2(\tau)\rangle_A|-\rangle_B + \sqrt{\frac{1}{24}}|v\rangle_a|A_v\rangle_a|\varepsilon_2(\tau)\rangle_a|A'_-\rangle_A|\varepsilon'_2(\tau)\rangle_A|-\rangle_B. \end{aligned} \quad (48)$$

Hence, tracing out agent I_A 's environment, and both agent E_A 's apparatus and environment, since agent E_A 's outcome is irrelevant, the state of agent I_A 's memory reads

$$\begin{aligned} \rho_3 = & \frac{5}{12}|h\rangle_a|A_h\rangle_a|+\rangle_B\langle h|_a\langle A_h|_a\langle +|_B \\ & + \frac{5}{12}|v\rangle_a|A_v\rangle_a|+\rangle_B\langle v|_a\langle A_v|_a\langle +|_B \\ & + \frac{1}{12}|h\rangle_a|A_h\rangle_a|-\rangle_B\langle h|_a\langle A_h|_a\langle -|_B \\ & + \frac{1}{12}|v\rangle_a|A_v\rangle_a|-\rangle_B\langle v|_a\langle A_v|_a\langle -|_B. \end{aligned} \quad (49)$$

And therefore, relying on assumption Q, agent I_A can make the following claims: (i) the system is in the state given by Eq. (48) just before agent E_B 's measurement; (ii) a certain outcome in the basis $\{|+(\tau)\rangle_B, |-(\tau)\rangle_B\}$ is going to be obtained; and (iii) neither $\langle\Psi_3(\tau)|\pi_{+,B}|\Psi_3(\tau)\rangle = 1$, nor $\langle\Psi_3(\tau)|\pi_{-,B}|\Psi_3(\tau)\rangle = 1$, conditioned to my memory record is v_a . Hence, the decoherence framework modifies fact 1, giving rise to the following:

New fact 1: If agent I_A obtains that the photon is vertically polarized (outcome v_a), then a further measurement of the laboratory B in the basis $\{|+(\tau)\rangle_B, |-(\tau)\rangle_B\}$ will lead to either $+_B$ (with $p = 5/6$) or $-_B$ (with $p = 1/6$).

It is worth noting that this prediction can be experimentally confirmed by simultaneously reading the memory records of agents I_A and E_B , as soon as the last outcome is fixed by the corresponding environmental monitoring. Even though it is reasonable to wonder if this inconclusive statement is a consequence of the changes induced in agent I_A 's memory by the external interference measurement performed by agent E_A , the key point is that no correlations between I_A and E_B perceptions exist before the action of agent E_A , so there is no other way to determine whether agent E_B 's outcome is bounded by agent I_A 's or not. In Sec. IV C we will see that the results are different if the decoherence framework is not taken into account.

As the inconsistency discussed in Ref. [2] is based on fact 1, the result we have obtained is enough to show that the decoherence framework is free from it. Notwithstanding, to delve in the interpretation of this remarkable thought experiment, we will discuss facts 2 and 3.

Again, we focus on the predictions that the involved agents can make by means of the decoherence framework and their experimental verification by reading their corresponding memory records. Fact 2 is made from agent I_B 's point of view, so we focus on the state of the system after both agents I_A and I_B have completed their measurements, which reads

$$\begin{aligned} |\Psi_2\rangle = & \sqrt{\frac{1}{3}}|v\rangle_a|A_v\rangle_a|\varepsilon_2(t)\rangle_a|h\rangle_b|A_h\rangle_b|\varepsilon_1(t)\rangle_b \\ & + \sqrt{\frac{1}{3}}|v\rangle_a|A_v\rangle_a|\varepsilon_2(t)\rangle_a|v\rangle_b|A_v\rangle_b|\varepsilon_2(t)\rangle_b \\ & + \sqrt{\frac{1}{3}}|h\rangle_a|A_h\rangle_a|\varepsilon_1(t)\rangle_a|v\rangle_b|A_v\rangle_b|\varepsilon_2(t)\rangle_b. \end{aligned} \quad (50)$$

To obtain their common view of the system, both their environments must be traced out. Hence, the memory records of both agents are compatible with the mixed state given by [24]

$$\begin{aligned} \rho_2 = & \frac{1}{3}|v\rangle_a|A_v\rangle_a|h\rangle_b|A_h\rangle_b\langle v|_a\langle A_v|_a\langle h|_b\langle A_h|_b \\ & + \frac{1}{3}|v\rangle_a|A_v\rangle_a|v\rangle_b|A_v\rangle_b\langle v|_a\langle A_v|_a\langle v|_b\langle A_v|_b \\ & + \frac{1}{3}|h\rangle_a|A_h\rangle_a|v\rangle_b|A_v\rangle_b\langle h|_a\langle A_h|_a\langle v|_b\langle A_v|_b. \end{aligned} \quad (51)$$

That is, if agent I_B relies on the decoherence framework to calculate agent I_A 's outputs conditioned to the one it has obtained, it can safely conclude fact 2 *at this stage of the experiment*.

Let us now proceed with fact 3. As is formulated from agent E_A 's point of view, we start from the state of the system after agent E_A 's measurement, which reads

$$\begin{aligned} |\Psi_3(\tau)\rangle = & \sqrt{\frac{2}{3}}|+(\tau)\rangle_A|A'_+\rangle_A|\varepsilon'_1(\tau)\rangle_A|v\rangle_b|A_v\rangle_b|\varepsilon_2(\tau)\rangle_b \\ & + \sqrt{\frac{1}{6}}|+(\tau)\rangle_A|A'_+\rangle_A|\varepsilon'_1(\tau)\rangle_A|h\rangle_b|A_h\rangle_b|\varepsilon_1(\tau)\rangle_b \\ & - \sqrt{\frac{1}{6}}|-(\tau)\rangle_A|A'_-\rangle_A|\varepsilon'_2(\tau)\rangle_A|h\rangle_b|A_h\rangle_b|\varepsilon_1(\tau)\rangle_b. \end{aligned} \quad (52)$$

Equation (52) represents the state of the whole system after the measurements performed by agents I_A , I_B , and E_A are completed. The way that these agents perceive this state depends again on the action of their respective environments, and therefore can be described by tracing out the corresponding degrees of freedom. A joint vision of agents I_B and E_A is obtained tracing out the environments ε_b and ε'_A , leading to

$$\begin{aligned} \rho_3(\tau) = & \frac{2}{3} | +(\tau) \rangle_A | A'_+ \rangle_A | v \rangle_b | A_v \rangle_b \langle +(\tau) |_A \langle A'_+ |_A \langle v |_b \langle A_v |_b \\ & + \frac{1}{6} | +(\tau) \rangle_A | A'_+ \rangle_A | h \rangle_b | A_h \rangle_b \langle +(\tau) |_A \langle A'_+ |_A \langle h |_b \langle A_h |_b \\ & + \frac{1}{6} | -(\tau) \rangle_A | A'_- \rangle_A | h \rangle_b | A_h \rangle_b \langle -(\tau) |_A \langle A'_- |_A \langle h |_b \langle A_h |_b. \end{aligned} \quad (53)$$

This state is fully compatible with fact 3a. At this stage of the experiment, the correlation between the memory records

of I_B and E_A is incompatible with the outcomes $-_A$ and v_b being obtained at the same run of the experiment. This means that agent E_A can use assumption Q to conclude (i) system is in state given by Eq. (52) after my measurement, (ii) a certain outcome was obtained by agent I_B in the basis $\{|h\rangle_b, |v\rangle_b\}$, and (iii) as $\langle \Psi_3(\tau) | \pi_{h,b} | \Psi_3(\tau) \rangle = 1$, conditions I have obtained $-_A$, then fact 3a is correct. Furthermore, agent I_B , relying only on its outcome, the details of the whole protocol, and the decoherence framework, can also predict fact 3a.

To follow with the argument, agent E_A performs a nested reasoning to determine the outcome obtained by agent I_A . A very relevant point is the time at which agent I_A 's memory record is evaluated. If we rewrite the current state of the system, Eq. (52), in a basis including $\{|h\rangle_a | h\rangle_b, |h\rangle_a | v\rangle_b, |v\rangle_a | h\rangle_b, |v\rangle_a | v\rangle_b\}$, we obtain

$$\begin{aligned} |\Psi_3(\tau)\rangle = & \sqrt{\frac{1}{3}} | h \rangle_a | A_h \rangle_a | \varepsilon_1(\tau) \rangle_a | A'_+ \rangle_A | \varepsilon'_1(\tau) \rangle_A | v \rangle_b | A_v \rangle_b | \varepsilon_2(\tau) \rangle_b + \sqrt{\frac{1}{3}} | v \rangle_a | A_v \rangle_a | \varepsilon_2(\tau) \rangle_a | A'_+ \rangle_A | \varepsilon'_1(\tau) \rangle_A | v \rangle_b | A_v \rangle_b | \varepsilon_2(\tau) \rangle_b \\ & + \sqrt{\frac{1}{12}} | h \rangle_a | A_h \rangle_a | \varepsilon_1(\tau) \rangle_a | A'_+ \rangle_A | \varepsilon'_1(\tau) \rangle_A | h \rangle_b | A_h \rangle_b | \varepsilon_1(\tau) \rangle_b + \sqrt{\frac{1}{12}} | v \rangle_a | A_v \rangle_a | \varepsilon_2(\tau) \rangle_a | A'_+ \rangle_A | \varepsilon'_1(\tau) \rangle_A | h \rangle_b | A_h \rangle_b | \varepsilon_1(\tau) \rangle_b \\ & - \sqrt{\frac{1}{12}} | h \rangle_a | A_h \rangle_a | \varepsilon_1(\tau) \rangle_a | A'_- \rangle_A | \varepsilon'_2(\tau) \rangle_A | h \rangle_b | A_h \rangle_b | \varepsilon_1(\tau) \rangle_b + \sqrt{\frac{1}{12}} | v \rangle_a | A_v \rangle_a | \varepsilon_2(\tau) \rangle_a | A'_- \rangle_A | \varepsilon'_2(\tau) \rangle_A | h \rangle_b | A_h \rangle_b | \varepsilon_1(\tau) \rangle_b. \end{aligned} \quad (54)$$

Thus, to determine the joint vision of agents I_A and I_B at this stage of the experiment we have just to trace out ε_a , ε_b , A'_A , and ε'_A from the density matrix arising from this wave function. This leads to

$$\begin{aligned} \rho_3 = & \frac{1}{3} | h \rangle_b | A_h \rangle_b | v \rangle_a | A_v \rangle_a \langle h |_b \langle A_h |_b \langle v |_a \langle A_v |_a \\ & + \frac{1}{3} | v \rangle_b | A_v \rangle_b | v \rangle_a | A_v \rangle_a \langle v |_b \langle A_v |_b \langle v |_a \langle A_v |_a \\ & + \frac{1}{6} | h \rangle_b | A_h \rangle_b | h \rangle_a | A_h \rangle_a \langle h |_b \langle A_h |_b \langle h |_a \langle A_h |_a \\ & + \frac{1}{6} | v \rangle_b | A_v \rangle_b | h \rangle_a | A_h \rangle_a \langle v |_b \langle A_v |_b \langle h |_a \langle A_h |_a. \end{aligned} \quad (55)$$

This is one of the most remarkable consequences of the decoherence framework. In Sec. III we have shown that external interference measurements generally change the memory records of measured agents. Equation (55) shows that such interference measurements also change the correlations between the memories of two distant agents. If the correlations between agents I_A 's and I_B 's outcomes are evaluated *before* the interference experiment performed by agent E_A , fact 2 is correct; if they are evaluated *afterwards*, it changes to: if agent I_B has observed h_b , then agent I_A 's memory record is compatible with both h_a and v_a . It is worth noting that the decoherence framework can be used by all the agents to calculate *both* situations.

The agents involved in the thought experiment devised in Ref. [2] use the time evolution corresponding to each measurement to track the system *back*, that is, in the language of the decoherence framework, to calculate what agents I_B and I_A *thought in the past*. Hence, agent E_A can rely on the decoherence framework to conclude (i) given Eq. (52), agent I_B obtained the outcome h_b *before* my own measurement,

since no changes in laboratory B have occurred in between; (ii) hence, independently of what agent I_A thinks now, it obtained the outcome v_a before my own measurement and conditioned to agent I_B 's outcome h_b ; and (iii) therefore, agent I_A 's memory record *was* v_a in the past, if I have obtained $-_A$, even though it can be either v_a and h_a now.

The previous paragraph illustrates one of the most significant features of the decoherence framework: it can be used to calculate both the past and the current state of all agents' memory records; no ambiguities arise as a consequence of external interference experiments.

Regarding the thought experiment devised in Ref. [2], a proper use of the decoherence framework, taking into account the exact times at which the agents make their claims, shows that both facts 3a and 3b are correct. But this framework also shows that fact 3c is *not* correct, because it relies on fact 1, which is incompatible with it. Hence, the reasonings discussed in this section invalidate the proof of the no-go theorem presented in Ref. [2]. If assumptions Q, S, and C are used within the decoherence framework, agents I_A , I_B , E_A , and E_B do not reach the contradictory conclusion that $-_A$ implies $+_B$. The key point in this argument is that one agent must predict a correlation which is fixed only after an external interference experiment on itself, if it wants to make a claim about the final outcome of the protocol. The standard interpretation of quantum measurements is ambiguous about this point. One can suspect that something weird might happen, but a calculation to confirm or to refute this thought cannot be done. On the contrary, the decoherence framework provides exact results that can be tested by means of a proper experiment.

Finally, it is worth remarking that we have *not* proved that the decoherence framework is free from inconsistencies. We have just shown that the proof of the theorem proposed in Ref. [2] is not valid if the decoherence framework is taken into account. But the main statement of the theorem can be still considered as a conjecture.

C. Discussion

The conclusions of the previous section are entirely based on the decoherence framework. Results of Ref. [2] are well substantiated if this framework is not taken into account, that is, if a correlation between a system and a measuring apparatus is considered enough to complete a measurement. In such a case, the final state of the protocol can be written in four different shapes:

$$|\Psi_\alpha\rangle = \sqrt{\frac{1}{3}}|v\rangle_A|h\rangle_B + \sqrt{\frac{1}{3}}|h\rangle_A|v\rangle_B + \sqrt{\frac{1}{3}}|v\rangle_A|v\rangle_B, \quad (56a)$$

$$|\Psi_\beta\rangle = \sqrt{\frac{2}{3}}|+\rangle_A|h\rangle_B - \sqrt{\frac{1}{6}}|-\rangle_A|h\rangle_B + \sqrt{\frac{1}{6}}|+\rangle_A|v\rangle_B, \quad (56b)$$

$$|\Psi_\gamma\rangle = \sqrt{\frac{2}{3}}|v\rangle_A|+\rangle_B - \sqrt{\frac{1}{6}}|h\rangle_A|-\rangle_B + \sqrt{\frac{1}{6}}|h\rangle_A|-\rangle_B, \quad (56c)$$

$$|\Psi_\delta\rangle = \sqrt{\frac{3}{4}}|+\rangle_A|+\rangle_B - \sqrt{\frac{1}{12}}|+\rangle_A|-\rangle_B + \sqrt{\frac{1}{12}}|-\rangle_A|-\rangle_B - \sqrt{\frac{1}{12}}|-\rangle_A|+\rangle_B, \quad (56d)$$

relying on four different basis. If the preferred basis for each measurement is *not* fixed by a unique mechanism, like the one coming from the decoherence framework, the conclusions of the involved agents become ambiguous. The following reasoning can be understood as a consequence of the basis ambiguity problem [5].

Equation (56c) can be used to establish a perfect correlation between the outcomes v_a and $+_B$: if laboratory A is in state $|v\rangle_A$, which can be understood as the state resulting from the outcome v_a obtained by agent I_A , then the outcome $+_B$ is guaranteed. Hence, fact 1 is well supported—the final state of the whole experiment can be written in a way compatible with it. In a similar way, Eq. (56b) establishes a perfect correlation between $|-\rangle_A$ and $|h\rangle_B$, which can be interpreted as follows: if agent E_A as obtained $-_A$, then agent E_B has obtained h_B . Again, the final state of the whole protocol is compatible with this fact. Finally, Eq. (56a) establishes a perfect correlation between $|h_B\rangle$ and $|v_A\rangle$, meaning that if agent I_B has obtained h_B , then agent I_A has obtained v_A . And this is again compatible with the final state of the experiment.

Hence, as a consequence of the basis ambiguity problem, agents I_A , I_B , E_A , and E_B can rely on Eqs. (56a), (56b), and (56c) to infer a conclusion incompatible with Eq. (56d). As we have discussed in Sec. IV B, the decoherence framework fixes this bug by ruling out the basis ambiguity, and by providing just one preferred basis for each outcome.

V. OBSERVER-INDEPENDENT FACTS

This section deals with the no-go theorem discussed in Ref. [3]. This theorem has been experimentally confirmed in Ref. [4]. A criticism is published in Ref. [22].

A. Original version of the experiment and no-go theorem

The structure of this experiment has been already discussed in Sec. IV A. The only difference is the initial state, which consists in a pair of polarized photons, spanned by $\{|h\rangle, |v\rangle\}$, and reads

$$|\Psi\rangle_\beta = \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} (|h\rangle_a|v\rangle_b + |v\rangle_a|h\rangle_b) + \sqrt{\frac{1}{2}} \sin \frac{\pi}{8} (|h\rangle_a|h\rangle_b - |v\rangle_a|v\rangle_b). \quad (57)$$

This state is used to illustrate a no-go theorem that establishes that the following four statements are incompatible, that is, are bounded to yield a contradiction:

Statement 1. Quantum theory is valid at any scale.

Statement 2. The choice of the measurement settings of one observer has no influence on the outcomes of other distant observer(s).

Statement 3. The choice of measurement settings is statistically independent from the rest of the experiment.

Statement 4. One can jointly assign truth values to the propositions about outcomes of different observers.

In Refs. [3,4] the thought experiment used to proof this theorem consists of the following steps:

(i) The internal agents, I_A and I_B , perform their (pre)measurements, that is, establish a correlation between the measured photons and their apparatuses given by Eq. (4).

(ii) The external agents, E_A and E_B , choose between performing interference experiments, or measuring the polarization of the internal photons.

(iii) A Bell-like test is performed on the four different combinations resulting from point (ii), to conclude that it is not possible to jointly assign truth values to the outcomes obtained by the internal and the external agents.

This protocol was experimentally performed in Ref. [4], validating the violation of the Bell-like test prediction in Ref. [3].

The state resulting from step (i) is

$$|\Psi_0\rangle = \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} (|h\rangle_a|A_h\rangle_a|v\rangle_b|A_v\rangle_b + |v\rangle_a|A_v\rangle_a|h\rangle_b|A_h\rangle_b) + \sqrt{\frac{1}{2}} \sin \frac{\pi}{8} (|h\rangle_a|A_h\rangle_a|h\rangle_b|A_h\rangle_b - |v\rangle_a|A_v\rangle_a|v\rangle_b|A_v\rangle_b). \quad (58)$$

To proceed with step (ii), agent E_A chooses between observables A_0 and B_0 ,

$$A_0 = |h\rangle_a|A_h\rangle_a\langle h|_a\langle A_h|_a - |v\rangle_a|A_v\rangle_a\langle v|_a\langle A_v|_a, \quad (59)$$

$$B_0 = |+\rangle_A|+\rangle_A - |-\rangle_A\langle -|_A, \quad (60)$$

where $|\pm\rangle_A = (|h\rangle_a|A_h\rangle_a \pm |v\rangle_a|A_v\rangle_a)/\sqrt{2}$. The first one, A_0 , can be interpreted as a simple reading of agent I_A 's memory, whereas the second one, B_0 , performs an external interference

experiment, and therefore can be linked to agent E_A 's memory. Following the same spirit, agent E_B chooses between A_1 and B_1 ,

$$A_1 = |h\rangle_b|A_h\rangle_b|A_h\rangle_b - |v\rangle_b|A_v\rangle_b|v\rangle_b|A_v\rangle_b, \quad (61)$$

$$B_1 = |+\rangle_B|+\rangle_B - |-\rangle_B|-\rangle_B, \quad (62)$$

where $|\pm\rangle_B = (|h\rangle_b|A_h\rangle_b \pm |v\rangle_b|A_v\rangle_b)/\sqrt{2}$.

Finally, the third step is performed taking into account that statements 1–4 imply the existence of a joint probability distribution $p(A_0, B_0, A_1, B_1)$ whose marginals satisfy the Claude-Horne-Shimony-Holt (CHSH) inequality [25,26]

$$S = \langle A_1B_1 \rangle + \langle A_1B_0 \rangle + \langle A_0B_1 \rangle - \langle A_0B_0 \rangle \leq 2. \quad (63)$$

In Ref. [3] is theoretically shown that the initial state given by Eq. (57) leads to $S = 2\sqrt{2}$; in Ref. [4] this result is confirmed by an experiment. The conclusion is that these results are incompatible with statements 1–4, and therefore, assuming that statements 2 (nonlocality) and 3 (freedom of choice) are compatible with quantum mechanics [3,4,26], quantum theory is incompatible with the existence of observer-independent well-established facts.

B. The role of the decoherence framework

Unfortunately, this simple protocol is not consistent with the decoherence framework. The previous analysis accounts neither for the structure of laboratories summarized in Table I, nor for the measuring protocol given in Table II. The decoherence framework postulates that a definite outcome does not emerge until an external environment monitorizes the state composed of the system, the apparatus, and the observer. Therefore, Eq. (58) does not represent the outcomes obtained by agents I_A and I_B , but just an entangled system composed of two photons and two apparatuses. And consequently, the fact that it violates a CHSH inequality does not entail the refutation of the fourth statement of the theorem, since definite outcomes have not still appeared—it just shows that the state (58) includes quantum correlations that cannot be described by means of a joint probability distribution, but such correlations involve neither definite outcomes nor observers' memory records.

As a first conclusion, the previous paragraph is enough to show that the thought experiment devised in Ref. [3] cannot refute the possibility of jointly assigning truth values to the propositions about the outcomes of different observers, if the decoherence framework is considered. Following the same line of reasoning that in Sec. IV C we can also state that the conclusion in Ref. [3] is well supported if a measurement is understood as a correlation between a system and its measuring apparatus. In such a case, the correlations between A and B observables represent the correlations between the outcomes obtained by the internal and the external observables, and therefore the CHSH proves that they are incompatible with a definite joint probability distribution.

The rest of the section is devoted to a variation of the setup devised in Ref. [3]. The idea is to follow the same spirit, but

making it suitable to challenge the decoherence framework. This modified experimental setup consists of two main steps:

(i) The internal agents measure their systems in the basis $\{|h\rangle, |v\rangle\}$, and the external ones perform interference experiment in the basis $\{|+\rangle, |-\rangle\}$.

(ii) Two superexternal agents choose between the operators A and B , given by Eqs. (65a)–(65d), to establish complementary facts about the outcomes obtained in step (i).

This variation allows to apply a CHSH inequality to the outcomes obtained by the internal and the external agents, and therefore to test if we can jointly assign truth values to them. To properly apply the decoherence framework to this experiment, it is mandatory to include in the protocol all the environments which determine the emergence of definite outcomes. This can be done in three different stages:

Stage 1. Agent I_A measures the state of photon a in a basis given by $\{|h\rangle_a, |v\rangle_a\}$, and I_B measures the state of photon b in a basis given by $\{|h\rangle_b, |v\rangle_b\}$. Without explicitly taking into account the external apparatuses and environments, which are not entangled with laboratories A and B at this stage, the resulting state is

$$\begin{aligned} |\Psi_1\rangle = & \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} |h\rangle_a|A_h\rangle_a|\varepsilon_1(t)\rangle_a|v\rangle_b|A_v\rangle_b|\varepsilon_2(t)\rangle_b \\ & + \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} |v\rangle_a|A_v\rangle_a|\varepsilon_2(t)\rangle_a|h\rangle_b|A_h\rangle_b|\varepsilon_1(t)\rangle_b \\ & + \sqrt{\frac{1}{2}} \sin \frac{\pi}{8} |h\rangle_a|A_h\rangle_a|\varepsilon_1(t)\rangle_a|h\rangle_b|A_h\rangle_b|\varepsilon_1(t)\rangle_b \\ & - \sqrt{\frac{1}{2}} \sin \frac{\pi}{8} |v\rangle_a|A_v\rangle_a|\varepsilon_2(t)\rangle_a|v\rangle_b|A_v\rangle_b|\varepsilon_2(t)\rangle_b. \quad (64) \end{aligned}$$

The decoherence framework establishes that agents I_A and I_B do not observe definite outcomes until this stage is reached. It is worth remembering that each of its environment is continuously monitorizing each of its apparatuses, by means of Hamiltonians like (12).

At this stage, an experiment equivalent to the one discussed in Refs. [3,4] could be done, by means of the following A_0, A_1, B_0 , and B_1 observables:

$$A_0 = |h\rangle_a|A_h\rangle_a|h\rangle_a\langle A_h|_a - |v\rangle_a|A_v\rangle_a\langle v|_a\langle A_v|_a, \quad (65a)$$

$$B_0 = |h\rangle_b|A_h\rangle_b|h\rangle_b\langle A_h|_b - |v\rangle_b|A_v\rangle_b\langle v|_b\langle A_v|_b, \quad (65b)$$

$$A_1(\tau) = |+(\tau)\rangle_A\langle +(\tau)|_A - |-(\tau)\rangle_A\langle -(\tau)|_A, \quad (65c)$$

$$B_1(\tau) = |+(\tau)\rangle_B\langle +(\tau)|_B - |-(\tau)\rangle_B\langle -(\tau)|_B, \quad (65d)$$

where $|+(\tau)\rangle_A$ is given by Eq. (38a); $|-(\tau)\rangle_A$ is given by Eq. (38b), and equivalent relations determine $|+(\tau)\rangle_B$ and $|-(\tau)\rangle_B$. Again, τ is the time at which the interference measurements are performed, according to points R1–R4 of Table II. In such a case, A_0 and B_0 can be properly interpreted as agents I_A 's and I_B 's points of view, but A_1 and B_1 are still not linked to agents E_A 's and E_B 's perceptions—their environments must act to determine the corresponding definite outcomes. Hence, the CHSH inequality applied to this state would allow us to get a conclusion about the compatibility of the internal agents' memories and the states of the laboratories in which they live, but they would tell us nothing about the outcomes obtained by the external agents.

Stage 2a. From Eq. (64), agent E_A measures the state of laboratory A in the basis $\{|+(\tau)\rangle_A, |-(\tau)\rangle_A\}$, considering requisites R1–R4 of Table II. The resulting state is

$$\begin{aligned} |\Psi_2\rangle = & \frac{1}{2} \left(\cos \frac{\pi}{8} - \sin \frac{\pi}{8} \right) |+(\tau)\rangle_A |A'_+\rangle_A |\varepsilon'_1(\tau)\rangle_A |v\rangle_b |A_v\rangle_b |\varepsilon_2(\tau)\rangle_b \\ & + \frac{1}{2} \left(\cos \frac{\pi}{8} + \sin \frac{\pi}{8} \right) |-(\tau)\rangle_A |A'_-\rangle_A |\varepsilon'_2(\tau)\rangle_A |v\rangle_b |A_v\rangle_b |\varepsilon_2(\tau)\rangle_b \\ & + \frac{1}{2} \left(\cos \frac{\pi}{8} + \sin \frac{\pi}{8} \right) |+(\tau)\rangle_A |A'_+\rangle_A |\varepsilon'_1(\tau)\rangle_A |h\rangle_b |A_h\rangle_b |\varepsilon_1(\tau)\rangle_b \\ & + \frac{1}{2} \left(\sin \frac{\pi}{8} - \cos \frac{\pi}{8} \right) |-(\tau)\rangle_A |A'_-\rangle_A |\varepsilon'_2(\tau)\rangle_A |h\rangle_b |A_h\rangle_b |\varepsilon_1(\tau)\rangle_b. \end{aligned} \quad (66)$$

At this stage, observable A_1 represents agent E_A 's point of view, but B_1 is still not linked to agent E_B 's memory.

Stage 2b. From Eq. (64) again, agent E_B measures the state of laboratory B in the basis $\{|+(\tau)\rangle_B, |-(\tau)\rangle_B\}$, considering requisites R1–R4 of Table II. The resulting state is

$$\begin{aligned} |\Psi_3\rangle = & \frac{1}{2} \left(\cos \frac{\pi}{8} - \sin \frac{\pi}{8} \right) |v\rangle_a |A_v\rangle_a |\varepsilon_2(\tau)\rangle_a |+(\tau)\rangle_B |A'_+\rangle_B |\varepsilon'_1(\tau)\rangle_B \\ & + \frac{1}{2} \left(\cos \frac{\pi}{8} + \sin \frac{\pi}{8} \right) |v\rangle_a |A_v\rangle_a |\varepsilon_2(\tau)\rangle_a |-(\tau)\rangle_B |A'_-\rangle_B |\varepsilon'_2(\tau)\rangle_B \\ & + \frac{1}{2} \left(\cos \frac{\pi}{8} + \sin \frac{\pi}{8} \right) |h\rangle_a |A_h\rangle_a |\varepsilon_1(\tau)\rangle_a |+(\tau)\rangle_B |A'_+\rangle_B |\varepsilon'_1(\tau)\rangle_B \\ & + \frac{1}{2} \left(\sin \frac{\pi}{8} - \cos \frac{\pi}{8} \right) |h\rangle_a |A_h\rangle_a |\varepsilon_1(\tau)\rangle_a |-(\tau)\rangle_B |A'_-\rangle_B |\varepsilon'_2(\tau)\rangle_B. \end{aligned} \quad (67)$$

As this stage has been obtained from Eq. (64), it is not subsequent to Eq. (66). Therefore, observable B_1 represents agent E_B 's point of view, but A_1 is still not linked to agent E_A 's memory.

Stage 3. Agent E_A measures the state of laboratory A in the basis $\{|+(\tau)\rangle_A, |-(\tau)\rangle_A\}$, considering requisites R1–R4 of Table II, and agent E_B measures the state of laboratory B in the basis $\{|+(\tau)\rangle_B, |-(\tau)\rangle_B\}$, following the same procedure. This case is subsequent to either stage 2a or stage 2b. The resulting state is

$$\begin{aligned} |\Psi_4\rangle = & \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} |+(\tau)\rangle_A |A'_+\rangle_A |\varepsilon'_1(\tau)\rangle_A |+(\tau)\rangle_B |A'_+\rangle_B |\varepsilon'_1(\tau)\rangle_B - \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} |-(\tau)\rangle_A |A'_-\rangle_A |\varepsilon'_2(\tau)\rangle_A |-(\tau)\rangle_B |A'_-\rangle_B |\varepsilon'_2(\tau)\rangle_B \\ & + \sqrt{\frac{1}{2}} \sin \frac{\pi}{8} |+(\tau)\rangle_A |A'_+\rangle_A |\varepsilon'_1(\tau)\rangle_A |-(\tau)\rangle_B |A'_-\rangle_B |\varepsilon'_2(\tau)\rangle_B + \sqrt{\frac{1}{2}} \sin \frac{\pi}{8} |-(\tau)\rangle_A |A'_-\rangle_A |\varepsilon'_2(\tau)\rangle_A |+(\tau)\rangle_B |A'_+\rangle_B |\varepsilon'_1(\tau)\rangle_B. \end{aligned} \quad (68)$$

At this stage, the four agents have observed definite outcomes, and therefore their four memories can be read to interpret these outcomes. Hence, if one wants to test statements 1–4 of the theorem formulated in Ref. [3], one must start from this state. So, let us imagine that we are running a quantum algorithm performing this experiment and we want to test if we can jointly assign truth values to the outcomes obtained by the four agents. One of us can choose between A_0 and B_0 , defined in Eqs. (65a) and (65b), to decide between reading agent I_A 's or agent E_A 's memories, and another one can choose between A_1 and B_1 , defined in Eqs. (65c) and (65d), to decide between reading agent I_B 's or agent E_B 's memories. Then we can run a large number of realizations of the same experiment to test if the CHSH inequality given in Eq. (63) holds. If it gives rise to $S > 2$ we can conclude that statement (4) of the theorem is violated; if not, we can conclude that it is possible to jointly assign truth values to the observations done by the four agents.

A straightforward calculations provides the following result:

$$\langle \Psi_4 | A_0 B_0 | \Psi_4 \rangle = 0, \quad (69)$$

$$\langle \Psi_4 | A_1 B_0 | \Psi_4 \rangle = 0, \quad (70)$$

$$\langle \Psi_4 | A_0 B_1 | \Psi_4 \rangle = 0, \quad (71)$$

$$\langle \Psi_4 | A_1 B_1 | \Psi_4 \rangle = 1/\sqrt{2}. \quad (72)$$

Therefore, the CHSH inequality, Eq. (63), applied to $|\Psi_4\rangle$ leads to $S = 1/\sqrt{2} < 2$.

Two main conclusions can be gathered from this section. First, the experiment devised in Ref. [3], and its experimental realization [4], are incompatible with the decoherence framework, because, according to it, they do not deal with proper outcomes; thus, they cannot be used to refute the possibility of jointly assigning truth values to the outcomes obtained by different observers. Second, a variation to the experiment in Ref. [3] designed to challenge the decoherence framework following the same spirit is compatible with the four statements discussed above—quantum theory is valid at any scale; the choice of the measurement settings of one observer has no influence on the outcomes of other distant observers; the choice of the measurement settings is independent from the rest of the experiment, and one can jointly assign truth values to the propositions about the outcomes of different observers. In other words, these statements do not imply a contradiction in this experiment, if the role of all the parts of

each laboratory, given in Table I, and the physical mechanisms giving rise to each outcome, are considered. This conclusion is fully compatible with the main idea behind the decoherence framework. As is clearly stated in the title of Ref. [5], the main aim of this formalism is to explain how classical results, like definite outcomes, can be obtained from quantum mechanics, without relying on a nonunitary wave-function collapse. This section shows that the memory records of all the agents are indeed classical as a consequence of the continuous monitoring by their environments, and therefore satisfy the corresponding CHSH inequality. Notwithstanding, the state after all these definite outcomes have emerged, Eq. (68), is quantum and has true quantum correlations. If one applies the CHSH inequality to the states of the two internal and the two external laboratories, the resulting equations are formally the same that those in Ref. [3]—confirming that the thought experiment discussed in this section follows the same spirit that the one in Ref. [3]—and therefore one recovers the original result, $S = 2\sqrt{2}$. This means that we cannot assign joint truth values to the state of these laboratories, but we can make this assignment to the state of the agents' memories. In Refs. [3,4] there is no distinction between the state of the laboratory in which an agent lives and the state of its memory; the decoherence framework is based precisely on this distinction.

Before ending this section, it is worth remarking that our result does not prove that the use of statements 1–4 is free from contradictions in any circumstances. We have just shown that the particular setup used to prove the no-go theorem in Ref. [3] does not lead to contradictions if the decoherence framework is properly taken into account. But, again, the main statement of the theorem still can be considered as a conjecture.

VI. CONCLUSIONS

The main conclusion of this work is that neither the original Wigner's friend experiment, nor the extended version proposed in Ref. [2], nor the one in Ref. [3] (and its corresponding experimental realization in Ref. [4]) entail contradictions if the decoherence framework is properly taken into account.

This framework consists in considering that a quantum measurement and the corresponding (apparent) wave-function collapse are the consequence of the interaction between the measuring apparatus and an uncontrolled environment, which must be considered as an inseparable part of the measuring device. In this work, we have relied on a simple model to show that a chaotic interaction is necessary to induce such an apparent collapse, but, at the same time, a quite small number of environmental qubits suffices for that purpose. This implies that any experiment on any quantum system can be modeled by means of a unitary evolution, and therefore all the time evolution, including the outcomes obtained by any observers, is univocally determined by the initial state, the interaction between the system and the measuring apparatuses, and the interaction between such apparatuses and the corresponding environments. Seeing the reality as if a random wave-function collapse had happened is due to the lack of information suffered by the observers—only the system as a

whole evolves unitarily, not a part of it. This is a somehow paradoxical solution to the quantum measurement problem: ignoring an important piece of information about the state in which the observer lives is mandatory to observe a definite outcome; taking it into account would lead to no observations at all. But, besides the ontological problems arising for such an explanation, the resulting framework is enough for the purpose of this work.

The first consequence of this framework is that the memory records of Wigner's friends—the internal agents in a Wigner's-friend experiment—change as a consequence of the external interference experiment performed by Wigner, these changes are univocally determined by the Hamiltonian encoding all the time evolution and therefore can be exactly predicted. Hence, if an agent has observed a definite outcome, then external interference experiments performed on the laboratory in which it lives change its memory records; if such changes do not occur is because the agent has not observed a definite outcome.

The second consequence of the decoherence framework is that the contradictions discussed in Refs. [2,3] are ruled out. If the agents involved in the thought experiment devised in Ref. [2] use the decoherence framework as the common theory to predict the other agents' outcomes, their conclusions are not contradictory at all. The analysis of the experiment proposed in Ref. [3] is a bit more complicated. Its original design is not compatible with the decoherence framework. Hence, it cannot be used to refute the possibility to jointly assigning truth values to the agents' outcomes, that is, it cannot be used to prove the no-go theorem formulated in Ref. [3]. Therefore, a variation of that experiment, following the same spirit, is proposed to show that, if the CHSH inequality is applied to the state at which the whole system is at the end of the protocol, that is, when the records in the memories of the four agents are fixed, the resulting value is compatible with the existence of observer-independent facts.

However, this is not enough to dismiss the main statements of the no-go theorems formulated in such references. The conclusion of this work is that the examples used to prove these theorems are not valid within the decoherence framework, but we have not proved that this framework is totally free from similar inconsistencies. Hence, these statements can be still considered as conjectures. Further work is required to go beyond this point.

It is also worth to remark that the decoherence formalism also narrows the conditions under which the external interference measurements, the trademark of Wigner's-friend experiments, are expected to work. This means that, if the decoherence framework results in being true, human beings acting as observers are almost free from suffering the strange effects of such experiments. Notwithstanding, the promising state of the art in quantum technologies may provide us, in the future, quantum machines able to perform these experiments.

Finally, the conclusion of this work must not be understood as a strong support of the decoherence framework. It just establishes that such a framework does not suffer from the inconsistencies typically ensuing Wigner's-friend experiments. However, there is plenty of space for theories in which the

wave-function collapse is real [7]. These theories predict a totally different scenario, since after each measurement the wave function of the whole system collapses and therefore becomes different from the predictions of the decoherence framework. Hence, experiments like the ones discussed in this work might be a way to test which of this proposals is correct—if any.

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- [10] One of the trademarks of the decoherence interpretation of quantum mechanics is that the pointer states of any apparatuses, that is, the states which appear as objective outcomes, are those which survive to the continuous monitorization by a complex environment. In other words, the environment, and its interaction with the measuring apparatus, is the origin of the classical perception of the reality. For a more detailed discussion, we refer the reader to Refs. [5,9].
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- [12] Of course, the agent cannot restore the complete state from a single measurement. If its outcome is, say, h , it can just conclude that the global state must be $|\Psi_2\rangle = \alpha|h\rangle|A_h\rangle|e_1(t)\rangle + \beta|v\rangle|A_v\rangle|e_2(t)\rangle$, with unknown coefficients α and β such that $|\alpha|^2 + |\beta|^2 = 1$, and $|\alpha| > 0$. It is very important to note that this conclusion, which is one of the trademarks of the decoherence formalism, is totally different from the standard interpretation of quantum measurements, following which the observer concludes that the state of the system is $|\Psi_2\rangle = |h\rangle|A_h\rangle$ as the aftermath of observing the outcome h .
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- [18] From this point of view, we can consider that the measurement is done by a small quantum machine, and that an amplification process is done by the environment afterwards. In this way, the premeasurement can be considered purely quantum, whereas the amplification, required for a large agent to see the outcome, is the responsible for the transition to a *classical* outcome. Nevertheless, the result is the same if the external environment-apparatus interaction is still present, provided that its characteristic time is larger than the time required to complete the premeasurement.
- [19] An interesting question is what happens if the nondiagonal elements, C_{hv} and C_{vh} , are clearly different from zero. In such a case, the interaction between the apparatus and the environment does not determine the pointer states of the apparatus, and therefore the agent cannot see a definite outcome. On the contrary, the instantaneous eigenbasis of the reduced density matrix resulting from tracing out the environmental degrees of freedom is continuously changing in time, and therefore the agent memory does not record a definite result for the measurement. As the fluctuations of these diagonal elements are expected to be tiny if the environment is large enough, the outcomes can be considered stable. A more detailed discussion can be found in Ref. [5].
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- [24] As a marginal note, it is revealing to notice that the perfect correlation between outcomes h_b and v_a , or, equivalently, between

h_a and v_b , occur *without* any kind of nonlocal collapse. The measurement performed on stage 1 does not affect photon b , and hence the fact that agent I_A observes the definite outcome h_a does not imply that the other photon, which can be far away, instantaneously collapses onto v_b . The result of such a measurement is Eq. (34). Equation (51) is just a practical representation of what both agents see as a consequence of

ignoring both environments, but not the result of a physical process involving a nonlocal collapse.

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