Efficient linear optical generation of a multipartite W state via a quantum eraser

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Entanglement among multiple particles is a keystone for not only fundamental research on quantum information but also various practical quantum information applications. In particular, the W state has attracted a lot of attention due to the robustness against particle loss and the applications in multiparty quantum communication. However, it is challenging to generate a photonic W state with a large number of photons, since the N-photon W state requires superposition among N probability amplitudes. In this paper, we propose an efficient linear optical scheme to generate N-photon W states via quantum erasure. The success probability of our protocol polynomially decreases as the number of photons increases. We also discuss the experimental feasibility of our protocol and anticipate that one can efficiently generate tens of photonic W states with our scheme using currently available quantum photonics technologies.

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I. INTRODUCTION

Entanglement is at the heart of quantum information [1,2]. In particular, entanglement among multiple qubits plays crucial roles in quantum information processing such as nonlocality tests [3], multiparty quantum communication [4], and quantum computation [5]. Two representative genuine multipartite entanglements, the Greenberger-Horne-Zeilinger (GHZ) states and the W states, have significantly different features. Note that the interchange between these two states via local operation and classical communication is forbidden [6].

The N-qubit GHZ sate is represented as

$$|\mathrm{GHZ}_N\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}). \quad (1)$$

Since the GHZ state maximally violates the Bell-type inequality, it is usually considered as a maximally entangled state. However, it is extremely fragile and thus easily looses entanglement. For instance, if one or more qubit particles are lost, the remaining M-qubit state becomes a statistical mixture of

$$\rho_M = \tfrac{1}{2}(|0\rangle\langle 0|^{\otimes M} + |1\rangle\langle 1|^{\otimes M}). \quad (2)$$

In experiment, GHZ states up to tens of qubits have been observed in photonic qubits [7–9], trapped ions [10], and superconducting qubits [11,12].

Another representative genuine multipartite entangled state, the W state, is represented as

$$|W_N\rangle = \frac{1}{\sqrt{N}}(|0\dots01\rangle + |0\dots010\rangle + \dots + |10\dots0\rangle). \quad (3)$$

It is notable that, in the W state, every qubit shares the optimal amount of entanglement with all other qubits [13,14]. This unique property suggests a weblike structure that every qubit is coupled with all other qubits. Therefore, even if one or more qubits are lost, the remaining M-qubit state can maintain entanglement with some probability.

Photonic qubit implementation of W states provides fundamental quantum information test platforms [15] as well as practical applications to multiparty quantum communication [16–20] and quantum metrology [21]. There have been a few theoretical proposals [22–28] and experimental implementations [29–33] of photonic W-state generation. However, W-state preparation with large numbers of photons is challenging, since the number of probability amplitudes increases as the photon number increases. Due to the difficulty, there are few proposals to generate W states with an arbitrary number of qubits from independent single photons [34–42]. Most of these proposals are based on quantum fusion operation where $|W_N\rangle$ can be obtained from smaller W states, $|W_M\rangle$, where $M < N$ [36–42]. Note, however, that these schemes require multiphoton gate operations such as CNOT gate, Fredkin gate, and Toffoli gate, which suffer from probabilistic implementation using linear optical elements. Therefore, these schemes suffer from low success probability, i.e., the typical success probability P_N to generate an N-qubit W state exponentially decreases as N increases, $P_N \sim \mathcal{O}^{-N}$.

In this paper, we propose an efficient linear optical protocol to generate N-photon W states from single photons via a quantum eraser. We found that the success probability of our scheme approaches $P_N \sim 1/N$ for a large N and is thus much higher than those of other conventional W-state generation schemes [34–42]. It is also remarkable that the qubit particles have never overlapped during the entangling process with each other, which supports a counterintuitive result that the qubit particles do not need to touch one another for entanglement generation [26,43–45].

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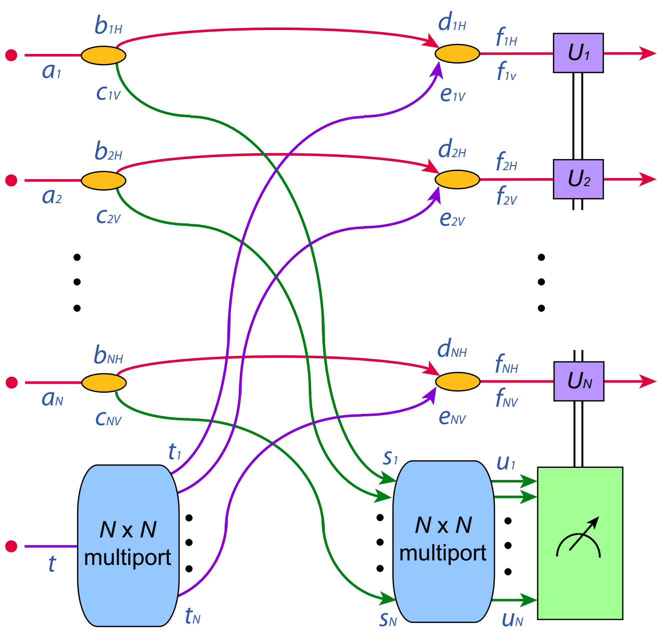

FIG. 1. Our scheme to generate an N-photon W state. The protocol begins with the $N+1$ single-photon inputs of $|\psi\rangle_{\rm in} = a_1^\dagger a_2^\dagger \cdots a_N^\dagger \otimes t^\dagger|0\rangle$. $a_i\,(i = 1, 2, \ldots, N)$ and t are divided by polarizing beamsplitters (PBSs) and a symmetrical $N \times N$ multiport, respectively. The single photons take the transformation of (6), and combined by N PBSs and an $N \times N$ multiport. The N-photon W state is generated at f_i when all f_i are occupied by one and only one photon, and a single photon is found at one of the outputs of the $N \times N$ multiport, u_k. The success probability can be increased by applying the feedforward operations U_i at f_i, which are determined by the single-photon detection event of u_k.

II. EFFICIENT MULTIPARTITE *W*-STATE GENERATION

A. *W*-state generation via quantum eraser

Figure 1 shows our schematic to generate an N-qubit W state. For simplicity, we present the scheme with photonic polarization qubits; however, it is valid for any type of bosonic particles. It can be also applied for other degrees of freedom of photons such as discrete-energy entanglement, which is preferred for long-distance communication [33]. The protocol begins with $N+1$ single-qubit inputs of $|\psi\rangle_{\rm in} = a_1^\dagger a_2^\dagger \ldots a_N^\dagger \otimes t^\dagger|0\rangle$, where $a_i^\dagger$ and $t^\dagger$ denote creation operators at a_i and t, respectively. Then, the input state of a_i and t are divided by polarizing beamsplitters (PBSs) and a symmetrical $N \times N$ multiport, respectively. The transformations can be presented as

$$a_i^\dagger \to \alpha_i b_{iH}^\dagger + \beta_i c_{iV}^\dagger,$$
$$t^\dagger \to \frac{1}{\sqrt{N}}(t_1^\dagger + t_2^\dagger + \cdots + t_N^\dagger), \tag{4}$$

where the subscript H (V) denotes horizontal (vertical) polarization, and α_i and β_i are complex coefficients which satisfy $|\alpha_i|^2 + |\beta_i|^2 = 1$. Here, we assume that all the relative phases between t_i are zero for simplicity. Note that the coefficients α_i and β_i can be controlled by changing the input polarization state of a_i. For simplicity, we will assume the probability

$$p_h = |\alpha_i|^2 \tag{5}$$

for all i. Then, as shown in Fig. 1, we take the transformation of

$$b_{iH}^\dagger \to d_{iH}^\dagger \to f_{iH}^\dagger,$$
$$c_{iV}^\dagger \to s_i^\dagger \to \sum_j \gamma_{ij} u_j^\dagger, \tag{6}$$
$$t_i^\dagger \to e_{iV}^\dagger \to f_{iV}^\dagger,$$

where γ_{ij} are the complex coefficients which satisfy $\sum_j |\gamma_{ij}|^2 = 1$. The transformation $s_i^\dagger \to \sum_j \gamma_{ij} u_j^\dagger$ presents N-port interference by a symmetrical $N \times N$ multiport with a s_i input. We note that the transformation equation, Eq. (6), implies the optical paths of single photons. For example, a horizontal single-photon state at b_{1H} goes to d_{1H} and eventually will be found at f_1 with a horizontal polarization state, i.e., f_{1H}.

For W-state generation, we postselect the case when all $N+1$ outputs, i.e., all f_i and one u_j, are occupied by single-photon states. From the transformation configuration, it happens only when one $a_i^\dagger$ takes $c_{iV}^\dagger$ while all other $a_j^\dagger$'s take $b_{jH}^\dagger$, and $t^\dagger$ takes $t_i^\dagger$, respectively. For example, if $a_1^\dagger$ takes $a_1^\dagger \to c_{1V}^\dagger$, $a_j^\dagger$ where $j \neq 1$ should take $a_j^\dagger \to b_{jH}^\dagger$ and $t^\dagger \to t_1^\dagger$. In this case, the output state is presented as

$$|\Psi_1\rangle = |\psi_1\rangle \otimes |s_1\rangle = f_{1V}^\dagger f_{2H}^\dagger \cdots f_{NH}^\dagger \otimes s_1^\dagger|0\rangle. \tag{7}$$

Note that in this state presentation, we do not take the transformation of $s_i^\dagger \to \Sigma_j \gamma_{ij} u_j^\dagger$, yet. Similarly, if the jth input takes $a_j^\dagger \to c_{jV}^\dagger$, the postselected output state is given as

$$|\Psi_j\rangle = |\psi_j\rangle \otimes |s_j\rangle$$
$$= f_{1H}^\dagger \cdots f_{(j-1)H}^\dagger f_{jV}^\dagger f_{(j+1)H}^\dagger \cdots f_{NH}^\dagger \otimes s_j^\dagger|0\rangle. \tag{8}$$

Since $|\psi_j\rangle$ are distinguishable from each other due to $|s_j\rangle$, the overall N-qubit state ρ_f is given as a mixture of $|\psi_j\rangle$ of

$$\rho_f = \frac{1}{N}(|\psi_1\rangle\langle\psi_1| + \cdots + |\psi_N\rangle\langle\psi_N|). \tag{9}$$

In order to make the overall state ρ_f pure, which corresponds to the coherent superposition of $|\psi_j\rangle$, we need to employ a quantum eraser to delete the spatial mode information of $|s_j\rangle$ [46,47]. By employing a symmetrical $N \times N$ multiport and measuring one of the outputs u_k, the final state becomes a coherent superposition of $|\psi_j\rangle$ as

$$|W_N^{(k)}\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{i\phi_j^{(k)}} |\psi_j\rangle$$
$$= \frac{1}{\sqrt{N}}\big(e^{i\phi_1^{(k)}} f_{1V}^\dagger f_{2H}^\dagger \cdots f_{NH}^\dagger$$
$$+ e^{i\phi_2^{(k)}} f_{1H}^\dagger f_{2V}^\dagger f_{3H}^\dagger \cdots f_{NH}^\dagger + \cdots$$
$$+ e^{i\phi_N^{(k)}} f_{1H}^\dagger \cdots f_{(N-1)H}^\dagger f_{NV}^\dagger\big)|0\rangle. \tag{10}$$

Here, $\phi_j^{(k)}$ denotes the relative phase between $|\psi_j\rangle$ when the ancillary qubit is measured at u_k. Note that $|W_N^{(k)}\rangle$ becomes an N-qubit W state as long as the relative phase $\phi_j^{(k)}$ is fixed. Since the relative phase can be fixed with the detection of the ancillary qubit at u_k, it heralds N-qubit W-state preparation at the output modes f_j. Note that the different ancillary qubit detection at u'_k implies a different relative phase $\phi_j^{(k')}$. The different relative phase can be adjusted with the local unitary operations of $U_1^{(k)} \otimes \cdots \otimes U_N^{(k)}$ to the N qubits. Therefore, one can increase the success probability of the W-state generation scheme by performing the feedforward unitary operations $U_1^{(k)} \otimes \cdots \otimes U_N^{(k)}$ at N qubits according to the ancillary qubit detection result u_k.

B. Success probability

Let us discuss the success probability of our W-state generation scheme. The simultaneous probability of $a_i^\dagger \to c_{iV}^\dagger$, $a_j^\dagger \to b_{jH}^\dagger$ for all other $j \neq i$ and $t^\dagger \to t_i^\dagger$ is given as

$$P = \frac{1}{N}(1-p_h)p_h^{N-1}. \tag{11}$$

Considering N qubits which can take c_{iV} and the probability $1/N$ for measuring u_k, the overall success probability of obtaining the W state remains the same. Therefore, with the condition of Eq. (5), the maximum success probability of generating an N-qubit W state (8) is given as

$$P_N^{(k)} = \max_{p_h} P = \frac{(N-1)^{N-1}}{N^{N+1}} \tag{12}$$

for $p_h = \frac{N-1}{N}$. Note that for a large N, the maximum success probability becomes

$$\lim_{N\to\infty} P_N^{(k)} \to \frac{1}{N^2}. \tag{13}$$

It is remarkable that the success probability can be further increased by performing feedforward operations of $U_i^{(k)}$ according to the measurement result u_k. In this case, we can obtain the unity probability for measuring u_k instead of $1/N$, and thus the success probability becomes

$$P_N^{\rm FF} = \frac{(N-1)^{N-1}}{N^N}. \tag{14}$$

For a large N, the maximum success probability with feedforward operation becomes

$$\lim_{N\to\infty} P_N^{\rm FF} \to \frac{1}{N}. \tag{15}$$

Overall, the success probability of our protocol to W-state generation polynomially decreases as the number of photons N increases.

We compare the success probabilities of representative protocols for generating an N-photonic qubit W state in Fig. 2. Note that the success probabilities of other protocols exponentially decrease ($P_N \sim \mathcal{O}^{-N}$) as the number of photons N increases, which is much faster than that of our protocol. It clearly shows that the success probability of our protocol is significantly higher than those of other representative protocols to generate a W state with tens of photons.

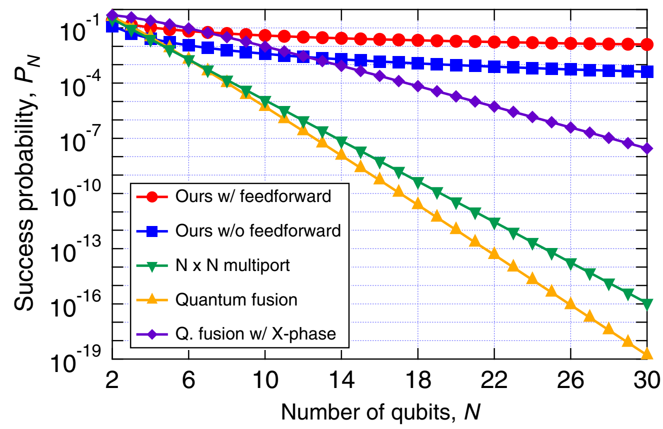

FIG. 2. The success probability of W-state generation with respect to the number of qubits. Ours with (w/) and without (w/o) feedforward stands for our protocol with (without) feedforward. The N × N multiport denotes the scheme using an N × N entanglement multiport [34]. Quantum fusion denotes N-photon W-state generation using a quantum fusion method which generates $|W\rangle_N$ from $|W\rangle_{N-1}$ [36]. Quantum fusion with X phase presents the quantum fusion method with cross-phase modulation, which is a nonlinear optical effect [42].

C. *W*-state generation among distant parties

The transformation and postselection of successful cases guarantee that the indistinguishable photons have never overlapped during the entangling process. The absence of photon overlap implies that the entangling process does not happen at a well-defined region. Rather, it can be shared by multiple separated regions [26,45].

As an application of the nonoverlapping particle property, one can implement informationally balanced W-state generation among distant parties [45,48]. Figure 3 shows the conceptual scheme for informationally balanced W-state generation among distant multiple parties. Each communication party possesses a single-photon source and detector. While the mode b_{iH} is kept, they send c_{iV} to the third party via quantum channels ($c_i \to s_i$). The third party, who also has a

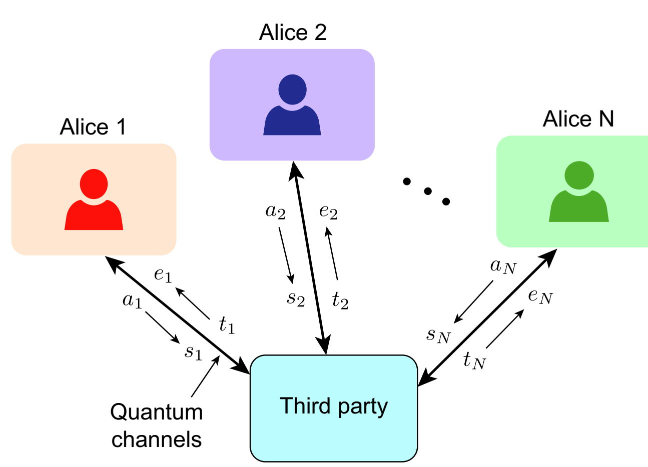

FIG. 3. Informationally balanced W-state generation among distant multiple parties.

single-photon source and detector, receives photonic modes c_i from the communication parties. At the same time, she transmits modes t_i to the communication parties via quantum channels ($t_i \rightarrow e_i$). After the necessary unitary transformations at the communication parties and the third party, the distant multiple communication parties share a W state only when all the communication parties and the third party have a single-photon each. Note that the success probability of the W-state generation can be increased with the announcement of a third-party detection result u_k and following unitary operations at the communication parties U_k. Unlike the GHZ state generation, it requires a third party to announce the measurement result of u_k; however, she does not participate in sharing qubit particles. Note that the information balance is a critical feature in most quantum communication applications [45,48,49].

III. EXPERIMENTAL FEASIBILITY

In order to generate a W state using our scheme, one needs $N + 1$ identical single-photon inputs and a linear optical network with excellent phase stability. We remark that it is already possible to generate tens of photonic qubit W-state generation with currently available quantum photonic technologies. For instance, generation of 12 identical single-photon states using spontaneous parametric downconversion has been reported [9]. Recently, 20 identical single-photon state generation using a quantum dot has been presented [50]. The complicated linear optical network can be implemented with rapidly developing integrated quantum photonics, which also provides a high level of phase stability [51]. Note that our protocol which is presented with photonic polarization modes can be easily converted to, for example, discrete-energy modes [33], so it can be directly applied to integrated quantum photonics.

IV. CONCLUSION

In summary, we have proposed an efficient linear optical protocol to generate a multiphoton W state via a quantum eraser. We have found that the success probability of our protocol polynomially decreases ($P_N \sim N^{-2}$ without feedforward and $P_N \sim N^{-1}$ with feedforward) as the number of photons N increases. Considering the success probability of other representative photonic W-state generation methods exponentially decreases ($P_N \sim \mathcal{O}^{-N}$) as N increases, our protocol provides a powerful tool to investigate quantum information with multipartite entanglement. We remark that our protocol can be implemented with current quantum photonics technologies [9,50,51].

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