Nonlocality claims are inconsistent with Hilbert-space quantum mechanics

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It is shown that when properly analyzed using principles consistent with the use of a Hilbert space to describe microscopic properties, quantum mechanics is a local theory: one system cannot influence another system with which it does not interact. Claims to the contrary based on quantum violations of Bell inequalities are argued to be incorrect. A specific example traces a violation of the CHSH Bell inequality in the case of a spin-$3/2$ particle to the noncommutation of certain quantum operators in a situation where (non)locality is not an issue. A consistent histories analysis of what quantum measurements measure, in terms of quantum properties, is used to identify the basic problem with derivations of Bell inequalities: the use of classical concepts (hidden variables) rather than a probabilistic structure appropriate to the quantum domain. A difficulty with the original Einstein-Podolsky-Rosen (EPR) argument for the incompleteness of quantum mechanics is the use of a counterfactual argument which is not valid if one assumes that Hilbert-space quantum mechanics is complete; locality is not an issue. The quantum correlations that violate Bell inequalities can be understood using local quantum common causes. Wave-function collapse and Schrödinger steering are calculational procedures, not physical processes. A general Principle of Einstein Locality rules out nonlocal influences between noninteracting quantum systems. Some suggestions are made for changes in terminology that could clarify discussions of quantum foundations and be less confusing to students.

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I. INTRODUCTION

The notion is widespread in popular articles, but also in many technical papers, review articles, and books, that quantum mechanics is "nonlocal," in some way that contrasts with the locality of classical physics. Here are a few almost random selections from this vast literature [1–7]. In particular, quantum mechanics is, we are told, inconsistent with "local realism" [8,9] because it predicts, and numerous experiments confirm, a violation of Bell inequalities, and this means that if the quantum-mechanical world is real there exist nonlocal influences which act instantaneously over arbitrarily large distances. And if two distant systems are in a suitable entangled quantum state, a measurement on one of them can instantaneously influence the other through a process known as "steering" [10–13].

To be sure, such claims have not gone unchallenged. Notable among more recent discussions is an interchange between a proponent of nonlocality, Tim Maudlin [14,15], and an advocate of quantum locality, Reinhard Werner [16,17], that appeared in a special issue of the Journal of Physics A published on the fiftieth anniversary of a famous paper [18] by John Bell; there was also a follow-up preprint [19] by Werner. It is of interest that neither protagonist in this debate actually applied quantum theory to properties and processes taking place in a microscopic quantum system. Instead, both used what might be called a "black box" approach: A macroscopic preparation of the quantum system is followed later by a measurement with a macroscopic output ("pointer position" in the antique but picturesque terminology of quantum foundations), with the discussion based upon quantum predictions of the relationship of input and output, without reference to what might be going on at the microscopic quantum level at an intermediate time. In Maudlin's case no reference to such goings on was needed for his arguments, whereas Werner employed an operational approach to quantum theory in which microscopic concepts are deliberately omitted. While a black box approach can sometimes be useful in this as in other areas of science, the claim of the present paper is that the locality issue is best addressed by opening the black box and examining what happens inside it, using consistent quantum principles. In particular, it is important to understand how quantum measurements can reveal microscopic quantum properties; something often assumed by experimenters who design and build apparatus, but not properly discussed in introductory (or advanced) quantum textbooks.

One source of the nonlocality idea is the widespread belief that measurements "collapse" quantum wave functions. If one of two (or more) separated quantum systems described by an entangled wave function is measured, then it is indeed possible to discuss the post-measurement situation using a "collapsed" wave function, and this no doubt contributes to the belief that there must be nonlocal influences in the quantum world. However, in this situation the wave function is merely a convenient tool for obtaining certain conditional probabilities that can be calculated by other methods that do not suggest any sort of nonlocal influence, as explained in Sec. VI.

To be sure, those who claim that instantaneous nonlocal influences are present in the quantum world will generally admit that they cannot be used to transmit information; this is known as the "no-signaling" principle, widely assumed in quantum information theory. This means that such influences

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(including wave-function collapse) cannot be directly detected in any experiment. The simplest explanation for their lack of influence is that such influences do not exist.

In classical physics two systems far apart can exhibit statistical correlations that allow a measurement on one of them to reveal some property of the other. No instantaneous nonlocal influences need be invoked if such correlations result from a local common cause at some time in the past. As explained in Sec. V C, an analogous kind of *quantum* common cause can be used to understand quantum correlations that violate Bell inequalities, thus removing any need for nonlocal influences.

The arguments that support these conclusions are carried out in several steps. First, in Sec. II the CHSH Bell inequality [20] is shown to be violated in a purely local situation where nonlocality plays no role. The key point is that the CHSH inequality employs *classical* hidden variables in a situation where a proper *quantum* description requires the use of *noncommuting* operators to represent physical quantities. That classical physics fails in the quantum domain is not at all surprising. What is surprising is that this fact [21] has been overlooked in much of the literature that claims quantum mechanics is nonlocal.

Next, Sec. III is devoted to an elementary discussion of projective quantum measurements and what they reveal about properties of measured systems *before* a measurement takes place. This is an essential part of opening the black box, and fills in a serious lacuna in textbooks. It justifies the belief of many experimental physicists that the apparatus they have carefully constructed and tested actually measures what it was designed to measure. A proper understanding of measurements disposes of another type of supposed quantum nonlocality: that quantum particles can simultaneously be in two different locations. The tools that allow quantum measurements to be understood in a rational manner are then applied in Sec. IV to a serious defect that enters many if not all derivations of Bell inequalities: a key factorization assumption that is supposed to represent the absence of nonlocal effects employs *classical* hidden variables that are inconsistent with Hilbert space quantum theory.

The much-discussed Einstein-Podolsky-Rosen (EPR) argument is examined in Sec. V, beginning in Sec. V A with Bohm's formulation in terms of two spin-half particles. If one assumes, contrary to EPR, that Hilbert space quantum mechanics is *complete*, this undermines a key counterfactual assumption about quantum measurements implicit in their work, an assumption that has nothing to do with locality. Following this, in Sec. V C it is shown that the experimentally observed correlations which violate Bell inequalities can be understood as arising from local *quantum* common causes, and hence in no need of explanations based upon instantaneous nonlocal influences. An analogy from the classical world helps understand why Alice's measurement of one of a pair of spin-half articles has not the slightest influence on the other particle, located far away in Bob's possession, though she is able to infer something about its properties. In no sense can she control or influence or "steer" Bob's particle.

In Sec. VI it is argued that wave-function collapse, while it can be used to calculate correlations, is simply a mathematical tool, and should *not* be understood as a nonlocal physical process. Indeed, a quite general *Principle of Einstein Locality* states that noninteracting systems cannot influence each other, whether or not they are in an entangled state. So in this respect discussions, as in [12,13], of Schrödinger steering are misleading.

A summary of the results of the paper are given in Sec. VII A. This is followed in Sec. VII B with suggestions for changes in terminology which might help clear up the confusion associated with long-standing, but unsupportable, claims of quantum nonlocality, thus making quantum theory less of an ordeal for students, and allowing more rapid progress in the study of quantum foundations.

The present paper incorporates, but also extends, material from some of the author's earlier publications [22–25]. The aim is to present a unified and comprehensive critique of quantum nonlocality claims, based in large part on a consistent analysis of quantum measurements. In order to understand in physical terms what is going on in the quantum world, measurements themselves must be described as physical processes governed by general quantum principles that apply to all processes. In particular, macroscopic measurement outcomes must be connected with the prior *quantum* properties, represented by Hilbert subspaces (as in Sec. III.5 of [26]), the apparatus was designed to reveal. The consistent histories (CH) approach[1] provides the precise rules needed to do this, and is the foundation of the discussions in Secs. III–VI.

In this paper a quantum *physical property* is represented by a Hilbert subspace or its projector, as distinct from a *physical variable* represented by a Hermitian operator; see Sec. III A. For the most part standard Dirac notation is employed, with the addition that $[\psi] = |\psi\rangle\langle\psi|$ denotes the projector onto a normalized pure state $|\psi\rangle$.

II. BELL INEQUALITIES

A common route to the belief that the world is nonlocal comes from the following sort of reasoning:

B1. Bell (and others) derived inequalities involving correlations of separated quantum systems, inequalities which will always be satisfied if a certain locality condition (local causality or local realism) is satisfied.

B2. Starting with the work of Freedman and Clauser [29], numerous experiments, among them [8,30–32], have shown, with ever increasing precision and control for errors and experimental loopholes, that experimentally measured correlations agree with the predictions of quantum mechanics and violate Bell inequalities.

B3. Therefore quantum mechanics, and the world it describes, must be nonlocal.

A. The Clauser, Horne, Shimony, and Holt inequality

To see what is wrong with this argument, consider the Clauser, Horne, Shimony, and Holt (CHSH) inequality [20],

[1]For an overview of consistent histories see [27]; a detailed treatment will be found in [28]. The material in [25] is of particular relevance to the present article.

one of the simplest Bell inequalities. It involves a quantity

$$S = A_0B_0 + A_0B_1 + A_1B_0 - A_1B_1$$
$$= (A_0 + A_1)B_0 + (A_0 - A_1)B_1, \tag{1}$$

where the A_j and B_k on the right-hand side are either classical random variables taking the values +1 and −1, or quantum observables (Hermitian operators) whose eigenvalues are +1 and −1, subject to the condition that each A_j commutes with every B_k.

In the classical case it is easy to see that because either $A_0 + A_1$ or $A_0 - A_1$ must be zero, S will lie between the limits

$$-2 \leqslant S \leqslant 2, \tag{2}$$

so its average $\langle S\rangle$ must fall in the same interval, whence the CHSH inequality:

$$|\langle S\rangle| \leqslant 2. \tag{3}$$

By contrast, if the A_j and B_k are quantum Hermitian operators with eigenvalues ±1 subject to the requirement that $[A_j, B_k] = 0$ for every j and k, it is easy to construct an example, see below, in which S has eigenvalues of $\pm 2\sqrt{2}$, 0, 0, and thus using the eigenstate for the largest eigenvalue to compute the average of S will yield $\langle S\rangle = 2\sqrt{2}$, in obvious violation of the inequality (3). A key feature of this example is that A_0 does not commute A_1, nor B_0 with B_1, and none of the four summands in (1) commute with the other three. There is no reason to expect the eigenvalues of a sum of noncommuting operators to bear any simple relationship with those of the summands, so the violation of (3) in the quantum case is not surprising. Nonlocality is irrelevant, as is shown by the following example.

B. Neon

The ^{21}Ne nucleus has a spin of 3/2, which is also the spin of a neutral neon atom of this isotope; it has a low but nonzero natural abundance. Thus the ground state of a ^{21}Ne atom is fourfold degenerate, and its quantum mechanical description uses a four-dimensional Hilbert space $\mathcal{H}$. Choose any orthonormal basis for this space, and let the basis vectors carry binary labels, thus $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$. These could, for example, be states in which the z component of angular momentum S_z for some (arbitrary) direction z takes on the values +3/2, +1/2, −1/2, −3/2 in units of $\hbar$, but any other choice would be equally good.

Next, as a matter of convenience, write $\mathcal{H}$ as a tensor product $\mathcal{H}_a \otimes \mathcal{H}_b$ of two two-dimensional spaces with orthonormal bases $|0\rangle_a$, $|1\rangle_a$, and $|0\rangle_b$, $|1\rangle_b$, respectively, related to the previously chosen basis of $\mathcal{H}$ through

$$|j,k\rangle = |j\rangle_a \otimes |k\rangle_b. \tag{4}$$

Finally, using this tensor product structure, define four operators

$$A_0 = Z \otimes I, \quad A_1 = X \otimes I, \quad B_0 = I \otimes X, \quad B_1 = I \otimes Z, \tag{5}$$

where I is a 2 × 2 identify matrix, while X, and Z are the Pauli x and z matrices. Define the products $M_{jk} = A_jB_k$:

$$M_{00} = Z \otimes X, \quad M_{01} = Z \otimes Z,$$
$$M_{10} = X \otimes X, \quad M_{11} = X \otimes Z \tag{6}$$

(where the subscripts label different operators, not matrix elements), and a quantum version of (1) takes the form

$$S = M_{00} + M_{01} + M_{10} - M_{11}. \tag{7}$$

Each M_{jk} has eigenvalues +1 and −1, both doubly degenerate, and each does not commute with any of the other three, even though each A_j commutes with each B_k.

Now imagine that a skilled experimenter is able to produce a beam consisting of neon atoms of this isotope, each in the same (pure) hyperfine state $|\psi\rangle$, and then, using a large number of runs, measures each M_{jk} and finds its average value. Note that four separate runs of the experiment are needed, since each M_{jk} does not commute with the others. Adding up the averages provides the average of S. Since the eigenvalues of the operator in (7) are $\pm 2\sqrt{2}$, 0, 0, if $|\psi\rangle$ is the eigenstate with the largest eigenvalue, the average $\langle S\rangle$ of S will be $2\sqrt{2}$, well outside the range (3).

In the case of this hypothetical ^{21}Ne experiment all the atoms belong to a single beam, and while the four different measurements require different apparatus settings, they can all be carried out in the same physical location in the same laboratory. Thus the violation of the CHSH inequality in this case has nothing to do with nonlocality. Instead it has everything to do with the fact that in quantum mechanics, unlike classical mechanics, physical properties and variables are represented by *noncommuting operators*. To be sure, doing the A and B measurements of photon polarizations at different locations, as in the usual Bell tests, guarantees that the A_j operators commute with the B_k operators, but this same requirement has simply been built into the protocol of the neon experiment.

Performing such an experiment would be difficult and expensive, and there is no reason to attempt it, since by now there is vast amount of experimental evidence that demonstrates, with high precision, the correctness of quantum mechanics. This includes the widely publicized experiments on correlated photon pairs, e.g., [8,31,32], which have confirmed that quantum theory violates Bell inequalities in the same way one would expect to be the case were the neon experiment actually carried out.

III. QUANTUM MEASUREMENTS

Quantum measurements of the sort we are interested in involve an amplification of a microscopic quantum property in such a way to produce a macroscopic result, the measurement *outcome*, a pointer position in the archaic but picturesque language of quantum foundations. Measurements of a beam of ^{21}Ne atoms could in principle be carried out in a similar way to the famous Stern-Gerlach experiment: use magnetic (perhaps assisted with electric) field gradients to separate the initial beam of particles into separate beams having different properties, each identified by quantum numbers referring to some observable. When far enough apart these beams would enter separate detectors where individual atoms are ionized and the electron fed to an electron multiplier resulting in a macroscopic current. Note that such a measurement determines the property of each atom *before* it is measured, not afterwards when the measurement is over and the detector has destroyed the atom.

A. Observables and properties

This simplest sort of *projective* measurement can be discussed in quantum mechanical terms as follows. Let $F = F^\dagger$ be the Hermitian operator corresponding to the physical variable (quantum observable) to be measured, and write it in the spectral form

$$F = \sum_j f_j P^j, \tag{8}$$

where the f_j are eigenvalues—we assume that $f_j \neq f_k$ for $j \neq k$—and the P^j projectors onto the corresponding eigenspaces. Here the j superscript of P^j is a *label*, not an exponent; this should cause no confusion, because a projector is equal to its square. These projectors satisfy the conditions

$$P^j = (P^j)^\dagger = (P^j)^2, \quad P^j P^k = \delta_{jk} P^j, \quad I = \sum_j P^j. \tag{9}$$

The first two equalities define a projector (orthogonal projection operator), while the last two define the collection $\{P^j\}$ to be a *projective decomposition of the identity* I (PDI). A measurement of F consists in determining which P^j represents the *quantum property* of the particle being measured at a time just before the measurement takes place. The term "property," following von Neumann, Sec. III.5 of [26], corresponds to a (closed) subspace of the Hilbert space, or its corresponding projector, and thus refers to something which can, at least potentially, be true or false. One should distinguish F, an observable or physical variable, from the property that F takes on a particular value or a range of values.[2] Thus a projector is the quantum counterpart of a set of points in the classical phase space, and a PDI is the quantum analog of a probabilistic *sample space*: a collection of mutually exclusive properties, one and only one of which can occur in any given run of an experiment. (For more details about measurement processes and their quantum description, see [25] and Chaps. 17 and 18 of [28].)

Suppose another observable $G = G^\dagger$ has the spectral form

$$G = \sum_k g_k Q^k, \tag{10}$$

where the g_k are its eigenvalues, and the properties $\{Q^k\}$ form a PDI. If F and G commute, $FG = GF$, then every Q^k commutes with every P^j and it is possible to measure F and G at the same time, using the PDI which is the common refinement of $\{P^j\}$ and $\{Q^k\}$, the collection of nonzero products $P^j Q^k$. However, if F and G are *incompatible*, $FG \neq GF$, then there will be some j and k such that $P^j Q^k \neq Q^k P^j$, and there is no common refinement of the two PDIs, so these observables cannot be measured in a single experiment; they must be determined in separate experimental runs. Note that if $P^j Q^k = Q^k P^j$ the product is itself a projector that represents the property "P^j AND Q^k," whereas if $P^j Q^k \neq Q^k P^j$ neither product is a projector, so the property P^j AND Q^k is not defined.[3] Textbooks tell us that two incompatible observables or properties cannot be measured simultaneously, and for this there is a simple explanation (not always given in textbooks): the simultaneous property is not represented by a projector, and thus does not exist. Even skilled experimenters cannot measure what is not there.

In the case of ^{21}Ne, none of the four operators in (6) commutes with any of the other three, which means that determining their averages requires four separate experiments. For example, M_{01} has eigenvalues $+1$ and -1, both doubly degenerate, so the PDI contains two projectors. To find the average $\langle M_{01}\rangle = \langle\psi|M_{01}|\psi\rangle$ the apparatus needs to separate the particles into two beams corresponding to these two eigenvalues, and after a large number of runs the experimental average will be $(N_+ - N_-)/(N_+ + N_-)$ if N_+ particles arrive in the $+1$ beam and N_- in the -1 beam. A *separate experiment*, which is to say a different arrangement for separating the incoming beams into separate beams, must be carried out for *each* of the M_{jk} in order to measure its average. And since S in (7) does not commute with any of the M_{jk}, an experimental check of this equality in the sense of equating the average $\langle S\rangle$ of S, as computed using quantum principles, with the sum of the experimental averages of the quantities on the right side would be a rather stringent test of the correctness of standard Hilbert space quantum mechanics.

B. Quantum measurement model

What follows is a simple quantum mechanical model of a projective measurement of an observable F, Eq. (8). Additional details will be found in Chaps. 17 and 18 of [28], and in [24,25]. In what follows we assume that F refers to a system, hereafter referred to as a "particle," with Hilbert space $\mathcal{H}_s$, while a much large Hilbert space $\mathcal{H}_m$ represents the measuring apparatus; together they constitute a closed system with Hilbert space $\mathcal{H} = \mathcal{H}_s \otimes \mathcal{H}_m$. At an initial time t_0 the particle is in a superposition of eigenstates of F—for simplicity assume the eigenvalues are nondegenerate—

$$|\psi_0\rangle = \sum_j c_j |\phi^j\rangle, \quad P^j = [\phi^j] = |\phi^j\rangle\langle\phi^j|, \tag{11}$$

and the apparatus is in the "ready-for-measurement" state $|\Phi_0\rangle$, so that the combined system is in the state

$$|\Psi_0\rangle = |\psi_0\rangle \otimes |\Phi_0\rangle = \sum_j c_j |\phi^j\rangle \otimes |\Phi_0\rangle. \tag{12}$$

Let t_1 be a time slightly later than t_0 during which there is negligible change under unitary time evolution, so at t_1 $|\Psi_1\rangle$ is the same as $|\Psi_0\rangle$. Next assume that during the time interval from t_1 to t_2 the particle and apparatus interact with each other in such a way that a measurement process takes place, so that by t_2 the macroscopic quantity representing the measurement

[2]In this usage "the energy of a harmonic oscillator is no greater than $(3/2)\hbar\omega$" is a property corresponding to a projector on a two-dimensional subspace, whereas "energy" by itself is a physical variable, not a property.

[3]The quantum logic of Birkhoff and von Neumann [33] does assign a property "P^j AND Q^k" when the projectors do not commute, but no one has yet turned their quantum logic into a useful tool for reasoning in physical terms about microscopic quantum properties. See Sec. 4.6 of [28] for a very simple example of one of the difficulties one runs into.

outcome, the "pointer position," has reached its final value. Let T be the unitary time development operator from t_1 to t_2 ($T = \exp[-i(t_2 - t_1)H]$ in the case of a time-independent Hamiltonian H), and let

$$\left|\Psi_2^j\right\rangle = T(|\phi^j\rangle \otimes |\Phi_0\rangle), \quad |\Psi_2\rangle = \sum_j c_j \left|\Psi_2^j\right\rangle = T|\Psi_1\rangle. \tag{13}$$

Next assume there is a PDI $\{M^k\}$ on $\mathcal{H}$, whose significance is that the property or projector M^k corresponds to the pointer (or whatever macroscopic variable indicates the measurement outcome) being in position k, and that

$$M^k\left|\Psi_2^j\right\rangle = \delta_{jk}\left|\Psi_2^j\right\rangle. \tag{14}$$

Thus if the particle is initially in the state $|\psi_j\rangle$ at t_1, its interaction with the apparatus will result in the pointer being in position j, i.e., possessing the property M^j, at time t_2, as one might expect in the case of a projective measurement. (Note that each M^k, since it represents a macroscopic quantum property, will project onto a subspace of very high dimension, compared to which 10 raised to the power 10^{10} is a relatively small number.)

To discuss the time dependence of the measuring process when the initial $|\psi_0\rangle$ is in a superposition—at least two of the c_j in (11) are nonzero—requires the use of *quantum histories*, sequences of quantum properties, represented by projectors, at successive times.[4] For our purposes it suffices to consider histories of the form

$$Y^{jk} = E_0 \odot E_1^j \odot E_2^k, \tag{15}$$

interpreted as meaning that the system at time t_0 has the property E_0, at time t_1 the property E_1^j, and at time t_2 the property E_2^k. (The symbol $\odot$ denotes a tensor product, a form of $\otimes$ used to separate properties at successive times. There is no assumption that events at successive times are related by a unitary time transformation.) Here j and k are labels, and the $\{E_1^j\}$ and $\{E_2^k\}$ are PDIs. The collection $\{Y^{jk}\}$ constitutes a *family of histories* or *framework*. Each history begins with the same property E_0 at time t_0, and different histories correspond to different events at later times. A family of histories constitutes a quantum sample space (analogous to a collection of random walks in classical physics) to which probabilities can be assigned using an extension of the Born rule, provided certain consistency conditions are satisfied. In our case the initial property is

$$E_0 = [\Psi_0] = [\psi_0] \otimes [\Phi_0], \tag{16}$$

and we shall consider three different families or frameworks based on different choices for the PDIs at t_1 and t_2.

The *unitary framework* $\mathcal{F}_u$ contains but a single history

$$\mathcal{F}_u : Y = [\Psi_0] \odot [\Psi_0] \odot [\Psi_2], \tag{17}$$

with the projectors at t_1 and t_2 corresponding to a unitary time development of the initial state. (Strictly speaking we should introduce a PDI $\{[\Psi_0], I - [\Psi_0]\}$, I the identity operator, at t_1, but the extended Born rule assigns zero probability to the second of these possibilities, so it can be ignored; similarly a PDI $\{[\Psi_2], I - [\Psi_2]\}$ at time t_2.) The trouble with the family $\mathcal{F}_u$ is that when two or more of the c_j are nonzero, the state $|\Psi_2\rangle$ is a coherent superposition of states that correspond to different pointer positions, and hence the corresponding property $[\Psi_2]$ does not commute with projectors representing different positions of the pointer; the two are incompatible, and trying to combine them will give a meaningless result, as noted earlier in the case of incompatible observable F and G. We have arrived at the infamous measurement problem of quantum foundations, or, in popular parlance, Schrödinger's cat.

This difficulty can be avoided by using in place of $\mathcal{F}_u$ a family

$$\mathcal{F}_1 : Y^k = [\Psi_0] \odot [\psi_0] \odot M^k, \tag{18}$$

where by physicists' convention $[\psi_0]$ at t_1 stands for $[\psi_0] \otimes I_m$ on the full Hilbert space, and histories with $I - [\psi_0]$ at t_1 have been omitted since they have zero probability. The use of $[\psi_0]$ rather than $[\Psi_0]$ as in (17) serves to focus attention on the particle at time t_1. The kth history Y^k ends in the pointer position M^k at time t_2, and the extended Born rule assigns to this outcome a probability

$$\Pr(Y^k) = \langle\Psi_2|M^k|\Psi_2\rangle = |c_k|^2 = \langle\psi_0|P^k|\psi_0\rangle = |\langle\phi^k|\psi_0\rangle|^2. \tag{19}$$

The final expression on the right is the formula students learn in an introductory course.

One can go a step further in opening the black box by using the framework

$$\mathcal{F}_2 : Y^{jk} = [\Psi_0] \odot [\phi^j] \odot M^k, \tag{20}$$

where $[\phi^j]$ (i.e., $[\phi^j] \otimes I_m$) at t_1 means the particle has the property $[\phi^j]$, while nothing is said about the state of the apparatus. It is easily shown that the consistency conditions for this family are satisfied, and the extended Born's rule assigns probabilities

$$\Pr(Y^{jk}) = \delta_{jk}|c_k|^2 = |\langle\phi^k|\psi_0\rangle|^2. \tag{21}$$

This agrees with (19), but provides additional information, namely the conditional probabilities (where subscripts 1 and 2 identify the time):

$$\Pr([\phi^j]_1 \,|\, [M^k]_2) = \delta_{jk} = \Pr([M^j]_2 \,|\, [\phi^k]_1), \tag{22}$$

assuming $c_k \neq 0$. The first says that if the measurement outcome (pointer position) is k at t_2, then at the earlier time t_1, *before* the measurement took place, the particle had the corresponding microscopic property $[\phi^k]$. In other words, a projective measurement of this sort reveals a prior property of the measured system when one uses an appropriate quantum description that allows for this possibility. Herein lies the key difference between $\mathcal{F}_2$, in which the different $[\phi^k]$ make sense at t_1, and $\mathcal{F}_1$, where they do not, since $[\psi_0]$, assuming at least two of the c_j in (13) are nonzero, does not commute with the relevant $[\phi^k]$.

In addition, since in $\mathcal{F}_2$ $[\phi^k]$ occurs at an *earlier time* than the measurement outcome M^k, the second equality in (22) allows one to identify the earlier $[\phi_k]$ as the *cause* of the later M^k. This is the way an experimenter will normally think

[4] See Sec. III of [25] for more details, and Chaps. 8 through 11 of [28] for an extended discussion of histories and their probabilities.

about the operation of a measurement apparatus; e.g., it is the arrival of a photon which caused the photodetector to produce a click, not vice versa. Note that the superposition state $|\psi_0\rangle$, whereas it does not appear at time t_1 in $\mathcal{F}_2$, can nonetheless be used, as in (21), for calculating the probabilities assigned to the different properties $[\phi^k]$ at this time. A wave function or ket used in this manner is referred to as a "preprobability" in Sec. 9.4 of [28]. This role as a calculational tool should be carefully distinguished from its use as a quantum property, as in (18).

A careful experimenter will want to check that the measurement apparatus built to measure a particular observable is functioning properly. One check is calibration: if the device has been built to measure F in (8), then for each j send in a stream of particles known to have the property P^j and check that the pointer always ends up at position j. Once the device has been calibrated, the experimenter will normally assume that if a particle whose property is unknown arrives at the detector and the pointer points at j, then the particle earlier had the property P^j. Thus the earlier property can be inferred or retrodicted from the measurement outcome.

But what if the particle was initially prepared in a superposition $|\psi_0\rangle$ of states corresponding to different values of j? The use of the framework $\mathcal{F}_2$ shows that such an inference remains valid. If the same initial state is used in a successive runs of the experiment, the outcomes will be different, with probabilities given by the usual formula (19). It is not meaningful to ask, "Did the particle have the property $[\psi_0]$ or the property $[\phi^k]$ prior to the measurement?," because the projectors do not commute. But if the question is: "Which among the $[\phi^j]$ was the property possessed by the particle just before it reached the apparatus," then the answer is given by using the framework $\mathcal{F}_2$ leading to the formula (22). Inferences of this sort are made all the time by experimenters, and it is to be regretted that this "common sense" understanding of quantum measuring processes is not explained in introductory textbooks.

We have employed three distinct frameworks or families of histories, $\mathcal{F}_u$, $\mathcal{F}_1$, and $\mathcal{F}_2$, in order to describe what goes on in a projective measurement. Which is the *right* framework? That depends on the question one wishes to address. If one is interested in relating the measurement outcome to the quantity it was designed to measure, $\mathcal{F}_2$ is the right framework, because it contains the corresponding microscopic events. These events are not simply absent from $\mathcal{F}_u$ and $\mathcal{F}_1$; in those families they have no meaning, because the $[\phi^j]$ are incompatible with the projectors used in $\mathcal{F}_u$ and $\mathcal{F}_1$ at time t_1. On the other hand, were one interested in whether the particle was perturbed on its way from an initial preparation to the time t_1 just before the measurement took place, a PDI at t_1 that included the state that evolved unitarily from the initial preparation would be appropriate. It is always a mistake to try and answer a question about a quantum property using a framework in which it is meaningless.

Different incompatible frameworks are used in quantum mechanics for answering different questions, and it is important to note that when a particular setup allows for several alternative incompatible frameworks, the answer provided by one of them to a question properly posed (in quantum terms) is not invalidated by the existence of alternative frameworks. Instead, there is a general consistency argument, see Chap. 16 of [28], that using alternative frameworks will never lead to contradictory results, i.e., some property P is true (probability 1) in one framework and false (probability 0) in another framework. Numerous quantum paradoxes represent apparent violations of this, but when examined they always involve some combination of arguments carried out by combining results from incompatible frameworks. Thus a central principle of CH is the *single framework rule*: valid quantum reasoning requires that different parts of an argument can all be embedded in, or expressed using, a single overall framework. The choice of which framework to use will depend upon which questions one wishes to answer. If one wants to assign probabilities to measurement outcomes it is necessary to employ a quantum description or framework in which the different macroscopic outcomes make sense: thus $\mathcal{F}_1$ or $\mathcal{F}_2$, rather than $\mathcal{F}_u$, for the example discussed above. If one wants to relate the measurement outcome to the corresponding prior microscopic property that was measured, the framework must be one in which those properties make sense, $\mathcal{F}_2$ rather than $\mathcal{F}_u$ or $\mathcal{F}_1$.

C. Quantum particle in different locations?

Can a quantum particle be in two different locations at the same time? To address this we first need to say what it means for a quantum particle to have the property that it is in some region of space R. That property is represented by a projector $\hat{R}$ whose action on the position-space wave function $\psi(\mathbf{r})$ is given by

$$\hat{R}\psi(\mathbf{r}) = \begin{cases} \psi(\mathbf{r}) & \text{if } \mathbf{r} \in R, \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

That is, it sets $\psi(\mathbf{r})$ to zero when $\mathbf{r}$ is not in R, but otherwise leaves it unchanged. The projector for the particle to be simultaneously in two regions R_1 and R_2 is $\hat{R}_1\hat{R}_2 = \hat{R}_2\hat{R}_1$. If the regions R_1 and R_2 do not overlap, this product is zero, which means the corresponding property cannot occur. Thus if "two places" is understood as two regions in space that do not overlap, the particle cannot be in both of them at the same time.

Once one understands that projective quantum measurements can be understood as measuring prior properties, the same conclusion follows from the textbook statement that even if a particle has a spread-out wave function, a measurement of position will find it in only one place. Thus if the support of the particle wave function is in the union $R = R_1 \cup R_2$ of two nonoverlapping regions R_1 and R_2, a position measurement will reveal its presence in one but not in the other, and its position just prior to measurement will be in the region indicated by the measurement outcome. Note that the *property* $[\psi]$, the Hilbert space projector that corresponds to the wave function $\psi(\mathbf{r})$, will not commute with either of projectors $\hat{R}_1$ or $\hat{R}_2$ associated with these two regions. assuming the support of $\psi(\mathbf{r})$ is not confined to one or the other. Thus in calculating the probabilities that the particle will be in (thus measured to be in) R_1 or R_2 one must understand $\psi(\mathbf{r})$ to be a *preprobability*; assuming it is normalized, $\rho(\mathbf{r}) = |\psi(\mathbf{r})|^2$ is a probability density which can be integrated over R_1 or R_2 to find the probability that the particle is in one of these regions, or that an appropriate measurement will find it there. Thus implicit in our discussion is a framework analogous to $\mathcal{F}_2$ in (20).

As a particular application one can think of the case of a double-slit experiment, and let R_1 and R_2 be nonoverlapping regions, where R_1 includes the first slit and its vicinity, but not the second slit, while R_2 is the vicinity of the second slit, but excludes the first. Suppose that at a particular time the wave representing the particle is in the union of R_1 and R_2. If detectors are placed immediately behind each slit, detection will show that the particle was in one of these regions, not both. If, on the other hand, the particle or wave emerging from the slits is undisturbed as it proceeds towards the distant interference region where there are a large number of detectors which detect its position at a later time, then it is correct to say that at the earlier time the particle was in the region R, but introducing the separate regions R_1 and R_2 into the quantum description at this time will violate the consistency conditions required to assign probabilities, making "Which slit did it pass through?" a meaningless question.[5] For more details see Chap. 13 of [28].

But, the reader may ask, if the particle was in $R = R_1 \cup R_2$, does that not immediately imply that it was either in R_1 or else it was in R_2? That would represent good classical reasoning, but it need not hold in the quantum world. To see why it can fail, consider a different situation: a quantum harmonic oscillator in which the possible energies are $(n + 1/2)\hbar\omega$ with corresponding (orthogonal) eigenstates $|n\rangle$, $n = 0, 1, \ldots$. Consider the two-dimensional subspace spanned by $|0\rangle$ and $|1\rangle$ whose projector is $P = [0] + [1]$. If the oscillator is in either of the two energy eigenstates $|0\rangle$ or $|1\rangle$, it possesses the property P. However, a superposition state $|\chi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ also lies in this two-dimensional subspace, but does *not* possess either property [0] or [1], as it does not have a well-defined energy. Similarly, a quantum particle passing through a double-slit system cannot, in general, be said to pass through a particular slit.

IV. CLASSICAL HIDDEN VARIABLES

There are a large number of published derivations of Bell inequalities, and it has even been claimed [6,7,34] that *any local* theory of the world, present or future, *must* lead to inequalities of this sort. That is, the experimental violations of Bell inequalities not only imply that the quantum world is nonlocal, but any future theory that gives results in agreement with these experiments will involve the same nonlocality. It is therefore useful to say a few words about what is wrong (from the perspective of Hilbert-space quantum mechanics) with the assumptions made in typical derivations of Bell inequalities, and why the aforementioned claim is false.

It will suffice to focus on the *factorization condition*, which always appears in some form or another in a derivation of the CHSH or other Bell inequalities:

$$\Pr(A, B|a, b) = \sum_\lambda \Pr(A|a, \lambda) \Pr(B|b, \lambda) \Pr(\lambda). \tag{24}$$

The symbols entering this expression have the following significance. Alice and Bob, who are far away from each other, are measuring pairs of particles produced at a common source. The outcome (pointer position) of Alice's measurement is A given the setting a of her apparatus, which determines the type of measurement being performed. Likewise, B and b refer to the outcome and setting for Bob's measurement. On the right side of (24) the "hidden variable" λ determines, in a probabilistic sense, the dependence of A on a and of B on b. (One can replace the sum over λ with an integral; it makes no difference.). Equation (24) expresses locality in the sense that if Alice and Bob are far from each other, the choice of a and the resulting outcome A should not influence B, nor the choice of b influence A, as long as λ, a "common cause," is held fixed.

To better understand the connection of such hidden variables with Hilbert space quantum mechanics, consider applying (24) to just one of the terms on the right side of (7), say $M_{00} = A_0B_0$, whose average in the state $|\psi\rangle$ we wish to evaluate using a proper quantum-mechanical calculation. Let us set $a = 0,\ b = 0$, and since they are fixed, drop them from both sides of (24), which becomes

$$\Pr(A_0, B_0) = \sum_\lambda \Pr(A_0|\lambda) \Pr(B_0|\lambda) \Pr(\lambda). \tag{25}$$

Here A_0 and B_0 are defined in (5), with eigenvalues ± 1, so can be written in the form, see (8):

$$A_0 = P_+ - P_-, \quad B_0 = Q_+ - Q_-, \tag{26}$$

using the two commuting PDIs $\{P_+, P_-\}$ and $\{Q_+, Q_-\}$. One can think of the arguments of $\Pr(A_0, B_0)$ as the eigenvalues of these operators, and thus (25) as the set of four equations, one for each p and q,

$$\Pr(P_pQ_q) = \sum_\lambda \Pr(P_p|\lambda) \Pr(Q_q|\lambda) \Pr(\lambda), \tag{27}$$

which assigns probabilities to the projectors P_pQ_q that together constitute the quantum sample space that is the common refinement of the PDIs used in (26). Identifying (24) with (25) is not completely trivial, since in the former A and B represent scalar quantities, measurement outcomes of $+1$ and -1, whereas in (25) A_0 and B_0 refer to the eigenvalues $+1$ and -1 of quantum operators, and (27) to the corresponding eigenspaces. This identification is correct provided projective measurements reveal pre-existing values, as explained in Sec. III.

The two sides of (27) will be equal if we let the hidden variable λ take on one of the four values $++, +-, -+, --$, given by the pair pq, and use conditional probabilities

$$\Pr(P_p|p'q) = \delta_{pp'}, \quad \Pr(Q_q|pq') = \delta_{qq'} \tag{28}$$

together with

$$\Pr(\lambda = pq) = \langle\psi|P_pQ_q|\psi\rangle. \tag{29}$$

Inserting these in the right side of (27) makes it equal to $\langle\psi|P_pQ_q|\psi\rangle$, the Born rule for $\Pr(P_pQ_q)$. Thus we have a

[5]There is an alternative framework in which the particle passes through a definite slit, but the detectors in the later interference region end up in a macroscopic quantum superposition (Schrödinger cat) state. One can understand why this framework has little appeal for understanding real experiments!

particular quantum application of (24) in the case $a = 0$, $b = 0$.

What works for $a = 0$, $b = 0$, the M_{00} term in (7), will work equally well for any of the other terms; one simply has to use appropriate choices for the PDIs $\{P_p\}$ and $\{Q_q\}$. But we know that the quantum average of the quantum S in (7) can exceed the classical CHSH bound in (3). Why is this? The trouble arises because if, for example, we consider M_{10} in place of M_{00} we will need a different choice for P_+ and P_- when A_0 in (26) is replaced with A_1, with which it does not commute, and this means changing the definition, or at least the physical meaning, of λ. But derivations of Bell inequalities always assume that λ, which is supposed to represent an earlier state of the measured particles, does *not* depend on the choices of a and b made by Alice and Bob, so changing it is not allowed.

Might there be some different choice for λ that evades this difficulty? Not likely, given the large number of careful analyses that show that (24), with λ independent of a and b, leads inexorably to the CHSH inequality, which is *not* satisfied by the correct quantum average of S in (7). What the foregoing analysis suggests is that the fundamental problem with such derivations is that they do not take proper account of the possible *noncommutativity* of quantum projectors representing the quantum properties of interest. And since this failure applies to ^{21}Ne, locality cannot be an issue.

To summarize, the fundamental difficulty with the factorization condition (24) is that it assumes a *single* sample space of mutually exclusive possibilities, independent of a and b, with elements labeled by λ. This would be quite appropriate for a classical system where there is a single phase space and the sample space employs nonoverlapping subsets of this phase space. But a quantum Hilbert space allows incompatible samples spaces, different PDIs with projectors that do not commute, and therefore lack a common refinement. Thus the usual derivations of CHSH and other Bell inequalities employ *classical* physics to discuss *quantum* systems, so it is not surprising when these inequalities fail to agree with quantum predictions, or the experiments that confirm these predictions.

V. EINSTEIN-PODOLSKY-ROSEN ARGUMENT

A. Bohm version of EPR

While the mistake associated with the claim that the violation of Bell inequalities implies nonlocality in the quantum world should be evident from the neon example of Sec. II B, and from the use of classical hidden variables for deriving these inequalities, Sec. IV, there are useful lessons to be learned from considering the original Einstein-Podolsky-Rosen (EPR) argument [35], where locality was simply assumed, using the simplified version introduced by Bohm, Chap. 22 of [36]. Two spin-half particles, a and b, are prepared in the spin-singlet state

$$|\psi_s\rangle = (|0\rangle_a \otimes |1\rangle_b - |1\rangle_a \otimes |0\rangle_b)/\sqrt{2}, \tag{30}$$

with $|0\rangle$ and $|1\rangle$ the $+1/2$ and $-1/2$ (in units of $\hbar$) eigenstates of S_z. Particle 1 is sent to Alice and 2 to Bob, who can then carry out measurements of the same or different components of spin angular momentum. If they measure the same component, say S_w, where w could be x or z or any other direction in space, the results will be opposite: if Alice observes $+1/2$ Bob will find $-1/2$, or $+1/2$ if Alice observes $-1/2$.

Recall that the Hilbert space of a spin-half particle is two dimensional, and thus the PDI associated with any spin component S_w consists of two projectors onto pure states. Neither projector associated with S_x commutes with either of the projectors associated with S_z, and consequently there is no subspace of the Hilbert space which can represent simultaneous values of both S_x and S_z. Hence expressions like "$S_x = +1/2$ AND $S_z = -1/2$" are meaningless,[6] and the same holds for any two distinct components of angular momentum.

B. The counterfactual argument

While, for reasons given above, S_x and S_z for a spin-half particle cannot be measured simultaneously, it is possible in principle to design an apparatus to measure *either* S_x *or* S_z, with the choice between the two made just before the particle enters the measuring device. (For example, a small region with a uniform magnetic field in the y direction placed just in front of the apparatus can cause $S_x = \pm 1/2$ to precess into $S_z = \pm 1/2$, turning an S_z into an S_x measurement; this field can be switched on or off just before the arrival of the particle.)

Suppose that with the S_z setting Alice finds $S_z = +1/2$ during a particular run. One can imagine that Alice *could* have chosen the S_x setting, and in that case *would have* obtained either $S_x = +1/2$ or $-1/2$, we do not know which. Does it not follow that the particle had *both* a definite S_z value revealed by the later measurement *and* a specific S_x component, the one that Alice *would* have learned *had* she measured S_x rather than S_z, a choice which she *could have made* at the very last instant before the particle reached the apparatus? The italicized words indicate that this is a *counterfactual* argument: it combines what actually happened with what *would have* happened in a similar but different situation. (For a discussion of consistent ways to discuss counterfactuals within quantum mechanics, see [37] or Chap. 19 of [28], and for an application to (non)locality issues, the interchange in [38,39].) Doesn't this prove that the quantum Hilbert space provides but an *incomplete* description of physical reality? The reader familiar with their original paper will notice the similarity with EPR's argument, which also contains the (implicit) assumption that if one measured one observable, one could very well have measured a different, incompatible observable.

However, one can just as well run the EPR argument in reverse. Given a classical situation where all observables (by definition) commute, or a quantum situation with two *commuting* observables, $FG = GF$, it makes perfectly good sense to ask: Suppose F, Eq. (8) was measured with the result indicating, say, the property P_2, what *would have happened* in this instance *if instead* G, Eq. (10) had been measured, i.e., what is the probability that the measurement would have revealed a property Q_k? Given some initial state the joint probability distribution corresponding to the common

[6]In quantum logic such a conjunction is a property that is always false (the zero-dimensional subspace).

refinement, the PDI composed of the nonzero P_jQ_k, can be computed, and from it a conditional probability $\Pr(Q_k \,|\, P_2)$, which is then a sensible (in general, probabilistic) answer to the counterfactual question. But when the projectors do not commute this cannot be done, and then, as noted earlier, F and G must be measured in separate experiments, and there is no reason to suppose that the value of F revealed in one experiment has anything to do with the value of G obtained in a different, independent experiment. In other words, the *completeness* of Hilbert space quantum mechanics, which makes *impossible* the simultaneous measurement of S_x and S_z, as there is nothing in the Hilbert space that corresponds to a joint property, undermines the counterfactual assumption that when Alice measured S_z she *could have* measured S_x *in the same run*. Were S_x to have been measured, it would have to have been in a different run, and there is no reason why the value of S_z measured in one run will somehow be related to the value of S_x measured in a different run.

Hence the counterfactual notion which enters, at least implicitly, the EPR argument is blocked as soon as one assumes, contrary to EPR, that Hilbert space quantum theory is complete, and there are no additional hidden variables. Given that attempts to supplement the quantum Hilbert space with hidden variables have thus far failed—as shown most clearly by experiments confirming the (Hilbert space) quantum violations of Bell inequalities [8,30–32], it would seem that the original EPR argument, that (Hilbert space) quantum mechanics is incomplete, fails. Locality, or its absence, has nothing to do with the matter: the issue is what measurements carried out on a *single* particle in a *single* location can tell one about the properties of *that* particle.

C. Quantum common cause

As noted in Sec. I, one reason for the belief in instantaneous nonlocal quantum influences is that quantum theory predicts, and experiment confirms, the existence of *correlations* which violate Bell inequalities, and thus cannot be explained by a common cause based on classical hidden variables. However, opening the black box and applying consistent quantum principles provides an explanation for the correlations in terms of local *quantum* common causes. Experiments that test Bell inequalities using entangled photon pairs already assume a common cause in the sense that pairs of photons produced at the source in the same, rather than a different, down conversion event are identified using their arrival times. All that is needed in addition is an argument that the polarizations measured later were also created in the same (local) event.

Here we employ the principle discussed in Sec. III that measurements of a suitable sort can be interpreted, by using a suitable framework, as revealing prior properties of the measured system. Reverting to spin-half language, if Alice's apparatus is set to measure S_z for particle a and the outcome corresponds to, say, $S_z = -1/2$, she can conclude that particle a possessed this property just before the measurement took place, and, assuming it was not perturbed on its way to her apparatus, at all previous times following the initial preparation. The same applies to Bob's measurement of S_z for particle b. Thus by applying the Born rule right after the two particles are prepared in the singlet state (30), one sees that the probability that particles 1 and 2 have the same z component of spin is zero, and the two possibilities for opposite S_z values each has a probability of 1/2. A similar argument using an appropriate framework applies to the case where Bob measures S_w for an arbitrary direction w. The probabilities for the correlations predicted by using what might be called a "measurement" framework, in which both measurement outcomes are traced all the way back to the source, are exactly the same as those predicted by textbook quantum theory using wave-function collapse in a "collapse" framework, Sec. VI, in which the entangled singlet state persists right up to the instant before one of the measurements. There is no reason that the Born rule can only be applied when a measurement takes place; this mistaken notion has been one of the reasons for the lack of progress in quantum foundations in resolving its infamous "measurement problem."

As noted in Sec. III B, inferences obtained in one framework are not invalidated by the existence of alternative frameworks. The collapse framework, which treats the entangled state as a property right up until the measurement takes place, precludes any discussion during that time period of spin states of the individual particles—see the comments in Sec. VI—thus concealing the fact made obvious in the "measurement framework," in which measurements reveal prior properties, that the quantum correlations between measurement outcomes have an explanation in terms of a (quantum) common cause. The reader may also find it helpful to consider the discussion of the measurement of M_{00} in Sec. IV, where proper use was made of a genuinely quantum "hidden variable" λ, as an example of a "quantum cause," in the same sense as that employed here.

Alice's choice of measurement on particle a has no influence at all on Bob's particle b and whatever measurements may be carried out on it. However, her knowledge of the outcome of a measurement of a particular component of angular momentum allows her to infer a property possessed by particle a before the measurement took place. Combined with what she knows about the preparations protocol, in particular the initial state $|\psi_s\rangle$, this allows her to infer something about particle b, from which she can also infer the probability of the outcome of a measurement of particle b. Thus if particle a is measured to have $S_z = -1/2$, Alice can assign an $S_z = +1/2$ property to particle b and predict with certainty the outcome of Bob's measurement of S_z, or assign a probability to the outcome if Bob instead measures some other component of spin angular momentum.

The following classical analogy may help in understanding this. Charlie inserts red and green slips of paper into two identical, opaque envelopes; then chooses one at random and mails it to Alice in Atlanta, and the other to Bob in Boston. From her knowledge of the preparation protocol Alice, upon opening her envelope and seeing the color of the slip of paper it contains, can immediately infer the color of the paper in Bob's envelope, whether or not he has already opened it or will open it at a later time. No magic or mysterious long-range influence is needed to understand how this works, and the same is true of its quantum analog.

Granted, this classical analogy does not cover all possibilities present in the quantum case; in particular the situation in which Alice measures one component of spin angular momentum and Bob a different component. However, it is still correct to say that from the S_z outcome of her measurement and her knowledge of the initial preparation, Alice can assign (conditional) probabilities to the outcomes of a measurement by Bob in the S_x or any other basis, and this possibility has nothing to do with her measurement having some mysterious effect upon Bob's particle.

VI. WAVE-FUNCTION COLLAPSE AND EINSTEIN LOCALITY

Spin-spin correlations in the Bohm version of EPR are usually calculated by one of two closely related methods. Let us suppose that Alice and Bob carry our measurements in the orthonormal bases $\{|a^0\rangle, |a^1\rangle\}$ and $\{|b^0\rangle, |b^1\rangle\}$, respectively. The joint probability distribution for an initial state $|\psi_s\rangle$ can be computed using the standard formula

$$\Pr(a^j, b^k) = \langle\psi_s|\,[a^j]\otimes[b^k]\,|\psi_s\rangle. \tag{31}$$

The discussion in Sec. III B justifies thinking of $|\psi_s\rangle$ as a preprobability, and identifying $[a^j]$ and $[b^k]$ as properties of the a and b particles prior to the measurement, the point of view adopted in the common cause discussion in Sec. V C.

An alternative approach which yields the same joint probabilities employs *wave-function collapse*. Assume that Alice's measurement is carried out first, and the outcome corresponds to $[a^0]$. This is thought of as "collapsing" the wave function $|\psi_s\rangle$ to a new state

$$\left|\psi_c^0\right\rangle = [a^0]\,|\psi_s\rangle/\sqrt{\langle\psi_s|\,[a^0]\,|\psi_s\rangle} \tag{32}$$

(where $[a^0]$ stands for $[a^0]\otimes I_b$). The (conditional) probability that Bob's measurement outcome will correspond to $[b^k]$ is then computed using the collapsed state:

$$\Pr(b^k\,|\,a^0) = \left\langle\psi_c^0\right|[b^k]\left|\psi_c^0\right\rangle. \tag{33}$$

When multiplied by $\Pr(a^0) = \langle\psi_s|\,[a^0]\,|\psi_s\rangle$ this gives the result in (31).

There is nothing wrong with this collapse procedure for obtaining the result in (31). However, as noted earlier in Sec. V C, Alice's measurement has no effect upon Bob's particle. Thus treating the collapse process in which $|\psi_s\rangle$ is replaced by $|\psi_c^0\rangle$, as an actual physical process in which Alice's measurement has somehow altered a property of particle b, is incorrect, and this error has given rise to a great deal of confusion, starting with EPR and extending up to more recent discussions of *steering*, e.g., [11–13], a term originating with Schrödinger [10] and expressing the idea that if Alice and Bob share an entangled state, Alice's measurement may be able to alter Bob's particle.

The mistake arises from a misunderstanding of the collapse framework. When $|\psi_s\rangle$ is employed as a preprobability, as in (31), it cannot be identified with a physical property of either particle a or b, since the corresponding projector $[\psi_s]$ does not commute with any nontrivial property of either particle. (The trivial properties are the identity projector I, always true, and the zero projector, always false.) Therefore its collapse to $|\psi_c^0\rangle$ in (32) cannot by itself indicate a change in some property of particle b. To discuss whether a measurement by Alice has a physical effect upon Bob's particle requires the use of a framework in which properties of the latter make sense, both before and after Alice's measurement takes place. This matter was studied in Chap. 23 of [28] for the Bohm version of EPR, showing that there is no such nonlocal effect as long as Alice's measurement apparatus does not directly interact with Bob's particle. This is a particular instance of a quite general *Principle of Einstein Locality*:

> Objective properties of isolated individual systems do not change when something is done to another noninteracting system.

Its proof will be found in [22]. Here "noninteracting" means that the two systems have independent dynamics: the unitary time-development operator for the combined systems is the tensor product of the individual time-development operators of the separate systems. Whether or not the systems are initially in an entangled state is irrelevant; entanglement should never be thought of as a mechanism by which one system can "influence" another. This result is hardly surprising given the widespread acceptance of the no-signaling principle, since if, contrary to Einstein locality, there were a change in some objective property, that change could be used to convey information, or at least this is how a physicist would tend to view the matter.[7]

VII. CONCLUSION

A. Summary

The central conclusion of this paper is the complete absence of nonlocal influences between quantum systems which are spatially separated and not interacting with each other: doing something to one system has no effect, instantaneous or otherwise, upon the other system. Experiments show no evidence of such effects, and the "no-signaling" principle, widely accepted in discussions of quantum information, assumes their absence. In brief, if physical reality is quantum mechanical, then quantum nonlocality, in the sense of nonlocal influences, is a myth.

Why, then, the widespread assumption, which often seems taken for granted without any need to defend it, that quantum mechanics is somehow "nonlocal" in a way in which classical physics is not? Wave-function collapse, produced by measurements when applied to a system in an entangled state with a distant system, is one source of the nonlocality notion, and this reflects the inadequate treatment of measurements in textbooks and much of the quantum foundations literature. As shown in Sec. VI, wave-function collapse is simply a method of computing a conditional probability, as in classical physics when two particles are statistically correlated. While this method of calculation might sometimes be useful in terms of intuitive insight, it does not correspond to a physical process.

The principal source of the current widespread belief in quantum nonlocality is undoubtedly the claim by Bell and

[7]For an alternative perspective by a philosopher, including a very clever construction of an influence that carries no information, see Chap. 4 of [6].

his successors that in a local world certain statistical correlations must satisfy some type of Bell inequality. The CHSH inequality, which belongs to this category, was studied in Sec. II where it was shown that it is violated by quantum correlations which have nothing to do with spatial separation, but are already exhibited by states associated with the spin-$3/2$ ground state of a ^{21}Ne atom. This was followed in Sec. IV with a discussion of the factorization formula which is central to derivations of Bell inequalities, and makes reference to a hidden variable or variables, typically denoted by λ. Such hidden variables are always assumed to be classical; they lack the structure of noncommuting projectors which are central to Hilbert space quantum mechanics. It is regrettable that so much attention has been paid to the locality assumptions involved in the derivation of Bell inequalities, and so little to the equally or more important assumption that quantum probabilities can be discussed using a classical sample space: in essence, assuming the microscopic world is not quantum mechanical but classical.

Much of the confusion surrounding discussions of nonlocality has to do with the absence from standard quantum mechanics, understood as what is found in textbooks, of a proper discussion of quantum *measurements*, and for this reason the essential principles have been summarized in Sec. III. The key to resolving what is generally referred to as the *measurement problem*, the possible appearance of superpositions of macroscopic "pointer" states (Schrödinger cats), is to use the consistent histories formulation of quantum theory in which time development is represented by stochastic histories rather than restricted to the unitary time development of a wave function. Using a *framework* (family of histories) with projectors for the pointer states gets rid of this measurement problem. Using a framework in which these macroscopic measurement outcomes are correlated with microscopic properties of the measured system at a time just before the measurement took place, resolves a second measurement problem: how the macroscopic outcomes can be used to infer (retrodict) the prior microscopic property that resulted in (caused) a particular outcome. Consistent reasoning using frameworks requires paying attention to the (possible) noncommutativity of quantum projectors as embodied in the *single framework rule*.

The tools used to analyze measurements in a fully quantum mechanical fashion made it possible to identify, in Sec. IV, the fundamental error, from the perspective of a consistent quantum theory, in derivations of the CHSH and other Bell inequalities. It is the assumption that the factorization condition (24) for probabilities can use *classical* hidden variables (parametrized by the symbol λ) associated with a *single* sample space, rather than appropriate quantum sample spaces, projective decompositions of the identity (PDIs).

Bell's work was motivated by the Einstein-Podolsky-Rosen (EPR) paper, in which locality was simply assumed, and the claim was made that quantum mechanics is incomplete. Their work was based on an inadequate understanding of quantum measurements, which at that time were assumed to simply collapse wave functions. In addition, their argument employs a counterfactual assumption which, translated into the Bohm version of the EPR paradox, is that while Alice actually measured (say) S_z, she could instead have measured and obtained a value for S_x during this particular run. But if one assumes, contrary to EPR, that Hilbert space quantum mechanics is complete, such a counterfactual assumption is misleading, since a spin-half particle cannot simultaneously possess an S_x and an S_z property. That has nothing to do, at least in any direct sense, with the EPR locality assumption. On the other hand, Einstein's belief that there are no ghostly nonlocal influences ("spukhafte Fernwirkungen") is fully justified, as noted in Sec. VI, by a consistent analysis employing Hilbert subspaces resulting in a Principle of Einstein Locality.

An additional argument, Sec. V C, undermines claims for quantum nonlocality based on correlations that violate Bell inequalities by showing that the relevant *quantum* correlations can be understood as arising from a *local quantum common cause*, something which, in the case of the polarization of down-converted photons, occurs at the source where they were created. This understanding makes use of the analysis of quantum measurements in Sec. III, in particular the fact that measurement outcomes reflect earlier microscopic properties of the measured system when analyzed using an appropriate framework.

B. Terminology: Some suggestions

Even the reader who agrees with the arguments presented in this paper may nonetheless, and with some justification, take the attitude that scientific terminology often acquires a technical meaning that is different from the way in which it was first used, and hence there is no difficulty if "local" and "nonlocal" continue to be used in the same way as in much of the current literature on quantum foundations and quantum information. After all, there are other examples: the term "heat capacity" is in common use in thermodynamics, and no one, except perhaps beginning students, is confused by the fact that "heat" is no longer regarded as a fluid, and heat capacities are typically measured by doing work on the system of interest, rather than connecting it to a thermal reservoir.

However, in the case of heat capacity there are at least some circumstances in which heat can, indeed, be treated as a conserved fluid, whereas in quantum mechanics "nonlocality" seems in almost every respect a misleading and confusing term. Granted, students who are setting up apparatus in the laboratory are, at least after a while, not likely to worry that an experiment setup at some distant location might suddenly make a photon disappear while on its way through an optical fiber to a detector, or perhaps suddenly appear out of nowhere. Theoreticians are more likely to be confused by nonlocality claims, and the appearance of such claims in textbooks and the popular literature can only add to the confusion felt by students learning quantum theory for the first time.

Those who agree with the author that clear thinking is a key part of good physics, and using appropriate terms is an aid to clear thinking, might at least wish to consider some alterations and/or clarifications in the use of various terms. Replacing nonlocal with "Bell nonlocal," a term already used in some publications, would be a useful clarification, and certainly appropriate, in that Bell himself believed (incorrectly) that violations of his inequalities indicated nonlocality. Similarly, replacing "steering" with "Schrödinger steering" would be a step in the right direction. However, in both cases adding a comment that the quantum world is in reality local—there are no instantaneous long-range influences—would help counter a widespread, but mistaken, belief to the contrary.

Replacing, or at least supplementing, "local" with "classical" in certain phrases would also be an improvement. Thus claims, e.g. [8,9], that recent experiments show that quantum mechanics is inconsistent with "local realism" lead to the strange conclusion that if quantum mechanics is local (as argued here) it must be unreal. But we have ample evidence that the real world is best described by quantum, not classical, mechanics, and so it is "*classical* realism" that is ruled out by experiments. Similarly, replacing "local causality" as used in [40,41] with "*classical* local causality" as a key ingredient in the derivation of Bell inequalities would help clarify their true nature.

These are simply offered as suggestions. The goal should be to use terms, including technical terms, which aid clear thinking rather than creating confusion.

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