Theory of two atoms in a chiral waveguide

P. R. Berman
Physics Department, University of Michigan, Ann Arbor, Michigan 48109-1040, USA

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A theory is presented that describes the atom and field dynamics for two atoms in a chiral waveguide. A source-field approach is used that enables one to identify the various physical processes contributing to these dynamics. Each atom is prepared in an arbitrary state at $t = 0$ and the field intensity and correlation functions are calculated, fully accounting for retardation. When the atoms are prepared in identical superposition states, the effects of constructive and destructive interference play a significant role on both the field intensity and second-order correlation function. It is also shown that the results can be taken over to provide a solution for the related problem of a single-photon pulse incident on an atom prepared in an arbitrary initial state.

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I. INTRODUCTION

The problem of two stationary atoms coupled by the vacuum radiation field represents a fundamental problem in quantum optics. If, at $t = 0$, the atoms are prepared in some arbitrary initial state and the field is in its vacuum state, a complete solution of the problem requires that the atom-field state vector be determined for all times $t > 0$. Since this problem is of such fundamental importance, it has been the subject of numerous studies dating back to the beginning of the quantum theory [1]. In many of these studies, the atoms are modeled as having a $J = 0$ ground state and a $J = 1$ excited state. The decay rate of the excited state for an isolated atom is denoted by γ_2. If $\gamma_2 R/c \ll 1$, where R is the separation of the atoms, retardation effects can be neglected insofar as they affect the atomic state dynamics. In this limit the problem admits to an analytic solution. For example, Lehmberg [2] has given a detailed description of the average field intensity radiated by the atoms for a number of different initial conditions. If retardation effects cannot be neglected, the solution can be expressed only as an infinite sum [3].

For the most part, calculations of the atomic state dynamics have been carried out assuming the atoms are at fixed positions in free space. More recently, however, motivated by developments in quantum information, there have been a number of studies of atoms confined to chiral waveguides, waveguides that allow for emission into one direction of the guide only [4–6]. Many of these studies have focused on the interaction of an incident pulse with ground-state atoms [5], but several authors have analyzed a problem in which a single-photon or n-photon pulse is incident on an atom prepared in its excited state [6]. By obtaining an analytic solution for the state vector or the state amplitudes, these authors [6] were able to show that, with a proper choice of the initial state for the field, the output field can mirror the input field to a good approximation, with one additional photon in the field. In such cases, the input field acts like a π pulse, driving the atom to its ground state. To achieve this result for a single-photon pulse, it is necessary that its temporal width be of the same order as the inverse lifetime of the atom. With increasing n, the pulse width needed for it to act as a π pulse is diminished. As a consequence the input pulse duration becomes much less that the atomic lifetime and radiative decay plays a negligible role. In this limit, the output pulse for an n-photon input pulse approaches that of an $(n + 1)$-photon pulse with approximately, but not exactly, the same spatiotemporal profile. Although accessible in their calculations, the authors do not focus on the output field intensity in the guide.

In this paper, I consider emission from two atoms located on the axis of a chiral waveguide. The radiation is produced solely by the atoms—there is no input field as in the previous studies mentioned above. One atom is located at $X = 0$ and the other at $X = X_2 > 0$. At time $t = 0$, the atoms are prepared in an arbitrary initial state, which can be entangled. A source-field approach [7] is used to obtain closed form analytic expressions for the field intensity, the integrated field intensity, the second-order correlation function, and the time-integrated second-order correlation function. The source-field approach is especially well suited to this calculation since it allows one to isolate and identify the various physical processes that contribute to the field radiated by this two-atom system. It will be seen that the second-order correlation function $g^{(2)}(X, t; X, t + \tau)$ at some position $X > X_2$ in the guide is a discontinuous function of t for $t = X/c$. In the Appendix, I show how the results can be used to obtain expressions for the field intensity and second-order correlation function in the complementary problem involving a single-photon pulse that is incident on an atom prepared in an arbitrary initial state.

II. HAMILTONIAN AND EQUATIONS OF MOTION

The atoms are modeled as two-level quantum systems having transition frequency ω_0, with the lower level denoted by 1 and the upper by 2. It is assumed that the atoms emit only z polarized radiation and I consider only z polarized electric fields in the guide. In the chiral guide under consideration, atoms can radiate only in the positive X direction. That is, for this two-atom system, the expectation value of the field intensity vanishes for all $X < 0$. An unusual feature of such

a guide is that the "dipole-dipole" interaction between the atoms is independent of their separation [8], a consequence of the strong coupling between the atoms and the modes of the chiral waveguide. The positive frequency operator of the electric field operator for this one-dimensional problem can be written as

$$E_+(X,t) = i\sum_k (\hbar\omega_k/2\epsilon_0 AL)^{1/2} a_k(t) e^{ikX} e^{-i\omega_k t}, \quad (1)$$

where L is a quantization length, A is the cross-sectional area of the guide, and $a_k(t)$ is a destruction operator (written in an interaction representation) for the mode having propagation constant $k = \omega_k/c$. All relevant field modes have frequencies that are sufficiently close to the atomic frequency to justify the replacement of $(\hbar\omega_k/2\epsilon_0 AL)^{1/2}$ that appears in the expression for the field operator by $(\hbar\omega_0/2\epsilon_0 AL)^{1/2}$.

For an atom-field interaction of the form $-\mu E$, where μ is an atomic dipole moment matrix element (assumed real), the Hamiltonian for the atom-field system is written in an interaction representation and in the rotating-wave approximation as

$$H^{(I)}(t) \approx \hbar g \sum_{j=1}^{2}\sum_k \sigma_+^{(j)}(t) a_k(t) e^{ikX_j} e^{-i(\omega_k-\omega_0)t} - a_k^\dagger(t) e^{-ikX_j}\sigma_-^{(j)}(t) e^{i(\omega_k-\omega_0)t}, \quad (2)$$

where $\sigma_+^{(j)}(t)$ $[\sigma_-^{(j)}(t)]$ is a raising (lowering) operator for atom j and

$$g = -i\left(\frac{\omega_0}{2\hbar\epsilon_0 AL}\right)^{1/2}\mu \quad (3)$$

is a coupling constant. The interaction representation is defined by

$$a_k^H(t) = a_k(t)e^{-i\omega_k t}; \quad (4a)$$

$$\sigma_\pm^{H(j)}(t) = \sigma_\pm^{(j)}(t)e^{\pm i\omega_0 t}, \quad (4b)$$

where the H superscript indicates an operator in the Heisenberg representation.

In the Wigner-Weisskopf approximation, it is straightforward to show that the excited state decay rate for a single atom in the guide is given by

$$\gamma_2 = \frac{\omega_0\mu^2}{2\hbar\epsilon_0 Ac}. \quad (5)$$

In terms of γ_2, the coupling constant g can be written as

$$g = -i\sqrt{\frac{\gamma_2 c}{L}}, \quad (6)$$

and the factor $(\hbar\omega_k/2\epsilon_0 AL)^{1/2} \approx (\hbar\omega_0/2\epsilon_0 AL)^{1/2}$ that appears in Eq. (1) as

$$\left(\frac{\hbar\omega_0}{2\epsilon_0 AL}\right)^{1/2} = \sqrt{\frac{\gamma_2 c}{L}}\frac{\hbar}{\mu}.$$

From Schrödinger's equation with the Hamiltonian given in Eq. (2), it then follows that the time evolution equations for the atomic and field operators are

$$\dot\sigma_+^{(j)}(t) = \sqrt{\frac{\gamma_2 c}{L}}\sum_k a_k^\dagger(t) e^{-ikX_j}\left[2\sigma_{22}^{(j)}(t) - 1\right]e^{i(\omega_k-\omega_0)t}, \quad (7a)$$

$$\dot\sigma_{22}^{(j)}(t) = -\sqrt{\frac{\gamma_2 c}{L}}\sum_k \sigma_+^{(j)}(t) a_k(t) e^{ikX_j} e^{-i(\omega_k-\omega_0)t} - \sqrt{\frac{\gamma_2 c}{L}}\sum_k a_k^\dagger(t) e^{-ikX_j}\sigma_-^{(j)}(t) e^{i(\omega_k-\omega_0)t}, \quad (7b)$$

$$\dot a_k(t) = \sqrt{\frac{\gamma_2 c}{L}}\sum_{j=1}^{2} e^{-ikX_j}\sigma_-^{(j)}(t) e^{i(\omega_k-\omega_0)t}, \quad (7c)$$

along with the adjoints of these equations. The operator $\sigma_{22}^{(j)}(t)$ is that associated with the Schrödinger operator $|2\rangle\langle 2|$ for atom j.

The formal solution for $a_k(t)$ is given by

$$a_k(t) = a_k + \sqrt{\frac{\gamma_2 c}{L}}\sum_{j=1}^{2}\int_0^t e^{-ikX_j}\sigma_-^{(j)}(t') e^{i(\omega_k-\omega_0)t'}, \quad (8)$$

containing a term equal to $a_k \equiv a_k(0)$ and a source term that depends on atomic operators. Substituting Eq. (8) back into Eqs. (7a) and (7b), converting the sum over k to an integral using the prescription,

$$\sum_k \to \frac{L}{2\pi c}\int_{-\infty}^{\infty} d\omega_k, \quad (9)$$

I find that, for $t > 0$, the atomic operators evolve as

$$\dot\sigma_+^{(1)}(t) = -\gamma\sigma_+^{(1)}(t) + i\frac{\hbar}{\mu}\tilde E_-^{(0)}(0,t)\left[2\sigma_{22}^{(1)}(t) - 1\right]; \quad (10a)$$

$$\dot\sigma_-^{(1)}(t) = -\gamma\sigma_-^{(1)}(t) - i\frac{\hbar}{\mu}\left[2\sigma_{22}^{(1)}(t) - 1\right]\tilde E_+^{(0)}(0,t); \quad (10b)$$

$$\dot\sigma_+^{(2)}(t) = -\gamma\sigma_+^{(2)}(t) + \gamma_2\sigma_+^{(1)}(t - X_2/c)\left[2\sigma_{22}^{(2)}(t) - 1\right]e^{-ik_0X_2} + i\frac{\hbar}{\mu}\tilde E_-^{(0)}(X_2,t)\left[2\sigma_{22}^{(2)}(t) - 1\right]; \quad (10c)$$

$$\dot\sigma_-^{(2)}(t) = -\gamma\sigma_-^{(2)}(t) + \gamma_2\left[2\sigma_{22}^{(2)}(t) - 1\right]\sigma_-^{(1)}(t - X_2/c)e^{ik_0X_2} - i\frac{\hbar}{\mu}\left[2\sigma_{22}^{(2)}(t) - 1\right]\tilde E_+^{(0)}(X_2,t); \quad (10d)$$

$$\dot\sigma_{22}^{(1)}(t) = -\gamma_2\sigma_{22}^{(1)}(t) + i\frac{\hbar}{\mu}[\sigma_+^{(1)}(t)\tilde E_+^{(0)}(0,t) - \tilde E_-^{(0)}(0,t)\sigma_-^{(1)}(t)]; \quad (10e)$$

$$\dot\sigma_{22}^{(2)}(t) = -\gamma_2\sigma_{22}^{(2)}(t) - \gamma_2\sigma_+^{(2)}(t)\sigma_-^{(1)}(t - X_2/c)e^{ik_0X_2} - \gamma_2\sigma_+^{(1)}(t - X_2/c)\sigma_-^{(2)}(t)e^{-ik_0X_2}; + i\frac{\hbar}{\mu}[\sigma_+^{(2)}(t)\tilde E_+^{(0)}(X_2,t) - \tilde E_-^{(0)}(X_2,t)\sigma_-^{(2)}(t)], \quad (10f)$$

where $k_0 = \omega_0/c$, $\gamma = \gamma_2/2$, and

$$\tilde{E}_+^{(0)}(X,t) = [\tilde{E}_-^{(0)}(X,t)]^\dagger = i\left(\frac{\hbar}{\mu}\right)\sqrt{\frac{\gamma_2 c}{L}}\sum_k e^{ikX} a_k e^{-i(\omega_k-\omega_0)t}. \quad (11)$$

In using the prescription (9), I have implicitly made the Wigner-Weisskopf approximation by setting $\omega_k = |k|c$ and neglecting the contributions from negative values of k. In one dimension, this approximation is a good one for a chiral guide, but *not* for a bi-directional guide. In a bi-directional guide atoms can radiate in both directions and negative values of k contribute to the decay rate in the same way as positive values—the decay rate in that case is twice that given in Eq. (5). Equation (10) reflects the underlying physics of this chiral guide. Atom 1 evolves as if atom 2 were absent. Since atom 2 cannot radiate in the backwards direction, it does not influence the decay of atom 1. On the other hand the evolution of atom 2 at time t depends on the state of atom 1 at the retarded time $t - X_2/c$. The additional $\tilde{E}_\pm^{(0)}(X_j,t)$ terms in Eqs. (10) are "fluctuation" terms that are needed to maintain the equal-time commutation relations between the atom-atom and atom-field operators as well as operator products such as $\sigma_+^{(j)}(t)\sigma_-^{(j)}(t) = \sigma_{22}^{(j)}(t)$; $\sigma_+^{(j)}(t)\sigma_{22}^{(j)}(t) = 0$. However, owing to the fact that the initial state for the field is the vacuum state and that [9]

$$[\tilde{E}_+^{(0)}(0,t), \sigma_+^{(1)}(t')] = 0 \quad t \geqslant t'; \quad (12a)$$

$$[\tilde{E}_+^{(0)}(X_2,t), \sigma_+^{(2)}(t')] = 0 \quad t \geqslant t', \quad (12b)$$

the fluctuation terms do not contribute to any of the expectation values of operators that are evaluated in this paper.

III. FIELD INTENSITY

I first calculate the field intensity $I(X,t)$ at position $X > X_2$ and time t, defined as

$$I(X,t) = 2\epsilon_0 cA\langle E_-(X,t)E_+(X,t)\rangle. \quad (13)$$

In this case, the position X is meant to correspond to the position of a photodetector that records the field intensity. When the solution given by Eq. (8) is substituted back into Eq. (1) and the sum over k is converted to an integral using Eq. (9), the field operator can be expressed:

$$E_+(X,t) = E_+(t_r) = E_+^{(0)}(t_r) + E_+^{(\text{Source})}(t_r), \quad (14)$$

where the "free-field" component is given by

$$E_+^{(0)}(t_r) = i\left(\frac{\hbar}{\mu}\right)\sqrt{\frac{\gamma_2 c}{L}}\sum_k a_k e^{-i\omega_k t_r}, \quad (15)$$

and the "source-field" component by

$$E_+^{(\text{Source})}(t_r) = i\left(\frac{\hbar\gamma_2}{\mu}\right)e^{-i\omega_0 t_r}[\sigma_-^{(1)}(t_r) + \sigma_-^{(2)}(t_r + X_2/c)e^{-ik_0X_2}]. \quad (16)$$

The time,

$$t_r = t - X/c, \quad (17)$$

appearing in Eq. (14) is the retarded time at the field point relative to the *origin*.

The $E_\pm^{(0)}(t_r)$ terms do not contribute to the expectation value in Eq. (13), since the field starts in the vacuum state. It then follows that

$$I(X,t) = I(t_r) = \hbar\omega_0\gamma_2\langle\sigma_{22}^{(1)}(t_r) + \sigma_{22}^{(2)}(t_r + X_2/c) + [\sigma_+^{(1)}(t_r)\sigma_-^{(2)}(t_r + X_2/c)e^{-ik_0X_2} + \text{adjoint}]\rangle, \quad (18)$$

where the expectation value is taken with respect to the initial state vector,

$$|\psi(0)\rangle = |i\rangle_A|0\rangle_F, \quad (19)$$

and $|i\rangle_A$ is the initial atomic state vector and $|0\rangle_F$ is the vacuum state of the field. The intensity at point X at time t depends only on the retarded time $t_r = t - X/c$ relative to the position of atom 1 and on the retarded time,

$$t_r(2) = t - (X - X_2)/c = t_r + X_2/c, \quad (20)$$

relative to the position of atom 2. From this point onwards, I drop the r subscript on t and t always refers to the retarded time t_r relative to the origin unless noted otherwise.

From Eqs. (10e) and (11), I find that the first term needed in Eq. (18) is simply

$$\langle\sigma_{22}^{(1)}(t)\rangle = e^{-\gamma_2 t}\rho_{22}^{(1)}(0)\Theta(t), \quad (21)$$

where $\rho_{22}^{(1)}(0)$ is the initial excited state density matrix element for atom 1 and $\Theta(t)$ is a Heaviside function defined by $\Theta(t) = 1$ for $t \geqslant 0$ and $\Theta(t) = 0$ for $t < 0$. Atom 1 decays as if atom 2 was not present. The second term needed in Eq. (18) is proportional to $\langle\sigma_{22}^{(2)}(t + X_2/c)\rangle$. To calculate this term, I start from Eq. (10f), use Eqs. (10a), (10c), and (11), and obtain

$$\langle\dot{\sigma}_{22}^{(2)}(t)\rangle = \dot{\rho}_{22}^{(2)}(t) = -\gamma_2\rho_{22}^{(2)}(t) - \gamma_2 G(t)f(t) - \gamma_2 G^*(t)f^*(t), \quad (22)$$

where

$$f(t) = e^{-\gamma(t-X_2/c)}e^{ik_0X_2}\Theta(t - X_2/c), \quad (23)$$

$$G(t) = \langle\sigma_+^{(2)}(t)\sigma_-^{(1)}(0)\rangle, \quad (24)$$

and $\rho_{22}^{(2)}(t)$ is the excited state density matrix element for atom 2.

Using Eqs. (10c) and (11), I find that $G(t)$ satisfies the differential equation,

$$\dot{G}(t) = -\gamma G(t) + \gamma_2 f^*(t)\langle\sigma_+^{(1)}(0)[2\sigma_{22}^{(2)}(t) - 1]\sigma_-^{(1)}(0)\rangle = -\gamma G(t) + 2\gamma_2 f^*(t)\langle\sigma_+^{(1)}(0)\sigma_{22}^{(2)}(t)\sigma_-^{(1)}(0)\rangle - \gamma_2 f^*(t)\rho_{22}^{(1)}(0). \quad (25)$$

It then follows from Eqs. (10f) and (12) that $\langle\sigma_+^{(1)}(0)\sigma_{22}^{(2)}(t)\sigma_-^{(1)}(0)\rangle$ obeys the differential equation,

$$d\langle\sigma_+^{(1)}(0)\sigma_{22}^{(2)}(t)\sigma_-^{(1)}(0)\rangle/dt = -\gamma_2\langle\sigma_+^{(1)}(0)\sigma_{22}^{(2)}(t)\sigma_-^{(1)}(0)\rangle. \quad (26)$$

In deriving this equation, I used the identities,

$$\sigma_-^{(1)}(t)\sigma_-^{(1)}(0)|\psi(0)\rangle = \langle\psi(0)|\sigma_+^{(1)}(0)\sigma_+^{(1)}(t) = 0, \quad (27)$$

which are a consequence of Eqs. (10a), (10b), (12), (11), and (19). The solution of Eq. (26) is

$$\langle\sigma_+^{(1)}(0)\sigma_{22}^{(2)}(t)\sigma_-^{(1)}(0)\rangle = e^{-\gamma_2 t}\langle\sigma_+^{(1)}(0)\sigma_{22}^{(2)}(0)\sigma_-^{(1)}(0)\rangle\Theta(t) = e^{-\gamma_2 t}T(0)\Theta(t), \tag{28}$$

with

$$T(0) = \langle\sigma_{22}^{(2)}(0)\sigma_{22}^{(1)}(0)\rangle. \tag{29}$$

When Eq. (28) is substituted into Eq. (25), the resulting equation can be integrated to obtain

$$G(t) = G(0)e^{-\gamma t}\Theta(t) + 2T(0)e^{-\gamma(t-X_2/c)}e^{-ik_0X_2}(e^{-\gamma_2X_2/c} - e^{-\gamma_2 t}) \times \Theta(t - X_2/c) - \gamma_2(t - X_2/c)e^{-ik_0X_2}e^{-\gamma(t-X_2/c)}\rho_{22}^{(1)}(0)\Theta(t - X_2/c). \tag{30}$$

It is now straightforward to insert Eq. (30) back into Eq. (22) and to integrate that equation to arrive at

$$\langle\sigma_{22}^{(2)}(t + X_2/c)\rangle = \rho_{22}^{(2)}(t + X_2/c) = \rho_{22}^{(2)}(0)e^{-\gamma_2(t+X_2/c)}\Theta(t + X_2/c) - \gamma_2 te^{-\gamma_2 t}e^{-\gamma X_2/c}[G(0)e^{ik_0X_2} + G^*(0)e^{-ik_0X_2}]\Theta(t) - 4T(0)e^{-\gamma_2 t}[e^{-\gamma_2X_2/c}(\gamma_2 t - 1) + e^{-\gamma_2(t+X_2/c)}]\Theta(t) + \rho_{22}^{(1)}(0)e^{-\gamma_2 t}\gamma_2^2t^2\Theta(t). \tag{31}$$

I now have the first two terms needed in Eq. (18).

The remaining terms in Eq. (18) are proportional to

$$\langle\sigma_+^{(2)}(t + X_2/c)\sigma_-^{(1)}(t)e^{ik_0X_2}\rangle + \text{c.c.} = G(t + X_2/c)e^{ik_0X_2}e^{-\gamma t}\Theta(t) + \text{c.c.} \tag{32}$$

Using Eq. (30), I find

$$G(t + X_2/c)e^{ik_0X_2}e^{-\gamma t}\Theta(t) = G(0)e^{-\gamma X_2/c}e^{ik_0X_2}e^{-\gamma_2 t}\Theta(t) + 2T(0)e^{-\gamma_2 t}(e^{-\gamma_2X_2/c} - e^{-\gamma_2(t+X_2/c)})\Theta(t) - \gamma_2 te^{-\gamma_2 t}\rho_{22}^{(1)}(0)\Theta(t). \tag{33}$$

Finally, by combining Eqs. (18), (21), (31), and (30), I obtain

$$I_N(t) = \frac{I(t)}{\hbar\omega_0\gamma_2} = \rho_{22}^{(2)}(0)e^{-\gamma_2X_2/c}e^{-\gamma_2 t}\Theta(t + X_2/c) + (1 - \gamma_2 t)e^{-\gamma_2 t}e^{-\gamma X_2/c}[G(0)e^{ik_0X_2} + G^*(0)e^{-ik_0X_2}]\Theta(t) + \rho_{22}^{(1)}(0)e^{-\gamma_2 t}(1 - \gamma_2 t)^2\Theta(t) + 4T(0)e^{-\gamma_2 t}e^{-\gamma_2X_2/c}[2(1 - e^{-\gamma_2 t}) - \gamma_2 t]\Theta(t), \tag{34}$$

where $I_N(t)$ is a dimensionless intensity defined such that its time integral is equal to the initial energy of the two-atom system.

The most general initial atomic state can be written as

$$|i\rangle_A = c_{11}|11\rangle + c_{12}|12\rangle + c_{21}|21\rangle + c_{22}|22\rangle, \tag{35}$$

where $|jk\rangle$ is a state in which atom 1 is in state j and atom 2 in state k. For this initial state vector,

$$\rho_{22}^{(1)}(0) = |c_{12}|^2 + |c_{22}|^2; \tag{36a}$$

$$\rho_{22}^{(2)}(0) = |c_{21}|^2 + |c_{22}|^2; \tag{36b}$$

$$T(0) = |c_{22}|^2; \tag{36c}$$

$$G(0) = c_{21}^*c_{12}. \tag{36d}$$

If the initial state is the factorized symmetric state,

$$|i_1\rangle_A = (\alpha|1\rangle_1 + \beta|2\rangle_1)(\alpha|1\rangle_2 + \beta|2\rangle_2), \tag{37}$$

$(|\alpha|^2 + |\beta|^2 = 1)$, then

$$\rho_{22}^{(1)}(0) = \rho_{22}^{(2)}(0) = |\beta|^2; \tag{38a}$$

$$T(0) = |\beta|^4; \quad G(0) = |\alpha\beta|^2. \tag{38b}$$

On the other hand, for the maximally entangled state,

$$|i_2\rangle_A = \frac{1}{\sqrt{2}}(|12\rangle + |21\rangle), \tag{39}$$

$$\rho_{22}^{(1)}(0) = \rho_{22}^{(2)}(0) = G(0) = 1/2; \tag{40a}$$

$$T(0) = 0. \tag{40b}$$

Equation (34) gives the intensity dynamics for any initial conditions, with retardation taken fully into account. It is easy to verify that

$$\int_0^\infty I(t)dt = \hbar\omega_0\big[\rho_{22}^{(1)}(0) + \rho_{22}^{(2)}(0)\big], \tag{41}$$

as it must from conservation of energy. I consider two limits, $\gamma X_2/c \ll 1$ and $\gamma X_2/c \gg 1$. For convenience, I define

$$d = \gamma X_2/c; \quad \theta = k_0X_2. \tag{42}$$

A. $d = \gamma X_2/c \ll 1$

In the limit that $\gamma X_2/c \ll 1$, interatomic retardation plays no role and

$$I_N(t) \sim \rho_{22}^{(2)}(0)\Theta(t)e^{-\gamma_2 t} + (1 - \gamma_2 t)e^{-\gamma_2 t}[G(0)e^{i\theta} + G^*(0)e^{-i\theta}]\Theta(t) + \rho_{22}^{(1)}(0)e^{-\gamma_2 t}(1 - \gamma_2 t)^2\Theta(t) + 4T(0)e^{-\gamma_2 t}[2(1 - e^{-\gamma_2 t}) - \gamma_2 t]\Theta(t). \tag{43}$$

For the symmetric factorized initial state [see Eq. (38)],

$$I_{N1}(t) \sim |\beta|^2e^{-\gamma_2 t}[1 + (1 - \gamma_2 t)^2]\Theta(t) + 2(1 - \gamma_2 t)e^{-\gamma_2 t}|\alpha\beta|^2\cos\theta\,\Theta(t) + 4|\beta|^4e^{-\gamma_2 t}[2(1 - e^{-\gamma_2 t}) - \gamma_2 t]\Theta(t), \tag{44}$$

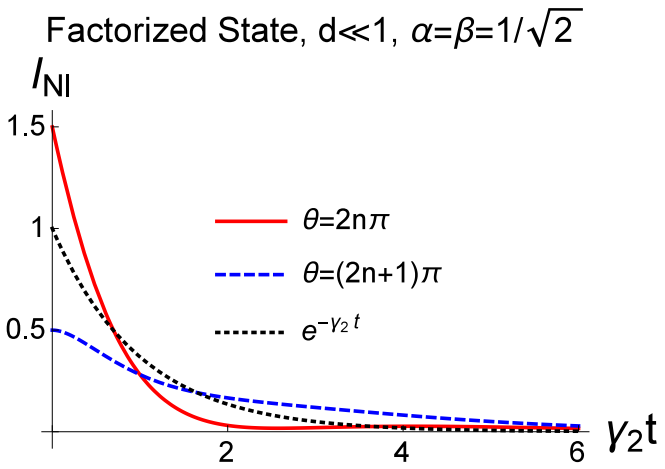

FIG. 1. Dimensionless intensity I_{N1} as a function of $\gamma_2 t$ for the symmetric factorized initial state with $d = \gamma_2 X_2/c \ll 1$, $\alpha = \beta = 1/\sqrt{2}$, and $\theta = k_0 X_2 = (2n+1)\pi, 2n\pi$.

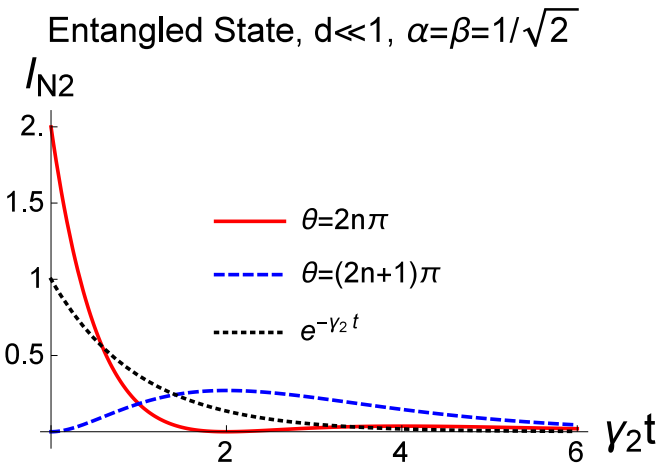

FIG. 2. Dimensionless intensity I_{N2} as a function of $\gamma_2 t$ for a maximally entangled initial state with $d = \gamma_2 X_2/c \ll 1$, $\alpha = \beta = 1/\sqrt{2}$, and $\theta = k_0 X_2 = (2n+1)\pi, 2n\pi$.

and for the maximally entangled initial state [see Eq. (40)],

$$I_{N2}(t) \sim \tfrac{1}{2}[1 + (1-\gamma_2 t)^2]\Theta(t)e^{-\gamma_2 t} + (1-\gamma_2 t)e^{-\gamma_2 t}\cos\theta\,\Theta(t). \tag{45}$$

At $t = 0$,

$$I_{N1}(0) = 2|\beta|^2[1 + |\alpha|^2\cos\theta]; \tag{46a}$$

$$I_{N2}(0) = 1 + \cos\theta. \tag{46b}$$

In both cases there can be constructive or destructive interference resulting from the relative spatial phase factor. For the factorized state, interference occurs only when the initial dipole moment of the atoms is nonvanishing ($\alpha\beta \neq 0$), with the spatial phase provided by the vacuum field. For the maximally entangled state, the atoms never acquire a dipole moment, but the relative spatial phase factor of the vacuum field at the two atomic sites at $t = 0$ leads to the interference. When $\theta = k_0 X_2$ is an odd multiple of π, $I_{N2}(0) \sim 0$, whereas the minimum value possible for $I_{N1}(0)$ is $2|\beta|^4$. In Fig. 1, I plot $I_{N1}(t)$ as a function of $\gamma_2 t$ for $d \ll 1$ and $\alpha = \beta = 1/\sqrt{2}$. The solid red curve is for $\theta = 2n\pi$ (constructive interference) and the dashed blue curve for $\theta = (2n+1)\pi$ (destructive interference). The dotted black curve is $e^{-\gamma_2 t}$, drawn for reference. The analogous curves for the maximally entangled state are shown in Fig. 2. Note that $I_{N2}(t) = 0$ if $\theta = 2n\pi$ and $\gamma_2 t = 2$.

In Fig. 3, I plot $I_{N1}(t)$ as a function of $\gamma_2 t$ for $d \ll 1$ and $\beta = 1$ (both atoms inverted). In this limit,

$$I_{N1}(t) \sim e^{-\gamma_2 t}[10 - 6\gamma_2 t + (\gamma_2 t)^2 - 8e^{-\gamma_2 t}]\Theta(t). \tag{47}$$

The dotted black curve is $2e^{-\gamma_2 t}$ and corresponds to what the intensity pattern would be for two noninteracting atoms. The fact that $I_{N2}(t) > 2e^{-\gamma_2 t}$ for early times and that the output field decays to zero more rapidly than $2e^{-\gamma_2 t}$ can be viewed as a signature of stimulated emission—the field from the first atom produces stimulated emission in the second atom. Alternatively, the output field can be interpreted as superradiance from the inverted two-atom system. It is interesting to compare Eq. (47) with the analogous result for the *spatially integrated* intensity of two, inverted atoms in *free space* having $d \ll 1$. In that case [2],

$$I_{N1}^{sr}(t) = 2e^{-2\gamma_2 t}(1 + 2\gamma_2 t)\Theta(t), \tag{48}$$

which is plotted as the dashed blue curve in the figure. As you can see, the chiral result is close, but not identical, to that of the corresponding free space result. I will return to this point in Sec. V.

B. $d = \gamma X_2/c \gg 1$

In the limit that $\gamma X_2/c \gg 1$, the field from atom 2 arrives at the detector, followed by the field from atom 1, which is modified by its interaction with atom 2. In that limit,

$$I_N(t) \sim \rho_{22}^{(2)}(0)e^{-\gamma_2(t+X_2/c)}\Theta(t + X_2/c) + \rho_{22}^{(1)}(0)e^{-\gamma_2 t}(1-\gamma_2 t)^2\Theta(t). \tag{49}$$

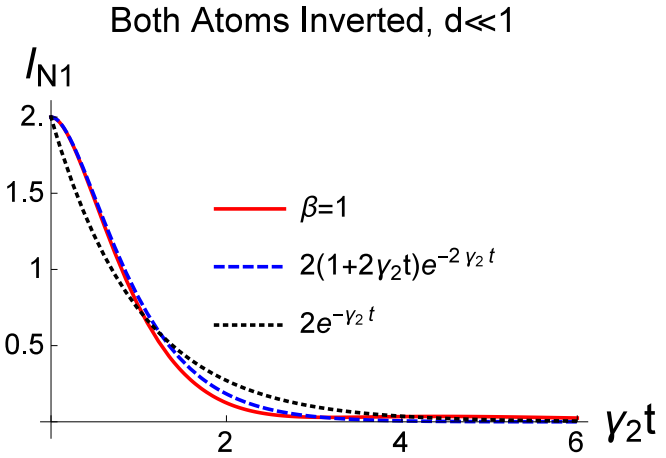

FIG. 3. Dimensionless intensity I_{N1} as a function of $\gamma_2 t$ for the symmetric factorized initial state with $d = \gamma_2 X_2/c \ll 1$ and $\beta = 1$ (both atoms inverted). The dashed blue curve corresponds to the spatially integrated intensity from two atoms in free space, that is, to two-atom superradiance.

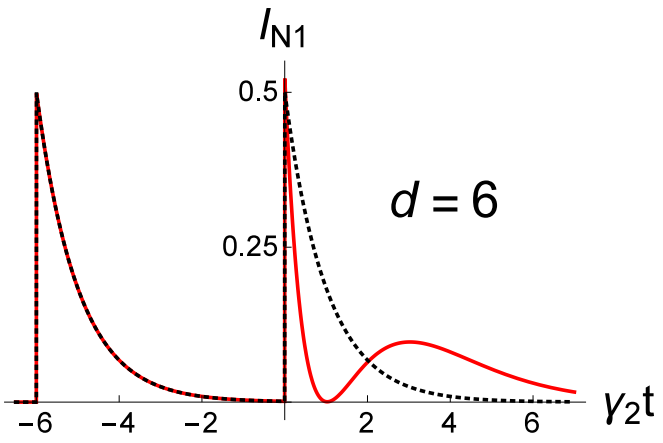

FIG. 4. Normalized intensity I_{N1} as a function of $\gamma_2 t$ for the symmetric factorized initial state with $d = \gamma_2 X_2/c = 6$, $|\beta|^2 = 1/2$, and $\theta = 2n\pi$. For $d \gg 1$, the intensity depends only on the initial state populations of the atoms. The black dotted curve represents the intensity at the detector neglecting any modification of the field from atom 1 produced by atom 2.

There is no longer any interference that depends on the spatial separation of the atoms. The only interference is between the scattered and unscattered components of the field radiated from atom 1 that is scattered by atom 2. This interference is totally destructive at $\gamma_2 t = 1$. In Fig. 4, the solid red curve is a plot of $I_{N1}(t)$ as a function of $\gamma_2 t$ for $d = 6$ and $|\beta|^2 = 1/2$ [$\rho_{22}^{(1)}(0) = \rho_{22}^{(2)}(0) = 1/2$]. The dotted black curve is $[e^{-\gamma_2(t+X_2/c)}\Theta(t + X_2/c) + e^{-\gamma_2 t}\Theta(t)]/2$, drawn for reference. It is seen that the detector initially records the field radiated by atom 2 whose wavefront arrives at $t = -X_2/c$, followed by the field from atom 1, modified by its interaction with atom 2, whose wavefront arrives at $t = 0$. For $t > 0$, the intensity at the detector no longer depends on the initial state of atom 2, since atom 2 has decayed by the time the field from atom 1 reaches atom 2.

IV. SECOND-ORDER CORRELATION FUNCTION

I now turn my attention to the second-order correlation function, which can be defined as

$$g^{(2)}(X, t, \tau) = \frac{\langle E_-(X, t)E_-(X, t + \tau)E_+(X, t + \tau)E_+(X, t)\rangle}{\langle E_-(X, t)E_+(X, t)\rangle\langle E_-(X, t + \tau)E_+(X, t + \tau)\rangle}. \quad (50)$$

For a photodetector located at position X, $g^{(2)}(X, t, \tau)$ is proportional to the joint probability of detecting one photon at time t and a second photon at time $t + \tau$. For $\tau > 0$, owing to Eq. (12), the contributions to $E_+(X, t)$ from $E_+^{(0)}(t)$ make no contributions to $g^{(2)}(X, t, \tau)$, and Eq. (50) reduces to

$$g^{(2)}(t, \tau) = \frac{\sum_{i,j,k,\ell=1}^{2}\langle\sigma_+^{(i)}(t^{(i)})\sigma_+^{(j)}(t^{(j)} + \tau)\sigma_-^{(k)}(t^{(k)} + \tau)\sigma_-^{(\ell)}(t^{(\ell)})\rangle e^{ik_0(ijk\ell)X_2}}{I_N(t)I_N(t + \tau)}, \quad (51)$$

where

$$t^{(1)} = t; \quad (52)$$

$$t^{(2)} = t + X_2/c; \quad (53)$$

$$k_0(ijk\ell) = k_0[\delta_{i,2} + \delta_{j,2} - \delta_{k,2} - \delta_{\ell,2}], \quad (54)$$

and $\delta_{i,2}$ is a Kronecker delta. Recall that t is the retarded time associated with the position of atom 1.

A time-integrated second-order correlation function can be defined by

$$g^{(2)} = \frac{\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2\langle E_-(X, t_1)E_-(X, t_2)E_+(X, t_2)E_+(X, t_1)\rangle}{\left[\int_{-\infty}^{\infty} dt\,\langle E_-(X, t)E_+(X, t)\rangle\right]^2}, \quad (55)$$

which will turn out to be independent of X and X_2, provided $X > X_2$, as is assumed. Using the fact that $g^{(2)}(X, t, -\tau) = g^{(2)}(X, t, \tau)$, it is possible to recast this equation as

$$g^{(2)} = \frac{2\gamma_2^2 \int_{-\infty}^{\infty} dt \int_0^{\infty} d\tau \sum_{i,j,k,\ell=1}^{2}\langle\sigma_+^{(i)}(t^{(i)})\sigma_+^{(j)}(t^{(j)} + \tau)\sigma_-^{(k)}(t^{(k)} + \tau)\sigma_-^{(\ell)}(t^{(\ell)})\rangle e^{ik_0(ijk\ell)X_2}}{\left[\rho_{22}^{(1)}(0) + \rho_{22}^{(2)}(0)\right]^2}. \quad (56)$$

The calculation of $g^{(2)}(t, \tau)$ reduces to an evaluation of the 16 terms in the sum,

$$M(t, \tau) = \sum_{i,j,k,\ell=1}^{2}\langle\sigma_+^{(i)}(t^{(i)})\sigma_+^{(j)}(t^{(j)} + \tau)\sigma_-^{(k)}(t^{(k)} + \tau)\sigma_-^{(\ell)}(t^{(\ell)})\rangle. \quad (57)$$

Using relationships of the type,

$$\sigma_+^{(i)}(t^{(i)})\sigma_-^{(i)}(t^{(i)}) = \sigma_{22}^{(i)}(t^{(i)}); \quad (58)$$

$$\sigma_{22}^{(i)}(t^{(i)})\sigma_-^{(i)}(t^{(i)}) = \sigma_+^{(i)}(t^{(i)})\sigma_{22}^{(i)}(t^{(i)}) = 0, \quad (59)$$

one can show that there are only nine nonvanishing terms in the sum, which can be rewritten as

$$M(t, \tau) = \sum_{j=1}^{9} A_j(t, \tau), \quad (60)$$

where

$$A_1(t,\tau) = \left\langle \sigma_+^{(2)}(t + X_2/c)\sigma_{22}^{(1)}(t+\tau)\sigma_-^{(2)}(t + X_2/c)\right\rangle; \tag{61a}$$

$$A_2(t,\tau) = \left\langle \sigma_+^{(1)}(t)\sigma_{22}^{(2)}(t+\tau + X_2/c)\sigma_-^{(1)}(t)\right\rangle; \tag{61b}$$

$$A_3(t,\tau) = \left\langle \sigma_+^{(1)}(t)\sigma_{22}^{(2)}(t+\tau + X_2/c)\sigma_-^{(2)}(t + X_2/c)\right\rangle e^{-ik_0X_2}; \tag{61c}$$

$$A_4(t,\tau) = \left\langle \sigma_+^{(2)}(t + X_2/c)\sigma_{22}^{(2)}(t+\tau + X_2/c)\sigma_-^{(1)}(t)\right\rangle e^{ik_0X_2} = A_3^*(t,\tau); \tag{61d}$$

$$A_5(t,\tau) = \left\langle \sigma_+^{(2)}(t + X_2/c)\sigma_{22}^{(2)}(t+\tau + X_2/c)\sigma_-^{(2)}(t + X_2/c)\right\rangle; \tag{61e}$$

$$A_6(t,\tau) = \langle \sigma_+^{(1)}(t)\sigma_+^{(2)}(t+\tau + X_2/c)\sigma_-^{(1)}(t+\tau)\sigma_-^{(2)}(t + X_2/c)\rangle; \tag{61f}$$

$$A_7(t,\tau) = \langle \sigma_+^{(2)}(t + X_2/c)\sigma_+^{(2)}(t+\tau + X_2/c)\sigma_-^{(1)}(t+\tau)\sigma_-^{(2)}(t + X_2/c)\rangle e^{ik_0X_2}; \tag{61g}$$

$$A_8(t,\tau) = \langle \sigma_+^{(2)}(t + X_2/c)\sigma_+^{(1)}(t+\tau)\sigma_-^{(2)}(t+\tau + X_2/c)\sigma_-^{(1)}(t)\rangle = A_6^*(t,\tau); \tag{61h}$$

$$A_9(t,\tau) = \langle \sigma_+^{(2)}(t + X_2/c)\sigma_+^{(1)}(t+\tau)\sigma_-^{(2)}(t+\tau + X_2/c)\sigma_-^{(2)}(t + X_2/c)\rangle e^{-ik_0X_2} = A_7^*(t,\tau). \tag{61i}$$

In these expressions, all the arguments of the operators must be positive.

Using Eqs. (12), (11), and (19), I find that, for $\tau > 0$, the $A_j(t,\tau)$ satisfy the differential equations:

$$\frac{\partial A_1(t,\tau)}{\partial\tau} = -\gamma_2 A_1(t,\tau)\Theta(t+\tau); \tag{62a}$$

$$\frac{\partial A_2(t,\tau)}{\partial\tau} = -\gamma_2 A_2(t,\tau); \tag{62b}$$

$$\frac{\partial A_3(t,\tau)}{\partial\tau} = -\gamma_2 A_3(t,\tau) - \gamma_2 A_6(t,\tau); \tag{62c}$$

$$\frac{\partial A_5(t,\tau)}{\partial\tau} = -\gamma_2 A_5(t,\tau) - \gamma_2\big[A_7(t,\tau) + A_7^*(t,\tau)\big]; \tag{62d}$$

$$\frac{\partial A_6(t,\tau)}{\partial\tau} = -\gamma_2 A_6(t,\tau); \tag{62e}$$

$$\frac{\partial A_7(t,\tau)}{\partial\tau} = -\frac{\gamma_2}{2}[\Theta(t+\tau+X_2/c) + \Theta(t+\tau)]A_7(t,\tau) - \gamma_2 A_1(t,\tau) + 2\gamma_2 B(t,\tau); \tag{62f}$$

$$\frac{\partial B(t,\tau)}{\partial\tau} = -\gamma_2[\Theta(t+\tau+X_2/c) + \Theta(t+\tau)]B(t,\tau), \tag{62g}$$

$$A_4(t,\tau) = A_3^*(t,\tau);\quad A_8(t,\tau) = A_6^*(t,\tau);\quad A_9(t,\tau) = A_7^*(t,\tau), \tag{62h}$$

where

$$B(t,\tau) = \left\langle \sigma_+^{(2)}(t + X_2/c)\sigma_+^{(1)}(t+\tau)\sigma_{22}^{(2)}(t+\tau + X_2/c)\sigma_-^{(1)}(t+\tau)\sigma_-^{(2)}(t + X_2/c)\right\rangle. \tag{63}$$

It follows immediately from the definitions given in Eq. (61) that

$$A_3(t,0) = A_4(t,0) = A_5(t,0) = A_7(t,0) = A_9(t,0) = 0. \tag{64}$$

Moreover, using Eqs. (10a), (10b), and (12), one can show that

$$[\sigma_-^{(1)}(t), \sigma_-^{(2)}(t + X_2/c)]|\psi(0)\rangle = \langle\psi(0)|[\sigma_+^{(2)}(t + X_2/c), \sigma_+^{(1)}(t)] = 0. \tag{65}$$

This relationship holds for both positive and negative t. As a consequence, I can deduce from Eqs. (62a), (62b), (62e), (62h), and (63) that

$$A_1(t,0) = A_2(t,0) = A_6(t,0) = A_8(t,0) = \left\langle \sigma_+^{(1)}(t)\sigma_{22}^{(2)}(t + X_2/c)\sigma_-^{(1)}(t)\right\rangle \equiv F(t), \tag{66}$$

and

$$B(t,0) = 0. \tag{67}$$

With these initial conditions the solution of Eq. (62) is

$$A_1(t,\tau) = F(t)e^{-\gamma_2\tau}[\Theta(t) + \Theta(-t)\Theta(t+\tau)e^{-\gamma_2 t}]; \tag{68a}$$

$$A_2(t,\tau) = F(t)\Theta(t)e^{-\gamma_2\tau}; \tag{68b}$$

$$A_3(t,\tau) = -\gamma_2\tau F(t)\Theta(t)e^{-\gamma_2\tau}; \tag{68c}$$

$$A_5(t,\tau) = \gamma_2^2\tau^2 e^{-\gamma_2\tau}F(t)\Theta(t) + \gamma_2^2(t+\tau)^2 e^{-\gamma_2(t+\tau)}F(t)\Theta(t+\tau)\Theta(-t); \tag{68d}$$

$$A_6(t,\tau) = F(t)\Theta(t)e^{-\gamma_2\tau}; \tag{68e}$$

$$A_7(t,\tau) = -\gamma_2\tau F(t)e^{-\gamma_2\tau}\Theta(t) - \gamma_2(t+\tau)F(t)e^{-\gamma_2(t+\tau)}\Theta(t+\tau)\Theta(-t), \tag{68f}$$

provided $t \geqslant -X_2/c$. For $t < -X_2/c$ and $\tau > 0$, all terms vanish since the earliest time that radiation can reach a detector located at position X is $t = -X_2/c$.

I can obtain a differential equation for $F(t)$ using Eqs. (10b) and (10f), but some care must be taken for negative t, that is, for $-X_2/c \leqslant t < 0$. Equation (10b) is valid only for $t \geqslant 0$; for $t < 0$, it must be replaced by

$$\dot{\sigma}_-^{(1)}(t) = -i\frac{\hbar}{\mu}\left[2\sigma_{22}^{(1)}(t) - 1\right]\tilde{E}_+^{(0)}(0,t). \tag{69}$$

It then follows from Eqs. (10b), (10f), (70), and (12) that

$$dF/dt = [-(2\gamma_2 + X_2/c)\Theta(t) - (\gamma_2 + X_2/c)\Theta(-t)]F, \tag{70}$$

implying that

$$F(t) = [e^{-\gamma_2(2t+X_2/c)}\Theta(t) + e^{-\gamma_2(t+X_2/c)}\Theta(-t)]T(0), \tag{71}$$

where $T(0) = F(0)$ is given by Eq. (29).

By combining Eqs. (51)–(71), I finally arrive at

$$M(t,\tau) = T(0)e^{-\gamma_2(2t+\tau)}e^{-\gamma_2X_2/c}\Theta(t+X_2/c) \times \{(2-\gamma_2\tau)^2\Theta(t) + [1-\gamma_2(t+\tau)]^2 \times \Theta(-t)\Theta(t+\tau)\}. \tag{72}$$

The quantity $M(t,\tau)$ is proportional to the joint probability that a detector placed at position X records one count at time t and a second count at time $t+\tau$, with $\tau > 0$. It can be seen that $M(t,\tau)$ undergoes a discontinuous jump at $t = 0$. That is, for $\gamma_2 t = \pm|\epsilon|$ and $|\epsilon| \ll 1$,

$$M(|\epsilon|/\gamma_2,\tau) \sim T(0)e^{-\gamma_2\tau}e^{-\gamma_2X_2/c}(2-\gamma_2\tau)^2; \tag{73a}$$

$$M(-|\epsilon|/\gamma_2,\tau) \sim T(0)e^{-\gamma_2\tau}e^{-\gamma_2X_2/c}[1-\gamma_2\tau]^2 \times \Theta(\tau - |\epsilon|/\gamma_2). \tag{73b}$$

For $t > 0$, the count at time t can be produced by radiation from *either* atom, regardless of the value of τ. However, once t is negative, the count at time t can be produced *only* by atom 2. The value of $M(\pm|\epsilon|/\gamma_2,\tau)$ is a maximum near $\tau = 0$, but it is about four times larger for $t > 0$. Mathematically, this result can be understood from the fact that $A_2(t,\tau)$, $A_3(t,\tau)$, $A_4(t,\tau)$, $A_6(t,\tau)$, and $A_8(t,\tau)$ no longer contribute to $M(t,\tau)$ for $t < 0$.

Other quantities of physical interest are the rate of delayed coincidences $R_c(X_2,\tau)$ which, for $\tau > 0$, is defined by

$$R_c(X_2,\tau) = \gamma_2^2\int_{-\infty}^{\infty} dt\, M(t,\tau) = \frac{\gamma_2 T(0)}{4}e^{-\gamma_2(\tau+X_2/c)}(7-6\gamma_2\tau) + \frac{\gamma_2T(0)}{4}e^{-\gamma_2(\tau-X_2/c)} \times \left[1 - 2\gamma_2\tau + 2\gamma_2^2\tau^2 + 2\frac{\gamma_2X_2}{c}\left(1 - 2\gamma_2\tau + \frac{\gamma_2X_2}{c}\right)\right]\Theta(\tau - X_2/c) + \frac{\gamma_2T(0)}{4}e^{-\gamma_2(X_2/c-\tau)}\Theta(X_2/c-\tau), \tag{74}$$

the time-integrated number of coincidence counts,

$$N_c = 2\int_0^{\infty} d\tau\, R_c(\tau) = 2T(0),$$

the second-order correlation function,

$$g^{(2)}(t,\tau) = \frac{M(t,\tau)}{I_N(t)I_N(t+\tau)}, \tag{75}$$

and the time-integrated second-order correlation function,

$$g^{(2)} = \frac{2P_{12}}{(P_1+P_2)^2} = \frac{2T(0)}{\left[\rho_{22}^{(1)}(0) + \rho_{22}^{(2)}(0)\right]^2}, \tag{76}$$

with

$$P_{12} = N_c/2 = T(0); \quad P_1 = \rho_{22}^{(1)}(0); \quad P_2 = \rho_{22}^{(2)}(0). \tag{77}$$

The time-integrated second-order correlation function is twice the number of pairs of excitations (P_{12}) divided by the square of the number of excitations. If there are exactly two excitations in the system, that is, when $\rho_{22}^{(1)}(0) = \rho_{22}^{(2)}(0) = T(0) = 1$, then $g^{(2)} = 1/2$, the same result that would be obtained for a two-photon, *single mode* field. Interestingly, for the symmetric factorized initial state with $\beta \neq 0$ [$\rho_{22}^{(1)}(0) = \rho_{22}^{(2)}(0) = |\beta|^2; T(0) = |\beta|^4$], $g^{(2)} = 1/2$ as well. For the maximally entangled initial state, $g^{(2)} = 0$, since there is only a single excitation in the system.

In Figs. 5 and 6 the dimensionless rate of delayed coincidences $\gamma_2R_c(X_2,\tau)$ is plotted as a function of $\gamma_2\tau$ for $T(0) =$

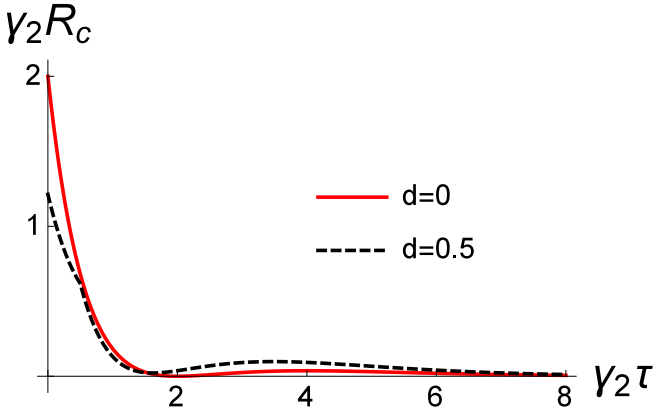

FIG. 5. Dimensionless rate of delayed coincidences $\gamma_2R_c(X_2,\tau)$ as a function of $\gamma_2\tau$ for $T(0) = 1$ (both atoms inverted) and $d = \gamma_2X_2/c = 0, 0.5$.

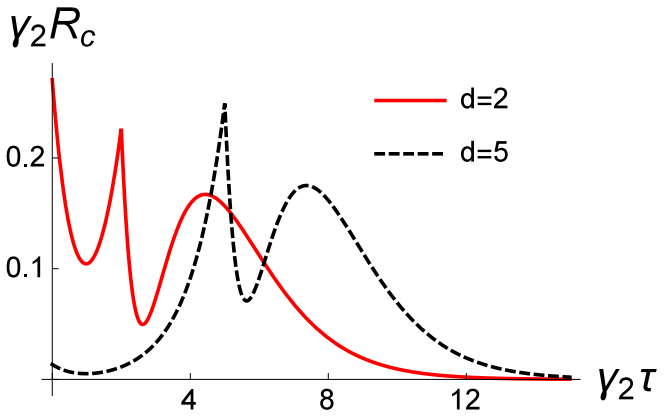

FIG. 6. Dimensionless rate of delayed coincidences $\gamma_2 R_c(X_2,\tau)$ as a function of $\gamma_2\tau$ for $T(0)=1$ (both atoms inverted) and $d=\gamma_2 X_2/c=2,5$.

1 and $d=\{0,0.5\}$ and $\{2,5\}$, respectively. Note that for $X_2=0$,

$$\gamma_2 R_c(0,\tau)=\frac{T(0)}{2}e^{-\gamma_2\tau}(2-\gamma_2\tau)^2 \tag{78}$$

is equal to 2 at $\gamma_2\tau=0$, vanishes identically for $\gamma_2\tau=2$, and has a secondary maximum before decaying away. When $X_2\neq 0$ and $d>1$, there is a local maximum at $\gamma_2\tau=d$ and a secondary maximum at $\gamma_2\tau\approx d+(3+\sqrt{3})/2$. If $X_2\neq 0$, the slope is always discontinuous at $\tau=X_2/c$; it varies from $-3T(0)$ to $-4T(0)$ if $d\ll 1$ and from $(1/4)T(0)$ to $(-3/4)T(0)$ if $d\gg 1$.

I restrict the discussion of the second-order correlation function to the symmetric factorized state with $\beta\neq 0$. For this initial state, I denote the second-order correlation function by

$$g_1^{(2)}(t,\tau)=\frac{M_1(t,\tau)}{I_{N1}(t)I_{N1}(t+\tau)}, \tag{79}$$

where

$$\begin{aligned}M_1(t,\tau)&=|\beta|^4 e^{-\gamma_2(2t+\tau)}e^{-\gamma_2 X_2/c}\Theta(t+X_2/c)\\&\quad\times[(2-\gamma_2\tau)^2\Theta(t)+[1-\gamma_2(t+\tau)]^2\Theta(-t)\Theta(t+\tau)],\end{aligned} \tag{80}$$

and $I_{N1}(t)$ is given by Eq. (34). Moreover, I consider only the two limiting cases, $\gamma_2 X_2/c\ll 1$ and $\gamma_2 X_2/c\gg 1$.

Recall that $T(0)=|\beta|^4$ for the symmetric factorized state. For classical fields, $g^{(2)}_{\rm classical}(t,\tau)=1$. For the two-atom system under consideration, $g_1^{(2)}(t,\tau)$ can vary between 0 and ∞ and can exhibit very different behavior as τ is scanned for different t. That is, both photon bunching and anti-bunching can occur, depending on the values of $\gamma_2\tau$ and $\theta=k_0X_2$. Only a few representative plots are given. For any value of γ_2X_2/c, $g_1^{(2)}(t\geqslant 0,\tau=2\gamma_2^{-1})=0$ and $g_1^{(2)}(t<0,\tau=-t+\gamma_2^{-1})=0$, nonclassical values resulting from intensity-intensity interference.

A. $d=\gamma_2X_2/c\ll 1$

Since $t\geqslant -X_2/c$ and $\gamma_2X_2/c\ll 1$, it is sufficient in this subsection to take $t\geqslant 0$. For the symmetric factorized initial state and $d\ll 1$,

$$g_1^{(2)}(t,\tau)\sim\frac{(2-\gamma_2\tau)^2}{W(\gamma_2 t)W[\gamma_2(t+\tau)]}, \tag{81}$$

where

$$\begin{aligned}W(x)&=1+(1-x)^2+2|\alpha|^2(1-x)\cos\theta\\&\quad+4|\beta|^2(-2e^{-x}+2-x).\end{aligned} \tag{82}$$

As a function of $\gamma_2 t$, the correlation function can become large if $\gamma_2\tau=0,\infty$, as $W(\gamma_2 t)$ goes through a minimum. For the symmetric factorized state with $|\beta|^2\ll 1$, this can occur at $\gamma_2 t=0$ or 2. Explicitly,

$$g_1^{(2)}(0,0)\sim\frac{1}{[1+|\alpha|^2\cos\theta]^2}; \tag{83}$$

$$g_1^{(2)}(0,\infty)\sim\frac{1}{2[1+|\alpha|^2\cos\theta]}, \tag{84}$$

$$g_1^{(2)}(2/\gamma_2,0)\sim\frac{1}{[1-|\alpha|^2\cos\theta-4|\beta|^2e^{-2}]^2}; \tag{85}$$

$$g_1^{(2)}(2/\gamma_2,\infty)\sim\frac{1}{2[1-|\alpha|^2\cos\theta-4|\beta|^2e^{-2}]}. \tag{86}$$

For $\theta=(2n+1)\pi$ (integer n), $g_1^{(2)}(0,0)\sim|\beta|^{-4}$, $g_1^{(2)}(0,\infty)\sim|\beta|^{-2}/2$, while for $\theta=2n\pi$, $g_1^{(2)}(2/\gamma_2,0)\sim 4.75|\beta|^{-4}$, $g_1^{(2)}(2/\gamma_2,\infty)\sim 1.09|\beta|^{-2}$. If both atoms are inverted initially,

$$g_1^{(2)}(t,\tau;\beta=1)\sim\frac{(2-\gamma_2\tau)^2}{\left[-8e^{-\gamma_2 t}+10-6\gamma_2 t+\gamma_2^2t^2\right]\left[-8e^{-\gamma_2(t+\tau)}+10-6\gamma_2(t+\tau)+\gamma_2^2(t+\tau)^2\right]} \tag{87}$$

and $g_1^{(2)}(t,0;\beta=1)$ reaches a maximum value of 13.2 for $\gamma_2 t=2.74$, with $g_1^{(2)}(t=2.74/\gamma_2,\infty;\beta=1)\sim 1.81$.

In Fig. 7, $g_1^{(2)}(t,\tau)$ is plotted as a function of $d=\gamma_2X_2/c\ll 1$ for $\theta=(2n+1)\pi$, $|\beta|^2=0.4$, and $\gamma_2 t=0$ (solid red curve) and $\gamma_2 t=2$ (dashed blue curve). Analogous curves are shown in Fig. 8 for $\theta=2n\pi$. The curves in Fig. 9 are drawn for $\beta=1$ and $\gamma_2 t=2,2.74$. Asymptotic values are indicated by dotted black lines.

B. $d=\gamma_2X_2/c\gg 1$

Radiation from atom 2 reaches the detector at $t=-X_2/c$ and that from atom 1 at $t=0$. Thus, to have coincidence

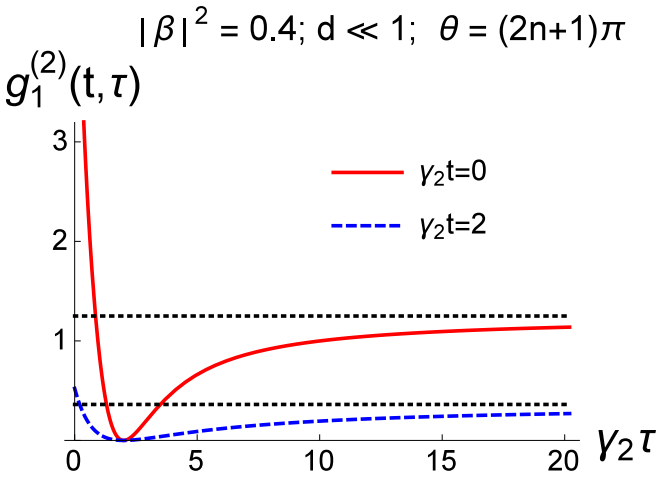

FIG. 7. Second order correlation function as a function of $\gamma_2\tau$ for a symmetric factorized initial state and $|\beta|^2 = 0.4$, $d \ll 1$, and $\theta = k_0X_2 = (2n+1)\pi$. The dotted black lines are the theoretical asymptotes.

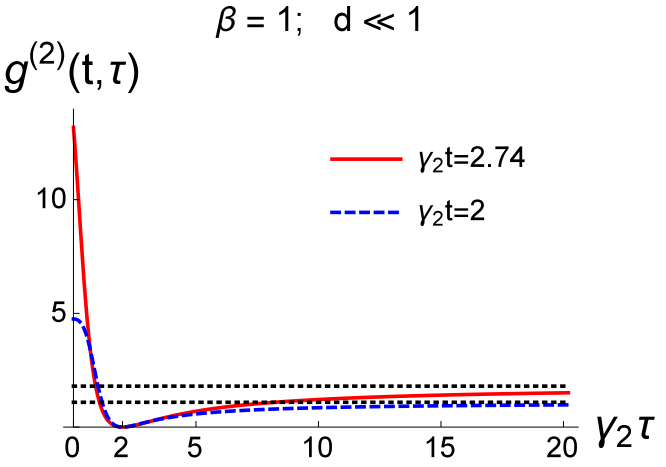

FIG. 9. Second order correlation function as a function of $\gamma_2\tau$ for a symmetric factorized initial state with $\beta = 1$ and $d \ll 1$. The dotted black lines are the theoretical asymptotes.

counts from the two atoms when $\gamma_2X_2/c \gg 1$, it is necessary that $t > -X_2/c$ and $\tau \gtrsim t$. If $t > 0$ and $\gamma_2X_2/c \gg 1$, then $g_1^{(2)}(t,\tau) \ll 1$, in general, so the examples that are given are restricted to $-X_2/c < t < 0$. For $-X_2/c < t < 0$,

$$g_1^{(2)}(t,\tau) = g(d,y) \sim \frac{(1-y)^2}{D(d,y)}\Theta(y), \tag{88}$$

where

$$d = \gamma_2X_2/c; \qquad y = \gamma_2(t+\tau), \tag{89}$$

and

$$D(d,y) = (1-y)^2 + 2|\alpha|^2(1-y)e^{-d/2}\cos\theta + [1+4|\beta|^2(2-y)] - 8|\beta|^2e^{-(d+y)}. \tag{90}$$

The second-order correlation function vanishes for $\tau < -t$ and at $y = \gamma_2(t+\tau) = 1$. For $-\gamma_2t < \gamma_2\tau < (-\gamma_2t+1)$, $g_1^{(2)}(t,\tau)$ rises for $\theta = (2n+1)\pi$, falls for $\theta = 2n\pi$, and is approximately constant for $\theta = (n+1/2)\pi$. For $\gamma_2\tau > (-\gamma_2t+1)$, $g_1^{(2)}(t,\tau)$ rises to an asymptotic value of unity for $\theta = (2n+1)\pi$, rises sharply and then falls to an asymptotic value of unity for $\theta = 2n\pi$, and is approximately equal to unity for $\theta = (n+1/2)\pi$. These features are illustrated in Figs. 10–12, in which $g_1^{(2)}(t,\tau)$ is plotted as a function of $\gamma_2\tau$ for $d = 7$, $\gamma_2t = -3, -6$, $\alpha = \sqrt{0.9}$, $\beta = \sqrt{0.1}$ and $\theta = (2n+1)\pi$ (Fig. 10), $\theta = 2n\pi$ (Fig. 11), and $\theta = (n+1/2)\pi$ (Fig. 12). Analogous curves for $\beta = 1$(not shown) are essentially the same as those shown in Fig. 12.

If $\gamma_2X_2/c \gg 1$ and t is close to zero, some special attention is needed. As long as $t < 0$, Eq. (88) remains valid and the dependence of $g_1^{(2)}(t,\tau)$ on $\gamma_2\tau$ mirrors that shown in Figs. 10–12. However, as soon as $t \geqslant 0$, the dependence changes dramatically since the count at time t can be produced by radiation from either atom. For $d \gg 1$ and $t = 0$, $g_1^{(2)}(0,\tau)$

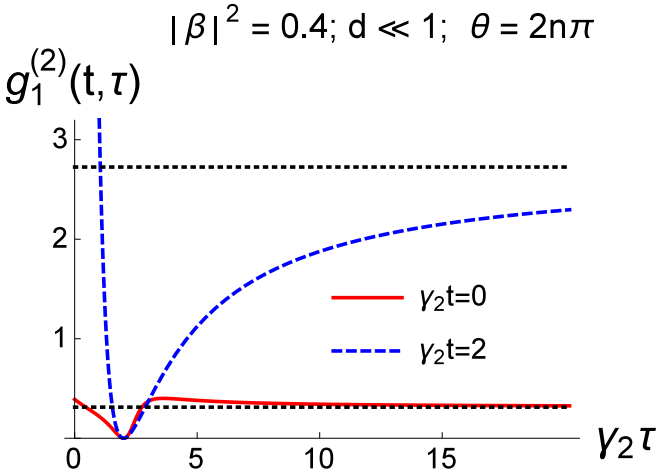

FIG. 8. Second order correlation function as a function of $\gamma_2\tau$ for a symmetric factorized initial state and $|\beta|^2 = 0.4$, $d \ll 1$, and $\theta = k_0X_2 = 2n\pi$. The dotted black lines are the theoretical asymptotes.

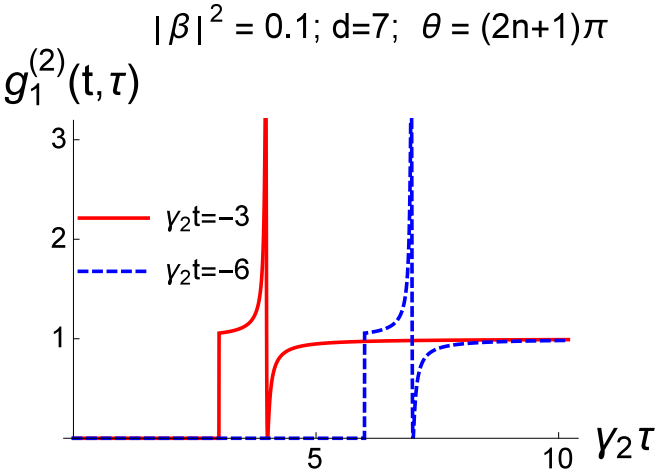

FIG. 10. Second order correlation function as a function of $\gamma_2\tau$ for a symmetric factorized initial state and $|\beta|^2 = 0.1$, $d = 7$, and $\theta = k_0X_2 = (2n+1)\pi$.

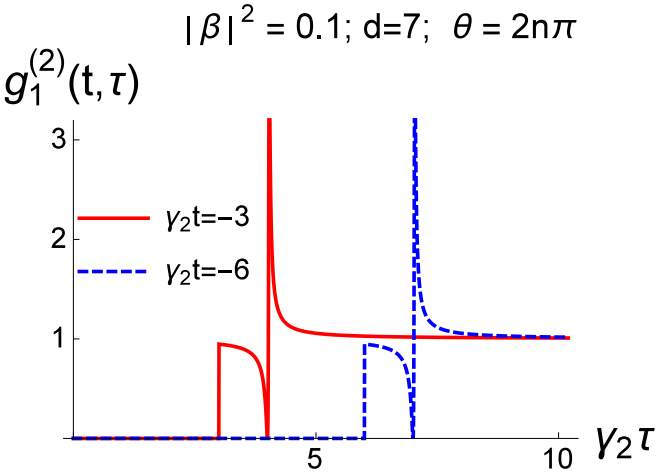

FIG. 11. Second order correlation function as a function of $\gamma_2\tau$ for a symmetric factorized initial state and $|\beta|^2 = 0.1$, $d = 7$, and $\theta = k_0 X_2 = 2n\pi$.

is nonvanishing only in a small range ϵ of $\gamma_2\tau$ about unity, for which

$$g_1^{(2)}[0,\, \tau = (1+\epsilon)/\gamma_2] \sim \frac{1}{4|\beta|^2\left(1-\frac{2}{e}\right)+1-2|\alpha|^2\epsilon e^{d/2}\cos\theta+\epsilon^2 e^d}. \tag{91}$$

That is, $g_1^{(2)}(0,\tau)$ is centered at

$$\gamma_2\tau = 1 + |\alpha|^2 e^{-d/2}\cos\theta, \tag{92}$$

has a width of order e^{-d}, and a maximum value

$$g_1^{(2)}(0,\tau)_{\max} = \frac{1}{4|\beta|^2\left(1-\frac{2}{e}\right)+1-|\alpha|^4\cos^2\theta}. \tag{93}$$

In Fig. 13, $g_1^{(2)}(0,\tau)$ is plotted as a function of $z = (\gamma_2\tau - 1)e^{d/2}/|\alpha|^2$ for $|\alpha|^2 = 0.9$, $d = 12$, and $\theta = \pi$ (solid red curve), $\theta = 2\pi$ (dashed blue curve), and $\theta = \pi/2$ (lower

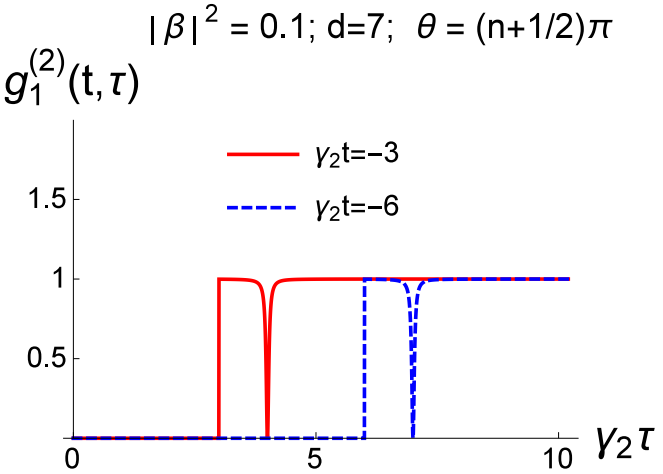

FIG. 12. Second order correlation function as a function of $\gamma_2\tau$ for a symmetric factorized initial state and $|\beta|^2 = 0.1$, $d = 7$, and $\theta = k_0 X_2 = (n+1/2)\pi$.

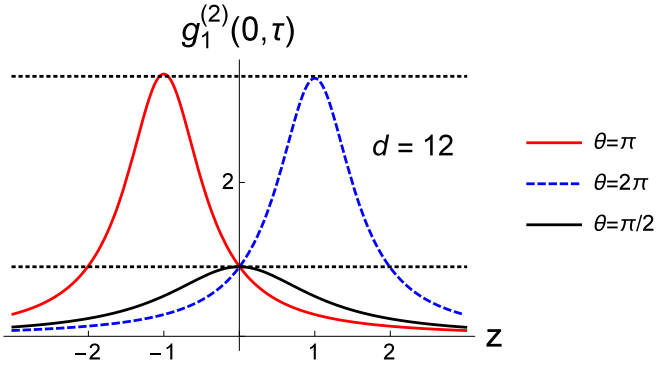

FIG. 13. Second order correlation function $g_1^{(2)}(0,\tau)$ as a function of $z = (\gamma_2\tau - 1)e^{d/2}/|\alpha|^2$ for a symmetric factorized initial state with $|\alpha|^2 = 0.9$ and $d = 12$. Dotted black lines give theoretical values for the maxima.

solid black curve), exhibiting all the features predicted above. The maximum values predicted in Eq. (93) are indicated by dotted black lines. The feature that $g_1^{(2)}(0,\tau)$ undergoes a qualitative change as t changes sign when $d \geqslant 1$ is illustrated in Fig. 14, drawn for $d = 4$, $\theta = \pi$, and $\gamma_2 t = -0.1, 0.1$.

V. CONCLUSIONS AND DISCUSSION

The problem of two atoms in a chiral waveguide has been studied in detail using a source-field approach. Analytic solutions were obtained for the field intensity and the second-order correlation function. In a chiral guide having cross-sectional area $A \gg \lambda^2$, the pulse area of the field radiated by each atom is much less than unity. As a consequence, when $\gamma_2 X_2/c \gg 1$, atom 2 does not undergo Rabi oscillations as it scatters the field from atom 1. Instead, it modifies this output field from atom 1, with total destructive interference in the output intensity at a time $t = \gamma_2^{-1}$ following the arrival of the wavefront at atom 2. When $\gamma_2 X_2/c \ll 1$ and both atoms are initially inverted, there is stimulated emission, but not into the same spatio-temporal modes as the input field produced

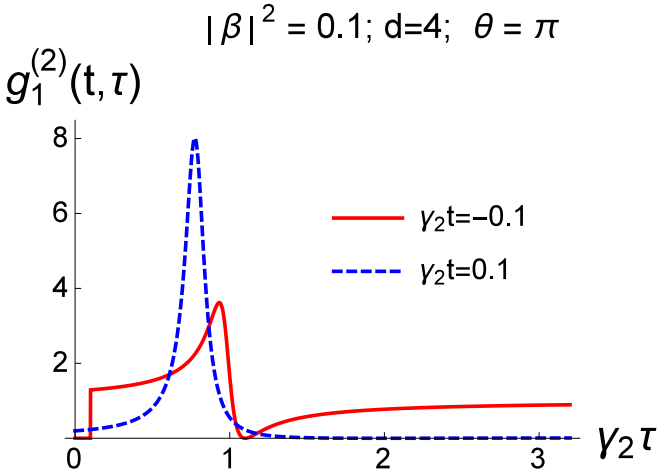

FIG. 14. Second order correlation function as a function of $\gamma_2\tau$ for a symmetric factorized initial state and $|\beta|^2 = 0.1$, $d = 4$, and $\theta = k_0 X_2 = \pi$, showing the change as t changes sign.

by atom 1. The second-order correlation function can take on nonclassical values ranging from zero to arbitrarily high limits, depending on the initial conditions. In the Appendix, it is shown how the results can be taken over to provide a solution for the related problem of a single-photon pulse incident on an atom prepared in an arbitrary initial state.

Admittedly, the solutions presented in this paper correspond to an idealized situation. Aside from the difficulty in designing chiral guides, preparing atoms in the desired initial state, and measuring the output field, there are always some losses present. The losses can be accounted for in a phenomenological manner in Eqs. (10) and (16) by assuming that the amplitude of the source field decays exponentially as it propagates down the guide. The effect of losses is to reduce both the output field intensity and the coupling between the atoms. As a consequence, when $\rho_{22}^{(1)}(0) = \rho_{22}^{(2)}(0) = T(0) = 1$, the output field is no longer a two-photon field and $g^{(2)} < 1/2$. It is not difficult to generalize Eqs. (10) and (16) to allow for additional atoms in the guide. However, the solution of the equations becomes increasingly complicated as the number of atoms increases. In effect, you need to first solve the operator equations for the first two atoms, then use this solution to obtain the operator solution for three atoms, and so forth.

If retardation is neglected and if all the atoms are prepared in their excited states, the problem corresponds to a unique type of superradiance, in which the atom-atom coupling induced by the vacuum field is independent of interatomic separation and does not result in a shift of the atomic energy levels, as in conventional superradiance. As we have seen [Eqs. (47) and (48)], even for two atoms separated by less than a wavelength, the total intensity radiated in the chiral guide differs from that of atoms in free space (two-atom superradiance). In effect, the chiral nature of the guide leads to coupling of Dicke states [10] that are *not* coupled for atoms in free space (states having the same m but different r in Dicke's notation). For example, in the two-atom case, there are four Dicke states,

$$|E\rangle = |r = 1, m = 1\rangle = |22\rangle; \tag{94a}$$

$$|S\rangle = |r = 1, m = 0\rangle = \frac{1}{\sqrt{2}}(|21\rangle + |12\rangle); \tag{94b}$$

$$|G\rangle = |r = 1, m = -1\rangle = |11\rangle; \tag{94c}$$

$$|A\rangle = |r = 0, m = 0\rangle = \frac{1}{\sqrt{2}}(|21\rangle - |12\rangle). \tag{94d}$$

Assuming the atoms are separated by much less than a wavelength and the atoms are prepared in state $|E\rangle$, the antisymmetric state $|A\rangle$ is not coupled to any other state for atoms in free space. However, in a chiral guide, while state $|A\rangle$ is still not coupled to states $|E\rangle$ and $|G\rangle$, it *is* coupled to state $|S\rangle$ and becomes populated as the two-atom system decays.

ACKNOWLEDGMENTS

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APPENDIX: SINGLE-PHOTON PULSE + ATOM

An analytic solution to the problem of a single-photon pulse interacting with an atom in a chiral guide was given by Valente *et al.* [6] by obtaining and solving a partial differential equation for field propagation in the guide. In this Appendix, I show that the source-field approach can be used as an alternative method for arriving at expressions for the field intensity and second-order correlation function for the atom-field problem by simply taking over the results of the main text for the atom-atom problem. It is not difficult to understand why the atom-atom results can be mapped into the atom-field results, since the atom-atom problem, in the limit that $\rho_{22}^{(1)}(0) = 1$, corresponds to a single-photon pulse (albeit a specific one) incident on an atom. In fact, this is the situation envisioned by Valente *et al.* in formulating their problem.

I take the atom located at $X = 0$ and the wavefront of the single-photon pulse located at $-ct_0 \leqslant 0$ at time $t = 0$. For an arbitrary single-photon input pulse, the initial state vector for the system is written as

$$|\psi(0)\rangle = |i\rangle_A \int_{-\infty}^{\infty} dk\, b(k)|k\rangle, \tag{A1}$$

where $|i\rangle_A$ is the initial atomic state vector, $b(k)$ is a field state amplitude, and $|k\rangle$ is a single-photon state of the field. I have assumed I can extend the k integral to $-\infty$ with little error. I define

$$q(t) = \left(\frac{1}{2\pi c}\right)^{1/2} \int_{-\infty}^{\infty} d\omega\, b(\omega/c) e^{-i(\omega-\omega_0)t}. \tag{A2}$$

The quantity $|q(t)|^2$ is proportional to the pulse intensity at $X = 0$ at time t; since the wavefront of the pulse arrives at the origin at $t = t_0$, $|q(t)|^2 = 0$ for $t < t_0$.

The calculation of the main text can then be used by setting $X_2 = 0$, $\rho_{22}^{(1)}(0) = 1$ and replacing $\sigma_-^{(1)}(t)$ with $q(t)\sigma_-^{(1)}(0)/\sqrt{\gamma_2}$. In this manner, I find

$$I(t) = \hbar\omega_0\gamma_2 I_N(t), \tag{A3}$$

where

$$I_N(t) = \left[\frac{|q(t)|^2}{\gamma_2} + Q(t) + Q^*(t) + \rho_{22}(t)\right]\Theta(t), \tag{A4}$$

where

$$\dot{Q}(t) = -\frac{\gamma_2}{2}Q(t) - \frac{dq/dt}{q(t)}Q(t) + |q(t')|^2[2e^{-\gamma_2 t'}\rho_{22}(0) - 1]; \tag{A5a}$$

$$\rho_{22}(t) = \rho_{22}(0)e^{-\gamma_2 t} - \gamma_2 \int_0^t dt'[Q(t') + Q^*(t')]e^{-\gamma_2(t-t')}. \tag{A5b}$$

Equation (A5a) can be solved formally as

$$Q(t) = \int_0^t dt' e^{-\frac{\gamma_2}{2}(t-t')} q(t)q^*(t')[2e^{-\gamma_2 t'}\rho_{22}(0) - 1]. \tag{A6}$$

As in the main text, all times in these equations are retarded times relative to the origin.

First consider the case where $\rho_{22}(0) = 0$ and

$$q(t) = q_1(t) = \sqrt{\gamma_2} e^{-\gamma(T-t)/2}\Theta(T - t), \tag{A7}$$

with $\gamma T \gg 1$ [5]. For this pulse, the intensity rises exponentially at the origin until time $t = T$, and then falls instantaneously to zero. The time constant of the intensity matches the excited state lifetime of the atom. In this case,

$$Q(t) = e^{-\gamma(t+T)}[1 - e^{\gamma_2 T} + (e^{\gamma_2 t} - e^{\gamma_2 T})\Theta(T-t)]; \quad \text{(A8a)}$$

$$\rho_{22}(t) = e^{-\gamma_2(t+T)}[(1 - e^{\gamma_2 T})^2 + (e^{\gamma_2 t} - e^{\gamma_2 T}) \times (-2 + e^{\gamma_2 t} + e^{\gamma_2 T})\Theta(T-t)], \quad \text{(A8b)}$$

and

$$\lim_{\gamma_2 T \gg 1} I_N(t) = e^{-\gamma_2(t-T)}\Theta(t-T). \quad \text{(A9)}$$

At $t = T$, $\rho_{22}(T) = 1$; the system is completely inverted when the tail end of the pulse reaches the origin [5]. For this result to be consistent with energy conservation, $I_N(t)$ must vanish identically for $t < T$, as it does [see Eq. (A9)]. The contribution to the radiated intensity from the first term in Eq. (A4) and that part of $\rho_{22}(t)$ associated with the first term in Eq. (A8b), is exactly canceled by the interference terms. In other words, as the pulse arrives at the atom, the interference of the incident and scattered radiation is totally destructive for times $t < T$. For $t > T$, the atom simply decays.

Next consider an atom prepared in its excited state [$\rho_{22}(0) = 1$] and an exponentially falling input pulse,

$$q(t) = q_2(t) = \sqrt{\Gamma_p}e^{-\Gamma_p t/2}\Theta(t), \quad \text{(A10)}$$

whose wavefront coincides with the atom at $t = 0$, for which

$$Q(t) = 2\Theta(t)\Gamma_p\left[\frac{e^{-\Gamma_p t} - e^{-(\gamma_2+\Gamma_p)t/2}}{\Gamma_p - \gamma_2} + 2e^{-(\gamma_2+\Gamma_p)t/2}\frac{1 - e^{-(\gamma_2+\Gamma_p)t/2}}{\gamma_2 + \Gamma_p}\right]; \quad \text{(A11a)}$$

$$\rho_{22}(t) = e^{-\gamma_2 t}\Theta(t) + \frac{4\gamma_2 e^{-(\gamma_2+\Gamma_p)t}K(t)\Theta(t)}{(\gamma_2 - \Gamma_p)^2(\gamma_2 + \Gamma_p)}, \quad \text{(A11b)}$$

and

$$I_N(t) = \frac{e^{-(\gamma_2+\Gamma_p)t}}{\gamma_2(\gamma_2 - \Gamma_p)^2}J(t)\Theta(t), \quad \text{(A12)}$$

where

$$K(t) = 2(\gamma_2)^2(-1 + e^{\Gamma_p t}) + \gamma_2\Gamma_p(4 + e^{\gamma_2 t} + e^{\Gamma_p t} - 6e^{(\gamma_2+\Gamma_p)t/2}) + (\Gamma_p)^2(-2 + e^{\gamma_2 t} - e^{\Gamma_p t} + 2e^{(\gamma_2+\Gamma_p)t/2}), \quad \text{(A13)}$$

and

$$J(t) = (\gamma_2)^3(-8 + 9e^{\Gamma_p t}) + (\gamma_2)^2\Gamma_p(16 + e^{\gamma_2 t} - 6e^{\Gamma_p t} - 12e^{(\gamma_2+\Gamma_p)t/2}) + \gamma_2(\Gamma_p)^2(-8 + 2e^{\gamma_2 t} + e^{\Gamma_p t} + 4e^{(\gamma_2+\Gamma_p)t/2}) + (\Gamma_p)^3 e^{\gamma_2 t}. \quad \text{(A14)}$$

The condition $\Gamma_p = 3\gamma_2$ is the optimal one for stimulated emission [6]. In this limit,

$$I_N(t) = 4(3e^{-3\gamma_2 t} - 2e^{-4\gamma_2 t})\Theta(t). \quad \text{(A15)}$$

Although the emission rate is increased, the temporal shape of the output pulse is *not* the same as the input pulse. If $\Gamma_p = \gamma_2$,

$$I_N(t) = e^{-\gamma_2 t}\left[10 - 6\gamma_2 t + (\gamma_2 t)^2 - 8e^{-\gamma_2 t}\right]\Theta(t), \quad \text{(A16)}$$

and we recover Eq. (48). For arbitrary $q(t)$,

$$\gamma_2\int_0^\infty I_N(t)dt = [1 + \rho_{22}(0)]; \quad \int_0^\infty I(t)dt = \hbar\omega_0[1 + \rho_{22}(0)]; \quad \text{(A17)}$$

the integrated intensity always corresponds to $\hbar\omega_0$ of energy from the field and $\hbar\omega_0\rho_{22}(0)$ of energy from the atom.

The second-order correlation function is given by

$$g^{(2)}(t,\tau) = \frac{C(t,\tau)}{I_N(t)I_N(t+\tau)}, \quad \text{(A18)}$$

with

$$C(t,\tau) = \rho_{22}(0)e^{-\gamma_2 t}\Theta(t)\sum_{j=1}^{9} C_j(t,\tau), \quad \text{(A19)}$$

and

$$C_1(t,\tau) = \frac{|q(t+\tau)|^2}{\gamma_2}; \quad \text{(A20a)}$$

$$C_2(t,\tau) = \frac{|q(t)|^2}{\gamma_2}e^{-\gamma_2\tau}; \quad \text{(A20b)}$$

$$C_3(t,\tau) = -q^*(t)\int_0^\tau d\tau' e^{-\gamma_2(\tau-\tau')}q(t+\tau')e^{-\gamma_2\tau'/2} = C_4^*(t,\tau); \quad \text{(A20c)}$$

$$C_6(t,\tau) = \frac{q^*(t)q(t+\tau)}{\gamma_2}e^{-\gamma_2\tau/2} = C_8^*(t,\tau), \quad \text{(A20d)}$$

with

$$\frac{\partial C_5(t,\tau)}{\partial\tau} = -\gamma_2 C_5(t,\tau) - \gamma_2[C_7(t,\tau) + C_7^*(t,\tau)]; \quad \text{(A21a)}$$

$$\frac{\partial C_7(t,\tau)}{\partial\tau} = -\frac{\gamma_2}{2}C_7(t,\tau) - \frac{\partial q(t+\tau)/\partial t}{q(t+\tau)}C_7(t,\tau) - \gamma_2 C_1(t,\tau), \quad \text{(A21b)}$$

from which it follows that

$$C_7(t,\tau) = -q(t+\tau)\int_0^\tau d\tau' \exp\left[-\frac{\gamma_2}{2}(\tau-\tau')\right] \times q^*(t+\tau'), \tag{A22a}$$

$$C_5(t,\tau) = -\gamma_2\int_0^\tau d\tau' e^{-\gamma_2(\tau-\tau')} \times [C_7(t,\tau') + C_7^*(t,\tau')]. \tag{A22b}$$

Given $q(t)$, it is now possible to use Eqs. (A3)–(A22) to calculate $C(t,\tau)$ and $g^{(2)}(t,\tau)$. Note that

$$(\gamma_2)^2\int_{-\infty}^{\infty} dt\int_0^\infty d\tau\, C(t,\tau) = \rho_{22}(0) \tag{A23}$$

for any $q(t)$.

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