Quantum correlation protection through spin self-rephasing in a one-dimensional Bose gas

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A system consisting of a number of trapped two-level atoms in the presence of external inhomogeneous magnetic field undergoes dephasing: classically, since each atom feels a different field along its trajectory, the pseudospin rotation rates differ; as a result the average spin decays. It has been demonstrated in recent years that in the case of systems initiated in the atomic coherent state such spin dephasing can be prevented by tuning the interaction between the atoms to induce so-called spin self-rephasing. While the effect has been studied theoretically from a semiclassical point of view, a quantum-mechanical description is limited. In this paper we provide a numerical simulation of an *ab initio* model and provide realistic examples of spin self-rephasing used to counteract the effect of inhomogeneous magnetic field. We found that the spin self-rephasing method can be also used to prevent atomic squeezed states from decoherence: while the dephasing inhibition is lower as compared with the case of coherent states, it is still comparatively large and may be useful in real interferometric scenarios.

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One of the fundamental limitations of precise measurements is noise introduced by measurement statistics: if one tries to estimate the expectation value of a classical random variable ϕ, the variance of an estimator diminishes with the number of independent experiments N as square root, $\Delta\langle\phi\rangle \propto N^{-1/2}$. Quantum information is highly successful in utilizing entangled states to beat this kind of limit [1]. Gravitational wave detectors soon are going to utilize the squeezed state of light to capture the tiny signal coming from the collision of massive astronomical objects; squeezed states were also used in the prototypes of magnetometers, capable of scanning magnetic-field fluctuations varying on the micrometer scale [2].

In principle, the system of N correlated subsystems, e.g., qubits, may be used to reduce the statistical uncertainty down to the so-called Heisenberg limit, $\Delta\langle\phi\rangle \propto N^{-1}$ [3]. The limit can be saturated by initiating the particles in the interferometer in a Schrödinger cat state [4]. However, the strongly correlated states are very fragile and due to their sensitivity to external disturbances even a small amount of noise can completely destroy the quantum correlations, making their applications questionable. In this respect there are very discouraging theoretical results showing that in the presence of any noise the precision scales asymptotically like $N^{-1/2}$ with at most constant factor improvement [5]. The larger the noise, the shorter the duration at which the highly entangled state can be of importance.

On the other hand, it has been demonstrated in experiments [6,7] that effects of certain noises can be strongly suppressed, by so-called spin self-rephasing—a subtle effect imposed by the quantum statistics and short-range interactions [8]. It has been observed that for atoms initiated in an uncorrelated classical-like state the spin self-rephasing increases coherence time by orders of magnitude. The effect was specifically used to overcome the noise coming from uncontrolled residual magnetic field and random shifts due to fluctuations of the total number of atoms [9].

The natural question arises whether these techniques can be applied to the correlated atomic sources, useful in the context of interferometry. In general, the question is very difficult, as one has to account for continuous degrees of freedom, typically omitted in theoretical treatments.

Here, we search for improvements in a specific microscopic *ab initio* model with decoherence caused by the external magnetic field, trying to elucidate the role of quantum statistics in preserving quantum correlations in systems of interacting, trapped two-level atoms (qubits) coupled to a magnetic field.

The paper is organized as follows. In Sec. I we present our model of N interacting two-level atoms trapped in the harmonic trap and in the presence of the inhomogeneous magnetic field. The first results are given in Sec. II, devoted to the simplest case of the ideal gas. In this section we discuss how the quantum state loses its coherence due to the inhomogeneous magnetic field and how the process depends on quantum statistics. In Sec. III we turn to the interacting case, in which the spin self-rephasing occurs. We show how the interaction keeps the spin state completely symmetric, therefore blocking the dephasing. On the other hand, the mechanism does not ensure that the desired properties (e.g., visibility and squeezing parameter) are preserved. This leads to the central part of the paper—the study of whether the

interactions can be employed to protect entanglement. We answer this question by giving an example of the squeezed states, the squeezing of which is preserved in a certain window of interaction. Discussion of the results follows in Sec. IV.

I. SYSTEM DESCRIPTION

We study a thermal cloud of N bosonic two-level atoms trapped in a harmonic one-dimensional (1D) trap, the internal degrees of freedom of which are spanned by two hyperfine states, which we denote by $|\downarrow\rangle$ and $|\uparrow\rangle$ (in physical realizations various atoms and internal states are used). The magnetic field is assumed to be nonuniform. We focus on dynamics in which an equilibrated system, a thermal cloud of N atoms in $|\downarrow\rangle$, is suddenly initiated in a certain state of the internal degrees of freedom $|\psi_{\rm spin}\rangle$, assuming unchanged spatial degrees of freedom:

$$\hat{\rho}(t=0) = \hat{\rho}_{\rm thermal} \otimes |\psi_{\rm spin}\rangle \langle\psi_{\rm spin}|\,, \tag{1}$$

where $\hat{\rho}_{\rm thermal}$ is the initial spatially thermal state [10]. Such a family of states covers the typical ones, in which by a $\pi/2$ pulse the atoms are in the spin coherent state with $|\psi\rangle$ equal to $\otimes_{i=1}^{N}(\frac{|\downarrow\rangle_i+|\uparrow\rangle_i}{\sqrt{2}})$ or squeezed states generated by two-axis countertwisting dynamics [11]. This is the standard initial state of atomic interferometers, where the fundamental quantity is the coherence

$$C := |\vec{S}| = \sqrt{\langle\hat{S}_x\rangle^2 + \langle\hat{S}_y\rangle^2}, \tag{2}$$

useful when the dynamics takes place on the equator of the Bloch sphere ($\langle S_z\rangle = 0$), which is the case in typical scenarios. The above is expressed with the help of the collective spin projection operators:

$$\hat{S}_x := \sum_{i=1}^{N} \hat{\sigma}_{x,i}, \quad \hat{S}_y := \sum_{i=1}^{N} \hat{\sigma}_{y,i}, \quad \hat{S}_z := \sum_{i=1}^{N} \hat{\sigma}_{z,i}. \tag{3}$$

The dynamics is generated by the Hamiltonian:

$$\hat{H} = \sum_{i=1}^{N} \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + V(\hat{x}_i) + \hat{H}_{\rm B}^{(i)} \right) + \sum_{i<j} \hat{V}_{\rm int}^{(i,j)}. \tag{4}$$

The first sum in the Hamiltonian (4) contains a sum of the single-particle contributions, with a harmonic external potential $V(\hat{x}) = \frac{1}{2}m\omega^2\hat{x}^2$ and a nonuniform magnetic field. We will show results for the case $B(x) = b_0 + b_1 x + b_2 x^2$ polarized along the Z direction. Therefore, the operator $\hat{H}_{\rm B}$ expressing the energy in magnetic field takes the form

$$\hat{H}_{\rm B}^{(i)} := \Delta\mu\, B(\hat{x}_i)\, \hat{\sigma}_{z,i}, \tag{5}$$

where the coefficient $\Delta\mu$ has the meaning of the differential magnetic moment between the states $|\downarrow\rangle$ and $|\uparrow\rangle$. The second sum in (4) expresses the energy arising from the interaction between atoms. We focus on the short-range interactions modeled by delta functions. There are three channels of such interaction, depending on the internal levels of the interacting atoms. Therefore, the interaction potential

$$\hat{V}_{\rm int}^{(i,j)} := \delta(\hat{x}_i - \hat{x}_j)\big(g_{00}\hat{P}_{i,j}^{00} + g_{11}\hat{P}_{i,j}^{11} + g_{01}\hat{P}_{i,j}^{01}\big) \tag{6}$$

depends on the projectors $\hat{P}_{i,j}^{\sigma\sigma'}$ which restrict the ith and jth atoms to the subspace with one atom in $|\sigma\rangle$ and one atom in $|\sigma'\rangle$.

The position dependent magnetic field, which results in the decay of coherence, has three contributions in our model. The uniform part $b_0\sigma_z$ only shifts the energy levels. This term alone would only rotate the whole state $|\psi(t)\rangle = e^{-i\Delta\mu\, b_0 t \hat{S}_z/\hbar}\,|\psi(0)\rangle$; since it commutes with every other part of the Hamiltonian, it may be removed by considering the state in the rotating reference frame. Because it introduces only trivial dynamics, we employ this procedure and set $b_0 = 0$.

The term $b_1 x\,\hat{\sigma}_{z,i}$ shifts the center of the trapping harmonic potential depending on the internal state. Finally $b_2 x^2\,\hat{\sigma}_{z,i}$ modifies the trap frequency ω into $\omega' := \sqrt{\omega^2 + 2\Delta\mu b_2\langle\hat{\sigma}_z\rangle}$ making it also state dependent. We restrict our consideration to small magnetic field, such that $\omega^2 \gg \Delta\mu b_2$—an often encountered scenario in the experiments.

II. NONINTERACTING CASE: ROLE OF THE QUANTUM STATISTICS

Due to the nonuniform quadratic fields one has to deal with two harmonic potentials $V_\sigma(x)$ associated with atoms in $|\sigma\rangle$, with $\sigma = \downarrow, \uparrow$. We will designate eigenstates (Hermite functions) for these two potentials as $|\phi_{n,\sigma}\rangle$. In the ideal gas case, the only effect of the linear term $\sum b_1\hat{x}\hat{\sigma}_{z,i}$ is the relative shift between the eigenstates $|\phi_{n,\uparrow}\rangle$ and $|\phi_{n,\downarrow}\rangle$. In the extreme case of strong magnetic field the two harmonic traps would be separated, and then the coherence C would vanish, just due to vanishing overlap between clouds of atoms in different internal states. For small magnetic field, being the subject of this paper, the linear term is less significant than the quadratic term $\sum b_2\hat{x}_i^2\sigma_{z,i}$, as discussed below. The quadratic term in the magnetic field is also an experimental issue [6] .

We first show in the example of one qubit how the presence of the state-dependent potentials leads to the contrast decay in time. For one particle, initially in a pure state $|\psi(x;\, t=0)\rangle\,(|\downarrow\rangle + |\uparrow\rangle)/\sqrt{2}$, dynamics can be easily evaluated. The spatial part of the state can be expanded in both bases of eigenstates (separately for $\sigma = \uparrow$ and $\downarrow$):

$$\psi(x;\, t=0) = \sum_n c_{n,\sigma}\,|\phi_{n,\sigma}\rangle\,, \tag{7}$$

which leads to a formula for the state at time t:

$$|\psi(t)\rangle = e^{-iHt/\hbar}\,|\psi(t=0)\rangle = \sum_n c_{n,\sigma}\, e^{-iE_{n,\sigma}t/\hbar}\;|\phi_{n,\sigma}\rangle\,|\sigma\rangle\,, \tag{8}$$

where $E_{n,\sigma} = \hbar\sqrt{\omega^2 + 2\,\Delta\,\mu\, b_2\,\langle\sigma|\hat{\sigma}_z|\sigma\rangle}(n+1/2)$ are the eigenstates of the modified harmonic potentials. The contrast (2) can be written as $C = |\langle S_x + iS_y\rangle|$. Let us define the term $s := \langle S_x + iS_y\rangle$; being an expectation value of the single-particle operator, it is extensible—contributions from independent atoms are simply added. Its value for a single atom in a state described by Eq. (8) is

$$s_{\rm sp} = \frac{1}{2}\sum_{n,m} c_{n,0}^* c_{m,1}\, e^{-i(E_{n,1}-E_{m,0})t/\hbar}\,\langle\phi_{n,0}|\phi_{m,1}\rangle. \tag{9}$$

We consider only weak magnetic fields. Then the phase evolution will be more important than the stationary overlaps, for

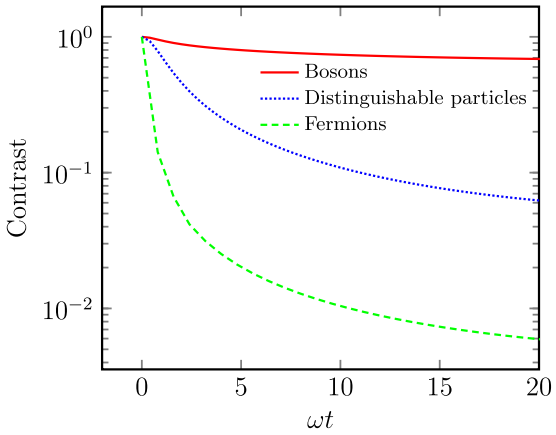

FIG. 1. Decay of the coherence (2) of ideal gas in a thermal state. The decay is due to a quadratic magnetic field with $b_2 = 0.05$ (in oscillatory units), without the linear term ($b_1 = 0$). Temperature is $k_B T = 10\hbar\omega$ and number of atoms is equal to $N = 100$ in all three considered cases.

which we assume $\langle\phi_{n,0}|\phi_{m,1}\rangle \approx \delta_{m,n}$. If the initial spatial state is the nth eigenstate of the bare harmonic potential $|n\rangle$, the value determining the constant rotates on the complex plane: $s_{\rm sp}^{|n\rangle} \approx \frac{1}{2}\exp[-i2\Delta\mu b_2 nt/(m\omega)]$. In this situation the contrast is constant—when more atoms are added, the situation is changed due to different rates of rotation.

The last formula is handy to determine the decay of the contrast. If the atom was initially in the thermal state of the bare potential, $s_{\rm sp}^{\rm th} \approx \frac{1-\xi}{1-\xi\, e^{-i\frac{2\Delta\mu b_2 t}{m\omega}}}$, where $\xi = e^{-\hbar\omega/(k_B T)}$. The computation may be applied to a thermal state of many noninteracting atoms: the composite contrast depends on single-particle contributions weighted according to the atom occupation,

$$C^{\rm th} \approx \left|\sum_{k=0}^{\infty} \frac{\langle n_k\rangle_{\rm T}}{N} s_{\rm sp}^{|k\rangle}\right|, \tag{10}$$

where $\langle n_k\rangle_{\rm T}$ is the thermal average occupation of the kth energy level.

Intuitively, the bigger the spread of atoms' energies, the bigger the differences between the phases the atoms accumulate during the evolution. By raising the gas temperature, we should observe faster decay of the contrast. An interesting situation is when we fix a temperature and increase the number of atoms. Then one observes qualitatively completely different behaviors of the contrast decays, depending on the atomic statistics one assumes. An example for ideal gas is given in Fig. 1, which shows the role of the atomic statistics—a key factor in preservation of the coherence.

In the fermionic case, the thermal state in the limit $N \to \infty$ converges to a many-body state in which each single-particle level up to the energy $N\hbar\omega$ is occupied by a single atom. The spatial and momentum distributions both widen, leading to quicker dephasing and coherence decay. In the case of bosons the reverse happens. In the limit $N \to \infty$ mostly the single-particle ground-state mode becomes occupied—a 1D reminiscence of Bose-Einstein condensation. The density is then dominated by the ground-state contribution, with all atoms being in a narrow region of space and momentum. Such gas dephases at a slower pace as compared to fermions and distinguishable particles.

III. INTERACTING CASE: SPIN SELF-REPHASING

The interacting case does not admit analytical treatment, therefore numerical simulations of the dynamics have been performed.

Due to the interaction three additional parameters do appear: the interaction strengths g_{00}, g_{01}, and g_{11} [see Eq. (6)]. To reduce the parameter space we decided for the following parametrization [12]:

$$\begin{aligned} g_{00} &= g - c, \\ g_{01} &= g, \\ g_{11} &= g + c. \end{aligned} \tag{11}$$

As a motivation for this choice, let us mention some results in the field of Bose-Einstein condensation of cold clouds of several two-level atoms. It is known that in the single mode approximation, when all atoms occupy a single common spatial orbital, the dynamics in the internal degrees of freedom is governed by the Hamiltonian $H_{\rm OAT} = \chi\,\hat{S}_z^2 + \tilde{\chi}\hat{S}_z\hat{N}$, where $\hat{N}$ is a particle number operator—a model similar to the one-axis twisting Hamiltonian known from [11]. The parameters χ and $\tilde{\chi}$, often called called "nonlinearities," in the one spatial mode approximation of the Bose-Einstein condensate are equal to

$$\chi = \frac{1}{2\hbar}\left(\frac{\partial^2 E}{\partial N_1} + \frac{\partial^2 E}{\partial N_2^2} - 2\frac{\partial^2 E}{\partial N_2 \partial N_1}\right), \tag{12}$$

$$\tilde{\chi} = \frac{1}{2\hbar}\left(\frac{\partial^2 E}{\partial N_0} - \frac{\partial^2 E}{\partial N_1^2}\right), \tag{13}$$

where E is the energy of the bimodal Bose-Einstein condensate, with N_0 atoms in one mode and N_1 atoms in the second one. The derivatives in Eqs. (12) have to be evaluated at the mean mode occupations, i.e., in the case of a half-half superposition studied in this paper, at $\langle N_0\rangle$ and $\langle N_1\rangle$ equal to $N/2$, where N is the total number of atoms.

This one-axis twisting Hamiltonian leads to nontrivial dynamics—a coherent state may become squeezed during the evolution. In the long-time regime (not yet realized in an experiment) a Schrödinger cat state—superposition of two orthogonal spin coherent states—appears [13]. This entangling dynamics would make the model more complex and it would be more difficult to draw conclusions. Therefore we want a system in which the nonlinear term $\chi\hat{S}_z^2$ is small, even though the interactions play an important role for spin self-rephasing. In the aforementioned approximation the parameter by $\hat{S}_z^2$ vanishes when interaction strengths are parametrized as in Eq. (11): $\chi \propto (g_{00} + g_{11} - 2g_{01})$. Avoidance of the nonlinearities is the primary cause of this parametrization.

In real experiment the system is quasi-1D—the gas is trapped in the strongly elongated harmonic trap. The corresponding effective 1D model has the coupling strength g equal to $2\hbar\omega_\perp a$, where $\omega_\perp$ is the high frequency of the

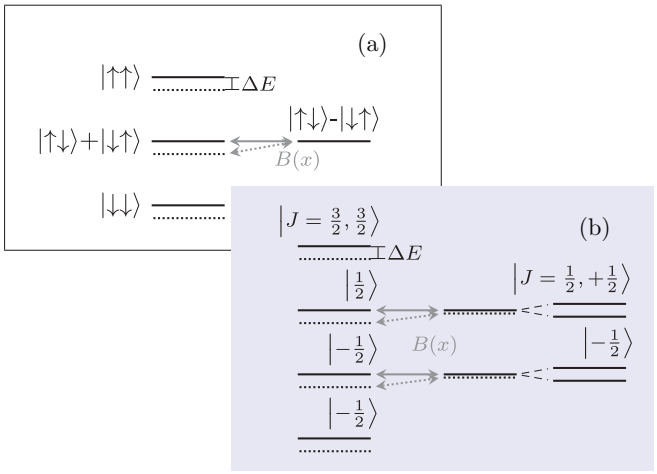

FIG. 2. Energy diagram illustrating the dephasing and the spin self-rephasing mechanisms for (a) two and (b) three bosons. Two (pseudo)spins can be either in the completely symmetric space (triplet) or in the singlet state; the nonuniform magnetic field couples the two subspaces. As the subspaces have the same energies, this coupling leads to a transfer of atoms out of the symmetric subspace, i.e., to the dephasing. In the case of higher number of atoms the nonuniform magnetic field acts similarly, coupling the maximal and lower total spin subspaces. Interactions between particles separate the total spin subspaces [$J = 1$ and 0 in (a), $J = 3/2$ and two copies of $J = 1/2$ in (b)], leading to smaller effective coupling. The figure is an adaptation of the figure and explanation given in [15].

transverse harmonic potential blocking the motion in the direction perpendicular to the X axis. As an example, in the case of rubidium atoms trapped in the potential with frequencies $(\omega_x,\ \omega_\perp) = 2\pi \times (17, 14\,000)$ Hz, as in [14], one has $g \approx 0.5$ in oscillatory units based on ω_x. Therefore we scan parameter space in the experimentally relevant interval $g \in [0,\ 1]$. The scattering lengths of ^{87}Rb are close to each other—the difference between g_{11} and g_{00} is around $0.05g$, so the parameter c for rubidium atoms trapped in the state of the art trap will be around 0.01.

In the next section we will show how the spin self-rephasing works for this parametrization and the experimentally relevant coupling strengths range.

The spin self-rephasing phenomenon relies on the proper tuning of the interaction strengths [6]—such that the energy-level shift in the symmetric subspace (see Fig. 2) overcomes the effects of the inhomogeneity of the magnetic field. This energy shift, known as the collisional shift δ_{col} is approximately equal to $|g_{11} - g_{00}|\bar{n}$, where $\bar{n}$ is the average density of the gas. The effect of the magnetic-field inhomogeneity Δ_B also introduces its own energy shift: Approximately the energy originating from the presence of the magnetic field $\langle\hat{B}(x)\rangle$.

Finally the so-called lateral elastic collision energy (see [6]), $E_{\mathrm{lat}} = 2g_{01}^2\bar{n}v_{\mathrm{T}}/(3\pi\sqrt{\pi}m)$ should be smaller than $\hbar\omega$ (Knudsen regime) and the collisional energy δ_{col}. We introduced v_{T} which is the thermal velocity equal to $\sqrt{k_{\mathrm{B}}T/m}$; T is the temperature. In the simulations presented in this paper, Δ_B and E_{lat} are of the order of 10^{-2} and δ_{col} is of the order of 1 in oscillatory units.

A. Coherent states

In this section the state $[|\psi_{\mathrm{spin}}\rangle$ in Eq. (1)] is initiated as maximal spin projection of S_x; the aim is to tune interactions in such a way that the coherence C [Eq. (2)] decays slower.

It has been experimentally demonstrated that the dephasing can be substantially blocked thanks to the interaction between atoms in the so-called spin self-rephasing effect [6,7,16–18]. In articles following the initial discovery of spin self-rephasing, the importance of total spin preservation has been studied as well [6]. The original explanation is illustrated by the diagrams in Fig. 2(a) involving analysis of two-particle subsystems: Initially, the particles are in the triplet spin state; the inhomogeneous magnetic field couples $|j = 1; m = 0\rangle := (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ to the antisymmetric singlet spin state $|j = 0; m = 0\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$, effectively lowering population of maximal total angular momentum. As we deal with bosons, the atoms can be in the singlet states in the internal degrees of freedom only if simultaneously their spatial wave function is antisymmetric. The interaction energy coming from the short-range potential is zero in the singlet subspace—a result of the spatial wave function being antisymmetric as well. On the other hand if the internal state of atoms is in the symmetric triplet manifold then also their spatial wave function has to be symmetric, and these atoms do interact. Therefore, in the case of interacting bosons energy levels of triplet spin substates are shifted by ΔE (see Fig. 2), but the energy of the singlet case is constant (the spatial state is antisymmetric). As a result, population transfer is diminished. In the case of the higher number of atoms the dephasing mechanism in this picture is slightly more complicated, as energies of all spin subspaces do change, but the core reasoning still applies [see Fig. 2(b)].

To visualize the aforementioned transfers of population, let us introduce the spin-density matrix of a system: After partial trace of spatial degrees of freedom, only internal degrees of freedom are left. We can rearrange them to form the block-diagonal spin-density matrix, containing all of the information useful in quantum metrology:

$$\hat{\rho}_{\mathrm{spin}} = \begin{pmatrix} \hat{\rho}_{j=N/2} & & \cdots & \\ & \hat{\rho}_{j=(N-1)/2} & & \vdots \\ \vdots & & \ddots & \vdots \\ & \cdots & & \hat{\rho}_{j=j_{\min}} \end{pmatrix}, \tag{14}$$

where $j_{\min} = 1/2$ for odd N, otherwise $j_{\min} = 0$. The subspace with the maximal collective (pseudo)spin, $j = N/2$, corresponds to completely symmetric states. Usually in metrological scenarios the state is initialized so that only the $\rho_{j=N/2}$ part is populated (it is the case for coherent and squeezed states). Population conservation of this part of the spin-density matrix is hence a natural goal.

As demonstrated in Fig. 3, in the simulations, the desired behavior is observed: When interactions are present, a large amount of maximal spin population is preserved. The number of simulations is summarized in the lower panel of Fig. 3, showing the map of weights of the maximal spin subspace at fixed time $\omega t = 30$. By this metric, total spin is preserved well if high enough interaction between particles is present; a wide range of interaction strengths works well.

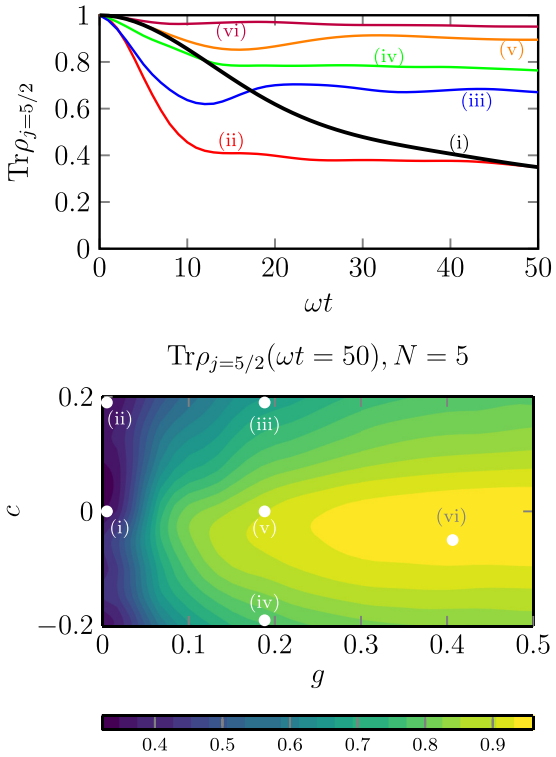

FIG. 3. Top: Evolution of maximal spin population as a function of time for several interaction strengths denoted in the bottom image by dots (the noninteracting case is denoted by thick black line), $N = 5$, $k_B T = 3\hbar\omega$, and quadratic inhomogeneity $b_2 = 0.01$. Quick decay followed by plateau is visible. Bottom: Preservation of trace for the same parameters for time $\omega t = 50$ as a function of interactions parametrized by mean interaction strength g and c.

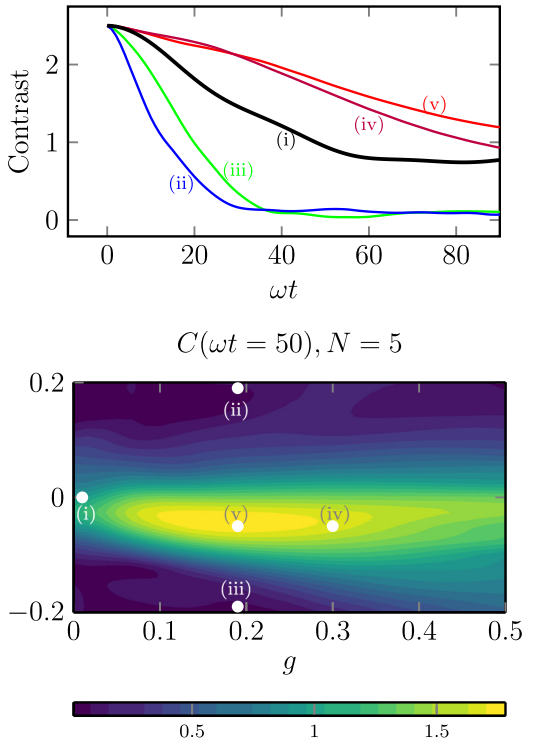

FIG. 4. Effect of interactions on preservation of the coherence C. Total number of particles $N = 5$, temperature $k_B T = 3\hbar\omega$, and quadratic inhomogeneity $b_2 = 0.01$. Top: typical short-time evolution of C for several interaction strengths denoted in the bottom image by dots; the noninteracting case is denoted by thick black line. As evidenced by the figure, it is possible to approximately double the time of decoherence (when the contrast attains half of its original value). Bottom: $\langle S_x \rangle$ at $\omega t = 50$ for various values of interaction strengths parametrized by g and c. Please note that the same interaction range is used in Fig. 3.

As shown above, interactions help in preservation of the total spin. To determine how the coherence is protected in the same scenario, Fig. 4 shows the dynamics and the map of values of C at a fixed instant of time $\omega t = 50$ in the broad range of interaction strengths. Although the state remained symmetric, apparently its dynamics is quite complex, and in many cases its coherence has been lost. This is caused in part by the small transfer between the symmetric and the other subspaces, but also the interactions may separate the clouds of atoms in different internal states (spin-position entanglement) or introduce strong, here unwanted, quantum correlations.

A natural question arises whether it is possible to use the spin self-rephasing to prevent not only the first-order coherence—already shown in the experiment—but also more subtle correlations. The simulations performed do not indicate usefulness in protection of the highly entangled states like the Schrödinger cat state ($|N0\rangle + |0N\rangle$); from now on we focus on squeezed states of practical importance in quantum metrology.

B. Squeezed spin states

Typical preparation of the squeezed spin state involves utilization of particle-particle interactions to produce the effective Hamiltonian quadratic in spin components. A few of the examples are one-axis twisting S_z^2 (implemented in the number of experiments [19–22]) or two-axis countertwisting (TACT) $S_y S_z + S_z S_y$. Spin squeezed states are produced as a result of the evolution. Here we use the TACT model of spin squeezing (although we expect the results to be essentially the same for OAT because of short squeezing times), in which the initial squeezed state is generated by evolution of coherent state $|+\rangle$ (the eigenstate to the maximal S_x eigenvalue):

$$|\psi_\theta\rangle = \exp[i\theta(S_y S_z + S_z S_y)]|+\rangle. \tag{15}$$

The TACT Hamiltonian is used only in the preparation of the spin state for further numerical evolution according to Hamiltonian Eq. (4). During the preparation only spin degrees of freedom are taken into account (in particular, inhomogeneous magnetic field does not affect the initial state). The squeezing parameter quantifies the usefulness of the state in quantum metrology, as compared with the spin coherent states [23]. The definition depends on the choice of direction of spin observation $\vec{n}$ and a perpendicular direction $\vec{n}_\perp$:

$$\xi_\psi^2 = \frac{N(\Delta^2 \vec{S} \cdot \vec{n}_\perp)_\psi}{\langle \vec{S} \cdot \vec{n} \rangle_\psi^2}. \tag{16}$$

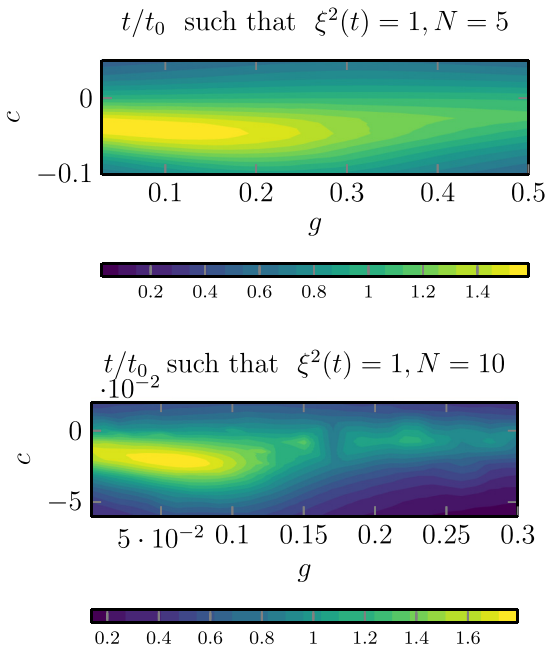

FIG. 5. Effect of interaction on preservation of squeezing parameter ξ^2 for various values of interaction strengths parametrized by g and c. The color is correlated with the time of decoherence t at which squeezing parameter ξ^2 becomes larger than 1 as compared to the time of decoherence without interactions t_0. The temperature $k_B T = 3\hbar\omega$, quadratic inhomogeneity $b_2 = 0.01$, and total number of particles $N = 5$ (top) and $N = 10$ (bottom). In both cases the squeezing time [θ in Eq. (15)] is 0.05, leading to $\xi^2(N = 10) = 0.44$ and $\xi^2(N = 5) = 0.69$ at the beginning of the evolution. In the case of $N = 10$ it is possible to increase the time at which ξ^2 becomes higher than 1 by a factor of 75%; for $N = 5$ this time can be increased by about 60%.

In our calculations we use $\vec{n} \propto \langle \vec{S} \rangle$. Since $\langle S_z \rangle = 0$ during the evolution and the spin dynamics is confined to the equator of the Bloch sphere, we choose $\vec{n}_\perp = (n_y, -n_x, 0)$. This variable is a measure of entanglement between individual spins: If $\xi^2 < 1$, the spins are entangled. In the following paragraphs we study the dynamics of the squeezing parameters, assuming the initial spin state prepared by the TACT Hamiltonian [Eq. (15)].

Our results are shown in Fig. 5. For ten particles even for relatively long evolution time, $\omega t = 30$, one can restore squeezing $\xi < 1$ in certain windows of parameters, which corresponds to delay of the decoherence time of about 75% (for $N = 5$ the decoherence time can be delayed by about 60%). The large interaction strengths are not desired: They are outside the parameter range required by spin self-rephasing (a result of E_{lat} increase) and induce strong correlations along with complicated spatial dynamics.

Our results indicate that the protection of squeezed states is lower as compared to coherent states, however still reasonably large to expect good results when the number of particles is higher. Schrödinger cat states—superpositions of two coherent states—on the other hand are known to be extremely fragile to any source of noise, which is reflected in our simulations: Their coherence is quickly lost and particle-particle interaction offers no protection.

IV. DISCUSSION AND CONCLUSIONS

In this paper we investigated *ab initio* a system of several two-level particles (pseudospins) in one-dimensional harmonic potential. In such an approach, all effects typically considered in metrology in an approximated way, such as collisional shift, noises, or identical spin-self rotation effect, are automatically and rigorously taken into account, as incorporated in the dynamics generated by a many-body Hamiltonian.

The subject of the paper is to show that the spin self-rephasing mechanism can be used to prevent decoherence of the entangled states induced by an inhomogeneous magnetic field. In the semiclassical description, each pseudospin would rotate around the magnetic-field polarization axis, with an angular velocity depending on the value of the local field $B(x)$. As a result, the collective spin decays in time. From one point of view, the dephasing is caused by populating a space with a total spin of less than $N/2$. This unwanted process can be inhibited by tuning the interactions between particles: In the interacting system, the subspaces with different total spin have different interaction energies, which stops the population transfer between them [6]. On the other hand, even if the state remains completely symmetric (total spin $j = N/2$), the interactions also contribute to the different type of dephasing, as shown in Fig. 4. One such effect is the introduction of collisional shift, and in the case of larger interactions even nonlinear effects leading to phase collapse. In the light of emerging quantum technologies, it is not clear whether the interactions can be used to protect entanglement. In this paper we have shown that there is a window of parameters in which the squeezed state obtained from the TACT model is stable despite the presence of nonlinear magnetic field. In the example for ten particles, the time scale at which the squeezing disappears can be extended by a factor of 75% using reasonably low interaction strengths.

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APPENDIX A: SIMULATION METHOD

Our implementation provides a way to determine the evolution of state vectors and expectation values of observables of a system consisting of several bosons in a weakly inhomogeneous harmonic trap, in both the high- and low-temperature limit. The numerical simulations assume unitary evolution generated by Hamiltonians composed of (1) pseudospin components, S_x, S_y, S_z, and polynomials of them, $S_z^2, S_x S_y + S_y S_x, \ldots$; (2) spin-position couplings, emerging from the trap inhomogeneities, $\sum \hat{x}_i \hat{\sigma}_{z,i}, \sum \hat{x}_i^2 \hat{\sigma}_{z,i}, \ldots$; and (3) particle-particle interaction [Eq. (6)].

In the high-temperature scenario we use a quantum Monte Carlo technique, which requires construction of sub-basis around the fixed spatial Fock state, which corresponds to the subspace traversed by the state under time evolution.

1. Initial state

The initial state is assumed to be thermal in spatial degrees of freedom and pure in spin: The spin part is in fact assumed to be in the minimal spin projection S_z eigenstate. The resulting state is subsequently rotated by S_y, so that it is now in maximal spin projection of S_x, with the spatial degrees of freedom left intact; if needed, the state can also be squeezed by the TACT Hamiltonian. This is motivated by the usual experimental realizations, in which trapped atoms are selected by their internal state and left to thermalize in the trap, then a sequence of pulses is applied to the trapped ensemble, and later evolution is studied.

2. Thermal state

The initial state thus depends on the properties of the Hamiltonian restricted to the subspace with minimal spin projection, $S_z = -N/2$. In the low-temperature limit, the relevant part of the Hamiltonian can be diagonalized and the eigenstates evolved according to the procedure described above with appropriate weights.

The simulation of high-temperature states is more intricate: The relevant part of the Hamiltonian (i.e., the subspace in which the majority of the population resides) has high enough dimension so that no direct diagonalization might be applied. There is, however, a method of random sampling from the thermal density matrix, often called quantum Monte Carlo, which greatly reduces the computational cost needed to determine the evolution of the initially thermal state at the expense of deterministic behavior. Consider the thermal density matrix,

$$\rho_T = \frac{\exp(-H/T)}{\mathrm{Tr}\exp(-H/T)}. \tag{A1}$$

By denoting $\mathrm{Tr}\exp(-H/T)$ by Z, it is possible to rewrite the above expression by inserting the identity matrix:

$$\rho_T = Z^{-1}\exp\left(-\frac{H}{2T}\right)\left[\sum_i |i\rangle\langle i|\right]\exp\left(-\frac{H}{2T}\right). \tag{A2}$$

Sampling from the density matrix can now be understood as approximating the identity matrix with finite number of elements $|i\rangle\langle i|$—this is the same as choosing random vectors from a (fixed) basis S with uniform probability and evolving them by $\exp(-\frac{H}{2T})$. This of course works the best if S is as close to the eigenbasis of H as possible. In the simulations we have chosen S to be the eigenbasis of the harmonic oscillator without inhomogeneities and interactions—since both of these terms of the Hamiltonian are small as compared to the bare harmonic oscillator energy, the assumption is fulfilled.

The sampled vectors are not normalized, since $\exp(-\frac{H}{2T})$ is not unitary. This is anticipated behavior: Length of the vector indicates the relative weight when the sampled ensemble is brought together in order to calculate the expectation values. Samples can be further evolved unitarily to determine the dynamics of a system, and this is the procedure employed in the simulations. In our case, samples are drawn until the appropriate expectation values in the relevant time span converge to a stable value.

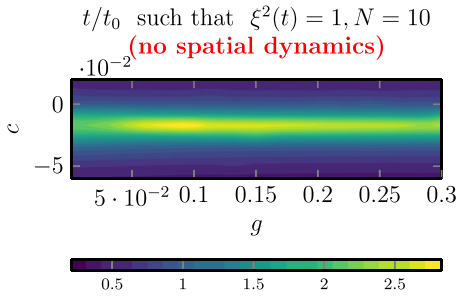

FIG. 6. Figure analogous to the bottom picture of Fig. 5 (preservation of squeezing parameter for $N = 10$); note the same range of interaction parameters. In order to prepare this picture, the simulation basis has been artificially restricted, effectively leading to ignoring of spatial dynamics. The oversimplified description leads to decoherence time delay of about 180%, far from 75% obtained with more accurate simulation.

APPENDIX B: IMPORTANCE OF SPATIAL DYNAMICS

Proper treatment of spatial degrees of freedom is important in order to obtain a meaningful result, especially when large interaction strengths are present. To determine the effect of spatial dynamics, we have performed additional simulations with an artificially restricted basis. The restriction effectively reduces the effect of particle-particle interactions to energy shift, which is not an accurate description for high interaction strengths. This is illustrated in Fig. 6, in which results analogous to those in Fig. 5 are presented. The discrepancy between the two pictures is clear and demonstrates the importance of full quantum description in the analysis presented in this paper.

However, such simplified simulations are convenient in analysis of noninteracting bosons. Provided the magnetic field is small, and little spatial dynamics is introduced, the description is sufficient and useful in calculation of e.g., evolution of the squeezing parameter [Eq. (16)], which depends on expectation values of two-particle operators though variance.

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