Many-body cooperative effects in an ensemble of pointlike impurity centers near a charged conductive surface

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On the basis of a quantum microscopic approach we study the cooperative effects induced by the dipole-dipole interaction in an ensemble of pointlike impurity centers located near a charged perfectly conducting surface. We analyze the simultaneous influence of the modified spatial structure of field modes near the conductive surface and the electric field on the transition spectrum of an excited atom inside an ensemble and on the radiation trapping. We show that the electric field modifies the cooperative Lamb shift, as well as the character of sub- and super-radiant decay. We also demonstrate that electro-optical effects in the presence of a surface are essentially different in comparison with those in free space. The nonmonotonic behavior of electro-optical effects in the presence of a surface with increasing of the electric field has been shown.

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I. INTRODUCTION

Since the seminal work of Purcell [1], the interaction of light with atoms localized inside a cavity or waveguide, as well as near its surface, has attracted considerable attention. Now it is well understood that a cavity modifies the spatial structure of the modes of the electromagnetic field. This leads to the modification of the radiative properties of atoms, and in particular to the enhancement and inhibition of the spontaneous decay rate [2–6]. This proposes an exciting tool for the preparation of media with given optical properties. For this reason, the study of atomic systems in the presence of a cavity or waveguide is one of the modern trends in atomic optics and quantum optics. Light interaction with atoms coupled to nanophotonic structures, such as nanofibers [7–9], photonic crystal cavities [10], and waveguides [11,12], may have future applications in quantum metrology, scalable quantum networks, and quantum information science [13–15].

Modification in the structure of field modes changes not only single-particle characteristics but also the nature of photon exchange between different atoms. In its turn this leads to an alteration of the dipole-dipole interatomic interaction [16,17], as well as associated cooperative effects [18–21]. Herewith, such an indirect influence on the properties of atomic media via modification of cooperative effects can be more significant than the direct influence via modification of single-atom properties. For this reason, cooperative properties of cold atomic gases coupled with dielectric nanoscale structures, in particular, nanofiber [22–27] and photonic crystals [28–30], are intensively discussed nowadays. Cooperative phenomena, such as super-radiance, attract considerable attention in a wide area of research, in particular, plasmonics [31–34]. Another relevant direction is the investigation of the influence of nonradiative effects on super-radiant light emission [35,36].

In fact, not only a cavity or waveguide can modify the spatial structure of the modes of the electromagnetic field. A single metallic surface also has this property. For this reason, the characteristics of the ensemble of atoms or quantum dots located near the conductive surface differ from ones in the case of the same ensemble in free space [37]. If the metallic surface is charged, an electrostatic field causes Stark shifts of the atomic energy levels, which leads to additional modification of the interatomic dipole-dipole interaction [38,39].

Increasing in the size of the atomic ensemble, i.e., increasing of the number of atoms, leads not only to quantitative changes but also qualitatively new behavior of atoms when interacting with the electromagnetic field, for instance, Anderson localization of light. For this reason, the study of many-body cooperative effects deserves special attention. However, many-body phenomena, including multiple and recurrent light scattering, induced by the dipole-dipole interaction in an ensemble of pointlike impurity centers near a charged conductive surface, have not been studied in detail yet. In this paper we will show that electro-optical effects in the presence of a surface are essentially different in comparison with those in free space. When an atomic ensemble is located near a surface, the character of electro-optical effects is affected by two physical mechanisms. The first is the modification of single-atom properties such as spontaneous decay rate; second is an alteration of the dipole-dipole interaction caused by the peculiarities of the spatial structure of the modes of the electromagnetic field near the surface. Moreover, these two mechanisms are nonadditive, i.e., they can enhance and/or compensate each other.

The goal of this paper is to describe theoretically polyatomic cooperative effects in a dense ensemble of pointlike impurity centers embedded in a solid dielectric and placed near a perfectly conductive charged plate. We simultaneously analyze two factors affecting the character of cooperative effects in the system with strong interatomic correlations: The peculiarities of the spatial structure of field modes near the conductive surface as well as Stark splitting of energy levels

induced by an electrostatic field. We show that the influence of the electric field on the collective effects in a dense polyatomic ensemble located near the conductive surface significantly differs from that in the case of an ensemble in free space, without a surface.

II. BASIC ASSUMPTIONS AND APPROACH

Let us consider an ensemble which consists of N motionless impurity atoms embedded into a transparent dielectric and placed near a charged perfectly conducting plate. The longitudinal sizes of the plate are assumed to be significantly larger than resonant transition wavelength λ_0 and the sizes of the atomic sample. We will suppose also that the temperature of the system is low enough to neglect the electron-phonon interaction. In this case the influence of the dielectric matrix on impurity atoms is restricted by random shifts of their energy levels caused by inhomogeneous internal fields in a dielectric. These approximations allow us to consider dynamics of the model system consisting of the set of motionless pointlike scatterers and the electromagnetic field.

At the present time, there are several approaches to the description of collective effects in the system under consideration [40–51]. In this paper we use the consistent quantum-posed theoretical approach. In the framework of this approach, the considered quantum system is described by the wave function, which can be found by the method proposed first in [52,53] and developed afterward in [49] for a description of the collective effects in dense and cold nondegenerate atomic gases. This method was successfully used for the analysis of the optical properties of dense atomic ensembles as well as for studying light scattering from such ensembles [54–59].

Further, this method was generalized on the case of atomic systems located in a Fabry-Perot cavity [19,20]. In the papers [38,39] it was used to analyze the dipole-dipole interaction between two motionless point atoms near a single perfectly conducting mirror.

The quantum microscopic approach was described at great length in several of our papers [20,39,49] and we will not reproduce the general theory in detail here. In the following paragraphs, we just provide a brief overview of it. The reader is referred to the mentioned papers for the theoretical developments and justifications.

The method employed is based on the solution of the nonstationary Schrödinger equation for the wave function of the joint system consisting of all impurity atoms and the electromagnetic field, including vacuum reservoir. Full Hamiltonian $\widehat{H}$ of the joint system can be presented as a sum of Hamiltonian $\widehat{H}_0$ of noninteracting atoms and variable field and operator $\widehat{V}$ of their interaction. The influence of the external constant electric field and static internal fields of the dielectric matrix is taken into account by shifts of the atomic energy levels.

We seek the wave function as an expansion in a set of eigenfunctions of the operator $\widehat{H}_0$. Using this representation of the wave function, we convert the Schrödinger equation to the system of linear differential equations for the amplitudes of the quantum states. The total number of equations in this system is equal to infinity.

The key simplification of the approach is in the restriction of the total number of states taken into account. We assume that the initial excitation is weak, and all nonlinear effects are negligible. With the accuracy up to the second order of the fine-structure constant, we can consider only the states with no more than one photon (see [60]).

Despite the restriction of the total number of quantum states, the set of equations remains infinite because of the infinity number of the single-photon field states. We can, however, formally solve it without any additional approximations. For this purpose we express the amplitudes of the quantum states with single photons via the amplitudes of the states corresponding to atomic excitation without photons. Then we put these expressions in the equations for the amplitudes of single-photon states. In this way we obtain a closed finite system of equations for the amplitudes b_e of the quantum states with one excited atom in the ensemble.

For Fourier components $b_e(\omega)$ we have (at greater length see [19,49])

$$\sum_{e'}[(\omega-\omega_e)\delta_{ee'}-\Sigma_{ee'}(\omega)]b_{e'}(\omega)=i\delta_{es}. \tag{2.1}$$

When deriving this expression, we assumed that at the initial time only one atom is excited (this state is denoted by index s), while all other atoms are in the ground state. The electromagnetic field at $t=0$ is in the vacuum state. The index s as well as the indices e and e' contain information both about the number a of the atom and about the specific atomic sublevel excited in the corresponding state.

The matrix $\Sigma_{ee'}(\omega)$ describes both spontaneous decay and photon exchange between the atoms. It plays a key role in the microscopic theory. The explicit expressions for the elements of this matrix corresponding to a Fabry-Perot cavity were derived in [19,20].

The size of the system (2.1) is determined by the number of atoms N and the structure of their energy levels. In this paper we consider the impurity atoms with the ground state $J=0$. Total angular momentum of the excited state is $J=1$. It includes three Zeeman sublevels $|J,m\rangle$, which differ by the value of angular momentum projection on the quantization axis z: $m=-1,0,1$. Therefore, the total number of onefold atomic excited states is $3N$. The quantization axis can be chosen arbitrarily. In the considered geometry there is a special direction—perpendicular to the surface. For convenience, further in this paper we assume that the quantization axis z is directed perpendicularly to the charged mirror and, consequently, along its electrostatic field.

Due to the external electrostatic field $\mathcal{E}$ of a charged plate and internal random field of the dielectric medium, resonant frequencies of different atomic transitions ω_{a_m} differ from those of an isolated atom in free space ω_0:

$$\omega_{a_m}=\omega_0+\Delta_{a_m}+\Delta\omega_m, \tag{2.2}$$

where Δ_{a_m} is the frequency shift of the sublevel m of atom a ($a=1,\ldots,N$), which depends on its spatial position due to inhomogeneity of internal fields in a dielectric, and $\Delta\omega_m$ is Stark shift caused by the electrostatic field of a plate, which is the same for similar transitions of different atoms.

The approach used in this paper is applicable for the description of dense atomic ensembles, when frequency shifts of collective states exceed the natural linewidth γ_0 of an isolated atom. Consequently, this method is applicable when

frequency shifts of atomic levels caused by different reasons are the same order. In particular, it is true when the shifts induced by the external electric field, Δ_{a_m} and $\Delta\omega_m$, are comparable with γ_0 or even more.

Frequency shifts Δ_{a_m} are caused by random fluctuations of the internal fields of a dielectric. Therefore, we consider Δ_{a_m} to be a random value. Its distribution depends on many factors, mainly the species of host and impurity atoms in a dielectric and the chemical bond. In this paper we approximate Δ_{a_m} by Gaussian distribution with zero mean value and rms deviation δ. Also we suppose that this distribution is the same for all Zeeman sublevels of the excited state.

We denote the Stark shift of the resonant frequency of the transition $J = 0 \leftrightarrow J = 1, m = \pm 1$ caused by the electrostatic field of a charged plate as $\Delta\omega_{m=\pm 1}$; for the transition $J = 0 \leftrightarrow J = 1, m = 0$ it is $\Delta\omega_{m=0}$. The influence of an electrostatic field on the character of photon exchange is significant in the case when Stark splitting $\Delta = \Delta\omega_{m=0} - \Delta\omega_{m=\pm 1}$ is comparable with the natural linewidth γ_0 of an isolated atom.

Numerical solution of the system (2.1) allows us to obtain the Fourier amplitudes of atomic states $b_e(\omega)$. Using $b_e(\omega)$ we can obtain the amplitudes of all states taken into account in our calculations (see [19,49]) and, consequently, the wave function of the considered system.

To analyze the dynamics of the atomic ensemble located near a single mirror on the basis of mathematical formalism developed for a cavity, we should go to the limit of infinite distance between the mirrors and consider atoms near the first mirror. In this case the influence of the second mirror on the dynamics of the atomic system can be neglected.

Note that any physical observables that we will analyze depend on the positions of all impurity atoms. In this paper we consider spatially disordered atomic ensembles with uniform (on average) distribution of atomic density, as it is the case in experiments. By this reason we average all the results over random spatial configurations of the ensemble by a Monte Carlo method. To take into account the inhomogeneous broadening we also perform Monte Carlo averaging over random shifts Δ_{a_m} of energy levels caused by the inhomogeneity of the internal fields of a dielectric.

In the next section, we use the general approach to investigate the simultaneous influence of the peculiarities of the spatial structure of field modes near the conductive surface as well as Stark splitting of energy levels induced by an electrostatic field on the character of many-body cooperative effects. We will calculate the transition spectrum of an excited atom surrounded by an ensemble of unexcited atoms, and spontaneous decay dynamics. On this basis, we will analyze the influence of the electrostatic field on radiation trapping in the considered system. All the results presented in the next section were obtained by Monte Carlo averaging over several thousand random configurations of the atomic ensemble. In this case the standard deviation for b_e is very small. Relative error of the calculated results does not exceed 1%.

III. RESULTS AND DISCUSSION

Some of the effects, caused by the influence of a charged conducting surface on an atomic ensemble, can be described within the framework of monatomic approximation and appear for dilute ensembles or even for single atoms. Some effects, caused by the modification of the interatomic dipole-dipole interaction due to simultaneous influence of the conducting surface and the electrostatic field, are essentially collective.

Monatomic effects are relatively simple and have been well studied to date. When a single atom is located close to the uncharged surface, the spectrum of atomic transition represents a Lorentz profile, like in the case of a free atom. But the linewidth γ differs from that of a free atom and depends on the distance z between the atom and the surface. If z is less than or comparable to the resonant wavelength λ_0, the difference is very significant. Accordingly, the dynamics of the spontaneous decay of the excited atom is described by a single-exponential law, $P_s(t) = \exp(-\gamma t)$. The function $\gamma(z)$ depends on the Zeeman sublevel, which is initially populated. Thus, for Zeeman sublevels $m = \pm 1$, $\gamma(z)$ converges to zero if the atom approaches the surface. For $m = 0$ this limit is equal to $2\gamma_0$. As z increases both values tend to γ_0. On the whole, the function $\gamma(z)$ has a nonmonotonic oscillating character (see, for example, [39]).

If the conducting surface is additionally charged, its electric field causes Stark shifts, which actually does not influence the monatomic effects. Only the frequencies of atomic resonances change. Their shapes remain the same. The amplitude and the width of the resonance change absolutely negligibly because the Stark shift is absolutely negligible in comparison with the frequency of any optical transition.

Collective effects in dense atomic ensembles under considered conditions have been studied in less detail. We begin our analysis with studying the shape of the atomic transition connected with spontaneous decay of an atom initially excited in the dense atomic ensemble. We assume that at the initial time all the other atoms of the ensemble are unexcited.

Such a spatially localized initial atomic excitation can be prepared by a two-photon resonance method [61]. In the framework of this method the sample is illuminated by two narrow and off-resonant orthogonally propagated light beams (both beams parallel to the conducting surface). Each beam does not cause single-photon excitation, but their simultaneous interaction with atoms in the crossing region causes two-photon excitation from the ground S to the high-energy excited D state if conditions of two-photon resonance are satisfied. Spontaneous relaxation of the D state leads to the population of the studied P state. This method allows obtaining a small cluster of excited atoms in the bulk region of a sample. For simplicity in this paper we consider that at the initial time only one atom is excited. Note that modern experimental techniques allow obtaining so precise spatial resolution to detect luminescence of a single atom or quantum dot [62,63].

A. Atomic transition spectrum

As it is clear from the aforesaid, the effect of the surface depends on the positions of all atoms and especially of the excited one z_{exc}. The most interesting phenomena are observed if z_{exc} does not exceed the wavelength of resonant light. By this reason, further, we will consider $z_{\text{exc}} = 1$ assuming that reference point $z = 0$ corresponds to the position of the

surface (hereafter, we take $\lambda\!\!\!^{-} = k_0^{-1} = \lambda_0/2\pi$ as the unit of length). Also for simplicity we assume at first that inhomogeneous broadening is negligible, so that $\Delta_{a_m} = 0$ (respectively, $\delta = 0$). In this case all the atoms are resonant to each other, so the role of the dipole-dipole interaction is manifested to the maximum extent.

In the general case, the specific type of transition spectrum for a given density depends not only on $z_{\rm exc}$ but also on the size of the atomic ensemble. We have previously analyzed size dependence of the transition spectrum. When the size is comparable with the mean free path of a photon, the changes of the transition spectrum with increasing in size are essential. As linear size increase, these changes become more and more weak. Size dependence has an evident tendency to saturation. Further, we present the results, which correspond to sufficiently large sample, when size dependence can be neglected. So it can be used for a description of the transition spectrum of an excited atom inside any macroscopic ensemble with reasonable accuracy.

The line shape of atomic transition corresponding to the decay of Zeeman sublevel $m = 0$ is shown in Fig. 1(a). Here we compare the shape of atomic resonance in four cases. For convenience of the comparison, the frequency is calculated from the resonant frequency taking into account Stark shift $\delta\omega = \omega - \omega_{a_m}$ [see Eq. (2.2)]. The first curve is obtained when both the surface and electric field are absent. The specific dimensionless atomic density is chosen equal to $n = 0.05$. We see that it is a sufficiently large value, so that the dipole-dipole interaction plays an important role and the shape is essentially different from the Lorentz contour typical for a free atom. We observe an asymmetry in the line shape, which is a typical manifestation of cooperative effects [64]. This asymmetry is inextricably linked to the line shift induced by the dipole-dipole interaction. Note that an asymmetry of the atomic transition spectrum is connected with asymmetries in the spectrum of dielectric permittivity and total cross section of light scattering [65]. The last is in good agreement with prediction of the Debye-Mie model, which gives the asymmetric spectrum of light scattering on the spherically symmetric atomic sample.

Curve 1 transforms into curve 2 when we switch on the electric field corresponding to the Stark splitting $\Delta = \gamma_0$. The electric field without a conducting surface causes an essential shift and essentially modifies the shape of the resonance. Here we see the influence of the electric field on collective effects, partially on collective Lamb shift caused by modification of resonant dipole-dipole interatomic interaction. Placing the atomic ensemble near an uncharged surface [curve 3 in Fig. 1(a)] changes amplitude but practically does not transform the shape of the resonance and the collective shift. Simultaneous influence of the electric field and surface causes the change of the collective Lamb shift, the width of the resonance, as well as its shape.

The presence of the electric field and/or the surface makes the system optically anisotropic. By this reason their influence on the shape of the transitions $J = 0 \leftrightarrow J = 1, m = \pm 1$ differs from that corresponding to the transition $J = 0 \leftrightarrow J = 1, m = 0$. This can be seen in Fig. 1(b). The surface causes essential narrowing of the resonance which is explained mainly by monatomic effects. For $z_{\rm exc} = 1$, $\gamma_{m=\pm1} = 0.65\gamma_0$. The

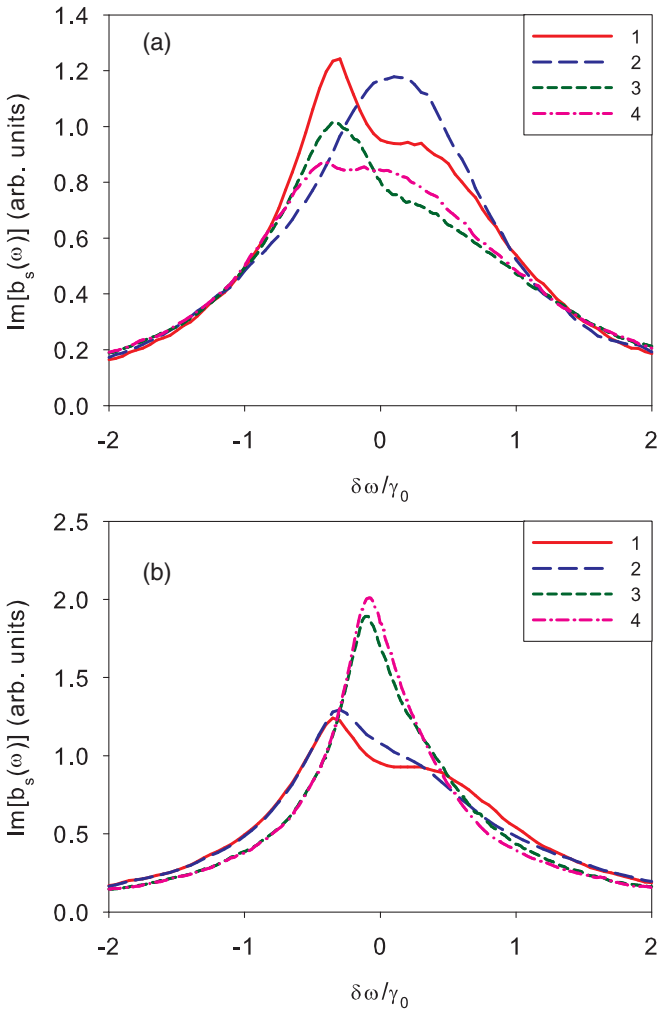

FIG. 1. Transition spectrum of an atom inside an atomic ensemble with $n = 0.05$, $\delta = 0$. (a) $m = 0$. (b) $m = \pm 1$. 1, electric field is absent, $\Delta = 0$, and there is no surface; 2, electric field is present, resulting in $\Delta = \gamma_0$; 3, there is a conducting surface $z_{\rm exc} = 1$ and the electric field is absent; 4, there are both the field resulting in $\Delta = \gamma_0$ and the surface. The detuning $\delta\omega$ is calculated from the resonant frequency taking into account Stark shift (if it is nonzero).

effect of the electric field is weak for the considered transition, which agrees with the previously obtained results of the calculation of the dielectric constant tensor [66,67]. Note, however, that near the surface different Zeeman sublevels not only decay in different ways but are also affected by the electric field in different ways. This can be understood if we compare Fig. 1(b) with Fig. 1(a).

The influence of the electric field changes with its magnitude. This dependence is most pronounced for transition $J = 0 \leftrightarrow J = 1, m = 0$. It is illustrated by Fig. 2 where we show the shape of the transition spectrum for different Stark splitting Δ. For clarity, in Fig. 2 the frequency is calculated from the resonant frequency of the transition $J = 0 \leftrightarrow J = 1$, $m = \pm 1$ of a free atom taking into account the Stark shift, $\omega_{m=1} = \omega_0 + \Delta\omega_{m=\pm1}$. Figures 2(a) and 2(b) correspond to the cases with and without conducting surface. In these figures we added reference vertical lines, which indicate all the considered values of Stark splitting.

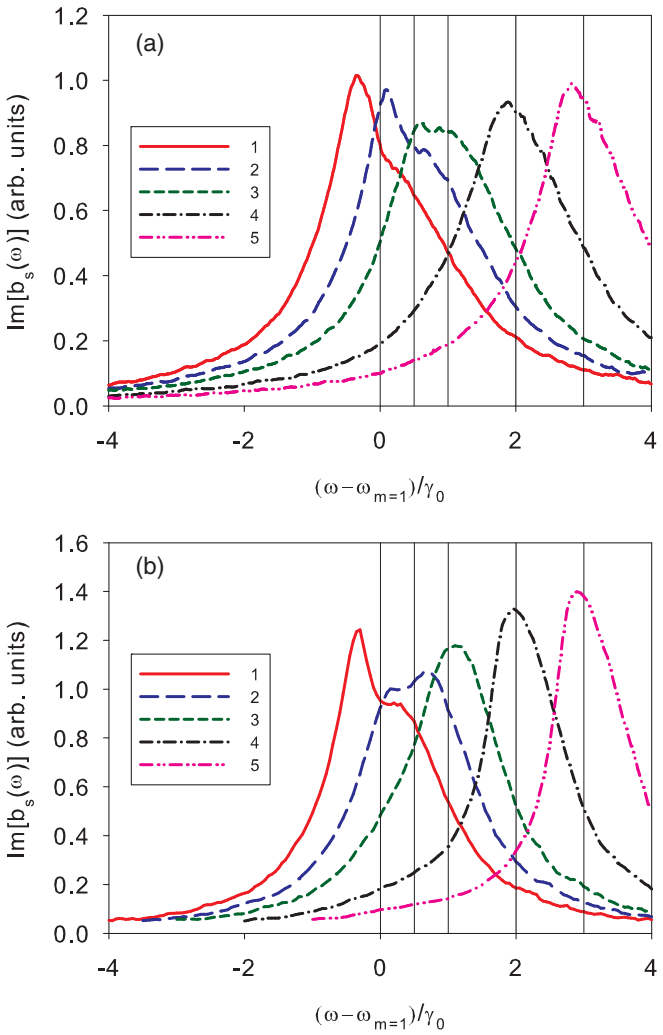

FIG. 2. Transition spectrum of an atom in the electric field. $m = 0$, $n = 0.05$, $\delta = 0$. (a) Near a conducting surface, $z_{\rm exc} = 1$. (b) In free space. 1, $\Delta = 0$; 2, $\Delta = 0.5\gamma_0$; 3, $\Delta = \gamma_0$; 4, $\Delta = 2\gamma_0$; 5, $\Delta = 3\gamma_0$.

In the case of the atomic ensemble near the surface, the parameters of the resonance such as its shape, amplitude, and width change with magnitude of constant field nonmonotonously. The collective Lamb shift is also nonmonotonic. The maximal distortion corresponds to the splitting, which is close to the natural linewidth of a free atom.

The influence of the electric field on collective effects in free space [see Fig. 2(b)] has some peculiarities. We see very strong distortion of the resonance shape for Stark splitting less than γ_0. For strong field, corresponding to $\Delta > \gamma_0$, the collective Lamb shift is less than that in the case of the atomic ensemble near the surface [compare with Fig. 2(a)]. Besides that, increasing of the field causes some line narrowing and increasing of the amplitude of the resonance.

In Fig. 2 we show the line shape up to Stark splitting equal to $\Delta = 3\gamma_0$. It is clear that the dependence of the observable spectrum on the Stark splitting Δ should disappear when this splitting becomes more than atomic level shifts caused by resonant dipole-dipole interaction. Our calculation indicates that for a considered density it takes place at $\Delta \sim 15\gamma_0$.

The main features observed in Figs. 1 and 2 can be explained by the fact that the discrepancy between spectral dependencies shown here from the Lorentz curve is caused by the collective effects, such as recurrent scattering and associated interatomic dipole-dipole interaction. The types of photons that atoms can exchange are determined by the structure of the field modes, that is, by the presence or absence of a surface. Herewith, the efficiency of this exchange is determined by the mutual resonance, that is, by the magnitude of the constant electric field. The combined effect of these two factors (constant electric field and electromagnetic field modulation due to the surface) is different for the transition $J = 0 \leftrightarrow J = 1, m = 0$ and for the transitions $J = 0 \leftrightarrow J = 1, m = \pm 1$. Our analysis shows that for the transition $J = 0 \leftrightarrow J = 1, m = 0$ these factors compensate each other, whereas for the transitions $J = 0 \leftrightarrow J = 1, m = \pm 1$ they enhance each other.

In conclusion of this section, we note that the solution of an algebraic system of equations (2.1) with a given right-hand side is equivalent, in essence, to finding its Green's function with given point source. Calculation of the amplitude $b_s(\omega)$ means determination of the Fourier component of the Green's function in the point of the source. In accordance with [68], knowledge of this function allows us to determine the local density of states of the atomic system, as well as to find a number of characteristics of this system, for example, the mean free path of photons inside it.

B. Light trapping

The influence of the charged conducting surface on an atomic ensemble can be detected in the experiment, for example, by measurement of its afterglow after initial excitation. The dynamics of the total intensity and, consequently, light trapping is determined by the dynamics of the atomic excited-state population.

In this subsection we analyze time dependence of the total population of the excited states of all atoms of the ensemble. As earlier, for simplicity we assume that initially only one atom is excited. Our analysis is based on the calculation of the inverse Fourier transform of $b_e(\omega)$. It allows us to obtain the time dependence of the quantum amplitudes of the onefold atomic excited states, $b_e(t)$. The time-dependent population of any Zeeman sublevel of any atom in an ensemble can be calculated in a standard way: $P_e(t) = |b_e(t)|^2$.

The total excited-state population $P_{\rm sum}(t)$ is given by a sum of $|b_e(t)|^2$ over all atoms in the ensemble. Figure 3 shows the time dependence of the total excited-state population in the case $\delta = 0$. The results are presented for the atomic ensemble of cylindrical shape: One of the planes of a cylinder coincides with the plane of a conducting surface, the radius of a cylinder is $R = 12$, and the length is $L = 13$, which is much more than the mean free path of a photon at the considered density $n = 0.05$. The initially excited atom is located at $z_{\rm exc} = 1$ on the central axis of a cylinder.

In Fig. 3, like in Fig. 1, we compare four main cases: The ensemble in free space, in the electric field, near the uncharged surface, and near the charged surface. In all the cases we see typical manifestation of collective effects. The dynamics of an atomic excitation cannot be described by a simple

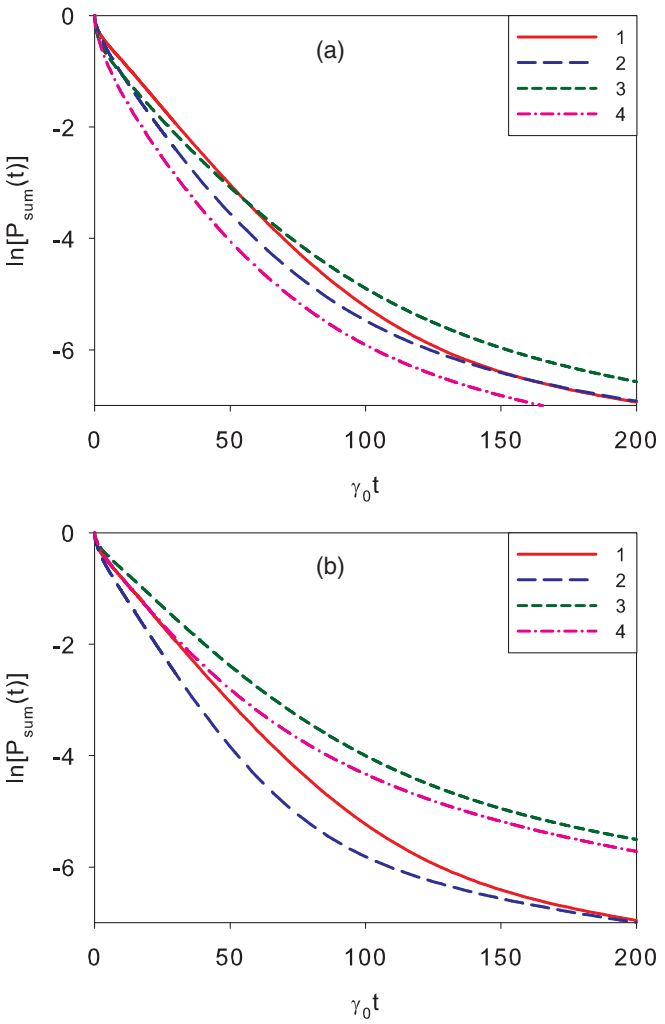

FIG. 3. Time dependence of the total excited-state population of an atomic ensemble with $n = 0.05$, $\delta = 0$. (a) $m = 0$. (b) $m = \pm 1$. 1, atomic ensemble in free space; 2, electric field resulting in $\Delta = \gamma_0$, and there is no surface; 3, there is a conducting surface $z_{\text{exc}} = 1$ and the electric field is absent; 4, there are both the field resulting in $\Delta = \gamma_0$ and the surface.

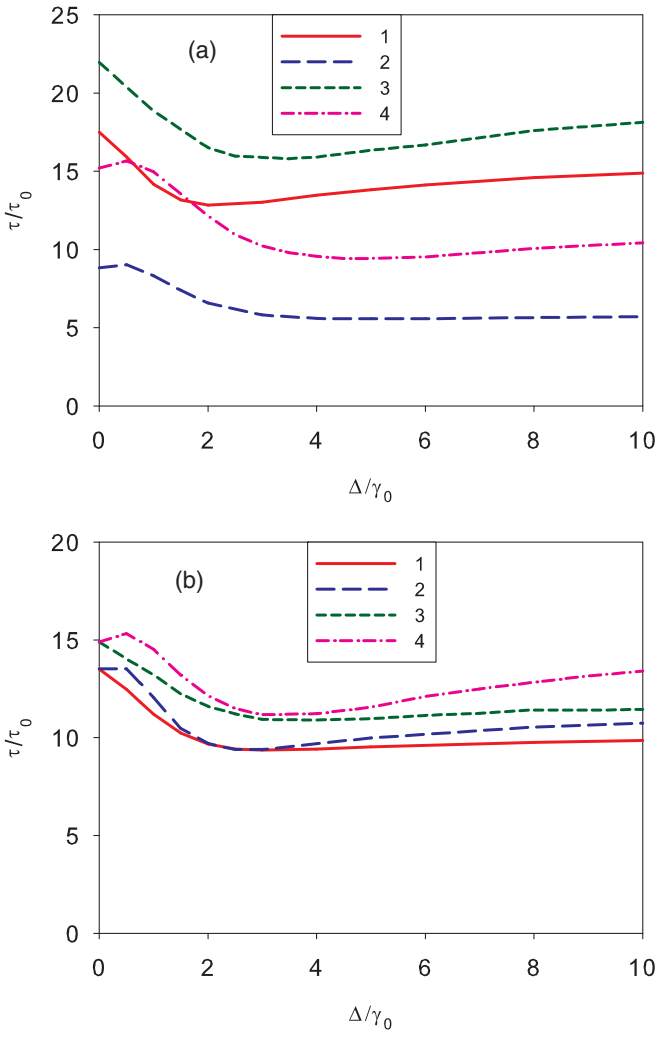

FIG. 4. The time of radiation trapping depending on the electrostatic field strength. $\delta = 0$. (a) Atomic ensemble near the conducting surface. (b) Atomic ensemble in free space. 1, $n = 0.05$, $m = \pm 1$; 2, $n = 0.05$, $m = 0$; 3, $n = 0.1$, $m = \pm 1$; 4, $n = 0.1$, $m = 0$. $\tau_0 = 1/\gamma_0$ is the natural lifetime of the excited states of a free atom.

one-exponential law like in the case of a single atom. It is explained by interatomic interaction caused by the photon exchange between different atoms. Among different collective quantum states formed as a result of this interaction in the considered ensemble there are both super- and subradiant ones. In such a case the spontaneous decay dynamics is described by a multiexponential law.

Besides these typical collective effects, Fig. 3 demonstrate some features determined by the surface and electric field. We see that both these factors separately influence the nature of the decay dynamics which is connected with the above-mentioned modification of the dipole-dipole interatomic interaction and, consequently, with changes in both sub- and super-radiant states.

It should be noted that these factors, when combined, can strengthen each other or compensate. Thus, for $m = 0$, their combined effect accelerates the decay of excitation. Curve 4 in Fig. 3(a), which describes the dynamics of the decay of the excitation near the charged surface, decreases most rapidly. In the case of the initial excitation of an atom to the level $m = \pm 1$, in contrast, both factors act in different directions, and curve 4 in Fig. 3(b) demonstrates the intermediate decay rate. Here, the physical nature of the combined effect of the constant electric field and electromagnetic field modulation due to the surface is the same as already discussed above in the analysis of the spectra in Sec. III. A.

Comparison of the curves 1 and 2, as well as 3 and 4, makes it possible to reveal the influence of the electric field on the nature of the afterglow of the ensemble in the absence of a conducting surface and near it. This comparison gives grounds to conclude that a change in the structure of the modes of the electromagnetic field, caused by the presence of a surface, leads to a change in the effect of the constant field, i.e., modifies electro-optical effects in dense atomic systems.

This circumstance is also confirmed by the analysis of the typical time of radiation trapping. We will estimate this time τ from the relation $P_{\text{sum}}(\tau) = 1/e$. Figure 4 demonstrates how

this time changes with increasing in the electrostatic field strength both for an ensemble in free space [Fig. 4(a)] and near the conducting surface [Fig. 4(b)]. The field strength, as earlier, we characterize by the Stark splitting Δ.

In Fig. 4 we see that the electric field can significantly affect the light trapping. The detailed analysis shows that the dependence of the time of radiation trapping on the Stark splitting is complex; in some cases it can even be nonmonotonic. So in Fig. 4 we see that this dependence predominantly decreases in the diapason from $\Delta = 0$ up to $\Delta \sim 3\gamma_0$. With further increasing of Δ, the time of radiation trapping slowly increases.

The physical reason for the nonmonotonic dependence of the time of radiation trapping on the magnitude of the constant electric field shown in Fig. 4 is caused by the nonmonotonic alteration of the mean free path of a photon with increasing of the electric field. The mean free path of a photon is affected by two warring factors. The first one is the increasing nonresonance between different atomic transitions, which leads to an increase of the mean free path of a photon at small Δ. With further increasing of Stark splitting, the second factor comes into play, namely, the weakening of the dipole-dipole interaction in dense media. At large values of Δ, the second factor becomes prevailing. Our analysis shows that this weakening causes a decrease of the cooperative Lamb shift and an increase of the coefficient of extinction, that, in turn, decreases the mean free path of a photon and increases the time of radiation trapping. Both factors described here manifest themselves differently at different atomic transitions, so the atomic medium is anisotropic even in the absence of a conducting surface, and the curves 1 and 2 do not coincide with each other even in Fig. 4(b). In the case of the presence of a conductive surface, the time of radiation trapping is significantly affected by an alteration of the dwell time, i.e., the lifetime of an atom in the excited state. When an atom is located near the surface, the dwell time for the sublevels $m = \pm 1$ is significantly larger than that for the sublevel $m = 0$.

Note that the position of the minimum on the curves shown in Fig. 4, its depth, as well as the slew rate of τ at large Δ significantly depend on the atomic density. When increasing the density, the line broadening caused by the dipole-dipole interaction increases, which weakens the effect of the first factor described above. At the same time, this enhances the second factor. Thus, when increasing the atomic density, the position of the minimum moves right, to larger values of Δ, and the slew rate increases.

To conclude this section of the paper, consider the influence of the inhomogeneous broadening caused by internal fields of the dielectric δ on the light trapping. Dependence of the trapping time τ on δ for different initial conditions of excitation is shown in Fig. 5. The calculations were performed for $\Delta = 0$. As δ increases, the mean free path of the photon also increases, because the role of cooperative multiple scattering becomes weaker. This leads to a monotonic decrease of the time of radiation trapping.

Note, however, that the suppression of collective effects with increasing in δ manifests itself more slowly as the density of impurity centers increases. Increasing of the density compensates negative influence of the inhomogeneous broadening on the collective effects. This fact is confirmed by comparison

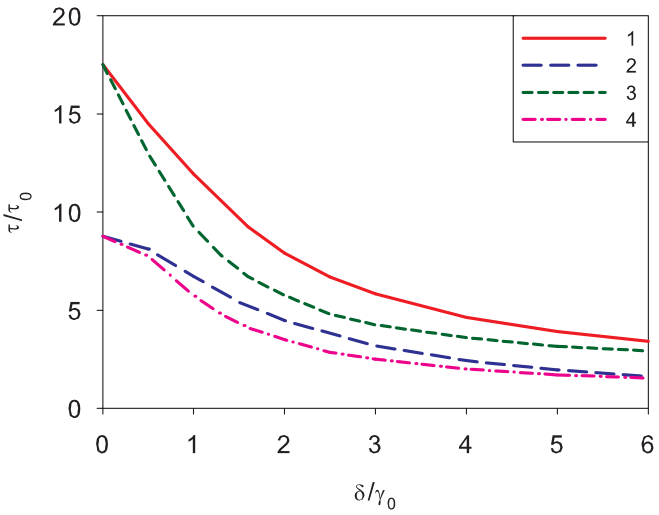

FIG. 5. The time of radiation trapping depending on the inhomogeneous broadening. 1 and 2, $n = 0.1$; 3 and 4, $n = 0.05$. 1 and 3, $m = \pm 1$; 2 and 4, $m = 0$.

of the curves shown in Fig. 5 which correspond to different atomic densities. Therefore, for high densities of impurities, very often used in experiments, influence of the dipole-dipole interaction can be significant even for large inhomogeneous broadening, $\delta \gg \gamma_0$.

When the sizes of a sample are fixed, increasing of the density obviously leads to an increase in the trapping time. For us now, it is not the absolute value of the trapping time that is important, but its decay rate with increasing in δ. For this reason, considering the ensembles of the density $n = 0.1$, we reduced the size so that for $\delta = 0$ the absolute value of the trapping time is the same as that for the density $n = 0.05$. Comparison of the curves 1 and 3, as well as 2 and 4, shows that, with density increasing, the mutual nonresonance of different impurities centers becomes less pronounced.

IV. CONCLUSION

We have studied many-body cooperative effects caused by the dipole-dipole interaction in an ensemble of pointlike impurity centers embedded into a transparent dielectric and located near a charged perfectly conducting surface. On the basis of the general quantum microscopic theory, we have analyzed the simultaneous influence of the surface and the electrostatic field on the transition spectrum of an excited atom inside an ensemble, as well as on the dynamics of the total excited-state population related to the whole ensemble. The cooperative Lamb shift depending on the electric field has been studied. The time of radiation trapping as a function of the electric-field strength and the inhomogeneous broadening has been investigated. We showed the nonmonotonic behavior of electro-optical effects in the presence of a surface with increasing of the electric field. We have demonstrated the nonadditive influence of the electric field and modified spatial structure of the field modes near the surface on the collective effects in dense atomic systems.

In our opinion, of special interest is the application of the theory described in the present paper for the investigation

of Anderson localization of light in quasi-two-dimensional ensembles of impurity centers embedded into a transparent dielectric and located near a charged conducting surface. This is associated with the fact that, in the systems of reduced dimensionality, cooperative phenomena have a number of nontrivial features that promote the Anderson localization. Moreover, the electrostatic field partially removes the degeneracy of the multiplet of the excited state, which additionally contributes to the strong localization of light [69]. Despite the absence of the Anderson localization in atomic ensembles in free space, even in the presence of the electric field, as it has been proved in [70], the combined effect of the surface and the electric field gives us hope to detect the Anderson localization.

One more promising direction for the development of the theory described in the present paper is its generalization to the analysis of the dipole-dipole interaction in atomic ensembles placed in a waveguide. The case when the resonant frequency of atomic transition is less than the cutoff frequency of the waveguide attracts particular interest due to spontaneous decay suppression of all the Zeeman sublevels. Moreover, the analysis of the atomic systems in a waveguide can be useful for the investigation of Anderson localization, because in quasi-one-dimensional systems all the collective states are localized [71,72].

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