

Damping of the Anderson-Bogolyubov mode in Fermi mixtures by spin and mass imbalancePiotr Zdybel^{✉*} and Pawel Jakubczyk[✉]*Institute of Theoretical Physics, Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland* (Received 22 August 2019; revised manuscript received 11 October 2019; published 25 November 2019)

We study the temporally nonlocal contributions to the gradient expansion of the pair fluctuation propagator for spin- and mass-imbalanced Fermi mixtures. These terms are related to the damping processes of soundlike (Anderson-Bogolyubov) collective modes and are relevant for the structure of the complex pole of the pair fluctuation propagator. We derive conditions under which damping occurs even at zero temperature for a large enough mismatch of the Fermi surfaces. We compare our analytical results with numerically computed damping rates of the Anderson-Bogolyubov mode.

DOI: [10.1103/PhysRevA.100.053622](https://doi.org/10.1103/PhysRevA.100.053622)**I. INTRODUCTION**

Progress in manipulating ultracold atomic setups [1–4] opens opportunities to experimentally study many-body problems hardly accessible in conventional solid-state systems. In particular, the ability to engineer fermionic mixtures with different particle masses [5–8] and populations [9–13] motivates detailed studies of the influence of imbalance on superfluidity. For instance, exotic superfluid phases such as the interior gap (Sarma-Liu-Wilczek) superfluids [14,15] or the nonuniform Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) states [16,17] are theoretically possible to realize in two-component mixtures with different spin populations and masses of particles. Of substantial interest is also the imbalance-induced phase transition between the uniform superfluid and normal phases. By manipulating the mass imbalance, the tricritical point may be shifted [18–20] or even expelled from the phase diagram, giving rise to a stable quantum critical point (QCP) [21,22]. Another aspect concerns the possible quantum phase transition to the FFLO state [23,24].

In addition to the thermodynamic properties, the excitation spectra of such superfluid systems and their evolution upon increasing imbalance are highly interesting. Generally one expects the occurrence of a gapless (Anderson-Bogolyubov) soundlike branch (discussed in the present paper) as well as a gapped amplitude mode. Note, however, that additional interesting features arise, for example, in two-band superfluids [25–28], or systems involving spin-orbit coupling [29–33].

One particularly interesting and generic problem concerns the properties of Anderson-Bogolyubov (AB) modes [34,35], also known as Nambu-Goldstone modes [36,37] in the low-momentum limit. According to the Goldstone theorem [38], the spontaneous breaking of continuous U(1) symmetry for the Fermi gas results in low-energy soundlike collective excitations. These modes have been successfully observed in several experiments [39–43] and studied extensively in numerous theoretical papers [44–53] in the last 20 years. However, most of these works pay relatively little attention to the

spin- and mass-imbalance influence on the excitation spectra and their damping in particular. The dominant mechanism of damping in such systems is related to the inelastic scattering of the Goldstone phonon from thermally excited fermionic quasiparticles [54]. The damping rate vanishes in the limit $T \rightarrow 0$ [54,55] because of the disappearance of the thermal cloud of quasiparticles [56]. This picture is consistent with the detailed analysis performed by Kurkjian and Tempere [57], which shows that the process of absorption and emission of the AB phonon by fermionic quasiparticles leads to exponentially suppressed damping at low temperatures in the presence of a gap. This temperature dependence is an essential characteristic of the so-called Landau damping [58] for gapped modes. However [59], large enough spin polarization of the Fermi gas leads to an enhancement of the damping factor even for relatively low temperatures. This suggests a relation between a mismatch of Fermi surfaces corresponding to the two-particle species forming the mixture and the mechanism of the damping process.

It is therefore worth taking a closer look at the problem of the impact of spin and mass imbalance on the Landau damping. The present paper contributes to an analytical understanding of the damping process by considering the structure of the Gaussian pair fluctuation (GPF) propagator in the low-momentum limit ($q \rightarrow 0$). We derive an inequality involving parameters of the system, giving a necessary condition to obtain a nonzero damping rate of the AB mode in a uniform s -wave superfluid in the presence of both spin and mass imbalance. Our central result indicates that for a large enough mismatch of the Fermi surfaces, the AB modes are damped even at $T = 0$. We formulate an intuitive interpretation of this result and relate it to the mechanism of Landau damping. We subsequently compare the conclusions drawn from the analytical results with numerically obtained complex poles ($z_q = \omega_q - i\Gamma_q/2$) of the GPF propagator, where the dispersion relation of the collective mode and its damping rate are given, respectively, by the real and imaginary part of z_q .

The paper is organized as follows. In Sec. II, we introduce the considered model using the path-integral formalism and discuss the structure of the GPF propagator. In Sec. III, we employ the gradient expansion to extract the leading terms of

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the GPF propagator responsible for damping. We formulate conditions under which Landau damping is active even for $T = 0$ and present an intuitive interpretation of this result. Section IV contains a numerical study of the poles of the GPF propagator. We discuss the obtained dispersion relations and damping rates of the AB modes for different realizations of spin and mass imbalance and compare the results with analytical expressions from Sec. III. We summarize the paper and give a perspective for future studies in Sec. V.

II. PAIR FLUCTUATION PROPAGATOR

We consider a two-component spin-polarized Fermi mixture with unequal masses in thermodynamic equilibrium. Particles with opposite spins interact via an attractive contact potential $\mathcal{V}(\mathbf{x}, \mathbf{y}) = g\delta(\mathbf{x} - \mathbf{y})$, where $g < 0$. Utilizing the path-integral formalism [60], we obtain the grand-canonical partition function represented as a functional integral over the Grassmann fields $\{\bar{\psi}_x^\sigma, \psi_x^\sigma\}$,

$$Z = \int \mathcal{D}[\bar{\psi}_x^\sigma, \psi_x^\sigma] \exp(-S_\psi), \quad (1)$$

where

$$S_\psi = \int_x \left\{ \sum_\sigma \bar{\psi}_x^\sigma [\partial_\tau + \hat{K}^\sigma] \psi_x^\sigma + g \bar{\psi}_x^+ \bar{\psi}_x^- \psi_x^- \psi_x^+ \right\} \quad (2)$$

is the fermionic action. Throughout the paper we put $\hbar = 1$ and $k_B = 1$. In the above equations, we use the following notation: $x = (\tau, \mathbf{x})$, $\int_x(\cdot) = \int_0^\beta d\tau \int d^d\mathbf{x}(\cdot)$, $\beta = 1/T$, and $\hat{K}^\sigma = -\nabla_x^2/2m_\sigma - \mu_\sigma$, where m_σ and μ_σ are the mass and chemical potential of a particle with spin $\sigma \in \{+, -\}$, respectively. The presence of two distinct Fermi surfaces in this problem makes it convenient to introduce the imbalance parameters. We define $\zeta = \frac{r-1}{r+1}$, where $r = m_-/m_+$. We also use $h = (\mu_+ - \mu_-)/2$ as the ‘‘Zeeman’’ field, which measures the spin polarization and $\mu = (\mu_+ + \mu_-)/2$ as the average chemical potential.

The standard procedure to analyze the Gaussian fluctuations is to integrate out the Grassmann fields ψ_x^σ by introducing an auxiliary field η_x via the Hubbard-Stratonovich transformation [44,48], which decouples the interaction term in Eq. (2) into the Cooper channel [60]. Afterwards, we expand η_q (in reciprocal space) around the mean-field value of the superfluid gap Δ in such a way that $\eta_q = \sqrt{\beta V} \Delta \delta_{q,0} + \phi_q$, where V is the volume of the system and ϕ_q describes fluctuations of the order parameter. Thus, the expansion to the quadratic order in ϕ_q leads to a Gaussian action. For a comprehensive discussion of this procedure, we refer to the paper by Iskin and Sá de Melo [48]. Alternatively, the same result can be obtained using diagrammatic theory by a resummation of the infinite subclass of ladder diagrams (the random phase approximation) [61,62].

As a result, we obtain the partition function within the GPF approximation, which is given by

$$Z = Z_{\text{MF}} \int \mathcal{D}[\phi] \exp\left(-\frac{\beta V}{2} \int_q \Phi_q^* \mathbb{F}_q^{-1} \Phi_q\right), \quad (3)$$

where Z_{MF} is the mean-field part of the partition function, $\Phi_q^* = [\phi_q^*, \phi_{-q}]$, $\Phi_q = [\phi_q, \phi_{-q}^*]^T$, and \mathbb{F}_q is the GPF propagator matrix. In Eq. (3), $q = (iq_m, \mathbf{q})$ collects a bosonic

Matsubara frequency [$q_m = 2\pi m/\beta$ ($m \in \mathbb{Z}$)] and the (d -dimensional) wave vector \mathbf{q} . We also introduce the fermionic analog $k = (ik_n, \mathbf{k})$, where $k_n = 2\pi(n + \frac{1}{2})/\beta$ ($n \in \mathbb{Z}$). We use the shorthand notation $\int_q(\cdot) = \frac{1}{\beta} \sum_{q_m} \int \frac{d^d\mathbf{q}}{(2\pi)^d}(\cdot)$. Matrix elements of the inverse GPF propagator are expressed by normal (\mathcal{G}_k^σ) and anomalous (\mathcal{F}_k) components of the Green’s-function matrix [63],

$$[\mathbb{F}_q^{-1}]_{1,1} \equiv M_{1,1}(q) = \frac{1}{g} - \int_k \mathcal{G}_{k+q}^+ \mathcal{G}_{-k}^-, \quad (4)$$

$$[\mathbb{F}_q^{-1}]_{1,2} \equiv M_{1,2}(q) = \int_k \mathcal{F}_{k+q} \mathcal{F}_{-k}, \quad (5)$$

where $M_{1,1}(q) = M_{2,2}(-q)$, $M_{1,2}(q) = M_{2,1}^*(q)$, while the BCS-like Green’s functions are given by

$$\mathcal{G}_k^+ = \frac{u_k^2}{ik_n - E_k^+} + \frac{v_k^2}{ik_n - E_k^-}, \quad (6)$$

$$-\mathcal{G}_{-k}^- = \frac{v_k^2}{ik_n - E_k^+} + \frac{u_k^2}{ik_n - E_k^-}, \quad (7)$$

$$\mathcal{F}_k = u_k v_k^* \left(\frac{1}{ik_n - E_k^-} - \frac{1}{ik_n - E_k^+} \right). \quad (8)$$

Here, we use the BCS coherence factors given by $u_k^2 = (1 + \xi_k/E_k)/2$ and $v_k^2 = 1 - u_k^2$, where $\xi_k = \mathbf{k}^2/2m - \mu$, $E_k = \sqrt{\xi_k^2 + |\Delta|^2}$, and $m = \frac{2r}{r+1}m_+$. E_k^σ is an excitation energy of quasiparticle branches in the superfluid phase,

$$E_k^\sigma = \zeta \xi_k - (h - \zeta \mu) + \sigma E_k. \quad (9)$$

It is worth noting that $h = \zeta \mu$ corresponds to a situation where the two Fermi surfaces coincide. Therefore, $h - \zeta \mu$ measures the mismatch of Fermi spheres due to spin and mass imbalance. For a further discussion of the matrix elements of \mathbb{F}_q^{-1} , see Appendix A.

In order to obtain the mean-field value of the superfluid gap Δ , we consider the contribution $\Omega_{\text{MF}} = -T \ln Z_{\text{MF}}$ to the grand-canonical potential. This is given by [22]

$$\Omega_{\text{MF}} = V \min_{\Delta} \left\{ -\frac{|\Delta|^2}{g} + T \int_{\mathbf{k}} \sum_{\sigma} \ln f(-E_k^\sigma) \right\}, \quad (10)$$

where $\int_{\mathbf{k}}(\cdot) = \int \frac{d^d\mathbf{k}}{(2\pi)^d}(\cdot)$ and $f(x) = 1/[\exp(\beta x) + 1]$. The order parameter minimizing the grand-canonical potential for a given set of parameters is identified as the expectation value of the superfluid gap Δ .

III. DAMPING OF COLLECTIVE MODES

We now set out to analyze the complex pole of the GPF propagator. For this purpose, we expand the matrix elements of \mathbb{F}_q^{-1} in the low-momentum limit and extract the relevant nonlocal terms, which are related to the damping process. An analogous strategy is applied to derive the Hertz-Milliss-Moriya action in the context of quantum phase transitions in itinerant electron systems [64–67]. In this case, the nonlocal term $|q_m|/\gamma_q$ appearing in the Gaussian action after the expansion for small $|\mathbf{q}|$ and $|q_m|/|\mathbf{q}|$ is related to the Landau damping of collective spin fluctuations by particle-hole excitations [66]. This term is responsible for the occurrence of the complex pole of the propagator of paramagnons and γ_q is its damping rate [64]. The noticeable structural resemblance

between the Hertz approach and our problem encouraged us to exploit this procedure to investigate the Landau damping of the AB mode in the spin- and mass-imbalanced Fermi mixture.

We begin with a brief discussion of the gradient expansion along the line of Refs. [50,51]. We obtain a low-momentum and low-frequency expansion of the matrix elements $M_{j,l}(iq_m; \mathbf{q})$ [see Eq. (A1) and (A2)] up to the second order in powers of q_m and \mathbf{q} ,

$$M_{1,1}(iq_m; \mathbf{q}) = M_{1,1}(0; \mathbf{0}) + A\mathbf{q}^2 + iBq_m + Cq_m^2, \quad (11)$$

$$M_{1,2}(iq_m; \mathbf{q}) = M_{1,2}(0; \mathbf{0}) + D\mathbf{q}^2 + Eq_m^2. \quad (12)$$

The expressions for the coefficients of the gradient expansion are presented in Appendix B. This procedure neglects terms proportional to $|q_m|/|\mathbf{q}|$, which are crucial in the description of damping. To identify them, we analyze the full expression for $M_{1,1}(iq_m; \mathbf{q}) - M_{1,1}(0; \mathbf{q})$. Using the notation described in detail in Appendix A, we start from the following form,

$$\begin{aligned} M_{1,1}(iq_m; \mathbf{q}) - M_{1,1}(0; \mathbf{q}) \\ = - \int_{\mathbf{k}} \sum_{\sigma, \sigma'} C_{k,q}^{\sigma, \sigma'} \frac{f_{k,q}^{\sigma, \sigma'}}{q_m^2 + (E_{k,q}^{\sigma, \sigma'})^2} \left(iq_m - \frac{q_m^2}{E_{k,q}^{\sigma, \sigma'}} \right). \end{aligned} \quad (13)$$

The leading contribution involving *both* small frequency and momentum [and therefore not included in the expansion of Eq. (11)] comes from the second term in the parentheses in Eq. (13) for the elements with $\sigma = \sigma'$ and is given by

$$- \int_{\mathbf{k}} u_k^2 v_k^2 \sum_{\sigma} f'(E_k^{\sigma}) \frac{q_m^2}{a_{\sigma}^2(|\mathbf{k}|) \cos^2 \theta |\mathbf{q}|^2 + q_m^2}, \quad (14)$$

where $\cos \theta = \frac{\mathbf{k} \cdot \mathbf{q}}{|\mathbf{k}| |\mathbf{q}|}$, $f'(x) = -\frac{\beta}{4} \cosh^{-2}(\beta x/2)$, and $a_{\sigma}(|\mathbf{k}|) = (\frac{\zeta}{m} + \sigma \frac{\xi_k}{m E_k}) |\mathbf{k}|$. All the other contributions (as long as $|\Delta| > 0$) in the expansion of Eq. (13) are either of higher order or included in Eq. (11).

In the next step we perform integration over the angular variable θ , considering separately the cases $d = 2$ and $d = 3$. We assume that the ratio $|q_m|/|\mathbf{q}|$ is small [64,65]. Note that this is possible only if the mode in question is gapless [50]. In Eq. (14) we make the change of variables, $|\mathbf{k}| \rightarrow \varepsilon = \mathbf{k}^2/2m \geq 0$, and carry out the integration. This yields

$$- \frac{|q_m|}{|\mathbf{q}|} \int d\varepsilon c_d D_d(\varepsilon) u_{\varepsilon}^2 v_{\varepsilon}^2 \sum_{\sigma} \frac{f'(E_{\varepsilon}^{\sigma})}{|a_{\sigma}(\varepsilon)|} = \frac{|q_m|}{\gamma_q}, \quad (15)$$

where c_d is equal 1 for $d = 2$ and $\pi/2$ for $d = 3$. We also introduce the density of states per spin $D_d(\varepsilon)$, where $D_2(\varepsilon) = m/2\pi$ and $D_3(\varepsilon) = \frac{\sqrt{2m^3}}{2\pi^2} \varepsilon^{1/2}$. Equation (15) defines the quantity γ_q . An analogous procedure performed for the matrix element $M_{1,2}(iq_m; \mathbf{q})$ results in the same expression as in Eq. (15). We conclude that relations (11) and (12) should be supplemented by the nonlocal contributions obtained in Eq. (15).

We now consider the form of γ_q^{-1} in the zero-temperature limit. Using $f'(E_{\varepsilon}^{\sigma}) \rightarrow -\delta(-E_{\varepsilon}^{\sigma})$ for $T \rightarrow 0$, we obtain

$$\gamma_q^{-1} = \frac{1}{|\mathbf{q}|} \int d\varepsilon c_d D_d(\varepsilon) u_{\varepsilon}^2 v_{\varepsilon}^2 \sum_{\sigma} \frac{\delta(-E_{\varepsilon}^{\sigma})}{|a_{\sigma}(\varepsilon)|}. \quad (16)$$

Taking advantage of the identity

$$\delta[h(x)] = \sum_i \frac{\delta(x - x_i)}{|h'(x_i)|}, \quad (17)$$

where x_i are roots of $h(x)$, we further simplify Eq. (16). The equation $E_{\varepsilon}^{\sigma} = 0$ has two solutions, which are identical for $\sigma = +$ and $\sigma = -$ and given by

$$\varepsilon_{1,2} = \frac{\mu - \zeta h \pm \sqrt{(h - \zeta \mu)^2 - \Delta^2(1 - \zeta^2)}}{1 - \zeta^2}. \quad (18)$$

Since $\varepsilon = \mathbf{k}^2/2m$ is non-negative, we pick only roots fulfilling $\varepsilon_i \geq 0$. Making use of Eq. (17), we integrate over ε , which leads to

$$\gamma_q^{-1} = \frac{1}{|\mathbf{q}|} \sum_{\substack{i=1,2; \\ \text{if } \varepsilon_i \geq 0}} c_d D_d(\varepsilon_i) u_{\varepsilon_i}^2 v_{\varepsilon_i}^2 \sum_{\sigma} L_{\sigma}^{-1}(\varepsilon_i), \quad (19)$$

where $\ell_i = \sqrt{2m\varepsilon_i}$ and $L_{\sigma}(\varepsilon) = \sqrt{m/2\varepsilon} |a_{\sigma}(\ell)|^2$. We now discuss the implications of Eq. (19). First of all, we observe that $\gamma_q^{-1} \sim u_k^2 v_k^2 = \Delta^2/4E_k^2$. Therefore, γ_q^{-1} vanishes in the limit $\Delta \rightarrow 0$. Moreover, $u_k^2 v_k^2$ takes a maximal value for $k = \sqrt{2m\mu}$.

As we mentioned above, ε_i in Eq. (19) should be non-negative. This leads to the necessary condition for the occurrence of Landau damping. Indeed, when the requirement

$$|h - \zeta \mu| > \Delta \sqrt{1 - \zeta^2} \quad (20)$$

is met, there are two real roots of $E_{\varepsilon}^{\sigma} = 0$. In particular, whenever the two Fermi spheres coincide ($h = \zeta \mu$) we see that $\gamma_q^{-1} = 0$ for $T = 0$ and the Goldstone mode is not damped. Nonetheless, compliance with the condition in Eq. (20) does not guarantee fulfillment of $\varepsilon_i \geq 0$. For simplicity, let us now focus on the situation, where $h - \zeta \mu \geq 0$ and then consider $\varepsilon \geq 0$. This leads to

$$\begin{cases} h - \zeta \mu \geq \Delta \sqrt{1 - \zeta^2} & \text{for } \mu \geq \frac{\zeta \Delta}{\sqrt{1 - \zeta^2}}, \\ h \geq \sqrt{\mu^2 + \Delta^2} & \text{for } \mu < \frac{\zeta \Delta}{\sqrt{1 - \zeta^2}}. \end{cases} \quad (21)$$

The first inequality, in the above condition, assures the existence of at least one positive zero of E_{ε}^+ (see Fig. 1) and the second one corresponds to exactly one zero (see Fig. 2).

We can now interpret the obtained results in the context of the mechanism of Landau damping. Let us for now assume that $r \in [1, \infty[$ so that $\zeta \in [0, 1[$. The quasiparticle spectrum has two branches [see Eq. (9)]. If the two Fermi spheres coincide ($h - \zeta \mu = 0$), then the lower branch E_{ε}^- is filled, and the upper one E_{ε}^+ is empty [56]. In this case, the Landau damping is present only at nonzero temperatures (and is exponentially suppressed). The Goldstone phonon inelastically scatters thermally excited quasiparticles in the upper branch of the spectrum. Cranking up the mismatch of the Fermi surfaces leads to nonzero occupancy of fermions on E_{ε}^+ even at $T = 0$. Therefore, the Landau damping is present also at $T = 0$. We depicted this situation in Figs. 1 and 2. The position of the minimum of E_{ε}^+ is given by $\varepsilon_{\min} = \mu - \zeta \Delta / \sqrt{1 - \zeta^2}$ and at that point E_{ε}^+ is equal to $\Delta \sqrt{1 - \zeta^2}$. Whenever $\varepsilon_{\min} \geq 0$, the minimal mismatch of the Fermi spheres, which leads to nonzero occupancy of quasiparticles in the upper branch, is

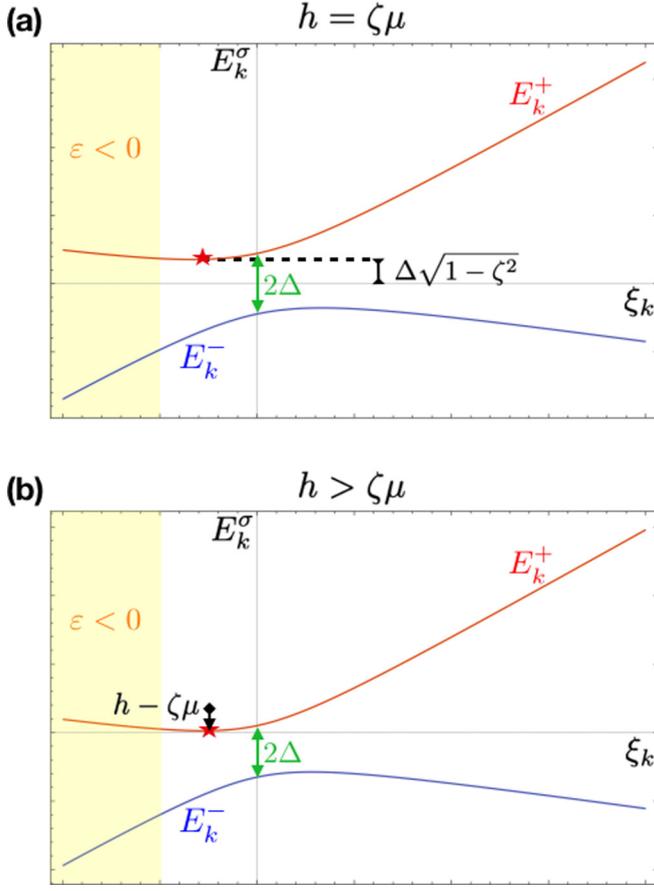


FIG. 1. Schematic illustration of the fermionic quasiparticle spectrum E_k^σ [see Eq. (9)] as a function of $\xi_k = \epsilon - \mu$, when the minimum of E_k^+ is located in the physical region ($\epsilon \geq 0$). The red star indicates the minimum of E_k^+ and in the shaded area we have $\epsilon < 0$. (a) In this case, the two Fermi spheres coincide ($h - \zeta\mu = 0$). The lower branch of the quasiparticle spectrum is fully occupied while the upper branch is empty. In this case, the Landau damping involves the inelastic scattering of the Goldstone phonon from thermally excited quasiparticles and becomes inactive for $T \rightarrow 0$. (b) The mismatch of the Fermi surfaces leads to nonzero occupation of the upper branch, whenever the condition $h - \zeta\mu \geq \Delta\sqrt{1 - \zeta^2}$ is fulfilled. In this case, the damping process occurs also at zero temperature.

given by $\Delta\sqrt{1 - \zeta^2}$ (see Fig. 1). If $\epsilon_{\min} < 0$, the minimal mismatch leading to nonzero occupancy in E_ϵ^+ is given by a value of E_ϵ^+ for $\epsilon = 0$ (see Fig. 2). This provides an interpretation of Eq. (21). We see that in the limit $r \rightarrow \infty$ the obtained condition is independent of ζ and is given by $h \geq \sqrt{\mu^2 + \Delta^2}$, whereas for $r = 1$ the considered condition is given by $h \geq \Delta$, which is consistent with the results of Ref. [59]. We emphasize that the above results require the presence of a superfluid gap ($\Delta > 0$). As we show in the next section for experimentally motivated choices of the mass-imbalance parameter r , the inequality (21) becomes fulfilled for h substantially lower than the critical value h_c , such that damping is present in a broad region of the phase diagram within the superfluid phase. We also note that the occurrence of damping is not interrelated with the order of the phase transition to the normal phase.

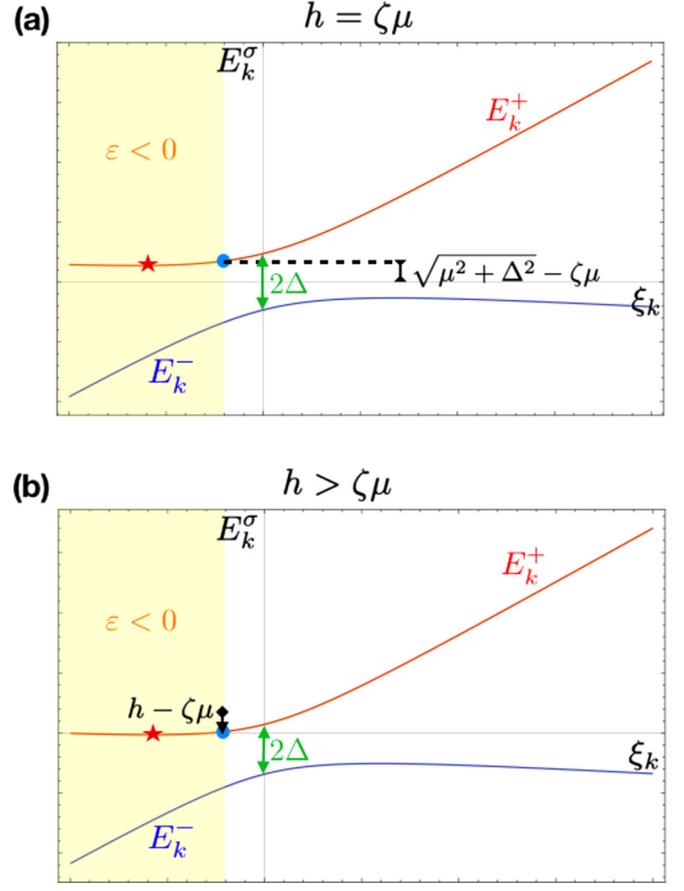


FIG. 2. The fermionic quasiparticle spectrum E_k^σ [see Eq. (9)] as a function of $\xi_k = \epsilon - \mu$, when the minimum of E_k^+ is located in the unphysical region ($\epsilon < 0$). The red star indicates the minimum of E_k^+ , the blue dot corresponds to $\epsilon = 0$, and in the shaded area we have $\epsilon < 0$. (a) In this case, the Fermi spheres coincide ($h - \zeta\mu = 0$). The upper branch of the quasiparticle spectrum is empty (the lower one is fully occupied), therefore the Landau damping is possible only due to the presence of thermal excitations from E_ϵ^- to E_ϵ^+ . (b) The mismatch of the Fermi surfaces leads to the nonzero occupation of the upper branch, when the condition $h - \zeta\mu \geq \sqrt{\mu^2 + \Delta^2} - \zeta\mu$ is met. In this case, Landau damping is active even at $T = 0$.

We close this section by considering the Landau damping in the proximity of a QCP [21,22], which can be generated for a wide range of system parameters. At the mean-field level, in the limit $h \rightarrow h_c^-$ (h_c is the critical value of h), the superfluid gap Δ goes continuously to 0. In the vicinity of the QCP, the condition $\epsilon_1 \geq 0$ yields

$$h + \mu - \frac{1 + \zeta}{2(h - \zeta\mu)} \Delta^2 + O(\Delta^4) \geq 0. \quad (22)$$

According to Ref. [22], the above condition is always fulfilled for $r > 3.01$ and $h > \mu \geq 0$. Therefore, in situations where the quantum phase transition is continuous, the Landau damping is unavoidably present in the proximity of the QCP.

IV. NUMERICAL RESULTS

In this section we numerically study the damping of the collective mode by analyzing the complex pole of the GPF

propagator. This amounts to finding complex roots of the following equation [44,53,59],

$$\det \mathbb{F}^{-1}(iq_m \mapsto z_q; \mathbf{q}) = 0, \quad (23)$$

where $z_q = \omega_q - i\Gamma_q/2$, ω_q is the dispersion relation, and Γ_q is the damping rate. The matrix elements $M_{j,l}(z_q, \mathbf{q})$ of \mathbb{F}_q^{-1} have a branch cut along the real axis [53]. We should perform the analytic continuation of $M_{j,l}(z_q, \mathbf{q})$ from the upper to the lower complex half plane, which results in a transition to another Riemann sheet. We proceed along the way described by Nozières [68]. We consider the quantity $A_{j,l}(\omega; \mathbf{q})$,

$$A_{j,l}(\omega; \mathbf{q}) = -\frac{1}{\pi} \text{Im} M_{j,l}^{(R)}(\omega; \mathbf{q}), \quad (24)$$

where the index (R) means that we take the retarded matrix element $M_{j,l}(iq_m \mapsto \omega + i0^+; \mathbf{q})$. Then the matrix elements $\tilde{M}_{j,l}$ analytically continued to the lower half plane are given by [53,59]

$$\tilde{M}_{j,l}^{(A)}(\omega; \mathbf{q}) = M_{j,l}^{(A)}(\omega; \mathbf{q}) - 2\pi i A_{j,l}(\omega; \mathbf{q}), \quad (25)$$

where the index (A) denotes the advanced counterpart of the matrix element ($iq_m \mapsto \omega - i0^+$). $\tilde{M}_{j,l}$ thus obtained can be extended in such a way that $\omega \mapsto z = \omega - i\Gamma/2$, where $\Gamma > 0$.

Using the procedure specified above (see Refs. [53,59,68]) we discuss the numerically obtained dispersion relations ω_q and damping rates Γ_q for $r = 1.0$ and $r = 6.67$, varying the Zeeman field h at $T \rightarrow 0$. We consider the three-dimensional case ($d = 3$). We *a posteriori* check the fulfillment of the condition $v_s = \lim_{|\mathbf{q}| \rightarrow 0} \omega_q/|\mathbf{q}| < 1$ (compare Sec. III). This condition ensures that the assumptions made in derivation of Eq. (16) are justified. We begin with the mass-balanced case ($r = 1$). The phase transition between the normal and superfluid phases is generically discontinuous for $r < 3.01$ and $T \rightarrow 0$ at the mean-field level [22]. Therefore, we expect that for $r = 1$ the change of Δ as a function of h should be modest up to the occurrence of the phase transition. In this case, the frequencies and damping factors of the Goldstone mode as a function of momentum $|\mathbf{q}|$ are shown in Fig. 3. The results are not affected by varying h between 0.0 and 0.4164, where the discontinuous phase transition takes place. The reason for this is a negligible change of Δ upon approaching the transition point. For all values of h considered in Fig. 3 the ratio $T/\Delta < 0.01$, which means that the thermal excitations should be negligible. For $r = 1$ ($\zeta = 0$) we always obtain $\mu > \zeta \Delta / \sqrt{1 - \zeta^2} = 0$. Therefore, the condition in Eq. (21) takes the form $h \geq \Delta$, which is never fulfilled for the discussed situation. That implies that the Landau damping is absent in the limit $T \rightarrow 0$ in compliance with the numerical results shown in Fig. 3 (the numerically obtained damping rates, in this case, are of the order of 10^{-11}). These results are consistent with Refs. [51,53].

We now examine the mass-imbalanced case, fixing the mass ratio $r = 6.67$ corresponding to a ${}^6\text{Li}$ and ${}^{40}\text{K}$ mixture [48]. We choose the parameters so that the system hosts a QCP in its phase diagram. Note, however, that this is of no relevance for the occurrence of Landau damping. In this case, the QCP at the mean-field level is located at $h_c = 1.9706$. The corresponding dispersion relations and damping rates of the

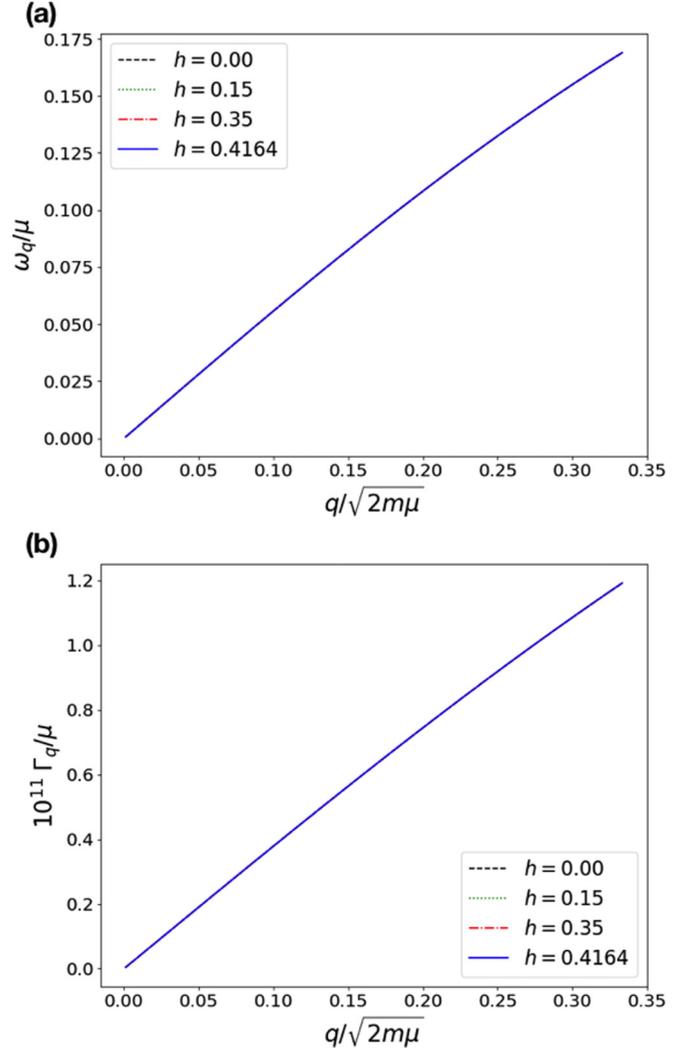


FIG. 3. (a) Dispersion relations ω_q of Goldstone modes (in units of μ) as a function of momentum for $r = 1$ and several values of h . For small enough values of $|\mathbf{q}|/\sqrt{2m\mu}$, we have $\omega_q = v_s|\mathbf{q}| + O(|\mathbf{q}|^3)$. (b) Analogous figure for damping rates Γ_q (in units of μ). All the curves coincide because of the weak dependence of the gap Δ on h . The damping factors are negligibly small. The plot parameters are $m_+ = 1$, $r = 1$, $\mu = 0.5$, $T = 0.005$, $g = -2.0$, and $\Lambda = 10$, where Λ is the upper momentum cutoff.

Goldstone phonons are shown in Fig. 4 for a few values of $h < h_c$. Since $T/\Delta < 0.04$ for all h in Fig. 4, we can reliably neglect thermal excitations. Furthermore, it turns out that for all the considered values of the Zeeman field we can apply the criterion $h \geq \sqrt{\mu^2 + \Delta^2}$ for the occurrence of the Landau damping [see Eq. (21)]. First, we observe that for $h = 1.4$ the above condition is not met. Thus, the upper branch of the quasiparticle spectrum is not populated, which means that the damping mechanism discussed in Sec. III is inactive. This is in agreement with the numerical results, which show that the damping rate is of the order of 10^{-10} . Second, for $h = 1.6$ we observe that $h \simeq \sqrt{\mu^2 + \Delta^2} = 1.59$. The obtained numerical values of Γ_q/μ are of the order of 10^{-5} , which is way larger than the value obtained for $h = 1.4$. In the remaining cases, the considered condition is fulfilled. In consequence, the

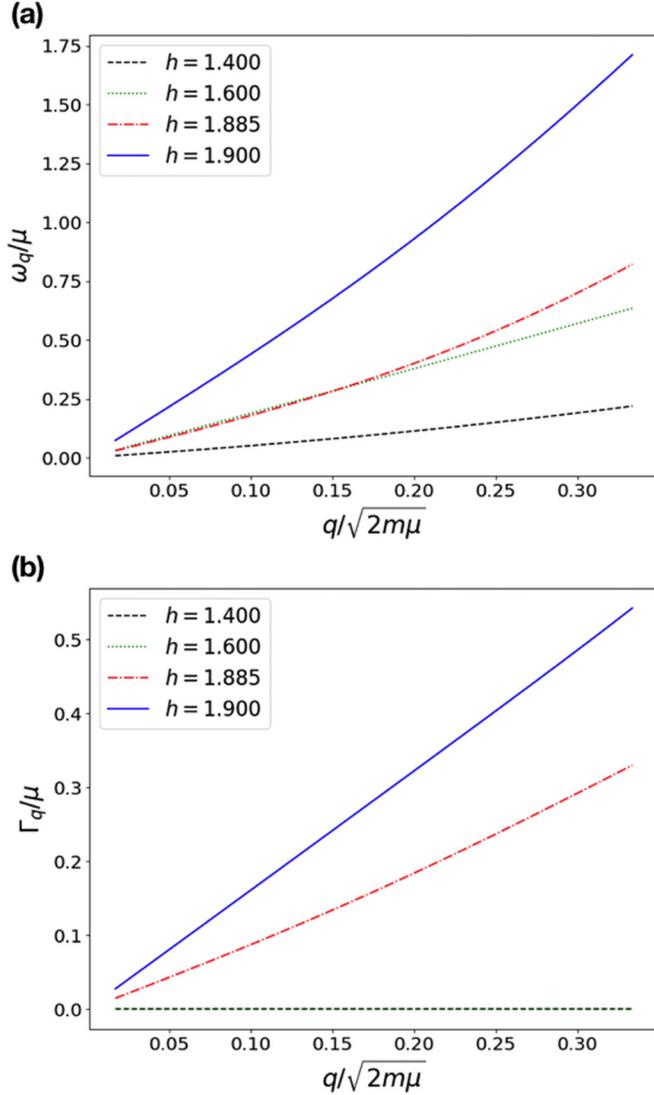


FIG. 4. (a) Dispersion relations ω_q of Goldstone modes (in units of μ) as a function of momentum for $r = 6.67$ and several values of h . For small enough values of $|\mathbf{q}|/\sqrt{2m\mu}$, we have that $\omega_q = v_s|\mathbf{q}| + O(|\mathbf{q}|^3)$. (b) Analogous figure for damping rates Γ_q (in units of μ). Landau damping becomes active above $h \approx 1.59$, which is substantially lower than the critical value $h_c \approx 1.97$. The plot parameters are $m_+ = 1$, $r = 6.67$, $\mu = 0.1$, $T = 0.04$, $g = -1.4$, and $\Lambda = 10$.

upper branch of the excitation spectrum is partially occupied by quasiparticles even for $T \rightarrow 0$. Therefore, Goldstone modes can be absorbed by fermionic excitations and the Landau damping is present. As we see in Fig. 4, this prediction is consistent with numerical results. Moreover, the dependence Γ_q/μ on h is shown in Fig. 5 for $|\mathbf{q}|/\sqrt{2m\mu} = 4.24 \times 10^{-3}$. We see that the activation of damping occurs precisely for the predicted value of h .

The choice of parameters discussed above corresponds to a situation where the phase transition between the normal and superfluid states is continuous. The obtained conclusion is, however, not sensitive to the order of the transition. To demonstrate this explicitly, we now fix the system parameters such that the phase diagram features a first-order phase

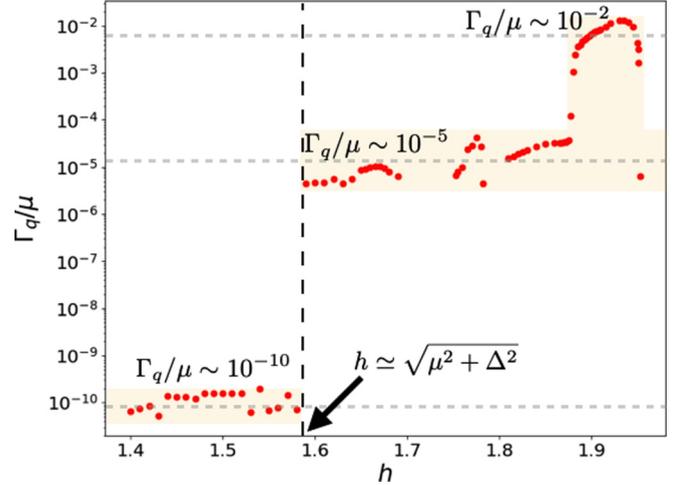


FIG. 5. Damping rates Γ_q of Goldstone modes (in units of μ and with a logarithmic scale) as a function of the Zeeman field for $r = 6.67$ and $|\mathbf{q}|/\sqrt{2m\mu} = 4.24 \times 10^{-3}$. Damping is active for $h \gtrsim \sqrt{\Delta^2 + \mu^2}$. Moreover, for values of h corresponding to the vicinity of the phase transition ($h \in [1.88, 1.95]$), damping becomes significantly stronger. The plot parameters are $m_+ = 1$, $r = 6.67$, $\mu = 0.1$, $T = 0.04$, $g = -1.4$, and $\Lambda = 10$. A continuous phase transition between the superfluid and normal phase occurs at $h_c = 1.9706$.

transition at low T . We set $r = 3.47$, $\mu = 0.1$, and $g = -1.4$. In this case a first-order quantum phase transition is located at $h_l = 0.59106$ (at the mean-field level). The numerically obtained dependence of Γ_q/μ on h is shown in Fig. 6 for $|\mathbf{q}|/\sqrt{2m\mu} = 0.0179$. According to the results of Sec. III, damping is expected to occur for $h \gtrsim \sqrt{\Delta^2 + \mu^2} = 0.563$, which very well agrees with the numerical data of Fig. 6.

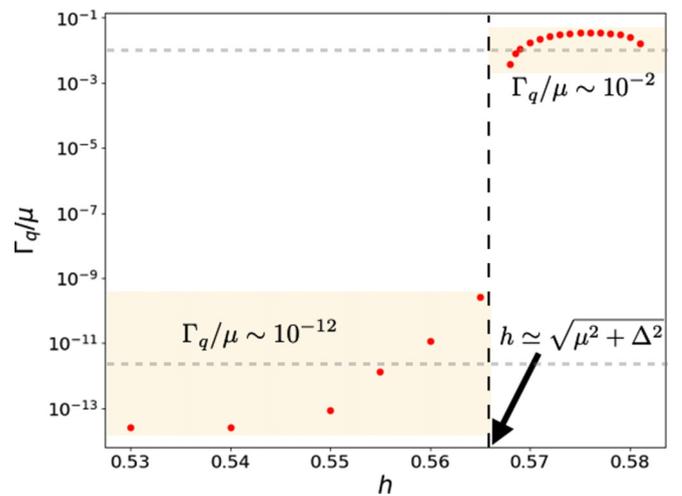


FIG. 6. Damping rates Γ_q of Goldstone modes (in units of μ and with a logarithmic scale) as a function of the Zeeman field for $r = 3.47$ and $|\mathbf{q}|/\sqrt{2m\mu} = 0.0179$. Damping is active for $h \gtrsim \sqrt{\Delta^2 + \mu^2}$. In this case, the phase diagram hosts a first-order phase transition for $h_l = 0.59106$. The plot parameters are $m_+ = 1$, $r = 3.47$, $\mu = 0.1$, $T = 0.002$, $g = -1.4$, and $\Lambda = 10$.

V. CONCLUSION AND OUTLOOK

We have studied the damping process of the Goldstone mode for spin- and mass-imbalanced Fermi mixtures by inspecting the structure of the pair fluctuation propagator. A detailed analysis based on the gradient expansion reveals the presence of a temporally nonlocal contribution in its matrix elements, giving rise to Landau damping. We have demonstrated that the Landau damping is activated by increasing the imbalance even for $T \rightarrow 0$ and is present for a large enough mismatch of the Fermi surfaces. We have derived an analytical criterion for its occurrence [see Eq. (21)]. We also provided an intuitive interpretation of the obtained analytical results. Finally, going beyond the gradient expansion, we have shown that our analytical predictions are in full agreement with the damping rates obtained numerically from complex roots of the analytically continued determinant of the inverse pair fluctuation propagator.

There are several interesting avenues for further research in this direction. The present analysis is performed at the Gaussian level (equivalent to the random-phase approximation), under the assumption of the presence of fully developed long-ranged order. It might be very interesting to investigate the evolution of the obtained physical picture after accounting for fluctuation effects. These should be of substantial relevance, in particular, in low dimensions, where the long-ranged ordered state becomes downgraded to the algebraic (Kosterlitz-Thouless) phase. Another question concerns the influence of the competing FFLO phase (characterized by a nonzero ordering wave vector) on the excitation spectra. Even though theoretical results [69–73] suggest that, in case of the neutral Fermi superfluids, these pair density wave states are unstable at $T > 0$, they are presumably still present as ground states. We finally note the interesting question concerning the impact of imbalance on damping of the amplitude mode, whose existence was recently experimentally established [43,74–77].

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APPENDIX A: MATRIX ELEMENTS OF \mathbb{F}_q^{-1}

In this Appendix, we present the explicit form of the matrix elements of \mathbb{F}_q^{-1} [see Eqs. (4) and (5)]. We assume without loss of generality that Δ , u_k and $v_k \in \mathbb{R}$. The considered matrix elements are given by

$$M_{1,1}(iq_m; \mathbf{q}) = \frac{1}{g} + \int_{\mathbf{k}} \sum_{\sigma, \sigma'} C_{k,q}^{\sigma, \sigma'} \frac{f_{k,q}^{\sigma, \sigma'}}{iq_m - E_{k,q}^{\sigma, \sigma'}}, \quad (\text{A1})$$

$$M_{1,2}(iq_m; \mathbf{q}) = \int_{\mathbf{k}} \sum_{\sigma, \sigma'} \mathcal{D}_{k,q}^{\sigma, \sigma'} \frac{f_{k,q}^{\sigma, \sigma'}}{iq_m - E_{k,q}^{\sigma, \sigma'}}, \quad (\text{A2})$$

where $E_{k,q}^{\sigma, \sigma'} = -(E_{-k}^{\sigma} - E_{k+q}^{\sigma'})$, $f_{k,q}^{\sigma, \sigma'} = f(E_{-k}^{\sigma}) - f(E_{k+q}^{\sigma'})$, $\mathcal{D}_{k,q}^{\sigma, \sigma'} = \sigma \cdot \sigma' \cdot u_{-k} v_{-k} u_{k+q} v_{k+q}$, and

$$C_{k,q}^{\sigma, \sigma'} = \begin{cases} u_{k+q}^2 v_{-k}^2, & \sigma = +, \sigma' = +, \\ u_{k+q}^2 u_{-k}^2, & \sigma = -, \sigma' = +, \\ v_{k+q}^2 v_{-k}^2, & \sigma = +, \sigma' = -, \\ v_{k+q}^2 u_{-k}^2, & \sigma = -, \sigma' = -. \end{cases}$$

In the above expressions, we performed a summation over fermionic Matsubara frequencies using standard textbook techniques [58,60,63].

APPENDIX B: GRADIENT EXPANSION OF \mathbb{F}_q^{-1}

In this Appendix, we present the coefficients of the gradient expansion, which appears in Eqs. (11) and (12). They are given by

$$\begin{aligned} A = \int_{\mathbf{k}} \frac{1}{24E_k^3} & \left[-4E_k^3 u_k^2 (3f''(E_k^-) \{a_k \alpha_k^- - \delta_k^- v_k^2\} \right. \\ & + 6b_k f'(E_k^-) - (\alpha_k^-)^2 f^{(3)}(E_k^-) v_k^2) \\ & + 4E_k^3 v_k^2 (3f''(E_k^+) \{a_k \alpha_k^+ + \delta_k^+ u_k^2\} \\ & + 6b_k f'(E_k^+) + (\alpha_k^+)^2 f^{(3)}(E_k^+) u_k^2) \\ & + 3u_k^2 (f(E_k^+) - f(E_k^-)) \{ -2\alpha_k^+ a_k E_k \\ & + 4b_k E_k^2 - u_k^2 (2\delta_k^+ E_k - (\alpha_k^+)^2) \} \\ & + 2\alpha_k^+ f'(E_k^+) E_k \{ 2a_k E_k - \alpha_k^+ u_k^2 \} \\ & + 2E_k^2 u_k^2 \{ \alpha_k^+ f''(E_k^+) + 2\delta_k^+ f'(E_k^+) \} \\ & + 3v_k^2 (f(E_k^-) - f(E_k^+)) \{ 2\alpha_k^- a_k E_k + 4b_k E_k^2 \\ & - v_k^2 ((\alpha_k^-)^2 + 2\delta_k^- E_k) \} \\ & + 2\alpha_k^- f'(E_k^-) E_k \{ 2a_k E_k - \alpha_k^- v_k^2 \} \\ & \left. - 2E_k^2 v_k^2 \{ (\alpha_k^-)^2 f''(E_k^-) + 2\delta_k^- f'(E_k^-) \} \right], \end{aligned}$$

$$B = \int_{\mathbf{k}} [f(E_k^+) - f(E_k^-)] \frac{u_k^4 - v_k^4}{4E_k^2},$$

$$C = \int_{\mathbf{k}} [f(E_k^-) - f(E_k^+)] \frac{u_k^4 + v_k^4}{8E_k^3},$$

$$\begin{aligned} D = \int_{\mathbf{k}} \frac{u_k v_k}{6} & \left[-3\alpha_k^- d_k f''(E_k^-) - 3\alpha_k^+ d_k f''(E_k^+) \right. \\ & - \frac{3}{4E_k^3} (2\alpha_k^- f'(E_k^-) E_k \{ 2d_k E_k - \alpha_k^- u_k v_k \} \\ & + [f(E_k^-) - f(E_k^+)] \{ 2\alpha_k^- d_k E_k + 4E_k^2 g_k \\ & - u_k v_k \{ (\alpha_k^-)^2 + 2\delta_k^- E_k \} \\ & - 2E_k^2 u_k v_k \{ (\alpha_k^-)^2 f''(E_k^-) + 2\delta_k^- f'(E_k^-) \} \} \\ & \left. + \frac{3}{4E_k^3} (2\alpha_k^+ f'(E_k^+) E_k \{ 2d_k E_k + \alpha_k^+ u_k v_k \} \right] \end{aligned}$$

$$\begin{aligned}
& + [f(E_k^-) - f(E_k^+)] \{ 2\alpha_k^+ d_k E_k - 4E_k^2 g_k \\
& - u_k v_k \{ 2E_k \delta_k^+ - (\alpha_k^+)^2 \} \\
& - 2E_k^2 u_k v_k \{ (\alpha_k^+)^2 f''(E_k^+) + 2\delta_k^+ f'(E_k^+) \} \\
& + 3u_k v_k \{ \delta_k^- f''(E_k^-) + \delta_k^+ f''(E_k^+) \} \\
& + u_k v_k \{ (\alpha_k^-)^2 f^{(3)}(E_k^-) + (\alpha_k^+)^2 f^{(3)}(E_k^+) \} \\
& - 6g_k \{ f'(E_k^-) - f'(E_k^+) \} \Big],
\end{aligned}$$

and

$$E = \int_{\mathbf{k}} [f(E_k^+) - f(E_k^-)] \frac{u_k^2 v_k^2}{4E_k^3}.$$

In the above equations we use the following abbreviations: $\alpha_k = \xi_k |\mathbf{k}| \cos \theta / m E_k$, $\delta_k = \xi_k \Delta^2 / 2m E_k^3 + \xi_k^3 / 2m E_k^3 + \Delta^2 |\mathbf{k}|^2 \cos^2 \theta / 2m E_k^3$, $\alpha_k^\sigma = \zeta |\mathbf{k}| \cos \theta / m + \sigma \alpha_k$, $\delta_k^\sigma = \zeta / 2m + \sigma \delta_k$, $a_k = \Delta^2 |\mathbf{k}| \cos \theta / 2m E_k^3$, $b_k = \Delta^4 / 4m E_k^5 + \Delta^2 \xi_k^2 / 4m E_k^5 - 3\Delta^2 \xi_k |\mathbf{k}|^2 \cos^2 \theta / 4m^2 E_k^5$, $d_k = \Delta \xi_k |\mathbf{k}| \cos \theta / 2m E_k^3$, and $g_k = \Delta (m \Delta^2 \xi_k + m \xi_k^3 + \Delta^2 |\mathbf{k}|^2 \cos^2 \theta - 2|\mathbf{k}|^2 \xi_k^2 \cos^2 \theta) / 4m^2 E_k^5$.

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