Explicit construction of nonadiabatic passages for stimulated Raman transitions

Hong Cao,[1,2] Shao-Wu Yao,[1] and Li-Xiang Cen[1,*]

[1]*Center of Theoretical Physics, College of Physics, Sichuan University, Chengdu 610065, China*

[2]*School of Material Science and Engineering, Chongqing Jiaotong University, Chongqing 400074, China*

(Received 21 August 2019; published 18 November 2019)

We propose a scheme which can produce desired nonadiabatic passages for the stimulated Raman transition in three-level systems. The state transfer in the protocol is realized by following the evolution of the dynamical basis itself and no additional coupling field is required. We also investigate the interplay between the present nonadiabatic passages and the shortcut to adiabaticity. By incorporating the latter technology, we show that alternative passages with less occupancy of the intermediate level could be designed.

DOI: 10.1103/PhysRevA.100.053410

I. INTRODUCTION

Stimulated Raman adiabatic passage (STIRAP) is an efficient technique for robust coherent population transfer in atomic and molecular systems, which has been extensively investigated over the past decades [1–6]. It can induce transitions between two levels that have the same parity, for which the direct coupling via the electric dipole radiation is forbidden. Specifically, the STIRAP applies two laser pulses, the pump and Stokes, to induce the coupling between each of the two levels and a common intermediate level under the condition of the two-photon resonance. The desired population transfer is realized through the adiabatic evolution of the dark state, which conventionally assumes a superposition form of the initial and the final target states.

Based on the transitionless tracking algorithm [7–9], the shortcut to adiabaticity [10–16] has been exploited to speed up the evolution of the STIRAP. In the initial proposals [11,12] of the stimulated Raman shortcut-to-adiabatic passage (STIRSAP), a compensating microwave field which couples the initial and target states is required to counteract the detrimental nonadiabatic effect. This additional microwave field indicates a critical disadvantage, especially considering that the direct coupling might be unfeasible in practical atomic levels with forbidden transition. In subsequent proposals [14,15], it was displayed that one can mimic the desired population transfer of the STIRSAP through modifying the pump and Stokes pulses, which removes the coupling term associated with the microwave field so as to avoid the former drawback. Strictly, in this modified STIRSAP scheme the evolution of the wave function (the so-called dressed state in Ref. [14]) differs from that in the former by a rotating transformation $V(t)$. The validity of the scheme then relies on the boundary condition of $V(t)$, that is, $V(t)$ should be a null operation at the initial (ending) time instant.

Successful design of the laser pulses for the STIRSAP is critically constrained by the above boundary condition of the dressed-state transformation $V(t)$. For example, this condition is not satisfied when the pump and the Stokes fields assume the commonly used Gaussian pulses [14,15]. Note that the rotating angle of $V(t)$ is correlated to the aforementioned microwave field that should have been applied in the initial STIRSAP protocol. The specified boundary condition of $V(t)$ can be fulfilled only when the strength of this additional interaction, or equivalently, the nonadiabatic effect induced by the initial pump and Stokes fields, should be negligible on the boundary. This actually requires that the driving protocol should satisfy the adiabatic condition at that time instant. At this stage, a promising design may rest on (but not be limited to) the prerequisite that the system should possess discrete energy levels at the initial (ending) time instant of the driving process.

On the other hand, valuable results have been obtained recently in understanding the nonadiabatic dynamics generated by several particular types of quantum driven models [17–20]. It has been shown that the nonadiabatic effect in some cases can play a positive role for the population transfer. For example, in the tangent-pulse-driven model with the matching frequency and amplitude, the nonadiabatic effect not only will not lead to unwanted transitions, but also can suppress the error caused by the truncation of the field pulse [17]. This feature of the nonadiabatic driving has also been found in a modified Landau-Zener model [18] and a special Allen-Eberly model [19]. Motivated by these results, it is natural to ask whether there exist such nonadiabatic passages that can be exploited directly to realize the stimulated Raman process.

In this paper we report the finding of a driving scheme via which the nonadiabatic passages for the stimulated Raman transition can be explicitly constructed. The scheme applies to the Λ-type three-level system with one-photon resonance in which the Stokes laser pulse can be of arbitrary analytical form but the pump pulse should be matched with the Stokes one. Several driving protocols generated by the scheme are illustrated and their corresponding features for the population transfer are characterized. Moreover, we explore the interplay between the present scheme and the STIRSAP protocol and elucidate how to reconstruct the nonadiabatic passages within the framework of the shortcut to adiabaticity. Incorporation with the latter technology enables us to obtain modified

*lixiangcen@scu.edu.cn

nonadiabatic passages with less occupancy of the intermediate level.

II. STIMULATED RAMAN NONADIABATIC PASSAGES WITH ONE-PHOTON RESONANCE

A. Description of the driving scheme

Consider a three-level system with the Λ configuration. The states $|1\rangle$ and $|3\rangle$ are ground or metastable levels, which are coupled to the excited level $|2\rangle$ via the pump pulse and the Stokes pulse, respectively. The Hamiltonian of the system under the rotating-wave approximation can be written as $H_{\rm tot}(t) = H_{\rm free} + H_{\rm int}(t)$, with the free Hamiltonian $H_{\rm free} = \sum_{i=1}^{3} E_i\sigma_{ii}$ and the interaction term

$$H_{\rm int}(t) = \Omega_p(t)(\sigma_{12}e^{i\omega_p t} + \sigma_{21}e^{-i\omega_p t}) + \Omega_s(t)(\sigma_{23}e^{-i\omega_s t} + \sigma_{32}e^{i\omega_s t}), \tag{1}$$

in which $\sigma_{ij} = |i\rangle\langle j|$ $(i, j = 1, 2, 3)$ and $\Omega_{p,s}(t)$ describe the Rabi frequencies of the pump and Stokes pulses, respectively. Under the condition of the one-photon resonance $\omega_p = E_2 - E_1$ and $\omega_s = E_2 - E_3$, one obtains the Hamiltonian in the interaction picture

$$H(t) = \Omega_p(t)(\sigma_{12} + \sigma_{21}) + \Omega_s(t)(\sigma_{23} + \sigma_{32}). \tag{2}$$

To implement fast population transfer from the state $|1\rangle$ to $|3\rangle$, we propose a nonadiabatic driving protocol in which the laser pulses satisfy

$$\Omega_p(t) = \frac{1}{2}\Omega_s(t)\sec\left[\frac{1}{2}\int_{t_0}^{t}\Omega_s(\tau)d\tau\right], \tag{3}$$

with the envelope of the Stokes laser $\Omega_s(t)$ being an arbitrary analytical function over $t \in (t_0, t_f)$. We will show below that, according to the Schrödinger equation $i\partial_t|\psi(t)\rangle = H(t)|\psi(t)\rangle$, complete population transfer $|1\rangle \to |3\rangle$ can be realized nonadiabatically by the protocol as long as the integral of the intercepted pulse $\int_{t_0}^{t}\Omega_s(\tau)d\tau \equiv 2\vartheta(t)$ goes from 0 to π.

To resolve the dynamics of the above stimulated Raman process, we note that the described model possesses a dynamical invariant [21,22]

$$I(t) = \sin^2\vartheta(t)(\sigma_{12} + \sigma_{21}) + \cos\vartheta(t)(\sigma_{23} + \sigma_{32}) - i\sin\vartheta(t)\cos\vartheta(t)(\sigma_{13} - \sigma_{31}), \tag{4}$$

which satisfies

$$\partial_t I(t) = -i[H(t), I(t)]. \tag{5}$$

It is recognized that the three operators $K_x \equiv \sigma_{12} + \sigma_{21}$, $K_y \equiv i(\sigma_{13} - \sigma_{31})$, and $K_z \equiv \sigma_{23} + \sigma_{32}$ satisfy the commutation relation $[K_\alpha, K_\beta] = i\epsilon_{\alpha\beta\gamma}K_\gamma$. By recording $I(t) = \vec{\chi}(t)\cdot\vec{K}$, Eq. (5) is readily verified through the following equations of the components:

$$\dot{\chi}_1(t) = -\Omega_s(t)\chi_2(t), \tag{6}$$

$$\dot{\chi}_2(t) = \Omega_s(t)\chi_1(t) - \Omega_p(t)\chi_3(t), \tag{7}$$

$$\dot{\chi}_3(t) = \Omega_p(t)\chi_2(t). \tag{8}$$

The eigenvalues of $I(t)$ are given by $\lambda_0 = 0$ and $\lambda_\pm = \pm 1$, and the eigenstate $|e_0(t)\rangle$ associated with the zero eigenvalue λ_0 is obtained as

$$|e_0(t)\rangle = \cos\vartheta|1\rangle - i\sin\vartheta\cos\vartheta|2\rangle - \sin^2\vartheta|3\rangle. \tag{9}$$

It is of interest to mention that $|e_0(t)\rangle$ itself constitutes the dynamical solution of the system. This can be recognized by substituting $|e_0(t)\rangle$ into the Schrödinger equation straightforwardly or by verifying that it has a vanishing Lewis-Riesenfeld phase [21,22] during the evolution:

$$\int_{t_0}^{t}\langle e_0(t)|i\partial_\tau - H(t)|e_0(t)\rangle d\tau = 0. \tag{10}$$

Moreover, it can be seen that the initial state $|1\rangle$ correlates exclusively with the basis state $|e_0(t)\rangle$. So the wave function will evolve along this dressed state for the time being. As $\vartheta(t_f) \to \frac{\pi}{2}$, the state transfer $|1\rangle \to |3\rangle$ is achieved up to a minus sign.

A particular feature one can recognize from the above stimulated Raman protocol is that at the initial $t = t_0$, $\Omega_p(t_0) = \frac{1}{2}\Omega_s(t_0)$. This is distinct from the delayed pulse sequence employed in the conventional STIRAP, which indicates that the nonadiabatic effect plays a decisive role in the present protocol. Also the Hamiltonian (2) and the dynamical invariant (4) are not commutative at $t = t_0$: $[H(t_0), I(t_0)] \neq 0$, which does not accord with the condition assumed in Ref. [11]. In addition, as $t \to t_f$ the asymptotic population on the target state $|3\rangle$ is specified by $P(t) = \sin^4\vartheta(t)$. One can show that the protocol is less sensitive to the truncation of the field pulses than the adiabatic protocol. Specifically, suppose that the field pulses are truncated at $t = t_{fc}$ with the Rabi frequencies $\Omega_p(t_{fc})$ and $\Omega_s(t_{fc})$. Let us define $\theta(t) \equiv \arctan\frac{\Omega_p(t)}{\Omega_s(t)}$. The population on $|3\rangle$ at $t = t_{fc}$ is given by

$$P(t_{fc}) = \sin^4\vartheta(t_{fc}) = \left[1 - \tfrac{1}{4}\cot^2\theta(t_{fc})\right]^2. \tag{11}$$

As $\cos\theta(t_{fc})$ is much less than 1, it is not difficult to verify that $P(t_{fc}) \geqslant \sin^2\theta(t_{fc})$. That is to say, in comparison with the conventional STIRAP, the nonadiabatic driving of the present protocol will reduce the loss of fidelity caused by the truncation.

B. Typical nonadiabatic passages

The above scheme offers an explicit way to construct stimulated Raman nonadiabatic passages via which the population transfer $|1\rangle \to |3\rangle$ can be realized. We present several typical examples in the following.

Example 1. The Rabi frequency of the Stokes laser is set to be a constant. According to Eq. (3), we have

$$\Omega_p(t) = \frac{\nu}{2}\sec\frac{\nu t}{2}, \quad \Omega_s(t) = \nu, \tag{12}$$

in which the time t goes from $t_0 = 0$ to $t_f \to \frac{\pi}{\nu}$. The dynamical invariant and the zero-eigenvalue dress state are given by

$$I(t) = \sin^2\frac{\nu t}{2}(\sigma_{12} + \sigma_{21}) + \cos\frac{\nu t}{2}(\sigma_{23} + \sigma_{32}) - \frac{i}{2}\sin\nu t(\sigma_{13} - \sigma_{31}) \tag{13}$$

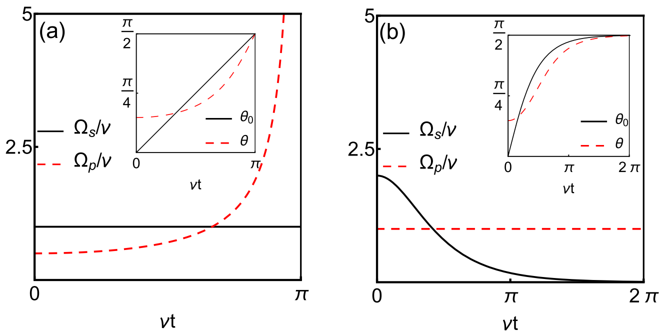

FIG. 1. Typical nonadiabatic passages of the driving scheme described by Eq. (3). (a) Pump and Stokes pulses [$\Omega_{p,s}(t)$ over the constant ν] specified by Eq. (12). (b) Field pulses specified by Eq. (15). The insets, with $\theta(t) \equiv \arctan \frac{\Omega_p(t)}{\Omega_s(t)}$ and $\theta_0(t) \equiv \arctan \frac{\Omega_{p0}(t)}{\Omega_{s0}(t)}$, show the difference between the field pulses of the present nonadiabatic passages and those in the initial Hamiltonian of the STIRSAP specified by Eqs. (26) and (27).

and

$$|e_0(t)\rangle = \cos\frac{\nu t}{2}|1\rangle - \frac{i}{2}\sin(\nu t)|2\rangle - \sin^2\frac{\nu t}{2}|3\rangle, \tag{14}$$

respectively. In this proposal the pump laser assumes a chirped pulse and the duration of the pulse $\nu t_f \approx \pi$ is much shorter than that of the usual adiabatic protocol.

Example 2. The passage is described by

$$\Omega_p(t) = \nu, \quad \Omega_s(t) = 2\nu\,\mathrm{sech}(\nu t), \tag{15}$$

in which $\Omega_p(t)$ of the pump laser is a constant. The two Rabi frequencies above are verified to satisfy Eq. (3) in view that half of the integration of $\Omega_s(t)$ gives rise to $\vartheta(t) = \arctan[\sinh(\nu t)]$. The eigenstate $|e_0(t)\rangle$ of the dynamical invariant is obtained as

$$|e_0(t)\rangle = \mathrm{sech}(\nu t)|1\rangle - i\tanh(\nu t)\mathrm{sech}(\nu t)|2\rangle - \tanh^2(\nu t)|3\rangle. \tag{16}$$

The field pulses in this protocol are defined in an infinite time domain and the truncation is inevitable. We define an effective pulse duration $t \in (t_0, t_{fc})$ in which t_{fc} is defined such that the population on $|3\rangle$ reaches $P(t_{fc}) \approx 0.9999$. For the current example we have $\nu t_{fc} \approx 1.8\pi$.

The schematic of the field pulses of the above two examples is depicted in Fig. 1. In principle, an unlimited number of nonadiabatic passages could be constructed by the scheme. Besides the above two, some other examples are displayed in Table I, including their field pulses, evolution of the population, and the effective pulse duration t_{fc}.

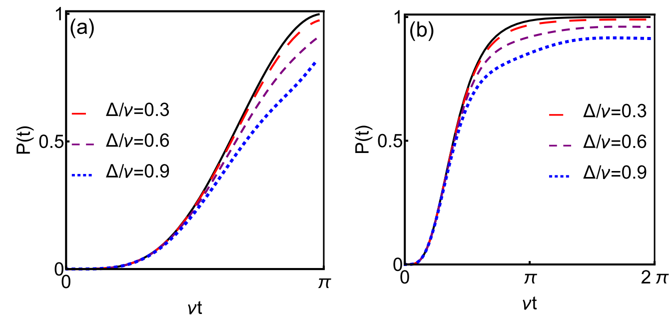

FIG. 2. Influence of the detunings on the desired stimulated Raman transitions. (a) Evolution of the population $P(t)$ on $|3\rangle$ through the nonadiabatic passage of Eq. (12), in which $\frac{\Delta}{\nu} = 0.3$, 0.6, and 0.9. (b) Corresponding evolution of $P(t)$ through the nonadiabatic passage of Eq. (15) with nonvanishing detunings. In both cases the system is in an initial state $|1\rangle$ and the solid curve represents the desired state transfer without detunings.

C. Detuning effects

We now consider the influence of the off-resonance on the population transfer of the above proposed scheme. Suppose that the detunings are described by $\Delta = E_2 - E_1 - \omega_p$ and $\Delta_s = E_2 - E_3 - \omega_s$. Instead of the expression of Eq. (2), the Hamiltonian in the rotating frame can be written as

$$\begin{aligned} H'(t) = {} & \Delta\sigma_{22} + \delta\sigma_{33} + \Omega_p(t)(\sigma_{12} + \sigma_{21}) \\ & + \Omega_s(t)(\sigma_{23} + \sigma_{32}), \end{aligned} \tag{17}$$

where $\delta \equiv \Delta - \Delta_s$. It can be seen that as $|\Delta| \ll |\Omega_{p,s}(t)|$ and $|\delta| \ll |\Omega_{p,s}(t)|$, the extra term $\Delta\sigma_{22} + \delta\sigma_{33}$ can be viewed as a small perturbation to the nonadiabatic passage. For the examples specified by Eqs. (12) and (15), the amplitudes of the field pulses $\Omega_{p,s}(t)$ are proportional to the driving frequency ν. That is to say, the influence of the off-resonance on the state transfer can be suppressed through increasing the scanning rate of the driving process. This result is straightforwardly verified by the exact numerical calculation shown in Fig. 2, in which we have set $\delta = 0$ for simplicity.

III. INTERPLAY WITH THE SHORTCUT TO ADIABATICITY

Control of the nonadiabatic evolution of quantum states based on the reverse-engineering approach has attracted much

TABLE I. Nonadiabatic passages generated by the scheme and the corresponding field pulses in the initial Hamiltonian of the STIRSAP.

Stokes pulse	Pump pulse	Target state population	νt_{fc}	$\Omega_{s0}(t)$	$\Omega_{p0}(t)$
$\Omega_s = \nu$	$\Omega_p = \frac{\nu}{2}\sec\frac{\nu t}{2}$	$\sin^4\frac{\nu t}{2}$	$\approx\pi$	$\frac{\nu}{2}$	$\frac{\nu}{2}\tan\left(\frac{\nu t}{2}\right)$
$\Omega_s = 2\nu\,\mathrm{sech}(\nu t)$	$\Omega_p = \nu$	$\tanh^4(\nu t)$	$\approx 1.80\pi$	$\nu\,\mathrm{sech}(\nu t)$	$\nu\tanh(\nu t)$
$\Omega_s = \frac{2\nu}{\sqrt{1-\nu^2t^2}}$	$\Omega_p = \frac{\nu}{1-\nu^2t^2}$	ν^4t^4	≈ 1	$\frac{\nu}{\sqrt{1-\nu^2t^2}}$	$\frac{\nu^2t}{1-\nu^2t^2}$
$\Omega_s = \nu^2t$	$\Omega_p = \frac{\nu^2t}{2}\sec\frac{\nu^2t^2}{4}$	$\sin^4\frac{\nu^2t^2}{4}$	$\approx 0.80\pi$	$\frac{\nu^2t}{2}$	$\frac{\nu^2t}{2}\tan\left(\frac{\nu^2t^2}{4}\right)$
$\Omega_s = 4\nu^2t\,\mathrm{sech}(\nu^2t^2)$	$\Omega_p = 2\nu^2t$	$\tanh^4(\nu^2t^2)$	$\approx 0.76\pi$	$2\nu^2t\,\mathrm{sech}(\nu^2t^2)$	$2\nu^2t\tanh(\nu^2t^2)$
$\Omega_s = 2\nu e^{\nu t}$	$\Omega_p = \nu e^{\nu t}\sec(e^{\nu t}-1)$	$\sin^4(e^{\nu t}-1)$	$\approx 0.30\pi$	$\nu e^{\nu t}$	$\nu e^{\nu t}\tan(e^{\nu t}-1)$

attention recently. Several strategies, including the counterdiabatic protocol [7,8], transitionless algorithm [9], and shortcut to adiabaticity [10], have been devoted to tracking the adiabatic trajectory through introducing an auxiliary field to counteract the nonadiabatic effect. For the control of the Λ-type three-level system, this strategy can be retrospected to the study of holonomic quantum computation [23]. To construct the nonadiabatic passages reversely for the stimulated Raman transition, various protocols have been proposed [11,12,14,15,24]. In this section we will first review the corresponding proposals of the STIRSAP. Then we will focus on the interplay between the present protocol and the STIRSAP and show how to reconstruct the proposed nonadiabatic passages by the shortcut-to-adiabatic technology.

A. Stimulated Raman shortcut-to-adiabatic passage and the dressed-state transformation

To be specific, let us review the STIRSAP protocol with the one-photon resonance of which the initial Hamiltonian in the interaction picture reads

$$H_0(t) = \Omega_{p0}(t)(\sigma_{12} + \sigma_{21}) + \Omega_{s0}(t)(\sigma_{23} + \sigma_{32}). \tag{18}$$

In general, the sequence of the delayed pulse interactions of the pump and the Stokes lasers are implemented in counterintuitive order, so the Rabi frequencies satisfy $\Omega_{s0}(t_0) \gg \Omega_{p0}(t_0)$ and $\Omega_{s0}(t_f) \ll \Omega_{p0}(t_f)$ at the initial and the ending time $t_{0,f}$, respectively. For the adiabatic evolution the state transfer $|1\rangle \to |3\rangle$ can be realized along the dark state $|d(t)\rangle = \cos\theta_0(t)|1\rangle - \sin\theta_0(t)|3\rangle$, with $\theta_0(t) = \arctan\frac{\Omega_{p0}(t)}{\Omega_{s0}(t)}$. Following the shortcut-to-adiabatic technology [11–15], the nonadiabatic effect of the evolution could be canceled by introducing a compensating microwave field $H_{cd}(t) = i\dot{\theta}_0(t)(\sigma_{13} - \sigma_{31})$ and the corrected Hamiltonian

$$H_{\rm corr}(t) = H_0(t) + H_{cd}(t) \tag{19}$$

can drive the system along the eigenstate $|d(t)\rangle$ of the initial $H_0(t)$ in a nonadiabatic manner. Note that $H_{cd}(t)$ represents a direct coupling between the levels $|1\rangle$ and $|3\rangle$ which might be unavailable for the control setup. To overcome this drawback, one can replace the above $H_{\rm corr}(t)$ by an alternative one [14,15]

$$\tilde{H}_{\rm corr}(t) = V(t)H_{\rm corr}(t)V^{\dagger}(t) - iV(t)\partial_t V^{\dagger}(t), \tag{20}$$

in which $V(t) = e^{i\phi(t)(\sigma_{23}+\sigma_{32})}$ accounts for a rotating transformation. When the rotating angle is set as $\phi(t) = \arctan\frac{\dot{\theta}_0(t)}{\Omega_{p0}(t)}$, the interacting term with respect to the direct coupling between $|1\rangle$ and $|3\rangle$ will disappear in the corrected Hamiltonian $\tilde{H}_{\rm corr}(t)$. The corresponding dynamical basis of $\tilde{H}_{\rm corr}(t)$ relates to $|d(t)\rangle$ via $|\tilde{d}(t)\rangle = V(t)|d(t)\rangle$. Therefore, as long as the boundary conditions $|\tilde{d}(t_{0,f})\rangle = |d(t_{0,f})\rangle$ are satisfied, the desired population transfer $|1\rangle \to |3\rangle$ could be realized via the evolution of the dressed state $|\tilde{d}(t)\rangle$.

It is worth mentioning that the condition $|\tilde{d}(t_0)\rangle = |d(t_0)\rangle$ is always fulfilled for the STIRSAP since the equality $V(t_0)|1\rangle = |1\rangle$ holds for any given $\phi(t_0)$. That is to say, the boundary condition of $V(t)$ at $t = t_0$ is irrelevant, but only the condition $V(t_f) = I$ is required in the design of the protocol. As will be shown in the below, this is the case, i.e., $V(t_0) \neq I$, when we reconstruct the nonadiabatic passages obtained previously within the framework of the STIRSAP.

B. Reconstructing the nonadiabatic passages via the shortcut-to-adiabatic technology

Let us move to consider the issue of constructing the nonadiabatic passages described in Sec. II through the STIRSAP scheme. The goal now is to determine reversely the initial Hamiltonian $H_0(t)$ based on the known target Hamiltonian $\tilde{H}_{\rm corr}(t) = H(t)$ specified by Eqs. (2) and (3). Note that the dynamical invariant of the system $\tilde{H}_{\rm corr}(t)$ is known to be the $I(t)$ of Eq. (4) and the corresponding dressed state $|\tilde{d}(t)\rangle \equiv |e_0(t)\rangle$ is shown in Eq. (9). It is readily seen that a rotating transformation $V^{\dagger}(t) = e^{-i\phi(t)(\sigma_{23}+\sigma_{32})}$ with $\phi(t) = \frac{\pi}{2} - \vartheta(t)$ can transform $|\tilde{d}(t)\rangle$ to the dark state $|d(t)\rangle$:

$$V^{\dagger}(t)|\tilde{d}(t)\rangle = \cos\vartheta(t)|1\rangle - \sin\vartheta(t)|3\rangle \equiv |d(t)\rangle. \tag{21}$$

Then the inverse transformation of Eq. (20) gives rise to

$$\begin{aligned} H_{\rm corr}(t) &= V^{\dagger}(t)\tilde{H}_{\rm corr}(t)V(t) - iV^{\dagger}(t)\partial_t V(t) \\ &= \Omega_p(t)\sin\vartheta(t)K_x + \frac{\Omega_s(t)}{2}(K_y + K_z). \end{aligned} \tag{22}$$

By comparing the above expression with Eq. (19), one recognizes that

$$H_0(t) = \Omega_p(t)\sin\vartheta(t)(\sigma_{12} + \sigma_{21}) + \frac{\Omega_s(t)}{2}(\sigma_{23} + \sigma_{32}) \tag{23}$$

and

$$H_{cd}(t) = \tfrac{1}{2}\Omega_s(t)K_y \equiv i\dot{\vartheta}(t)(\sigma_{13} - \sigma_{31}). \tag{24}$$

More explicitly, the form of the laser pulses of $H_0(t)$ can be expressed as

$$\Omega_{p0}(t) = \frac{\Omega_s(t)}{2}\tan\vartheta(t), \quad \Omega_{s0}(t) = \frac{\Omega_s(t)}{2}, \tag{25}$$

where $\vartheta(t) \equiv \arctan\frac{\Omega_{p0}(t)}{\Omega_{s0}(t)} \equiv \theta_0(t)$.

So far, we have completed the reconstruction of the nonadiabatic driving scheme through the STIRSAP; that is, the state transfer process realized by the control Hamiltonian of Eqs. (2) and (3) can be understood as the transitionless algorithm tracking the evolution of the instantaneous eigenstate $|d(t)\rangle$ of the Hamiltonian $H_0(t)$, corrected by a dressed-state transformation $V(t) = e^{i\phi(t)(\sigma_{23}+\sigma_{32})}$. As $\phi(t)$ here goes from $\phi(t_0) = \frac{\pi}{2}$ to $\phi(t_f) = 0$, it confirms the aforementioned statement that only the boundary condition $V(t_f) = I$ is necessary. Furthermore, for the concrete nonadiabatic passages specified by Eqs. (12) and (15), one obtains

$$\Omega_{p0}(t) = \frac{\nu}{2}\tan\frac{\nu t}{2}, \quad \Omega_{s0}(t) = \frac{\nu}{2} \tag{26}$$

and

$$\Omega_{p0}(t) = \nu \tanh(\nu t), \quad \Omega_{s0}(t) = \nu\, \text{sech}(\nu t), \tag{27}$$

respectively [see Fig. 1 for the evolution of the angle $\theta_0(t)$]. For more examples, the corresponding expressions of $\Omega_{p0}(t)$ and $\Omega_{s0}(t)$ are displayed in the last column of Table I.

IV. STRATEGY TO REDUCE THE INTERMEDIATE-LEVEL OCCUPANCY

Differing from the original STIRAP in which the population transfer is realized along the dark state, the occupancy on the excited level $|2\rangle$ will occur in the present nonadiabatic protocol when the system evolves along the dressed state $|\tilde{d}_0(t)\rangle$. Note that the detrimental effects on both the dark STIRAP and the bright STIRAP [25–27] due to the dissipation of this auxiliary intermediate level have been intensively investigated [28–33]. For instance, it was revealed that in the case of the strong-damping limit the population transfer through the dark eigenstate could be immune to the dissipation owing to the Zeno-like effect [30,31]. In addition, the detrimental effect on the bright STIRAP and its interplay with the detuning have also been explored [32]. The nonadiabatic passages proposed here will be sensitive to the decay of the intermediate level and will be somewhat similar to the bright STIRAP. Nevertheless, in view of the fact that the pulse durations of the passages here (see Table I) are much shorter than those of the adiabatic passages, it is expected that the detrimental effect caused by the dissipation of the intermediate level should be less serious than that in the bright STIRAP.

In addition, the intermediate-level occupancy of the present scheme can be reduced by following the approach of Ref. [14], that is, one can adjust the dressed state $|\tilde{d}(t)\rangle$ by constructing alternative control Hamiltonians. Essentially, this strategy makes use of the multiplicity of the control Hamiltonian of the tracking algorithm when aiming at the desired state evolution [9]. In detail, one can use the series of Hamiltonians $\{H_0'(t) = \eta(t)H_0(t)\}$ to replace $H_0(t)$ specified by Eqs. (23) and (25) with $\eta(t)$ an adjustable factor. Subsequently, the shortcut-to-adiabatic protocol should give rise to

$$H'_{\rm corr}(t) = \eta(t)H_0(t) + H_{cd}(t), \tag{28}$$

with $H_{cd}(t)$ being the same as in Eq. (24). Following the protocol, one can find that the rotating angle of the dressed-state transformation $V(t)$ should change as

$$\phi(t) \to \phi'(t) = \arctan[\eta^{-1}(t)\cot\vartheta(t)]. \tag{29}$$

Accordingly, the new Hamiltonian $\tilde{H}'_{\rm corr}(t)$ and the dressed state $|\tilde{d}'(t)\rangle = e^{i\phi'(t)(\sigma_{23}+\sigma_{32})}|d(t)\rangle$ can be formulated as

$$\tilde{H}'_{\rm corr}(t) = \frac{1}{2}\Omega_s\sqrt{1+\eta^2\tan^2\vartheta}K_x + \left(\frac{\eta}{2}\Omega_s - \dot{\phi}'\right)K_z \tag{30}$$

and

$$|\tilde{d}'(t)\rangle = \cos\vartheta|1\rangle - i\sin\vartheta\sin\phi'|2\rangle - \sin\vartheta\cos\phi'|3\rangle, \tag{31}$$

respectively.

Now it is straightforward to see that the population on the intermediate level $|2\rangle$ changes from $P_2(t) = \sin^2\vartheta(t)\cos^2\vartheta(t)$ [see Eq. (9)] to

$$P_2'(t) = \frac{\cos^2\vartheta(t)}{\eta^2(t) + \cot^2\vartheta(t)}. \tag{32}$$

As long as $|\eta(t)| > 1$, the above strategy will lead to reduction of the population on $|2\rangle$ with the ratio

$$\frac{P_2'(t)}{P_2(t)} = \frac{1}{\eta^2(t)\sin^2\vartheta(t) + \cos^2\vartheta(t)}. \tag{33}$$

For the case that η is a constant, the maximal reduction is always achieved at a time point with $\vartheta = \frac{\pi}{4}$ where the population $P_2(t)$ reaches its maximal value. The corresponding ratio there is given by $P_2'/P_2 = 2/(1+\eta^2)$. It should be noted that the reduction of the occupancy on $|2\rangle$ here is limited by the amplitude of the applied field in the experimental system. As is illustrated in Fig. 3, a sizable reduction of $P_2(t)$ could be obtained for the driving protocol, e.g., its maximal value could be dropped to $\frac{1}{13}$ if one increases the amplitude of the pulse to about fivefold ($\eta = 5$).

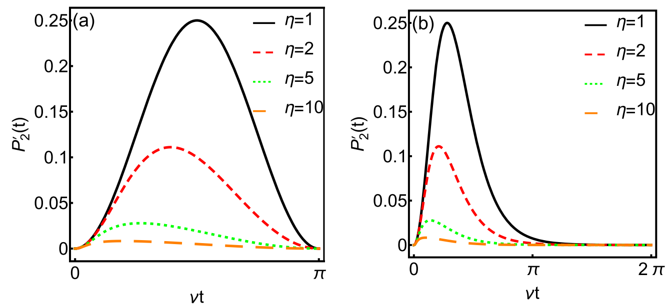

FIG. 3. Reduction of the intermediate-level occupancy by modifying the dressed state [see Eqs. (31)–(33)]. (a) Population on $|2\rangle$ of the nonadiabatic Raman passage specified by Eq. (12) and the modified passages with $\eta = 2, 5, 10$. (b) Population on $|2\rangle$ of the passage specified by Eq. (15) and the modified ones.

V. CONCLUSION

In summary, we have proposed a nonadiabatic driving scheme to realize the stimulated Raman transition in the three-level system. The main advantages of the scheme include the following: (i) It applies no auxiliary coupling field but only the pump and the Stokes lasers to the Λ-type system with the one-photon resonance, (ii) it can produce a variety of nonadiabatic passages with pulse durations much shorter than the usual adiabatic passages, and (iii) the nonadiabatic effect of the driving scheme is shown to be able to reduce the loss of fidelity caused by the truncation of the field pulses. Furthermore, we have investigated the interplay between the present nonadiabatic driving protocol and the STIRSAP based on the transitionless tracking algorithm. By incorporating the latter technology, we show that modified nonadiabatic passages with less occupancy of the intermediate level could be constructed.

ACKNOWLEDGMENT

This work was supported by the NSFC, China, under Grant No. 11847305.

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