Laser-assisted binary-encounter emission in relativistic ion-atom collisions

Z. Wang,[1,2,3] B. Najjari,[1] S. F. Zhang,[1] X. Ma,[1] and A. B. Voitkiv[3]
[1]*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*
[2]*University of Chinese Academy of Sciences, Beijing 100049, China*
[3]*Institute for Theoretical Physics I, Heinrich-Heine-Universität Düsseldorf, Universitätsstrasse 1, 40225 Düsseldorf, Germany*

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We consider binary-encounter electron emission in relativistic ion-atom collisions in the presence of a monochromatic low-frequency plane-wave electromagnetic field of circular polarization. The parameters of the field—its intensity and frequency—are chosen in such a way that the field *per se* has a negligible effect on the initial (ground) state of the atom. The results show, however, that this field may have a profound effect on the electron emission spectra, which is mainly caused by an enormous energy exchange between the emitted electron and the field.

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I. INTRODUCTION

There are just a few basic atomic collision processes which occur when atoms are bombarded by high-energy bare ions: atomic excitation and ionization, capture of an atomic electron by the incident ion, electron-positron pair production, and emission of radiation (bremsstrahlung). Fast ion-atom collisions have been intensively studied, both experimentally and theoretically (see, e.g., [1–7] and references therein).

With an increase of the impact energy (per nucleon), the role of electron capture diminishes (essentially vanishing at asymptotically high energies) and pair production becomes more and more important, whereas the behavior of atomic ionization is nonmonotonous: the initial decrease of the total ionization cross section with the impact energy is eventually replaced by a slow (logarithmic) growth when the impact energy enters the relativistic domain, in which the collision velocity v approaches the speed of light c (see, e.g., [3] and references therein).

Electron emission is the most important signature of atomic ionization and the study of its characteristics (such as the energy and angular distributions of the emitted electrons) provides much information and offers a deeper insight into the physics of the ionization process. One of the basic features of electron emission in high-energy ion-atom collisions is represented by the so-called binary-encounter emission (see, e.g., [8]). It originates from collisions, in which the momentum transferred to the atom by the projectile ion is "absorbed" by an "active" atomic electron, while the atomic residue acts merely as a spectator. As a result of this—essentially two-body—ionization mechanism, the electron leaves the atom with an energy which may greatly exceed that typical for (outer-shell) atomic electrons. In particular, the maximum energy $\varepsilon_{\max}$ of the electrons emitted due to the binary-encounter collisions is given by $\varepsilon_{\max} = mc^2\gamma_e = mc^2(2\gamma^2 - 1)$, where m is the electron mass, and γ_e and $\gamma = 1/\sqrt{1 - v^2/c^2}$ are the Lorentz factor of the electron and the projectile, respectively.

It is well known that various atomic collision or scattering processes involving incident photons and electrons can be substantially modified when they occur in the presence of an external electromagnetic field. For instance, such a field can strongly influence Compton scattering of a high-frequency photon on a bound atomic electron [9] and the radiative recombination of a free electron with an ion [10–16]. An external electromagnetic field also influences inelastic as well as elastic electron scattering on atoms (for recent results on this topic, see, e.g., [17,18]).

The presence of an external electromagnetic field in ion-atom collisions, by introducing new important degrees of freedom into the processes of ionization and electron transfer, may substantially influence the collision dynamics and eventually the outcome of these processes [19–21]. By using short "assisting" pulses of the electromagnetic field, one could also get an additional insight into the development of (slow) collisional processes in time (see, e.g., [22]). The studies of field-assisted ion-atom collisions are of interest for two main reasons. First, they are interesting from the point of view of basic atomic collision physics since the appearance of new parameters, such as field intensity, frequency, and polarization, enriches the corresponding physics. Second, these processes can be of importance and interest for applied physics areas, such as plasma heating or laser-driven fusion.

The field-assisted binary-encounter emission in fast but nonrelativistic ion-atom collisions was considered in [19,20]. The parameters of the field were chosen there in such a way that the field itself would have practically no effect on a free atom. Nevertheless, it was shown in [19,20] that even such electromagnetic fields may have a strong effect on the binary-encounter emission qualitatively changing the energy and angular distributions of the emitted electrons.

The main goal of the present article is to extend the studies of [19,20] to the relativistic domain of ion-atom collisions. The article is organized as follows. In the next section, we give a detailed description of our treatment for field-assisted ion-atom collisions occurring at relativistic collisions. Section III contains results and discussion. The main conclusions are formulated in Sec. IV.

Atomic units are used throughout, except where otherwise indicated.

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II. GENERAL CONSIDERATION

Let us consider a collision between an atom with one active electron and a high-energy bare ion. The collision occurs in the presence of an external electromagnetic field, which will be taken as the classical monochromatic plane wave of circular polarization. We shall use the semiclassical approximation in which the relative motion of the heavy particles (the nucleus of the atom and the ion) is treated classically and approximate it by a straight-line trajectory. The collision is considered in a reference frame where the nucleus of the atom is at rest and taken as the origin. In this frame, the trajectory of the projectile ion is described by $\boldsymbol{R}(t) = \boldsymbol{b} + \boldsymbol{v}t$, where $\boldsymbol{b} = (b_x, b_y, 0)$ is the impact parameter and $\boldsymbol{v} = (0, 0, v)$ is the collision velocity.

Within the semiclassical picture of the collision, the transition amplitude $a_{fi}(\mathbf{b})$ can be written as (see, e.g., Chap. 5 of [7])

$$a_{fi}(\mathbf{b}) = \left[-\frac{i}{c^2}\int d^4\mathsf{x}\, J_{el\,\mu}(\mathsf{x}) A_{\rm ion}{}^{\mu}(\mathsf{x})\right]_{fi}. \qquad (1)$$

Here, $J_{el\,\mu}(\mathsf{x})$ ($\mu = 0, 1, 2, 3$) is the electromagnetic transition four-current density of the electron at a space-time point x [$\mathsf{x}^{\mu} = (ct, \mathbf{r})$, $\mathsf{x}_{\mu} = (ct, -\mathbf{r})$] and $A_{\rm ion}{}^{\mu}(\mathsf{x})$ is the four potential of the electromagnetic field created by the projectile ion at the same point x. In (1), the summation over repeated Greek indices is implied.

In what follows, we assume that the projectile represents a comparatively weak perturbation for the electron so that the first order of perturbation theory in the projectile-electron interaction—the first Born approximation—can be used for the description of the binary-encounter emission. This is the case provided that

$$\frac{Z_p}{v} \ll 1, \qquad (2)$$

where Z_p is the atomic number of the bare projectile ion.

In the first order of perturbation theory in the projectile-electron interaction, the transition four-current density for the electron is given by

$$\left[J_{\mu}^{el}(\mathsf{x})\right]_{fi} = -c\, \bar{\psi}_f(\mathsf{x})\gamma_{\mu}\psi_i(\mathsf{x}). \qquad (3)$$

Here, ψ_i and ψ_f are the initial and final states of the electron and γ_{μ} ($\gamma_0 = \beta$, $\boldsymbol{\gamma} = \beta\boldsymbol{\alpha}$) are the Dirac matrices (see, e.g., [23]). The initial and final states are solutions of the following Dirac equation:

$$i\frac{\partial}{\partial t}\psi = \left[c\,\boldsymbol{\alpha}\cdot\left(\hat{\mathbf{p}} + \frac{1}{c}\mathbf{A}\right) + V_{at}(\mathbf{r}) - A^0 + \beta c^2\right]\psi. \qquad (4)$$

In this equation, $\hat{\mathbf{p}}$ is the electron momentum operator, $V_{at}(\mathbf{r})$ is the interaction of the electron with the atomic core, and A_0 and $\mathbf{A}$ represent the scalar and vector potential, respectively, of the electromagnetic wave.

In order to describe the field of the electromagnetic wave, we choose a gauge in which the scalar potential vanishes,

$$A^0 = 0, \quad \mathbf{A} = \frac{cF_0}{\omega_0}(\mathbf{e_1}\cos\varphi + \mathbf{e_2}\sin\varphi)$$
$$= \frac{cF_0}{2\omega_0}[\boldsymbol{\varepsilon}\exp(-i\varphi) + \boldsymbol{\varepsilon}^*\exp(-i\varphi)]. \qquad (5)$$

Here, F_0 is the amplitude of the electric component of the electromagnetic wave, $\mathbf{e}_1$ and $\mathbf{e}_2$ are real unit polarization vectors, $\boldsymbol{\varepsilon} = \mathbf{e}_1 + i\,\mathbf{e}_2$, and $\boldsymbol{\varepsilon}^* = \mathbf{e}_1 - i\,\mathbf{e}_2$. Further, $\varphi = \mathsf{k}_0\cdot\mathsf{x} = \omega_0 t - \mathbf{k_0}\cdot\mathbf{r}$, where $\mathsf{k}_0 = (\omega_0/c, \mathbf{k}_0)$, is the phase with ω_0 and $\mathbf{k}_0$ the field frequency and wave vector, respectively ($k_0 = |\mathbf{k}_0| = \omega_0/c$). Here and below, $\mathsf{a}\cdot\mathsf{b}$ denotes the Lorentz invariant scalar product of two four-vectors a and b, whereas $\mathbf{a}\cdot\mathbf{b}$ denotes the scalar product of two space three-vectors.

Since the field is transverse, one has $\mathbf{k_0}\cdot\mathbf{A} = 0$. Assuming that the field is circularly polarized, we also have $\mathbf{e_1}\cdot\mathbf{e_2} = 0$. We note that in the nonrelativistic (dipole) limit, the choice of the potentials in the form (5) corresponds to the so-called velocity gauge.

Initially the electron is in a bound state. In order to ensure that, already before the collision with the projectile, this state is not substantially depleted by the action of the electromagnetic wave, we shall assume the parameters of the electromagnetic field to be such that the field itself very weakly influences this state. Namely, following [19,20], we suppose that (i) the field strength F_0 is much less than the typical intra-atomic field F_{at} acting on the electron in the bound state, and (ii) the field frequency is much smaller than the ionization potential (this is discussed in more detail in Sec. III).

Although we have supposed that the field is relatively weak, $F_0 \ll F_{at}$, the term with the vector potential in the Dirac equation, which is proportional to F_0/ω_0, for low-frequency fields is not necessarily small compared to the other terms. This is, however, a formal problem which is resolved by making the following ansatz for the initial state of the electron (see [19,20]):

$$\psi_i(\mathbf{r}, t) = \exp\left(-\frac{i}{c}\mathbf{A}\cdot\mathbf{r}\right)\phi_0(\mathbf{r}, t)\exp(-i\varepsilon_b t), \qquad (6)$$

where ε_b is the energy of the (undistorted) initial atomic state and $\phi_0(\mathbf{r}, t)$ is a function to be determined. Inserting this ansatz into (4), we obtain the Dirac equation for $\phi_0(\mathbf{r}, t)$, where the interaction with the electromagnetic wave is now represented by terms containing $\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}$, $\boldsymbol{\alpha}\cdot(\mathbf{r}\cdot\nabla)\mathbf{A}$, and $\boldsymbol{\alpha}\cdot[\mathbf{r}\times(\nabla\times\mathbf{A})]$. Due to the condition $F_0 \ll F_{at}$, in that region of space where the atomic state is located, they are much smaller than V_{at} and can be neglected. This results in $\phi_0(\mathbf{r}, t) \approx \phi_b(\mathbf{r})$, where $\phi_b(\mathbf{r})$ is the (undistorted) initial atomic state, and

$$\psi_i(\mathbf{r}, t) = \exp\left(-\frac{i}{c}\mathbf{A}\cdot\mathbf{r}\right)\phi_b(\mathbf{r})\exp(-i\varepsilon_b t)$$
$$= \phi_b(\mathbf{r})\exp\left[-i\frac{F_0}{\omega_0}(\mathbf{e_1}\cdot\mathbf{r}\cos\varphi + \mathbf{e_2}\cdot\mathbf{r}\sin\varphi)\right]$$
$$\times \exp(-i\varepsilon_b t)$$
$$= \phi_b(\mathbf{r})\exp[-id\sin(\varphi + \delta_r)]\exp(-i\varepsilon_b t), \qquad (7)$$

where

$$d\,\sin(\varphi + \delta_r) = \frac{F_0}{\omega_0}(\mathbf{e_1}\cdot\mathbf{r}\cos\varphi + \mathbf{e_2}\cdot\mathbf{r}\,\sin\varphi), \qquad (8)$$

with

$$d = \frac{F_0}{\omega_0}\sqrt{(\mathbf{e_1}\cdot\mathbf{r})^2 + (\mathbf{e_2}\cdot\mathbf{r})^2}, \quad \delta_r = \arctan\left(\frac{\mathbf{e_1}\cdot\mathbf{r}}{\mathbf{e_2}\cdot\mathbf{r}}\right). \quad (9)$$

Let us now turn to the consideration of the final state of the electron. Since we are interested in those binary-encounter collisions, which are characterized by very large (on the atomic scale) transfers of energy and momentum to the electron, in the description of the final electron state we may now neglect the interaction V_{at} between the electron and the atomic core. The corresponding Dirac equation reads

$$i\frac{\partial}{\partial t}\psi_f = \left[c\boldsymbol{\alpha}\cdot\left(\hat{\mathbf{p}}+\frac{1}{c}\mathbf{A}\right) - A^0 + \beta c^2\right]\psi_f. \qquad (10)$$

This equation has a well-known solution—the so-called Volkov (or Gordon-Volkov) states ψ_V [24,25],

$$\psi_f = \psi_V = \sqrt{\frac{c^2}{V\,\varepsilon_p}}\exp(-i\mathsf{p}\cdot\mathsf{x}) \times \exp\left\{\frac{i}{\mathsf{k}_0\cdot\mathsf{p}}\int_{-\infty}^{\varphi}\left[\frac{\mathsf{A}^2(\varphi')}{2c^2}+\frac{\mathsf{A}(\varphi')\cdot\mathsf{p}}{c}\right]d\varphi'\right\} \times\left[1-\frac{1}{2c}\frac{\not{\mathsf{k}}_0\not{\mathsf{A}}}{\mathsf{k}_0\cdot\mathsf{p}}\right]u_f(\mathbf{p},s_f). \qquad (11)$$

Here, $\mathsf{p} = (\varepsilon_p/c,\ \mathbf{p})$ is the four-momentum of the electron and we use the Feynman slash notation $\not{a} = \gamma_\mu a^\mu$. Further, $u_f(\mathbf{p}, s_f)$ is the free Dirac bispinor for an electron with a (three-) momentum $\mathbf{p}$ and spin projection $s_f = \pm 1/2$, and $\sqrt{\frac{c^2}{V\varepsilon_p}}$ is the normalization factor, where ε_p is the final energy of the electron and V is the normalization volume. We note that the state described by Eq. (11) is obtained by assuming that the interaction with the electromagnetic field is adiabatically switched on and off at $t\to-\infty$ and $t\to+\infty$, respectively.

Taking into account the explicit form of the scalar and vector potentials of the electromagnetic field, given by (5), the Volkov state (11) becomes

$$\psi_f = \sqrt{\frac{c^2}{V\,\varepsilon_p}}\exp\left(-\,i\,\mathsf{p}\cdot\mathsf{x}-\frac{i}{\mathsf{k}_0\cdot\mathsf{p}}\frac{F_0^2}{2\omega_0^2}\varphi\right) \times\exp\left[-\frac{i}{\mathsf{k}_0\cdot\mathsf{p}}\frac{F_0}{\omega_0}(\mathbf{e_1}\cdot\mathbf{p}\sin\varphi-\mathbf{e_2}\cdot\mathbf{p}\cos\varphi)\right] \times\left[1-\frac{1}{2c}\ \frac{\not{\mathsf{k}}_0\not{\mathsf{A}}}{\mathsf{k}_0\cdot\mathsf{p}}\right]u_f(\mathbf{p},s_f). \qquad (12)$$

This state, in turn, can be put into the form

$$\psi_f = \sqrt{\frac{c^2}{V\,\varepsilon_p}}\exp(-i\varepsilon_p t - i\zeta\varphi)\exp(i\,\mathbf{p}\cdot\mathbf{r}) \times\ \exp[-i\,\lambda\ \sin(\varphi-\delta)][1+\eta^-\exp(i\,\varphi) + \eta^+\exp(-i\,\varphi)]u_f(\mathbf{p},s_f)\ , \qquad (13)$$

which is more convenient for further calculations, and where

$$\zeta = \frac{1}{\mathsf{k}_0\cdot\mathsf{p}}\frac{F_0^2}{2\,\omega_0^2}, \qquad (14)$$

$$\eta^\pm = \frac{F_0}{4\omega_0\ \mathsf{k}_0\cdot\mathsf{p}}\left(\frac{\omega_0}{c}+\boldsymbol{\alpha}\cdot\mathbf{k_0}\right)(\boldsymbol{\alpha}\cdot\mathbf{e_1}\pm i\,\boldsymbol{\alpha}\cdot\mathbf{e_2}), \qquad (15)$$

$$\lambda = \frac{F_0}{\omega_0\ \ \mathsf{k}_0\cdot\mathsf{p}}\sqrt{(\mathbf{e_1}\cdot\mathbf{p})^2+(\mathbf{e_2}\cdot\mathbf{p})^2}, \qquad (16)$$

and

$$\cos\delta = \frac{\mathbf{e_1}\cdot\mathbf{p}}{\sqrt{(\mathbf{e_1}\cdot\mathbf{p})^2+(\mathbf{e_2}\cdot\mathbf{p})^2}},\qquad \sin\delta = \frac{\mathbf{e_2}\cdot\mathbf{p}}{\sqrt{(\mathbf{e_2}\cdot\mathbf{p})^2+(\mathbf{e_2}\cdot\mathbf{p})^2}}. \qquad (17)$$

Our choice of the initial and final states of the electron, which was discussed above, can be summarized as follows. Since the interaction between the electron and the relativistic projectile ion is treated within the first order of perturbation theory, this interaction is not involved in the construction of these states where the electron is supposed to move only in the fields of the atomic core and the electromagnetic wave.

The latter is a low-frequency field with strength that is much smaller than the typical strength of the atomic field acting on the bound electron. Therefore, the initial electron state is just very weakly influenced by the electromagnetic field and, as a result, can be approximated by Eq. (7).

In the final state, where the electron has energy and momentum that are much larger than their typical values in the bound atomic state, the interaction between the emitted electron and the atomic core has essentially no influence on its motion and can be neglected. However, the effect of the low-frequency electromagnetic field on a high-energy electron can be effectively strong and has to be taken into account to all orders. Therefore, the emitted electron is described by the Volkov state.

One should note that such a description of the electron states has a shortcoming. Namely, since different Hamiltonians are used to treat the initial and final states of the electron, the results for cross sections will, in general, depend on a gauge which is used to represent the potentials of the field of the relativistic projectile. However, because the choice of the initial and final states of the electron seems to be quite reasonable from the physical point of view, we would not expect any noticeable gauge dependence for our results. Nevertheless, in order to keep this point under control, below two different gauges will be employed to represent the potentials of the projectile's field.

A. Calculations with the Lienard-Wichert potentials

In the description of the various aspects of relativistic ion-atom collisions, the Lorentz family of gauges, in which the equations for the scalar and vector potentials decouple from each other and which is relativistically covariant, is most frequently used. One particular choice of a gauge belonging to this family results in the so-called Lienard-Wichert potentials, which have an especially compact form and are, therefore, widely employed (see, e.g., [3,5]). Remembering that in the rest frame of the target atom the projectile ion moves along a classical straight-line trajectory, $\mathbf{R}(t) = \mathbf{b} + \mathbf{v}t$ with $\mathbf{b} = (b_x, b_y, 0)$ and $\mathbf{v} = (0, 0, v)$, these potentials read

$$A^0_{\text{ion}} = \gamma\frac{Z_p}{s},\qquad \mathbf{A}_{\text{ion}} = \frac{\mathbf{v}}{c}A^0_{\text{ion}}. \qquad (18)$$

Here, $s = |\mathbf{s}|$, where $\mathbf{s} = [\mathbf{b}-\boldsymbol{\rho}, \gamma\,(vt - z)]$ with $\boldsymbol{\rho}$ being the transverse part ($\boldsymbol{\rho}\cdot\mathbf{v} = 0$) of the electron coordinate vector

$\mathbf{r} = (x, y, z)$. (We note that $s = \sqrt{\gamma^2(vt - z)^2 + (\mathbf{b} - \boldsymbol{\rho})^2}$ can be viewed as the distance between the projectile and the electron as seen in the rest frame of the projectile.)

With the potentials (18), the transition amplitude reads

$$a_{fi}(\mathbf{b}) = i\,\gamma\, Z_p \int d^3\mathbf{r} \int_{-\infty}^{\infty} dt\ \psi_f^{\dagger}(\mathbf{r}, t)\Big(1 - \frac{v}{c}\alpha_z\Big)\frac{1}{s}\ \psi_i(\mathbf{r}, t). \tag{19}$$

We now introduce the Fourier transform $S_{fi}(\mathbf{Q})$ of the amplitude $a_{fi}(\mathbf{b})$, which is more convenient to work with when cross sections are calculated,

$$S_{fi}(\mathbf{Q}) = \frac{1}{2\pi} \int d^2\mathbf{b}\ \exp\,(i\mathbf{Q}\cdot\mathbf{b})a_{fi}(\mathbf{b}). \tag{20}$$

Using the explicit form of the initial and final states of the electron given by (7) and (13), respectively, we obtain

$$\begin{aligned} S_{fi}(\mathbf{Q}) = {}& \frac{i\,\gamma\, Z_p}{2\pi}\sqrt{\frac{c^2}{V\,\varepsilon_p}} \int d^2\mathbf{b}\exp\,(i\mathbf{Q}\cdot\mathbf{b}) \int_{-\infty}^{\infty} dt \\ & \times \int d^3\mathbf{r} u_f^{\dagger}(\mathbf{p}, s_f)\big[1 + \eta^{-}\exp(i\varphi) + \eta^{+}\exp(-i\varphi)\big]^{\dagger} \\ & \times \Big(1 - \frac{v}{c}\,\alpha_z\Big)\frac{1}{s}\phi_b(\mathbf{r})\exp\,(-i\varepsilon_b t)\ \exp\,[id\,\sin(\varphi + \delta_r)] \\ & \times \exp\big(i\varepsilon_p t + i\zeta\varphi\big)\exp\,(-i\,\mathbf{p}\cdot\mathbf{r})\exp\,[-i\lambda\,\sin(\varphi - \delta)]. \end{aligned} \tag{21}$$

Using the series expansion

$$\exp(iZ\sin\Theta) = \sum_{n=-\infty}^{\infty} J_n(Z)\exp(in\Theta), \tag{22}$$

where Z and Θ are real quantities and J_n are the cylindrical Bessel functions (see, e.g., [26]), we obtain

$$\begin{aligned} & \exp\,[i(\lambda\,\sin(\varphi - \delta) - d\,\sin(\varphi + \delta_r)] \\ & \quad = \sum_N \exp\,[iN(\varphi - \delta)] \sum_m J_m(d)J_{N+m}(\lambda) \\ & \qquad \times\ \exp\,[-im(\delta + \delta_r)]. \end{aligned} \tag{23}$$

In order to perform, in (23), the summation over m, we employ the Graf's addition theorem (see, e.g., [26]),

$$\sum_m J_m(d)J_{N+m}(\lambda)\exp\,[-im(\delta + \delta_r)] = J_N(w)\exp\,(iN\theta), \tag{24}$$

where

$$\begin{aligned} w &= \sqrt{\lambda^2 + d^2 - 2\lambda d\,\cos(\delta + \delta_r)}, \\ w\cos\theta &= \lambda - d\,\cos(\delta + \delta_r), \\ w\sin\theta &= -d\,\sin(\delta + \delta_r). \end{aligned} \tag{25}$$

Since for a low-frequency (and not too weak) field one has $\lambda \gg \frac{F_0}{\omega_0}\sqrt{(\mathbf{e_1}\cdot\mathbf{r})^2 + (\mathbf{e_2}\cdot\mathbf{r})^2} \simeq \frac{F_0}{\omega_0}a_0$, where

$$\lambda = \frac{F_0}{\omega_0\ \mathsf{k}_0\cdot\mathsf{p}}\sqrt{(\mathbf{e_1}\cdot\mathbf{p})^2 + (\mathbf{e_2}\cdot\mathbf{p})^2}\Big(\simeq \frac{cF_0}{\omega_0^2}\Big) \tag{26}$$

and $a_0 \simeq 1/Z_t$ is the typical size of the initial state of the electron (Z_t is the effective charge of the atomic core), we approximately obtain

$$\begin{aligned} w &\simeq \lambda - d\,\cos(\delta + \delta_r) \\ &= \lambda - \mathbf{u}\cdot\mathbf{r}, \end{aligned} \tag{27}$$

with

$$\mathbf{u} = \frac{F_0}{\omega_0}\frac{(\mathbf{e}_1\cdot\mathbf{p})\,\mathbf{e}_2 - (\mathbf{e}_2\cdot\mathbf{p})\,\mathbf{e}_1}{\sqrt{(\mathbf{e}_1\cdot\mathbf{p})^2 + (\mathbf{e}_2\cdot\mathbf{p})^2}}. \tag{28}$$

Besides, from the last expression in (25), it follows that $|\sin\theta| \ll 1$. Therefore, $\sin\theta \approx \theta$ and

$$\begin{aligned} \theta &= \mathbf{a}\cdot\mathbf{r}, \\ \mathbf{a} &= -\frac{\mathsf{k}_0\cdot\mathsf{p}}{(\mathbf{e}_1\cdot\mathbf{p})^2 + (\mathbf{e}_2\cdot\mathbf{p})^2}[(\mathbf{e}_1\cdot\mathbf{p})\,\mathbf{e}_1 + (\mathbf{e}_2\cdot\mathbf{p})\,\mathbf{e}_2]. \end{aligned} \tag{29}$$

Now we can perform, in (21), the integrations over the time t and the impact parameter $\mathbf{b}$. The corresponding result can be written in the following form:

$$\begin{aligned} S_{fi}(\mathbf{Q}) = {}& -2i\frac{Z_p}{v}\sqrt{\frac{c^2}{V\,\varepsilon_p}}\sum_N \exp(iN\delta)u_f^{\dagger}(\mathbf{p}, s_f) \\ & \times\ \frac{1}{Q^2 + q_{\min}^2/\gamma^2}\mathcal{G}_N. \end{aligned} \tag{30}$$

In this expression,

$$\mathcal{G}_N = [\Phi_0\mathcal{J}_N + \Phi_{-}\mathcal{J}_N^{+}\exp(-i\,\delta) + \Phi_{+}\mathcal{J}_N^{-}\exp(i\,\delta)], \tag{31}$$

where

$$\Phi_0 = \Big(1 - \frac{v}{c}\alpha_z\Big);\quad \Phi_{\pm} = \eta_{\pm}^{\dagger}\Phi_0, \tag{32}$$

$$\begin{aligned} \mathcal{J}_N &= \int d^3\mathbf{r}\exp(i\boldsymbol{\kappa}_N\cdot\mathbf{r})\phi_b(\mathbf{r})J_N(w), \\ \boldsymbol{\kappa}_N &= \mathbf{Q} + q_{\min}\frac{\mathbf{v}}{v} - (\zeta + N)\,\mathbf{k}_0 - \mathbf{p} + N\mathbf{a}, \\ \mathcal{J}_N^{\pm} &= \int d^3\mathbf{r}\exp(i\boldsymbol{\kappa}_N^{\pm}\cdot\mathbf{r})\phi_b(\mathbf{r})J_{N\pm1}(w), \\ \boldsymbol{\kappa}_N^{\pm} &= \boldsymbol{\kappa}_N \pm \mathbf{a}, \end{aligned} \tag{33}$$

and

$$q_{\min} = \frac{\varepsilon_p - \varepsilon_b + (\zeta + N)\omega_0}{v}. \tag{34}$$

The fully differential cross section reads

$$\frac{d\sigma}{d^2\mathbf{Q}\,d^3\mathbf{p}} = |S_{fi}|^2\frac{V}{8\pi^3}. \tag{35}$$

By summing over the final spin projections and integrating over the transverse momentum transfer $\mathbf{Q}$, we obtain the differential cross section, which determines the spectra of the emitted electrons,

$$\begin{aligned} \frac{d\sigma}{d^3\mathbf{p}} = {}& \frac{1}{2\pi^3}\frac{Z_p^2\,c^2}{v^2\,\varepsilon_p}\sum_{s_f}\sum_N \\ & \times \int d^2\mathbf{Q}\Big|u_f^{\dagger}(\mathbf{p}, s_f)\frac{1}{Q^2 + q_{\min}^2/\gamma^2}\mathcal{G}_N\Big|^2. \end{aligned} \tag{36}$$

B. Calculations in a gauge with vanishing scalar potential

Let us now calculate the emission cross sections using another gauge for the potentials of the projectile. A known disadvantage of the potentials in the Lienard-Wichert form is a very delicate near-cancellation between the contributions of the scalar and vector potentials to the component of the electric field parallel to the projectile velocity. This near-cancellation, in particular, does not hold if the initial and final states of the electron belong to different Hamiltonians.

In order to avoid this problem, we choose a gauge in which the scalar potential vanishes. To this end, we perform the gauge transformation of the Lienard-Wichert potentials, with the gauge function f given by

$$f = \frac{c}{v} Z_p \ln(s + s_z), \tag{37}$$

which eliminates the scalar potential. The new potentials then read

$$A^{0\,ng}_{\rm ion} = 0,$$
$$\mathbf{A}^{ng}_{\rm ion} = -\frac{c}{v}\frac{Z_p}{s}\left(\frac{s_x}{s+s_z};\ \frac{s_y}{s+s_z};\ \frac{1}{\gamma}\right). \tag{38}$$

With the potentials (38), the transition amplitude is given by

$$a_{fi}(\mathbf{b}) = -i\,\frac{c}{v} Z_p \int d^3\mathbf{r} \int_{-\infty}^{\infty} dt\ \psi_f^\dagger(\mathbf{r},t) \times \left[\frac{\alpha_x\, s_x}{s(s+s_z)} + \frac{\alpha_y\, s_y}{s(s+s_z)} + \frac{\alpha_z}{\gamma s}\right]\psi_i(\mathbf{r},t). \tag{39}$$

Using the same techniques as before, we obtain

$$S_{fi}(\mathbf{Q}) = 2i\frac{Z_p c}{v^2}\sqrt{\frac{c^2}{V\,\varepsilon_p}}\sum_N \exp(iN\delta)u_f^\dagger(\mathbf{p},s_f) \times \frac{1}{(Q^2 + q_{\rm min}^2/\gamma^2)}\frac{1}{q_{\rm min}}\mathcal{G}_N^{ng}, \tag{40}$$

where

$$\mathcal{G}_N^{ng} = \left[\Phi_0^{ng}\mathcal{J}_N + \Phi_-^{ng}\mathcal{J}_N^+ \exp(-i\delta) + \Phi_+^{ng}\mathcal{J}_N^- \exp(i\delta)\right], \tag{41}$$

$$\Phi_0^{ng} = \left(\alpha_x Q_x + \alpha_y Q_y + \alpha_z\frac{q_{\rm min}}{\gamma^2}\right),$$
$$\Phi_\pm^{ng} = \eta_\pm^\dagger\Phi_0^{ng}, \tag{42}$$

and the other notations in these expressions are the same as when using the Lienard-Wichert potentials. The differential cross section determining the spectra of emitted electrons now reads

$$\frac{d\sigma}{d^3\mathbf{p}} = \frac{1}{2\pi^3}\frac{Z_p^2\,c^4}{v^4\,\varepsilon_p}\sum_{s_f}\sum_N \times \int d^2\mathbf{Q}\left|u_f^\dagger(\mathbf{p},s_f)\,\frac{1}{(Q^2+q_{\rm min}^2/\gamma^2)}\frac{1}{q_{\rm min}}\mathcal{G}_N^{ng}\right|^2. \tag{43}$$

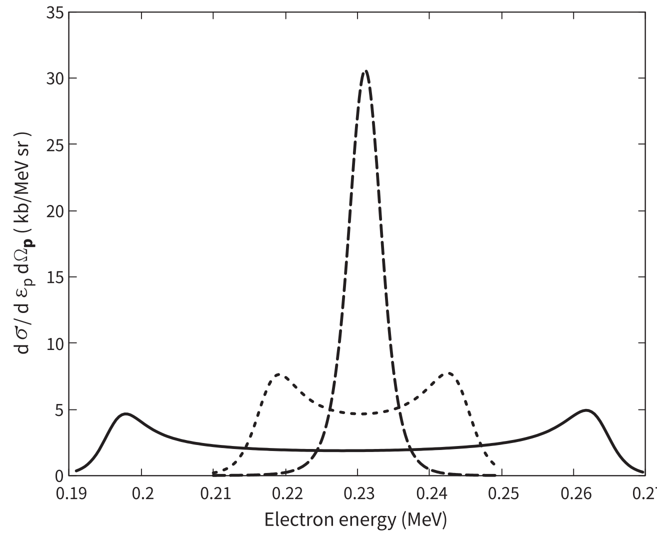

FIG. 1. The energy spectrum of electrons emitted under $\vartheta_p = 1^\circ$ in 0.1 GeV/u Ne^{10+}+ He → Ne^{10+} + He^+ + e^- collisions. Dashed curve: field-free collisions; dotted curve: collisions in the presence of the electromagnetic field with $F_0 = 0.02$ a.u. and $\omega_0 = 0.004$ a.u. Solid curve: same as in the dashed curve, but for $F_0 = 0.05$ a.u. The calculations were performed using the Lienard-Wichert potentials.

III. RESULTS AND DISCUSSION

In this section, we discuss some results for electron emission computed by using Eqs. (36) and (43). The electrons produced in binary-encounter emission reach the largest energies when they are ejected under small angles with respect to the projectile velocity, and in what follows we focus on this particular angular emission domain.

We choose helium atoms as a target and suppose that they are bombarded by high-energy bare neon nuclei Ne^{10+} in the presence of a low-frequency electromagnetic field of moderate intensity. The helium atoms are initially in the ground state. With our assumptions (see the previous section) that the electromagnetic field has almost no influence on the ground state of the atom and that $Z_p/v \ll 1$, it becomes obvious that double ionization in such collisions will be just a tiny fraction of single ionization and can, therefore, be ignored. Since we are interested, first of all, in the qualitative changes in the emission spectra caused by the action of the electromagnetic field, for simplicity in our calculations helium is regarded as a hydrogenlike ion with an effective nuclear charge $Z_t = 1.345$ which yields a good value for the single-ionization potential of helium.

Figures 1–3 show the energy spectra of electrons emitted under the angle $\vartheta_p = 1^\circ$ from helium atoms in collisions with Ne^{10+} projectiles having energy of 0.1 GeV/u ($v \approx 58$ a.u., $\gamma \approx 1.1$), 0.5 GeV/u ($v \approx 104$ a.u., $\gamma \approx 1.5$), and 1 GeV/u ($v \approx 120$ a.u., $\gamma \approx 2.08$), respectively. The collisions occur in the presence of a circularly polarized electromagnetic wave with frequency $\omega_0 = 0.004$ a.u. (≈0.11 eV), which propagates perpendicular to the projectile velocity (for definiteness, we choose the propagation direction to be along the x axis). The field strength F_0 is equal to 0, 0.02, and 0.05 a.u., and the corresponding field intensity is 0, 1.4×10^{13}, and 8.75×10^{13} W/cm^2, respectively. We note that the lifetime

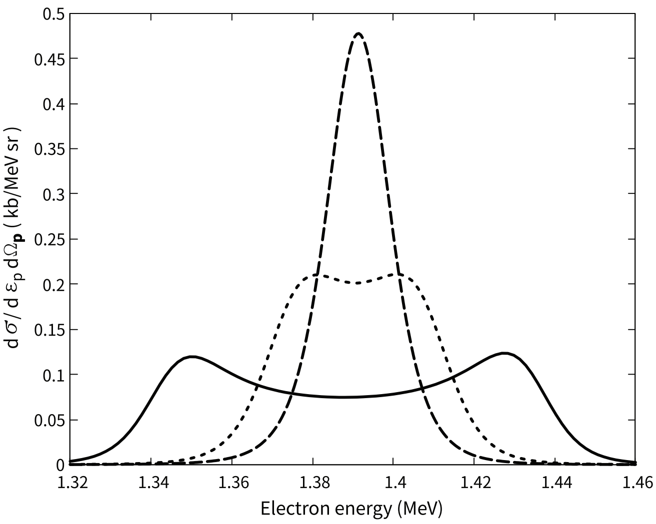

FIG. 2. Same as in Fig. 1, but for the projectile energy of 0.5 GeV/u.

of the ground state of helium with respect to photoionization by the field with $F_0 = 0.05$ a.u. is $\simeq 10^{-5}$ sec (for instance, a nanosecond laser pulse would leave the atoms essentially unaffected). We also note that the chosen collision geometry—the electrons are emitted (almost) parallel to the projectile velocity whereas the electromagnetic wave propagates perpendicular to it—provides favorite conditions for the coupling between the electron and the electromagnetic field.

Let us first briefly discuss the binary-encounter emission in the field-free case ($F_0 = 0$ a.u.). The inspection of the integrals (33), which enter the cross section (36) [or (43)], shows that the position of the binary peak is determined by the condition $p_\parallel = q_{\min}$, where $p_\parallel$ is the component of the momentum of the electron parallel to the projectile velocity. Indeed, when this condition [27] is fulfilled, the two highly oscillating factors, $\exp(-ip_\parallel z)$ and $\exp(iq_{\min} z)$, in the integrands of (33)

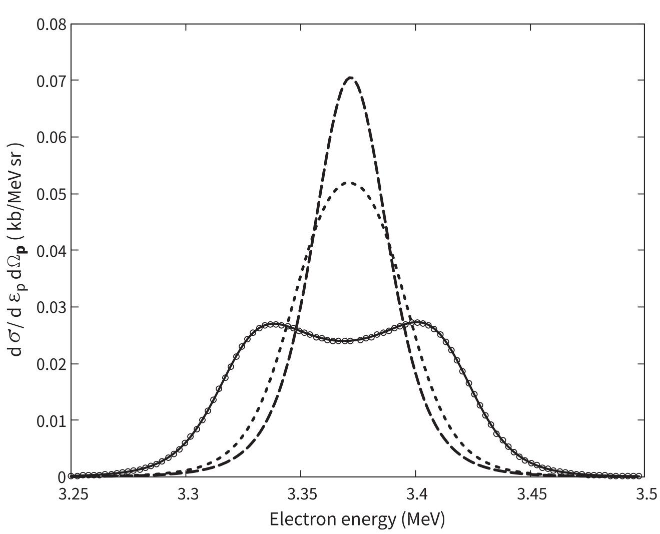

FIG. 3. Same as in Fig. 1, but for the projectile energy of 1 GeV/u. In addition, the corresponding results obtained by using the gauge with the zero scalar potential are shown by circles.

compensate each other, which leads to a strong increase in the magnitude of the cross section. By solving the equation $p_\parallel = q_{\min}$, we obtain that the binary peak, considered as a function of the projectile velocity v and the electron emission angle ϑ_p, is centered at the electron momentum p_c given by

$$p_c = \frac{2\,v\,\cos\vartheta_p}{1 - \frac{v^2}{c^2}\cos^2\vartheta_p} - \frac{I_b}{v\,\cos\vartheta_p}\frac{1 + \frac{v^2}{c^2}\cos^2\vartheta_p}{1 - \frac{v^2}{c^2}\cos^2\vartheta_p}, \qquad (44)$$

where I_b ($I_b > 0$) is the binding energy of the (active) electron in the initial target state [28,29]. Expression (44) is valid for ϑ as long as $v^2\cos^2\vartheta_p \gg I_b$ (and only if $\cos\vartheta_p > 0$). For relativistic collisions ($v \sim c$) with light targets, the latter is very well fulfilled for almost all emission angles (unless they become close to $\pi/2$). In the case of the forward emission from the helium target ($I_b \approx 24.6$ eV), the term in (44) containing the binding energy is comparatively very small and can simply be ignored. The corresponding central energy, $\varepsilon_{p_c} = \sqrt{c^2\,p_c^2 + c^4}$, reads

$$\begin{aligned}\varepsilon_{p_c} &= 2\,c^2\left(\frac{1}{1 - \frac{v^2}{c^2}\cos^2\vartheta_p} - \frac{1}{2}\right)\\ &= c^2\,\frac{\gamma^2 + (\gamma^2 - 1)\cos^2\vartheta_p}{1 + (\gamma^2 - 1)\sin^2\vartheta_p}.\end{aligned} \qquad (45)$$

The inspection of the same integrals (33) also shows that the effective energy width $\delta\varepsilon_p$ of the field-free binary peak can be determined from the equation $|p_\parallel - q_{\min}| \simeq p_t$, where $p_t \simeq Z_t$ is the typical orbiting momentum of the bound atomic electron in its initial state. Indeed, by writing $p_\parallel \approx (p_c + \delta p)\cos\vartheta_p$ and $\varepsilon_p \approx \varepsilon_{p_c} + (\partial\varepsilon_p/\partial p)_{p=p_c}\delta p$, we obtain that

$$\begin{aligned}\delta\varepsilon_p &\simeq \left.\frac{\partial\varepsilon_p}{\partial p}\right|_{p=p_c} |\delta p|\\ &= \frac{2p_t v\gamma^2}{1 + (\gamma^2 - 1)\sin^2\vartheta_p}.\end{aligned} \qquad (46)$$

It follows from (45) and (46) that the position and the width of the binary peak for forward emission rapidly increase with the impact energy, with $\delta\varepsilon_p$ always remaining much smaller than ε_{p_c} (this, in particular, is illustrated in Figs. 2 and 3). We also note that in extreme relativistic collisions ($\gamma \gg 1$), the central energy and the width of the binary peak scale as γ^2.

Let us now turn to the field-assisted collisions. There are a few main conclusions which can be drawn from the results shown in Figs. 1–3. First, like in the nonrelativistic case (see [19,20]), a relatively weak low-frequency electromagnetic field, which has almost no effect on the free target, may have a strong impact on the binary-encountered emission. Second, the main effect of the field is a qualitative change to the shape of the binary peak, which splits into two maxima. Third, keeping the field parameters unchanged while increasing the impact energy weakens the influence of the field on the binary peak (this is especially clearly seen by comparing Figs. 1 and 3). The last point, which is absent in the nonrelativistic collisions, becomes especially pronounced in extreme relativistic collisions where the effect of the electromagnetic field on the binary peak essentially vanishes (not shown in the figures).

The effect of the electromagnetic field on the binary emission is caused, first of all, by an energy exchange between the emitted electron and the field. Taking into account the properties of the Bessel functions, the energy exchange $\Delta\varepsilon$ can be roughly estimated as $\Delta\varepsilon \simeq \lambda\,\omega_0 \approx F_0\, v_e/\omega_0$ [see Eq. (26)], where v_e is the velocity of the emitted electron. This exchange can be quite large and when it substantially exceeds the width $\delta\varepsilon_p$ of the field-free binary peak given by (46), two maxima appear in the field-assisted binary peak.

In nonrelativistic collisions, when the impact energy (per nucleon) grows, both the energy exchange with the field and the width of the field-free binary peak are proportional to the collision velocity. Therefore, the relative effect of the electromagnetic field on the binary emission remains a constant. In relativistic collision, however, the situation is different. Now with an increase of the collision energy, the velocity v_e of the emitted electron, which enters $\Delta\varepsilon$, grows slower than the quantity γ^2, which enters the width $\delta\varepsilon_p$ [see Eq. (46)]. This is why, in the relativistic case, the effect of the field on the binary emission weakens with an increase of the impact energy. Moreover, it also becomes obvious that in extreme relativistic collisions ($v \to c$, $\gamma \gg 1$), where the electron velocity remains already essentially a constant ($v_e \simeq c$) but γ continues to increase, the effect of the field tends to fully vanish.

Let us now briefly touch upon the point of gauge (in)dependence of the calculated results. In Fig. 3, in addition to the results of calculations in which the projectile field is described by the Lienard-Wichert potentials, we also display spectra obtained when the projectile potentials are chosen in the form (38). An excellent agreement between the two sets of calculations indicates that the initial and final electron states are properly described. It is worth noting, however, that had we tried to apply our approximation for the final electron state (13) to treat electrons of relatively low kinetic energy ($\varepsilon_p - c^2 \lesssim Z_t^2$), this excellent agreement would turn into a very strong disagreement [30], clearly showing that the Volkov states are not appropriate for describing electrons emitted with low kinetic energies (see, also, [29]).

Concerning the description of the interaction with the electromagnetic wave, we note that the theoretical approach used in this paper is gauge invariant with respect to the transformation between the two most popular gauges: the "velocity gauge," given by (5), and the gauge where

$$A'_0 = F_0(\mathbf{e}_2\cdot\mathbf{r}\cos\varphi - \mathbf{e}_1\cdot\mathbf{r}\sin\varphi),$$
$$\mathbf{A}' = A'_0\, c\,\mathbf{k}_0/\omega_0, \tag{47}$$

which in the nonrelativistic (dipole) limit reduces to the so-called length gauge. In this sense, the situation here is similar to those in [9,16], where field-assisted Compton scattering on a bound atomic electron and radiative recombination of an electron with a highly charged ion, respectively, were studied.

In order to put the approach of this paper into a broader perspective of the theory of atomic systems interacting with a laser field, we complete this section by comparing it with the so-called strong-field approximation (SFA) [31–34], which is a well-known method widely used for describing ionization of atoms (and ions) by a strong laser field.

Like some other theories in the field of atomic, molecular and optical physics, a treatment of atomic ionization by a laser field can be based on either of the following two expressions:

$$a_{fi}^{(+)} = -i\int_{-\infty}^{+\infty} dt \Big\langle \Big[\hat{H}(t) - i\frac{\partial}{\partial t}\Big]\chi_f(t) \mid \psi_i^{(+)}(t)\Big\rangle,$$
$$a_{fi}^{(-)} = -i\int_{-\infty}^{+\infty} dt \Big\langle \psi_f^{(-)}(t)\Big|\Big[\hat{H}(t) - i\frac{\partial}{\partial t}\Big]\varphi_i(t)\Big\rangle, \tag{48}$$

which represent the so-called post, $a_{fi}^{(+)}$, and prior, $a_{fi}^{(-)}$, forms of the transition amplitude (see, e.g., [5], pp. 66–67). In (48), $\psi_i^{(+)}(t)$ and $\psi_f^{(-)}(t)$ are solutions of the full Schrödinger (or Dirac) equation,

$$i\frac{\partial\psi^{(\pm)}(t)}{\partial t} = \hat{H}(t)\psi^{(\pm)}(t), \tag{49}$$

which satisfy the "in" [$\psi_i^{(+)}(t)$] and "out" [$\psi_f^{(-)}(t)$] boundary conditions, and the Hamiltonian $\hat{H}(t)$ includes all the interactions. Further, $\varphi_i(t)$ and $\chi_f(t)$ are the initial and final asymptotic states, respectively, which are solutions of the wave equation with the corresponding asymptotic Hamiltonians $\hat{H}_i$ and $\hat{H}_f$.

In [33], a treatment of atomic ionization by a laser field was formulated based on (48). The amplitude $a_{fi}^{(-)}$ (as more physically appealing for an approximate consideration of this process) was used, $\varphi_i(t)$ was taken as the undistorted initial atomic state (consequently, the "driving" interaction, $[\hat{H}(t) - i\partial/\partial t]\varphi_i(t)$, is the interaction with the laser field), and $\psi_f^{(-)}(t)$ was approximated by a Volkov state; the "velocity" gauge was used.

One can look for the exact state $\psi_f^{(-)}(t)$ by expanding it in the interaction with the atomic potential. Therefore, the result of [33] represents the first term of the corresponding series. It is this first term which is often referred to as the SFA (in the velocity gauge). If it is obtained by using another gauge for the potentials of the laser field, the result will, in general, be different. An expression for the transition amplitude similar to that obtained in [33] was earlier employed in [31], where the only essential difference was the use of the "length" gauge (the SFA in the length gauge).

An attempt to formulate the SFA, based on using the post form $a_{fi}^{(+)}$ of the transition amplitude (48), was undertaken in [32]. The Kramers-Henneberger transformation [35] was employed to construct an approximate expression for $\psi_i^{(+)}(t)$. The final asymptotic state $\chi_f(t)$ was the unperturbed (continuum) atomic state, correspondingly, the term $(\hat{H} - i\frac{\partial}{\partial t})\chi_f(t)$ represented the interaction with the laser field. The velocity gauge was used. The resulting amplitude, in general, does not coincide with that derived in [33]. However, if the continuum atomic state $\chi_f(t)$ is approximated by a plane wave, the expression for the transition amplitude given in [32] goes over into that obtained in [33].

Thus, in the SFA, the field, which drives the transition between the initial and final electron state, is the laser field [36] and the initial state is the undistorted atomic state (in any gauge). In contrast, in our case, the field, which drives the transition between the initial and final state, is the field of the projectile. Besides, our initial state (6) is not the undistorted atomic state, but also involves an important exponential factor depending on the laser field. Also, as was already mentioned, the present approach leads to results which are invariant with respect to the transformation between the velocity and length gauges, whereas the SFA is known to be rather strongly gauge dependent with respect to such a transformation.

Therefore, the only common point is that both the approach of the present paper and the SFA employ the Volkov state as the final state of the electron. However, even within this common point, there is an important difference between the use of the Volkov state in the problem, which we consider, and in atomic ionization by a laser field.

In our case, the emitted electron has very high energy and, therefore, the use of the Volkov state is very well justified. Contrary to that, in atomic ionization by a laser field, the overwhelming majority of the emitted electrons has relatively low energies and the use of the Volkov state becomes rather questionable if applied to the ionization of neutral atoms where the field of the residual ion has a strong effect on the motion of such electrons.

IV. CONCLUSION

We have considered the emission of high-energy electrons, which occurs in relativistic collisions of bare ions with atoms assisted by a circularly polarized low-frequency electromagnetic field. The field intensity was chosen to be rather modest such that the field itself has almost no effect on the initial (ground) state of the atom. Nevertheless, as our consideration has shown, such a field can qualitatively change the shape of the spectra of the emitted high-energy electrons due to a very large energy exchange between the electron and the field.

Compared to nonrelativistic field-assisted ion-atom collisions, for which similar effects were predicted earlier [19,20], in relativistic collisions the influence of the field on the energy spectrum was found to be comparatively weaker. The latter is mainly due to the fact that the coupling (and thus the energy exchange) between the emitted electron and the field increases with the impact energy (much) slower than the width of the energy distribution of electrons emitted in the field-free collisions.

Throughout the paper, we used the term "binary encounter" both for the field-free and field-assisted collisions. It should, however, be remarked that in the field-assisted collisions, the process of emission can be thought of as consisting of two steps. First, the electron bound in the atom experiences a violent binary collision with the projectile-ion, acquiring, as a result, very high energy momentum. Second, this high-energy electron interacts with the low-frequency electromagnetic field. Since the appearance of the electron in the field is "sudden," the energy exchange between them is nonzero and can, in fact, reach quite large values (for instance, tens of keV in the examples considered in Figs. 2 and 3). Thus, strictly speaking, in the field-assisted collisions, the binary-encounter character of electron emission ceases to exist because the field becomes an important "third body" in this process.

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[27] It is worth noting that the condition $p_\parallel = q_{\rm min}$ corresponds to the momentum conservation (along the z axis) for a binary-encounter collision and, since $q_{\rm min} = (\varepsilon_p - \varepsilon_b)/v$, already includes the energy conservation in the collision.

[28] The equation $p_\parallel = q_{\rm min}$ has two solutions. The other, given (at $v^2 \cos^2 \vartheta_p \gg I_b$) by $p \approx I_b/(v \cos \vartheta_p)$, corresponds to electrons emitted with low energy and is not of interest for the present study [29].

[29] Besides, such low-energy electrons cannot be described by plane waves and, therefore, our treatment, which uses the Gordon-Volkov states, could not be applied in this case.

[30] At large γ's, the difference between the results reaches orders of magnitude. The use of the Lienard-Wichert potentials in this case yields, as expected, clearly meaningless results.

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[36] Since in the SFA the initial state is the undistorted atomic state and the final state is the Volkov state, the SFA can also be reformulated in such a way that it will (formally) be the atomic field which drives the transition. In this respect, the SFA is similar to the so-called Oppenheimer-Brinkman-Kramers approximation used to describe electron capture in fast ion-atom collisions (see, e.g., [3,4] and references therein) in which either of the nuclei (atomic or ionic) can be considered as driving the transition.