Resonant photoproduction of high-energy electron-positron pairs in the field of a nucleus and a weak electromagnetic wave

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The resonant photoproduction of ultrarelativistic electron-positron pairs (PPPs) in a nuclear field and a weak laser field is theoretically studied. Under resonance conditions, the intermediate virtual electron (positron) in the laser field becomes a real particle. As a result, the initial process of the second order in the fine structure constant in the laser field effectively reduces into two successive processes of the first order: single-photon production of electron-positron pair in the laser field (laser-stimulated Breit-Wheeler process) and laser-assisted process of electron (positron) scattering on a nucleus. Resonant kinematics of PPPs is studied in detail. It is shown that for the considered laser intensities resonance is possible only for the initial photon energies greater than the characteristic threshold energy. At the same time, the ultrarelativistic electron and positron propagate in a narrow cone along the direction of the initial photon momentum. The resonant energy of the positron (electron) can has two values for each outgoing angle that varies from zero to some maximum value determined by the energy of the initial photon and the threshold energy. Resonant differential cross section of the studied process was obtained. It is shown that the resonant differential cross section of the PPP can significantly exceed the corresponding cross section of the PPP without an external field. The project calculations may be experimentally verified by the scientific facilities of pulsed laser radiation (SLAC, FAIR, XFEL, ELI, XCELS).

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I. INTRODUCTION

Due to the use of the powerful laser sources in modern applied and fundamental research [1–5] a theoretical study of quantum electrodynamics (QED) processes in strong light fields seems to be one of the priorities that is developing intensively (see, for example, Refs. [6–51]). The main research results were systematized in monographs [11–14] and reviews [15–21].

In particular, the process of photoproduction of an electron-positron pair (PPP) on the nucleus is of great interest since starting from energies above 3 MeV this process is a prevailing mechanism of the interaction of γ radiation with matter. A lot of works are devoted to this problem [12–14,18–20,26–28,35,38,42]. Separately, an article should be noted [50] where authors theoretically analyzed the formation of the electron-positron pair by the photon in a multifrequency electromagnetic field as well as an article [51], which is dedicated to influence of a strong laser field on the Bethe-Heitler photoproduction process by a relativistic nucleus.

It is important to emphasize that QED processes of higher than the first order in the fine structure constant in a laser field (laser-assisted QED processes) can occur in both nonresonant and resonant channels. In the laser field so-called Oleinik's resonances [9,10] can take place associated with the fact that in the light field lower-order processes in the fine structure constant are allowed (laser-stimulated QED processes) [15]. It is important to note that the probability of the resonant QED processes in a laser field can significantly (by several orders of magnitude) exceed the corresponding probability of the process without an external field.

We underline that the Oleinik's resonances for the PPP on the nucleus in the wave field were studied only for one of the possible channels, where the initial γ-quantum decay into a positron and an intermediate electron in the wave field, which is then scattered by the nucleus. The second channel, where the initial photon decay into an electron and an intermediate positron, which is then scattered by the nucleus has not been studied. We also emphasize that for the first channel, only the case of an ultrarelativistic pair was considered, when the positron propagates in a narrow cone with the direction of the initial photon momentum, and the electron is scattered at large angles [12–14,19,20].

In this paper, the theory of the resonant PPP of ultrarelativistic energies is developed for the case where the positron and the electron propagate in a narrow cone along the direction of momentum of the initial photon, i.e., with taking into account channels a and b (see Fig. 2). The process is considered in the Born approximation for interaction with the field of the nucleus. ($v/c \gg Z/137$, v is the electron velocity, c is a speed of light in vacuum, Z is the nuclear charge number).

We point out that there are two characteristic parameters in a problem of PPP: classical relativistically invariant parameter [15,19–22],

$$\eta = \frac{eF\lambda\!\!\!^{-}}{mc^2}, \tag{1}$$

numerically equal to the ratio of the work of the field at a wavelength to the rest energy of the electron (where e and m

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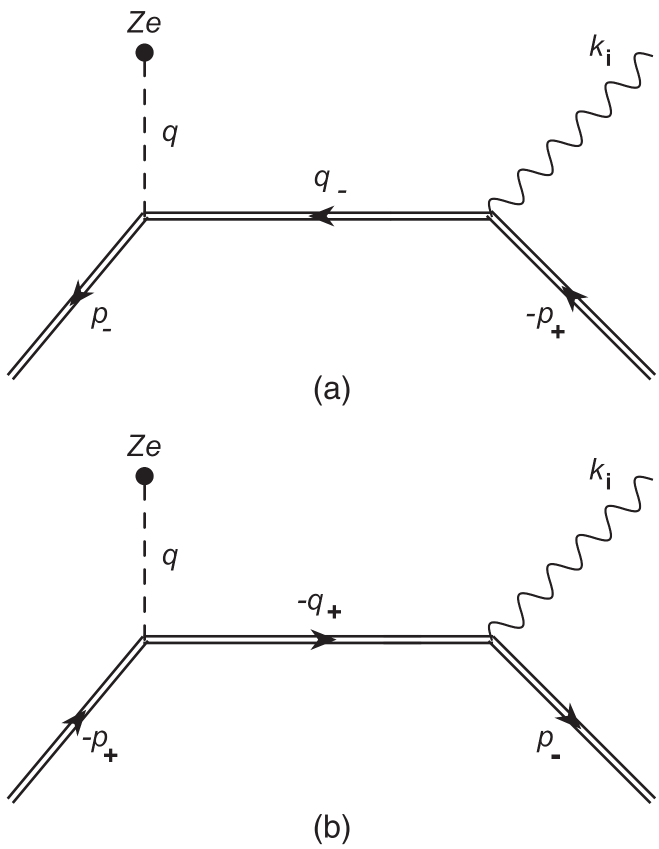

FIG. 1. Feynman diagrams of the PPP process on the nucleus in the field of a plane electromagnetic wave. Double incoming and outgoing lines correspond to the Volkov functions of the electron and positron in the initial and final states, the inner line corresponds to the Green's function of the fermion in the field of a plane wave (4). Wavy line corresponds to four-momentum of the initial gamma quantum. Dashed line stands for four-momentum of pseudophoton of recoil

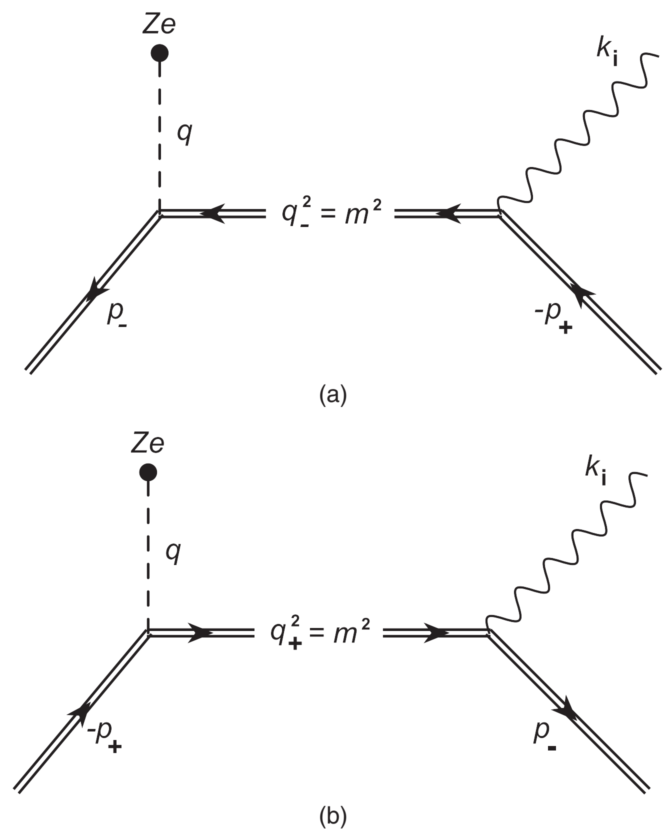

FIG. 2. Resonant photoproduction of the electron-positron pair in the field of the nucleus and a plane electromagnetic wave.

are the charge and mass of the electron, F and $\lambda\!\!\!^{-} = c/\omega$ are the strength of the electric field and wavelength, correspondingly, ω is the frequency of wave) and quantum multiphoton parameter (Bunkin-Fedorov parameter) [8,11,19–22]:

$$\gamma_i = \eta \frac{m v_i c}{\hbar\omega}. \tag{2}$$

Herein v_i is the velocity of the initial electron. Within the optical range of the frequencies ($\omega \sim 10^{15} s^{-1}$) the classical parameter is $\eta \sim 1$ for the fields of $F \sim 10^{10}/10^{11}\ V/cm$, quantum multiphoton parameter is $\gamma_i \sim 1$ for the fields of $F \sim (10^5/10^6)(c/v_i)\ V/cm$. Therefore, when $\eta \ll 1$ the quantum multiphoton parameter γ_i may be large. However, this is true only when the electrons (positrons) are scattered on the nucleus at large angles. In this case, the main parameter determining multiphoton processes is the Bunkin-Fedorov quantum parameter. Accordingly, the problem is usually studied in the intensity of moderately strong fields, in which these parameters satisfy the following conditions: $\eta \ll 1,\ \gamma_i \gg 1$. It is important that for the process of PPP with the scattering of electrons (positrons) on the nucleus at small angles, the Bunkin-Fedorov quantum parameter does not appear [44]. In this case, the main parameter of multiphoton processes is the classical parameter η and the laser field can be considered as weak [44]:

$$\eta \ll 1. \tag{3}$$

Equation (3) is a basic condition in the present paper. Further in this paper we use the relativistic system of units: $\hbar = c = 1$.

II. AMPLITUDE OF THE PPP ON A NUCLEUS IN A LIGHT FIELD

Let us take a four-potential of the external elliptically polarized light wave propagating along the axis z in the following form:

$$A(\phi) = \frac{F}{\omega}(e_x \cos\phi + \delta e_y \sin\phi),\ \ \phi = kx = \omega(t - z). \tag{4}$$

Here δ is the ellipticity parameter ($\delta = 0$ is the linear polarization, $\delta = \pm 1$ is the circular polarization), $e_{x,y} = (0, \mathbf{e}_{x,y})$ and $k = \omega n = \omega(1, \mathbf{n})$ are four-vectors of polarization and the momentum of the electromagnetic wave, particularly: $k^2 = 0,\ e_{x,y}^2 = -1,\ (e_{x,y}k) = 0$.

We study the problem of PPP on the nucleus in the field of a plane electromagnetic wave in the Born approximation on the interaction of electrons and positrons with the field of the nucleus. This is a second-order process in the fine structure constant and it is described by two Feynman diagrams (see Fig. 1).

The amplitude of such process after simple calculations can be represented as follows (see [12–14,18,19,27]):

$$S_{fi} = \sum_{l=-\infty}^{\infty} S_l, \tag{5}$$

where the partial amplitude with emission and absorption of $|l|$ photons of the wave has the following form:

$$S_l = i\frac{8Ze^3\pi^{5/2}}{\sqrt{2E_-E_+\omega_i}}\exp(i\tilde{\phi})[\bar{u}M_l v]\frac{\delta(q_0)}{\mathbf{q}^2}, \tag{6}$$

$$M_l = \sum_{l_1=-\infty}^{\infty}\left[H^0_{l+l_1}(\tilde{p}_-, q_-)\frac{\hat{q}_- + m_*}{q_-^2 - m_*^2}\varepsilon_\mu H^\mu_{l_1}(q_-, \tilde{p}_+)\right.$$
$$\left. + \varepsilon_\mu H^\mu_{l_1}(\tilde{p}_-, q_{+,})\frac{\hat{q}_+ + m_*}{q_+^2 - m_*^2}H^0_{l+l_1}(q_+, \tilde{p}_+)\right]. \tag{7}$$

In the relations (6)–(7) $\tilde{\phi}$ is the independent from the summation indexes phase, ε_μ is the four-vector of polarization of initial photon $v, \bar{u}$ and $\tilde{p}_\pm = (\tilde{E}_\pm, \tilde{\mathbf{p}}_\pm)$ are the Dirac bispinors and the four-quasimomenta of the positron and electron. The four-momenta of the intermediate positron and electron $q_\pm$ and the transmitted four-momentum q are determined by the expressions:

$$q_- = k_i + k - l_1\tilde{p}_+,\ q_+ = k_i + k - l_1\tilde{p}_-, \tag{8}$$

$$q = \tilde{p}_+ + \tilde{p}_- - k_i - lk. \tag{9}$$

Here $k_i = \omega_i(1, \mathbf{n_i})$ is the four-momentum of the initial photon, m_* is the effective mass of the electron in the light field [15,47,48]:

$$\tilde{p}_\pm = p_\pm + (1+\delta^2)\eta^2\frac{m^2}{4(kp_\pm)}k, \tag{10}$$

$$\tilde{p}^2_\pm = m_*^2,\ m_* = m\sqrt{1 + \frac{1}{2}(1+\delta^2)\eta^2}. \tag{11}$$

Here $p_\pm = (E_\pm, \mathbf{p}_\pm)$ are four-momenta of positron and electron. Notation with the hat in relation (7) and further stands for the dot product of the corresponding four-vector with Dirac γ matrices: $\tilde{\gamma}^\mu = (\tilde{\gamma}^0, \tilde{\gamma}), \mu = 0, 1, 2, 3$. For example, $\hat{q}_+ = q_+^\mu\tilde{\gamma}_\mu = q_+^0\tilde{\gamma}_0 - \mathbf{q}_+\tilde{\gamma}$. The amplitudes $H^\mu_{l_1}$ and $H^0_{l+l_1}$ (see Fig. 1) in relation (7) have the following form:

$$H^\mu_{l_1}(p_2, p_1) = a^\mu L_{l_1}(p_2, p_1) + b^\mu_- L_{l_1-1}$$
$$+ b^\mu_+ L_{l_1+1} + c^\mu(L_{l_1+2} + L_{l_1-2}), \tag{12}$$

$$H^0_{l+l_1}(p_2, p_1) = a^0 L_{l+l_1}(p_2, p_1) + b^0_- L_{l+l_1-1}$$
$$+ b^0_+ L_{l+l_1+1} + c^0(L_{l+l_1+2} + L_{l+l_1-2}), \tag{13}$$

matrices $a^\mu, b^\mu_\pm, c^\mu$ are determined by the expressions:

$$a^\mu = \tilde{\gamma}^\mu + \left(1+\delta^2\right)\eta^2\frac{m^2}{4(kp_1)(kp_2)}k^\mu, \tag{14}$$

$$b^\mu_\pm = \frac{1}{4}\eta m\left[\frac{\hat{\varepsilon}_\pm\hat{k}\tilde{\gamma}^\mu}{(kp_2)} + \frac{\tilde{\gamma}^\mu\hat{k}\hat{\varepsilon}_\pm}{(kp_1)}\right],\ \hat{\varepsilon}_\pm = \hat{e}_x \pm i\delta\hat{e}_y, \tag{15}$$

$$c^\mu = -(1-\delta^2)\eta^2\frac{m^2}{8(kp_1)(kp_2)}k^\mu. \tag{16}$$

Special functions L_l and their arguments have the following form [46]:

$$L_l(\gamma, \beta, \chi) = e^{-il\chi}\sum_{n'=-\infty}^{\infty} e^{2in'\chi}J_{l-2n'}(\gamma)J_{n'}(\beta), \tag{17}$$

$$\tan\chi = \delta\frac{(e_yQ)}{(e_xQ)},\ Q = \frac{p_2}{(kp_2)} - \frac{p_1}{(kp_1)}, \tag{18}$$

$$\gamma = \eta m\sqrt{(e_xQ)^2 + \delta^2(e_yQ)^2}, \tag{19}$$

$$\beta = \frac{1}{8}(1-\delta^2)\eta^2m^2\left[\frac{1}{(kp_2)} - \frac{1}{(kp_1)}\right]. \tag{20}$$

For the amplitudes $H^0_{l+l_1}(\tilde{p}_-, q_-)$ and $H^0_{l+l_1}(q_+, \tilde{p}_+)$ in the expressions (13), (14)–(20) we have to assume $p_1 \to q_-,\ p_2 \to \tilde{p}_-$, and $p_1 \to \tilde{p}_+,\ p_2 \to q_+$, and for the amplitudes $H^\mu_{l_1}(q_-, \tilde{p}_{+,})$ and $H^\mu_{l_1}(\tilde{p}_-, q_{+,})$ in the relations (12), (14)–(20) we have to make a transform $p_1 \to \tilde{p}_+,\ p_2 \to q_-$, and $p_1 \to q_+,\ p_2 \to \tilde{p}_-$.

III. POLES OF THE PPP AMPLITUDE

Resonant behavior of the studied process is explained by the fact that the low-order processes in the fine structure constant in the field of the electromagnetic wave are allowed, since intermediate particle enters the mass shell [12–14,18,19,27] (see Fig. 2):

$$q_-^2 = m^2, \quad q_+^2 = m^2, \tag{21}$$

where four-momenta of the intermediate electron q_- and positron q_+ under conditions (3) are defined as follows:

$$q_- = k_i + k - p_+, q_+ = k_i + k - p_-. \tag{22}$$

It is easy to verify that with the coincidence of the directions of propagation of the initial photon and the external field the simultaneous fulfillment of the resonant conditions (21) and (22) is impossible. We also note that for the process of electron-positron pair production by a photon in the field of a plane electromagnetic wave the main parameter is the classical relativistically invariant parameter η. Therefore for the fields (3) the processes with absorption of one photon from the external field $l_1 = 1$ are the most probable. At the same time, for the process of electron or positron scattering on the nucleus at large angles in the field of a wave, the main parameter is the Bunkin-Fedorov quantum parameter $\gamma_i \gtrsim 1$. Because of this, electron or positron can emit and absorb a large number of photons of the external field. However, if electron or positron is scattered at small angles, then the classical parameter becomes the main parameter for the process of electron or positron scattering on the nucleus and under conditions (3) the process with one photon of the laser field is the most probable.

Considering expressions (21) and (22) it is not difficult to obtain the relations for the initial photon frequency at resonance for Figs. 2(a) and 2(b). Under conditions (3) we have:

$$\omega_{i(a)} = l_1\omega\frac{\kappa_+}{\kappa_{i+} + 2l_1\omega\sin^2(\theta_i/2)}, \tag{23}$$

$$\omega_{i(b)} = l_1\omega\frac{\kappa_-}{\kappa_{i-} + 2l_1\omega\sin^2(\theta_i/2)}. \tag{24}$$

Herein

$$\kappa_{i\pm} = E_\pm - |\mathbf{p}_\pm|\cos\theta_{i\pm},\ \ \kappa_\pm = E_\pm - |\mathbf{p}_\pm|\cos\theta_\pm, \tag{25}$$

$$\theta_\pm = \angle(\mathbf{k}, \mathbf{p}_\pm), \quad \theta_{i\pm} = \angle(\mathbf{k}_i, \mathbf{p}_\pm), \quad \theta_i = \angle(\mathbf{k}_i, \mathbf{k}). \tag{26}$$

Further, we study the case of high energies of the initial γ quantum, when the electron-positron pair is ultrarelativistic and propagates in a narrow cone along the momentum of the initial photon.

$$E_\pm \gg m, \tag{27}$$

$$\theta_{i\pm} \ll 1, \quad \theta = \angle(\mathbf{p}_+, \mathbf{p}_-) \ll 1, \quad \theta_i \sim 1. \tag{28}$$

Under conditions (27), (28) and also by virtue of the law of conservation of energy in the external field (3)

$$\omega_i \approx E_+ + E_- \tag{29}$$

we can get the following relations:

$$\kappa_\pm \approx 2E_\pm \sin^2\frac{\theta_\pm}{2}, \quad \kappa_{i\pm} \approx \frac{m^2}{2E_\pm}(1+\tilde{\delta}^2_\pm), \tag{30}$$

Denoted here

$$\tilde{\delta}_\pm = \frac{E_\pm\theta_{i\pm}}{m} = 2x_\pm\delta_\pm,\ x_\pm = \frac{E_\pm}{\omega_i},\ \delta_\pm = \frac{\omega_i\theta_{i\pm}}{2m}. \tag{31}$$

Taking into account (25)–(30), and (23), (24) it is easy to obtain possible values of the resonant energies of the electron and positron for channels a and b:

$$x_{(a)+}(\delta^2_+) = \frac{1}{2(\delta^2_+ + \varepsilon_i)}\cdot[\varepsilon_i \pm \sqrt{\varepsilon_i(\varepsilon_i-1)-\delta^2_+}], \tag{32}$$

$$x_{(b)-}(\delta^2_-) = \frac{1}{2(\delta^2_- + \varepsilon_i)}\cdot[\varepsilon_i \pm \sqrt{\varepsilon_i(\varepsilon_i-1)-\delta^2_-}], \tag{33}$$

where

$$x_{(j)} = \frac{E_{(j)}}{\omega_i}, \quad j = a+, b-,$$

$$\varepsilon_i = \frac{\omega_i}{\omega_{\rm thr}}, \quad \omega_{\rm thr} = \frac{m^2}{\omega\sin^2(\theta_i/2)}. \tag{34}$$

Note that the energies of the electron (channel a) and the positron (channel b) might be obtained from the energy conservation law $x_{(a)-}(\delta^2_+) \approx 1 - x_{(a)+}(\delta^2_+)$, $x_{(b)+}(\delta^2_-) \approx 1 - x_{(b)-}(\delta^2_-)$.

From the relations (32) and (33) it follows that the minimum energy of the initial γ quantum at resonance will be at $\varepsilon_{i\,\min} = 1$ and $\delta^2_\pm = 0$, i.e., $\omega_{\rm thr}$ is the threshold energy. The threshold energy is determined by the electron rest energy, the frequency of the electromagnetic wave and the angle between the momenta of the initial photon and the electromagnetic wave. For the frequencies from the optical range the threshold energy is of the order of magnitude $\omega_{\rm thr} \sim 10^5/10^6$ MeV, and for x-ray laser $\omega_{\rm thr} \sim 10^2/10^3$ MeV.

Thus, the resonant energies of the positron and the electron are determined by two parameters: parameter ε_i is the energy of the initial photon in units of the threshold energy, and also ultrarelativistic parameters $\delta^2_\pm$ (31) determining possible outgoing angles of the positron and the electron. It is important to emphasize that the resonant energies of the positron and the electron for channel a are determined only by the outgoing angle of the positron (δ^2_+) and for channel b only by the outgoing angle of the electron (δ^2_-). Moreover, the resonant energy of the electron-positron pair can take two different values for each angle. Possible values of angles of the pair substantially depend on the value of the parameter ε_i and are enclosed in the interval

$$0 \leqslant \delta^2_\pm \leqslant \varepsilon_i(\varepsilon_i - 1), \quad \varepsilon_i \geqslant 1. \tag{35}$$

This shows that if the energy of the initial photon is equal to the threshold energy ($\varepsilon_i = 1$) then the electron-positron pair propagates exactly along the momentum of the initial γ quantum ($\delta^2_\pm = 0$) and the possible two values of the energy of the positron (electron) differ as much as possible from each other. With the increase of the outgoing angles of the electron-positron pair due to the relation (35) the difference in the two possible energies of the positron (electron) decreases. With the maximum possible outgoing angle

$$\delta^2_{\max} = \varepsilon_i(\varepsilon_i - 1) \tag{36}$$

the energy of the positron (electron) takes a single value [see (32), (33), also Fig. 3]. It is also important to note that energies of the electron and positron (for channels a and b) can be equal to each other ($E_{(a)+} = E_{(a)-}$, $E_{(b)+} = E_{(b)-}$) for one of the two possible values of energies [with "+" in front of square root in the relations (32) or (33)]. It takes place for the outgoing angle of positron (electron) $\delta^2_\pm = \delta^2_*$, where

$$\delta^2_* = \frac{1}{2}[\sqrt{1+4\varepsilon_i(\varepsilon_i-1)} - 1]. \tag{37}$$

Thus, if the outgoing angles of the positron (for channel a) or the electron (for channel b) are enclosed in the interval

$$0 < \delta^2_\pm < \delta^2_* \tag{38}$$

then the energy of the positron (electron) is always greater than the energy of the electron (positron). If

$$\delta^2_* < \delta^2_\pm \leqslant \delta^2_{\max} \tag{39}$$

then the energy of the positron (electron) is always less than the energy of the electron (positron) (see Fig. 3, the solid lines). If we choose an another value of the energy of the positron (electron) [the "–" sign in expressions (32) or (33)], then the energy of the electron (positron) will always be greater than the energy of the positron (electron) in the range of outgoing angles (35) (see Fig. 3 the dashed lines).

Note that in the papers [12–14,18,19,27] resonances were studied only for channel a, when the positron propagates along the initial photon momentum ($\delta^2_+ = 0$), and the electron is scattered on the nucleus at large angles. Here we study the resonances for channels a and b in the case of high energies of the initial γ quantum [$\varepsilon_i > 1$ ($\omega_i > \omega_{\rm thr}$)], when the ultrarelativistic electron-positron pair propagates in a narrow cone along the momentum of the initial photon. The interference of the channels under the resonant conditions will occur when the energies of the positron (electron) for channels a and b are equal ($E_{(a)+} = E_{(b)+}$) (see Fig. 3). If we exclude these cases, the interference of channels a and b in the conditions of resonance will not take place. This case will be considered in this paper, when

$$E_{(a)+} \neq E_{(b)+} \quad (E_{(a)-} \neq E_{(b)-}). \tag{40}$$

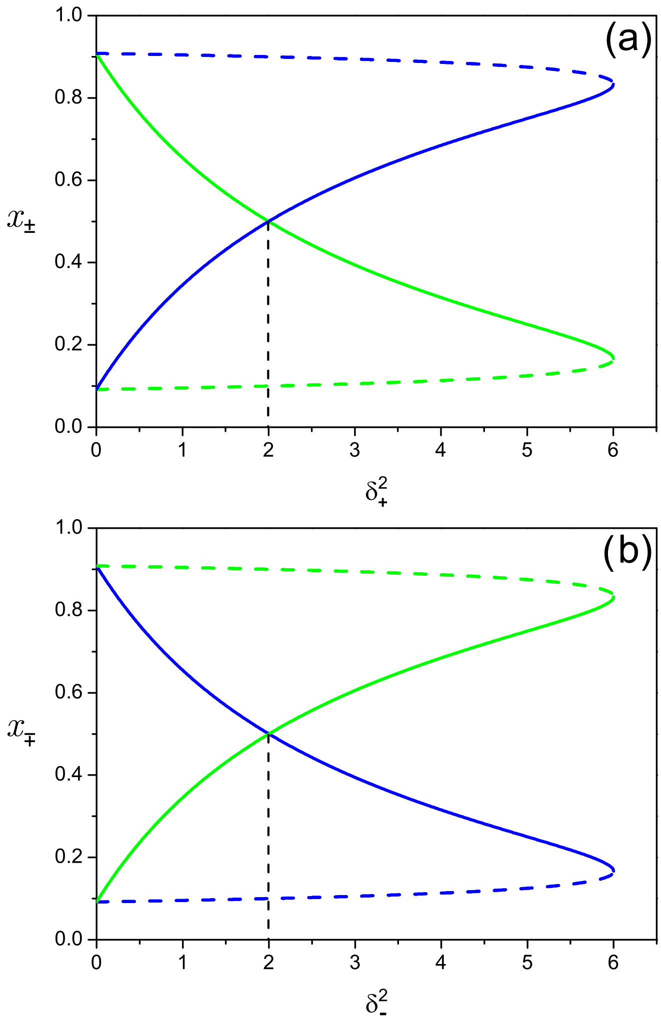

FIG. 3. Dependencies of the electron and positron resonant energies (in the units of energy of the initial photon) on the ultrarelativistic parameters δ_+^2 and δ_-^2. The energy of the initial γ quantum $\omega_i = 249.9$ GeV ($\omega_{\rm thr} = 83.3$ GeV). (a) channel a, (b) channel b. Solid lines represent energies of particles with sign "+", dashed lines with "–" [see (32) and (33)]. Green (light gray) color stands for positrons, blue color stands for electrons.

It is important to emphasize that in the nonresonant case, the energies of the positron and the electron change independently according to the law of energy conservation (29). In this case, the outgoing angles of the positron and the electron do not affect the energies of these particles. In the resonant case, we have a fundamentally different situation. In this case, the energies of the positron and the electron are determined from two equations: the energy conservation law (29) and the resonant equations [see (21)]. At the same time, the outgoing angle of a positron or electron relative to the momentum of the initial γ quantum determine the possible energy spectrum of the particles for channels a (32) or b (33).

IV. RESONANT DIFFERENTIAL CROSS SECTION

The resonant differential cross section of the PPP for unpolarized electrons, positrons, and the initial γ quantum in the field of a plane electromagnetic wave of weak intensity (3) with $l_1 = 1$ is obtained in the standard way (see Ref. [52] and Eqs. (37)–(46) in Ref. [49]). As a result, the resonant differential cross section of the PPP at the nucleus in the field of a plane electromagnetic wave for channels a ($d\sigma_{(a)\rm res}$) and b ($d\sigma_{(b)\rm res}$) takes the form:

$$d\sigma_{\rm res} = d\sigma_{(j)\rm res}, \quad j = a, b, \tag{41}$$

where

$$d\sigma_{(a)\rm res} = d\sigma_{l+1}(p_-, q_-)\frac{m^2|\mathbf{q}_-|}{2\pi|q_-^2 - m^2|^2}dW_1(q_-, p_+), \tag{42}$$

$$dW_1(q_-, p_+) = \frac{\alpha}{\omega_i E_+}\eta^2\left[\frac{\gamma^2_{p_+q_-}}{\eta^2} + (1+\delta^2)(2u_{(a)} - 1)\right]d^3p_+, \tag{43}$$

$$d\sigma_{l+1}(p_-, q_-) = 2Z^2r_e^2\frac{|\mathbf{p}_-|}{|\mathbf{q}_-|}\frac{m^2(m^2 + p_-^0q_-^0 + \mathbf{p}_-\mathbf{q}_-)}{(\mathbf{p}_- + \mathbf{p}_+ - \mathbf{k}_i - (l+1)\mathbf{k})^4}\cdot |L_{l+1}(p_-, q_-)|^2 d\Omega_-, \tag{44}$$

$$d\sigma_{(b)\rm res} = dW_1(p_-, q_+)\frac{m^2|\mathbf{q}_+|}{2\pi|q_+^2 - m^2|^2}d\sigma_{l+1}(q_+, p_+), \tag{45}$$

$$dW_1(p_-, q_+) = \frac{\alpha}{\omega_i E_-}\eta^2\left[\frac{\gamma^2_{p_-q_+}}{\eta^2} + (1+\delta^2)(2u_{(b)} - 1)\right]d^3p_-, \tag{46}$$

$$d\sigma_{l+1}(q_+, p_+) = 2Z^2r_e^2\frac{|\mathbf{p}_+|}{|\mathbf{q}_+|}\frac{m^2(m^2 + p_+^0q_+^0 + \mathbf{p}_+\mathbf{q}_+)}{(\mathbf{p}_- + \mathbf{p}_+ - \mathbf{k}_i - (l+1)\mathbf{k})^4}\cdot |L_{l+1}(q_+, p_+)|^2 d\Omega_+. \tag{47}$$

Here in the relations (44) and (47) solid angles of positron and electron are denoted by $d\Omega_\pm$ and r_e stands for classical radius of electron. In expressions (43) and (46) parameters $\gamma_{p_+q_-}$ and $\gamma_{p_-q_+}$ are determined by the relations (18), (19), where it is necessary to replace: $p_1 \to p_+$, $p_2 \to q_-$ for $\gamma_{p_+q_-}$ and $p_1 \to q_+$, $p_2 \to p_-$ for $\gamma_{p_-q_+}$. Relativistic-invariant parameters $u_{(a)}$ and $u_{(b)}$ equals to:

$$u_{(a)} = \frac{(kk_i)^2}{4(kq_-)(kp_+)}, \quad u_{(b)} = \frac{(kk_i)^2}{4(kq_+)(kp_-)}. \tag{48}$$

From the relations (42)–(47) we can see that for channels a and b, the resonant differential cross section of the PPP at the nucleus in the field of the plane electromagnetic wave effectively reduce into two first-order processes in the fine structure constant. For channel a, laser-stimulated Breit-Wheeler process first takes place ($dW_1(q_-, p_+)$ is the probability of this process per unit of time [15,48]), and then laser-assisted process of scattering of the intermediate electron on the nucleus ($d\sigma(p_-, q_-)$ is the differential cross section of this process [8,11]). A similar situation exists for channel b. However, here we have laser-assisted process of scattering of the intermediate positron on the nucleus [$d\sigma(q_+, p_+)$ is the differential cross section of this process].

We transform relativistic resonant cross sections (42) and (45) into resonant kinematics (27)–(31). In this case, we consider that for scattering of the electron or the positron on the nucleus at small angles, the following relations take a place [44]:

$$|L_{l+1}(p_-, q_-)|^2 = J_{l+1}^2(\gamma_{p_-q_-}) \approx 1,$$
$$|L_{l+1}(q_+, p_+)|^2 = J_{l+1}^2(\gamma_{q_+p_+}) \approx 1. \quad (49)$$

Since ($\gamma_{p_-q_-} \sim \eta \ll 1$) and ($\gamma_{q_+p_+} \sim \eta \ll 1$) in the process of scattering of electron or positron on the nucleus, the process with absorption of a one photon from electromagnetic wave is more probable, i.e., $l = -1$.

Further we consider circular polarization of the electromagnetic wave ($\delta^2 = 1$). Elimination of the resonant infinity in the channels a and b can be accomplished by adding of an imaginary term to the mass of the intermediate electron and positron. So, for channel a we have:

$$m \rightarrow \mu = m + i\Gamma_{(a)}, \quad \Gamma_{(a)} = \frac{q_-^0}{2m} W_{(a)}. \quad (50)$$

Here $W_{(a)}$ is the total probability (per unit time) of laser-stimulated Breit-Wheeler process [15].

$$W_{(a)} = \frac{\alpha m^2}{8\pi\omega_i}\eta^2 K_i, \quad (51)$$

$$K_i = \left(2 + \frac{2}{\varepsilon_i} - \frac{1}{\varepsilon_i^2}\right)\mathrm{Artanh}\left(\sqrt{\frac{\varepsilon_i - 1}{\varepsilon_i}}\right) - \left(\frac{\varepsilon_i + 1}{\varepsilon_i}\right)\sqrt{\frac{\varepsilon_i - 1}{\varepsilon_i}}. \quad (52)$$

Here in the relation (51) α is the fine structure constant. Taking into account the relations (50), the resonant denominator can be represented as:

$$|q_-^2 - \mu^2|^2 = 16m^4\left[x_{(a)+}^2\left(\delta_+^2 - \delta_{(a)+}^2\right) + \frac{4\Gamma_{(a)}^2}{m^2}\right] \quad (53)$$

the parameter $\delta_{(a)+}^2$ is related to the resonant frequency of the initial γ quantum for channel a by the expression:

$$\delta_{(a)+}^2 = \frac{4\varepsilon_i x_{(a)+}(1 - x_{(a)+}) - 1}{4x_{(a)+}^2}. \quad (54)$$

Similarly, we can define the expression for the resonant denominator of channel b:

$$|q_+^2 - \mu^2|^2 = 16m^4\left[x_{(b)-}^2\left(\delta_-^2 - \delta_{(b)-}^2\right) + \frac{4\Gamma_{(b)}^2}{m^2}\right] \quad (55)$$

the parameter $\delta_{(b)-}^2$ is related to the resonant frequency of the initial γ quantum for channel a by the expression:

$$\delta_{(b)-}^2 = \frac{4\varepsilon_i x_{(b)-}(1 - x_{(b)-}) - 1}{4x_{(b)-}^2}. \quad (56)$$

After the simple transformations of (42) and (45) we obtain the following expressions for the resonant differential cross sections of PPP in the case of an ultrarelativistic pair:

$$d\sigma_{(a)\mathrm{res}} = \frac{Z^2\eta^2\alpha r_e^2}{[d(x_{(a)+})]^2}\frac{(1 - x_{(a)+})^3 G(x_{(a)+})}{\left[\left(\delta_+^2 - \delta_{(a)+}^2\right)^2 + \Gamma_{\delta_+}^2\right]} \times \frac{dx_{(a)+}}{x_{(a)+}} d\delta_+^2 d\delta_-^2 d\varphi, \quad (57)$$

$$d\sigma_{(b)\mathrm{res}} = \frac{Z^2\eta^2\alpha r_e^2}{[d(x_{(b)-})]^2}\frac{(1 - x_{(b)-})^3 G(x_{(b)-})}{\left[\left(\delta_-^2 - \delta_{(b)-}^2\right)^2 + \Gamma_{\delta_-}^2\right]} \times \frac{dx_{(b)-}}{x_{(b)-}} d\delta_-^2 d\delta_+^2 d\varphi. \quad (58)$$

Here φ is an angle between planes ($\mathbf{k}_i, \mathbf{p}_+$) and ($\mathbf{k}_i, \mathbf{p}_-$). Γ_{δ_+} and Γ_{δ_-} are the angular radiative resonant width for channels a and b

$$\Gamma_{\delta_+} = \frac{\alpha\eta^2}{32\pi}\frac{\left(1 - x_{(a)+}\right)}{x_{(a)+}}K_i, \quad \Gamma_{\delta_-} = \frac{\alpha\eta^2}{32\pi}\frac{\left(1 - x_{(b)-}\right)}{x_{(b)-}}K_i, \quad (59)$$

$$G(x_\pm) = \frac{4\tilde{\delta}_\pm^2}{\left(1 + \tilde{\delta}_\pm^2\right)^2} + \left(\frac{x_\pm}{1 - x_\pm} + \frac{1 - x_\pm}{x_\pm}\right), \quad (60)$$

$$d(x_\pm) = d_0(x_\pm) + \left(\frac{m}{2\omega_i}\right)^2\left[d_1^2(x_\pm) + \frac{4\varepsilon_i}{\sin(\theta_i/2)}\{4\varepsilon_i - d_1(x_\pm)\}\right], \quad (61)$$

$$d_0(x_\pm) = \tilde{\delta}_+^2 + \tilde{\delta}_-^2 + 2\tilde{\delta}_+\tilde{\delta}_-\cos\varphi, \quad (62)$$

$$d_1(x_+) = \frac{1 + \tilde{\delta}_+^2}{x_+} + \frac{1 + \tilde{\delta}_-^2}{1 - x_+}, \quad d_1(x_-) = \frac{1 + \tilde{\delta}_-^2}{x_-} + \frac{1 + \tilde{\delta}_+^2}{1 - x_-}. \quad (63)$$

For the same energies of the positron and the electron for the resonant process in the wave field, the differential cross section of the PPP without an external field $d\sigma_0$ has form [52]:

$$d\sigma_0 = \frac{128}{\pi}Z^2 r_e^2\alpha x_\pm^3(1 - x_\pm)^3\frac{M_0 + (m/\omega_i)^2 M}{\left[d_0 + (m/2\omega_i)^2 d_1^2\right]^2} \times d\delta_+^2 d\delta_-^2 dx_\pm d\varphi, \quad (64)$$

$$M_0(x_\pm) = -\frac{\tilde{\delta}_+^2}{\left(1 + \tilde{\delta}_+^2\right)^2} - \frac{\tilde{\delta}_-^2}{\left(1 + \tilde{\delta}_-^2\right)^2} + \frac{1}{2x_\pm(1 - x_\pm)}\frac{\tilde{\delta}_+^2 + \tilde{\delta}_-^2}{(1 + \tilde{\delta}_+^2)(1 + \tilde{\delta}_-^2)} + \left(\frac{x_\pm}{1 - x_\pm} + \frac{1 - x_\pm}{x_\pm}\right)\frac{\tilde{\delta}_+\tilde{\delta}_-\cos\varphi}{(1 + \tilde{\delta}_+^2)(1 + \tilde{\delta}_-^2)}, \quad (65)$$

$$M(x_\pm) = \left(\frac{1}{x_\pm^2} + \frac{1}{(1 - x_\pm)^2}\right)b_\pm, \quad (66)$$

$$b_\pm(x_\pm) = \frac{\tilde{\delta}_\pm^2}{12\left(1 + \tilde{\delta}_\pm^2\right)^3}\left[2(1 - \tilde{\delta}_\pm^2)(3 - \tilde{\delta}_\pm^2) - \frac{1}{x_\pm(1 - x_\pm)}(9 + 2\tilde{\delta}_\pm^2 + \tilde{\delta}_\pm^4) + \left[\frac{x_\pm}{1 - x_\pm} + \frac{1 - x_\pm}{x_\pm}\right](9 + 4\tilde{\delta}_\pm^2 + 3\tilde{\delta}_\pm^4)\right\}. \quad (67)$$

Note that in the formulas for the differential cross section of the PPP without a laser field (64)–(67) for channel a we must select the plus sign (x_+) and for channel b the minus sign (x_-). It is important to emphasize that in differential cross sections (57), (58), and (64) introduced small corrections, which are proportional to the value $\sim (m/\omega_i)^2 \ll 1$. We note that these corrections make the dominant contribution to the corresponding differential cross sections under conditions:

$$|\varphi - \pi| \lesssim \frac{m}{\omega_i}, \left|\tilde{\delta}_+ - \tilde{\delta}_-\right| \lesssim \frac{m}{\omega_i}. \tag{68}$$

In this case, the values $M_0 \to 0$ and $d_0 \to 0$ and corresponding differential cross sections has sharp maximum. So, the differential cross sections without field (64) and in the field of wave (57), (58) in the kinematic range (68) will have the following order of magnitude:

$$d\sigma_0 \sim Z^2\alpha r_e^2\left(\frac{\omega_i}{m}\right)^2,\ d\sigma_{(a)\mathrm{res}} \lesssim \frac{Z^2\alpha r_e^2}{(\alpha\eta)^2}\left(\frac{\omega_i}{m}\right)^4. \tag{69}$$

We integrate the resonant differential cross sections, as well as cross section in the absence of the field with the respect to the azimuth angle φ. After a simple calculations we obtain:

$$d\sigma_{(a)\mathrm{res}} = 2\pi\left(Z^2\alpha r_e^2\right)\eta^2\frac{(1-x_{(a)+})^3}{x_{(a)+}} \times \frac{H_{1(+)}G(x_{(a)+})}{\left[\left(\delta_+^2 - \delta_{(a)+}^2\right)^2 + \Gamma_{\delta_+}^2\right]}dx_{(a)+}d\delta_+^2 d\delta_-^2, \tag{70}$$

$$d\sigma_{(b)\mathrm{res}} = 2\pi\left(Z^2\alpha r_e^2\right)\eta^2\frac{(1-x_{(b)-})^3}{x_{(b)-}} \times \frac{H_{1(-)}G(x_{(b)-})}{\left[\left(\delta_-^2 - \delta_{(b)-}^2\right)^2 + \Gamma_{\delta_-}^2\right]}dx_{(b)-}d\delta_+^2 d\delta_-^2, \tag{71}$$

$$d\sigma_0 = 64\left(Z^2\alpha r_e^2\right)x_\pm^3(1-x_\pm)^3 H_0 dx_\pm d\delta_+^2 d\delta_-^2, \tag{72}$$

$$H_0 = \frac{(\tilde{\delta}_+^2 + \tilde{\delta}_-^2)[D_0 + (m/\omega_i)^2 D]}{\left[\left(\tilde{\delta}_+^2 - \tilde{\delta}_-^2\right)^2 + \frac{1}{2}\left(\frac{m}{\omega_i}\right)^2(\tilde{\delta}_+^2 + \tilde{\delta}_-^2)d_1^2\right]^{3/2}}, \tag{73}$$

$$D_0 = -\frac{\tilde{\delta}_+^2}{(1+\tilde{\delta}_+^2)^2} - \frac{\tilde{\delta}_-^2}{(1+\tilde{\delta}_-^2)^2} + \frac{1}{2x_\pm(1-x_\pm)}\frac{\tilde{\delta}_+^2 + \tilde{\delta}_-^2}{(1+\tilde{\delta}_+^2)(1+\tilde{\delta}_-^2)} + \left(\frac{x_\pm}{1-x_\pm} + \frac{1-x_\pm}{x_\pm}\right)\frac{2\tilde{\delta}_+^2\tilde{\delta}_-^2}{(\tilde{\delta}_+^2 + \tilde{\delta}_-^2)(1+\tilde{\delta}_+^2)(1+\tilde{\delta}_-^2)}, \tag{74}$$

$$D = M(x_\pm) + d_1^2(x_\pm)\left(\frac{x_\pm}{1-x_\pm} + \frac{1-x_\pm}{x_\pm}\right) \times \frac{\tilde{\delta}_+^2\tilde{\delta}_-^2}{2(\tilde{\delta}_+^2 + \tilde{\delta}_-^2)(1+\tilde{\delta}_+^2)(1+\tilde{\delta}_-^2)}, \tag{75}$$

$$H_{1(\pm)} = (\tilde{\delta}_+^2 + \tilde{\delta}_-^2)\left\{(\tilde{\delta}_+^2 - \tilde{\delta}_-^2)^2 + \frac{1}{2}\left(\frac{m}{\omega_i}\right)^2(\tilde{\delta}_+^2 + \tilde{\delta}_-^2) \times \left[d_1^2(x_\pm) + \frac{4\varepsilon_i}{\sin(\theta_i/2)}\{4\varepsilon_i - d_1(x_\pm)\}\right]\right\}^{-3/2}. \tag{76}$$

It is important to emphasize that under the condition

$$|\tilde{\delta}_+^2 - \tilde{\delta}_-^2| \lesssim \frac{m}{\omega_i} \tag{77}$$

the resonant cross sections (70), (71), and also differential cross section of PPP in the absence of the laser field (72) have sharp maximum, which is associated with small angels of scattering. So we need to take into account small relativistic corrections $\sim (m/\omega_i)^2 \ll 1$. In this case, these differential cross sections have the following order of magnitude:

$$d\sigma_0 \sim Z^2\alpha r_e^2\left(\frac{\omega_i}{m}\right), \quad d\sigma_{(j)\mathrm{res}} \lesssim \frac{Z^2\alpha r_e^2}{(\alpha\eta)^2}\left(\frac{\omega_i}{m}\right)^3. \tag{78}$$

Note that the resonant denominators of the expressions (70), (71) have a characteristic Breit-Wigner form. When $\delta_+^2 \to \delta_{+(a)}^2$ (for channel a) and $\delta_-^2 \to \delta_{-(b)}^2$ (for channel b) the resonant differential cross sections has a sharp maximum:

$$R_{(j)}^{\max} = \frac{d\sigma_{(j)\mathrm{res}}^{\max}}{d\sigma_0} = f_0 R_{(j)},\ f_0 = \frac{32\pi^3}{(\alpha\eta)^2},\ j = a, b, \tag{79}$$

$$R_{(a)} = \frac{G(x_+)}{x_+^2(1-x_+)^2}\frac{H_{1(+)}}{H_0 K_i^2}, \tag{80}$$

$$R_{(b)} = \frac{G(x_-)}{x_-^2(1-x_-)^2}\frac{H_{1(-)}}{H_0 K_i^2}. \tag{81}$$

Expressions (79), (80), and (81) determine the magnitude of the resonant differential cross section of the PPP (in units of the corresponding differential cross section of the PPP without laser field) for channels a and b with simultaneous registration of the outgoing angles of the positron and the electron (parameters δ_+^2 and δ_-^2), as well as the positron energy in the interval from $E_{(a)+}$ to $[E_{(a)+} + dE_{(a)+}]$ (for channel a) and the electron energy from $E_{(b)-}$ to $[E_{(b)-} + dE_{(b)-}]$ (for channel b). It is important to emphasize that for channel a, the positron outgoing angle relative to the initial photon momentum (parameter δ_+^2) defines both the resonant energy of positron $E_{(a)+}$ (32) and the energy of electron $E_{(a)-} \approx \omega_i - E_{(a)+}$. At the same time, these quantities do not depend on the electron outgoing angle (parameter δ_-^2). For channel b we have the opposite situation. Here outgoing angle of the electron relative to the initial photon momentum (parameter δ_-^2) defines both the resonant energy of electron $E_{(b)-}$ and the energy of positron $E_{(b)+} \approx \omega_i - E_{(b)-}$. And these quantities do not depend on the positron outgoing angle (parameter δ_+^2).

From the expression (79) we can see that the magnitude of the maximum resonant cross section, is determined by functions: f_0, $R_{(a)}$, and $R_{(b)}$. From the one hand the magnitude of the function f_0, is mainly determined by the radiation width of the resonance, and is quite large. For the laser wave intensities $\eta = 0.1$ ($I \sim 10^{16}/10^{17}$ W/cm^2) function $f_0 \sim 10^9$. In a real experiment, the width of the resonance will be much larger than the radiation width, and the magnitude of the function f_0 will be much less. On the other hand, the magnitude of the functions $R_{(a)}$ and $R_{(b)}$ are determined mainly by the small transmitted momentum when the condition (77) is met and may also be large enough. Dependencies of the functions $R_{(a)}$ and $R_{(b)}$ on parameters δ_+^2 (outgoing angle of positron) and δ_-^2 (outgoing angle of electron) for channel a and b with

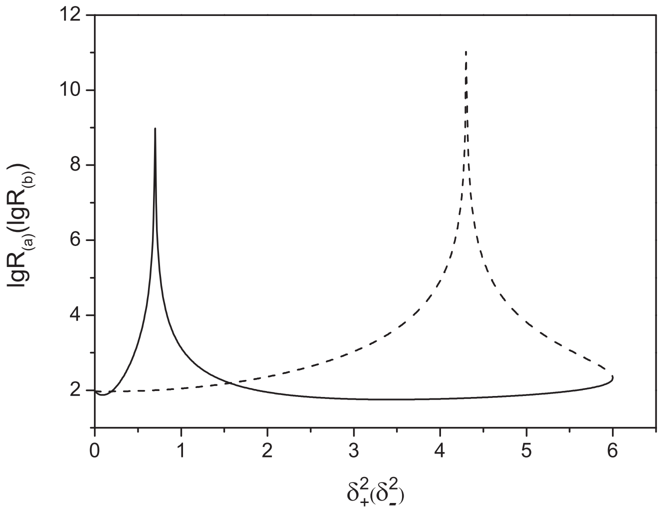

FIG. 4. Dependencies of the function $R_{(a)}(\delta_+^2)$ (80) on outgoing angle of positron ($\tilde{\delta}_-^2 = 1$ for channel a) and function $R_{(b)}(\delta_-^2)$ (81) on outgoing angle of electron ($\tilde{\delta}_+^2 = 1$ for channel b). Solid and dashed lines represent two possible values of energies of positron and electron [see (32) and (33)]. Energy of the initial photon $\omega_i = 249.9$ GeV ($\varepsilon_i = 3,\ \omega_{\text{thr}} = 83.3$ GeV).

fixed parameters $\tilde{\delta}_-^2$ and $\tilde{\delta}_+^2$ are represented on Fig. 4 and Fig. 5. Solid lines correspond to the energy of the positron (electron) with the sign "+" in front of square root, dashed lines correspond to the energy of a positron (electron) with the sign "–" in front of square root [see (32) and (33)]. We can see from the figures that when the parameters match $\tilde{\delta}_+^2 = \tilde{\delta}_-^2$ functions $R_{(a)}$ and $R_{(b)}$ have a sharp maximum and can reach 8–11 orders of magnitude. We note that the physical nature of these resonances is determined by the high energy of photon and very small transmitted momenta.

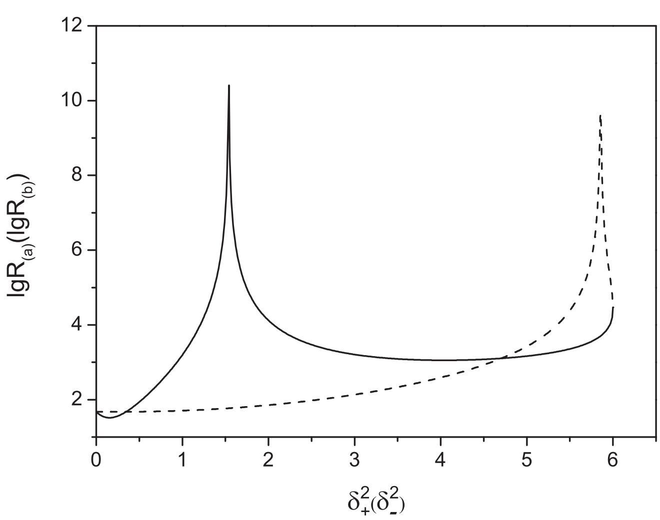

FIG. 5. Dependencies of the function $R_{(a)}(\delta_+^2)$ (80) on outgoing angle of positron ($\tilde{\delta}_-^2 = 3$ for channel a) and function $R_{(b)}(\delta_-^2)$ (81) on outgoing angle of electron ($\tilde{\delta}_+^2 = 3$ for channel b). Solid and dashed lines represent two possible values of energies of positron and electron [see (32) and (33)]. Energy of the initial photon $\omega_i = 249.9$ GeV ($\varepsilon_i = 3,\ \omega_{\text{thr}} = 83.3$ GeV).

Integrate the resonant differential cross section for channel a (70) with the respect to δ_-^2 (outgoing angle of electron) and resonant differential cross section for channel b (71) with the respect to δ_+^2 (outgoing angle of positron). After simple calculations we get:

$$d\sigma_{(a)}^{\text{res}} = \frac{1}{8}\pi\left(Z^2\alpha r_e^2\right)\left(\frac{\omega_{\text{thr}}}{m}\right)^2\frac{(1-x_{(a)+})}{x_{(a)+}} \times \frac{\eta^2 G(x_{(a)+})}{\left[\left(\delta_+^2-\delta_{(a)+}^2\right)^2+\Gamma_{\delta_+}^2\right]}dx_{(a)+}d\delta_+^2, \tag{82}$$

$$d\sigma_{(b)}^{\text{res}} = \frac{1}{8}\pi\left(Z^2\alpha r_e^2\right)\left(\frac{\omega_{\text{thr}}}{m}\right)^2\frac{(1-x_{(b)-})}{x_{(b)-}} \times \frac{\eta^2 G(x_{(b)-})}{\left[\left(\delta_-^2-\delta_{(b)-}^2\right)^2+\Gamma_{\delta_-}^2\right]}dx_{(b)-}d\delta_-^2. \tag{83}$$

It takes into account that the function $d_1(x_\pm) = 4\varepsilon_i$ for considered resonant kinematics. The resulting expression for the resonant differential cross section (82) determines the angular distribution [and energy (32)] of positron (irrespective to the directions of electron propagation). And the expression for the resonant differential cross section (83) determines the angular distribution [and energy (33)] of electron (irrespective to the directions of positron propagation).

When $\delta_+^2 \to \delta_{(a)+}^2$ (for channel a) and $\delta_-^2 \to \delta_{(b)-}^2$ (for channel b) resonant differential cross sections (82) and (83) have sharp maximum and take maximum values.

$$d\sigma_{(a)\text{res}}^{\max} = \left(Z^2\alpha r_e^2\right)g_i F_+(x_{(a)+})dx_{(a)+}d\delta_+^2, \tag{84}$$

$$d\sigma_{(b)\text{res}}^{\max} = \left(Z^2\alpha r_e^2\right)g_i F_-(x_{(b)-})dx_{(b)-}d\delta_-^2, \tag{85}$$

$$g_i = \frac{128\pi^3}{(\alpha\eta)^2}\left(\frac{\omega_{\text{thr}}}{m}\right)^2, \tag{86}$$

$$F_+(x_{(a)+}) = \frac{x_{(a)+}}{(1-x_{(a)+})K_i^2}G(x_{(a)+}),$$

$$F_-(x_{(b)-}) = \frac{x_{(b)-}}{(1-x_{(b)-})K_i^2}G(x_{(b)-}). \tag{87}$$

The resulting expression (84) determines the maximum possible values of the resonant differential cross section for channel a with simultaneous registration of the outgoing angle, as well as the positron energy. At the same time, the expression (85) determines the maximum possible values of the resonant differential cross section for channel b with simultaneous registration of the outgoing angle, as well as the electron energy.

In Fig. 6 represented functions $F_\pm(\delta_\pm^2)$ (87) that define angular and energy distributions of positrons (channel a) and electrons (channel b). It is seen from the figures that distributions of positrons and electrons are symmetric with the respect to the replacing of the positron with the electron. It is important to note that for each outgoing angle δ_+^2 (channel a) or δ_-^2 (channel b) from the interval (35) energy of electron-positron pair can take two values (32) or (33) with different probabilities (solid and dashed lines Fig. 6). So, for channel a, the pairs for which positrons have greater energy than electrons $E_{(a)+} > E_{(a)-}$ are more probable in the range of outgoing angles (38) [in expression (32) need to choose sign "+" in front of square root, also see solid lines Fig. 6 for

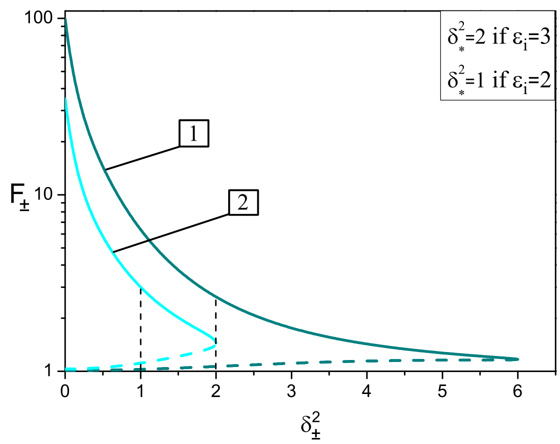

FIG. 6. Dependencies of the functions F_+ and F_- (81) on outgoing angle of positron δ^2_+ (for channel a) and outgoing angle of electron δ^2_- (for channel b). Solid lines represent sign "+" for energies of positron (32) or electron (33). Dashed lines stands for sign "–". Line 1 corresponds to the energy of the initial photon $\omega_i = 249.9$ GeV ($\varepsilon_i = 3$) and line 2 corresponds to the energy $\omega_i = 166.6$ GeV ($\varepsilon_i = 2$).

$\delta^2_+ < \delta^2_*$]. For channel b for the same outgoing angles of electron we have the opposite situation. With more probability electrons have greater energy than positrons $E_{(b)-} > E_{(b)+}$ [in expression (33) need to choose sign "+" in front of square root]. Upon transition from channel a to channel b, for the same outgoing angles of particles, the electron and positron energies transform into each other $E_{(a)+} \to E_{(b)-}$, $E_{(a)-} \to E_{(b)+}$. We emphasize that the cross section for the production of electron-positron pairs with such energies for channels a and b can be up to two orders of magnitude larger than the corresponding cross section for the production of pairs with other possible energies.

As can be seen from Fig. 6, the magnitude of the functions $F_\pm$, can vary over a wide range $F_\pm \sim 1/10^2$ determining the most probable values of the electron-positron pair energies for channels a and b in the range of outgoing angles (38). The maximum resonant cross section for channel a (84) and channel b (85) is mainly determined by the function g_i (86). This function in the field of optical frequencies for the intensity of the laser wave $\eta = 0.1$ ($I \sim 10^{16}/10^{17}$ W/cm^2) and the threshold energy of the process $\omega_{\rm thr} = 83.3$ GeV is equal to:

$$g_i \approx 2.4 \times 10^{20}. \tag{88}$$

Thus, the maximum resonant differential cross section for channel a (84) and channel b (85) can be a very large value

$$\frac{d\sigma^{\max}_{(j)\mathrm{res}}}{dx_\pm d\delta^2_\pm} \sim \left(Z^2\alpha r_e^2\right)(10^{20}/10^{22}), \quad j = a, b. \tag{89}$$

Note that a sufficiently large value of the resonance differential cross section is associated not only with the small radiation width of the resonance, which contributes to the order $\sim 10^6/10^8$, but also with very small transmitted momenta in this resonance process, the contribution of which is decisive.

V. CONCLUSION

The study of the resonant PPP process by high-energy γ quanta in the field of the nucleus and a weak laser wave allows us to formulate the main results. In this problem, there is a threshold energy of the process $\omega_{\rm thr}$ (34), which is of the order of magnitude $\omega_{\rm thr} \sim 10^2$ GeV in the optical frequency region. We studied the resonant production of the ultrarelativistic electron-positron pairs by high-energy γ quanta with energies $\omega_i > \omega_{\rm thr}$ when all particles (the initial photon, electron, and positron) propagate in a narrow cone.

Resonant angular and energy distributions of positrons from the ultrarelativistic parameter δ^2_+ (regardless of the outgoing angles of electrons, channel a) and resonant angular and energy distributions of electrons from the ultrarelativistic parameter δ^2_- (regardless of the outgoing angles of positrons, channel b) are symmetrical [see (84)–(87)]. The production of the electron-positron pairs in the range of outgoing angles (38) is more probable. Here, for channel a, the positron energy exceeds the electron energy, and for channel b we have the opposite situation: the electron energy exceeds the positron energy.

The maximum values of the resonant differential cross section of the PPP can be quite large and in units $(Z^2\alpha r_e^2)$ have the order of magnitude $\sim (10^{20}/10^{22})$. Moreover, the main contribution to this quantity is made by very small transmitted momenta.

The project calculations may be experimentally verified by the scientific facilities of pulsed laser radiation (SLAC, FAIR, XFEL, ELI, XCELS).

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