Reflectionless phenomenon in $\mathcal{PT}$-symmetric periodic structures of one-dimensional two-material optical waveguide networks

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In this paper, we construct a $\mathcal{PT}$-symmetric periodic structure with one-dimensional two-material waveguide networks and primarily investigate reflectionlessness. The formation conditions for reflectionlessness is presented in terms of the incident wave frequency and the number of unit cells. We show that, for this optical system, the orientation of unidirectional reflectionlessness can be controlled by tuning the spacing between waveguides with loss and gain. We also show the energy distribution when reflectionless light propagates through this $\mathcal{PT}$-symmetric optical structure. Finally, we find that unidirectional invisibility of the $\mathcal{PT}$-symmetric periodic structure could be achieved easily by adjusting the number of unit cells. Our results could be potentially important for developing a new generation of functional optical devices with singular directional responses.

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I. INTRODUCTION

In the past few years, non-Hermitian parity-time ($\mathcal{PT}$) symmetric systems have attracted considerable interest [1,2]. In close analogy to the wave equation in optics and the Schrödinger equation in quantum mechanics, $\mathcal{PT}$ symmetry in optics demands that the complex refractive index obey the condition $n(r) = n^*(-r)$, which implies that the real and imaginary parts of $n(r)$ be even and odd functions of position, respectively [3–6]. Rigorously balanced gain and loss are required in this condition. A variety of extraordinary phenomena can arise in $\mathcal{PT}$-symmetric structures when electromagnetic waves propagate through it, including double refraction [6], power oscillations [7], loss-induced transparency [8], perfect absorption [9,10], and other nonlinear effects [11,12]. Specifically, significant progress on unidirectional light reflectionlessness in $\mathcal{PT}$-symmetric periodic optical structures has been made in recent years [13–16]. According to the Lorentz reciprocity theorem, reflection and transmission in linear optical structures are bidirectional and symmetrical [17]. However, the introduction of gain and loss modulation in the $\mathcal{PT}$-symmetric optical system breaks the symmetry of reflection, and unidirectional reflectionlessness can be observed at the exceptional points (EPs) [18] via appropriate methods [15]. Intuitively, the vanishing of unidirectional reflection originates from absorption of the loss medium in $\mathcal{PT}$-symmetric optical systems [19]. However, the realization of unidirectional reflectionlessness in the loss and gain sides indicates that this phenomenon cannot be adequately explained in terms of absorption [20,21].

Most studies on optical $\mathcal{PT}$ symmetry focus on photonic crystals because they are more convenient for theoretical calculations. However, optical waveguide systems are also suitable for constructing optical $\mathcal{PT}$-symmetric structures. Optical waveguide networks are a kind of artificial photonic band-gap structure and have been widely investigated [22–25]. Compared with photonic crystals, optical waveguide networks are much more flexible as each waveguide segment can be bent and even folded freely. Consequently, more symmetries, local structures, and special defects can be constructed experimentally.

In this paper, we investigate the reflectionlessness in $\mathcal{PT}$-symmetric periodic structures of a one-dimensional two-material optical waveguide network (OTOWN); this optical structure is described in Sec. II A. The generalized eigenfunction method [25,26] is used to determine the transmissivity, reflectivity, and photonic localization [27], which are the bases of this paper. We interpret the reflectionlessness as a phase singularity [28,29] produced by complete destructive interference inside the $\mathcal{PT}$-symmetric structure, and we deduce the sufficient conditions for the reflectionlessness in terms of the incident wave frequency and the number of unit cells. The results show that, within a certain frequency range, the corresponding number of unit cells can always be found and has obvious periodicity, which makes the system exhibit reflectionlessness. Moreover, we propose and demonstrate a method for controlling the orientation of unidirectional reflectionlessness by tuning the spacing between waveguides with loss and gain in the $\mathcal{PT}$-symmetric optical structure. We also discuss energy changes when reflectionless light propagates through the $\mathcal{PT}$-symmetric optical structure. Finally, the relationship between unidirectional invisibility and unidirectional reflectionlessness was investigated. We find that unidirectional invisibility in our model could be achieved easily by adjusting the number of unit cells, and the corresponding numbers of unit cells are closely related with the periodicity of unidirectional reflectionlessness.

The remainder of this paper is organized as follows. The $\mathcal{PT}$-symmetric OTOWN model and the involved theory and formulations are introduced in Sec. II. In Sec. III A, we investigate the physical mechanism and formation conditions of the

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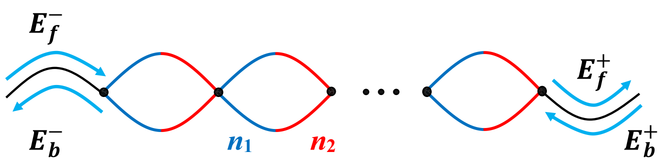

FIG. 1. Schematic of the OTOWN model with N unit cells, where E_f and E_b are the amplitude of the forward and backward traveling waves, respectively. The superscripts + and − indicate the field amplitude to the left and right of the scattering element, respectively. The upper and lower arms in the unit cell are composed of a pair of subsegments with length d and refractive indices $n_1 = n_{\rm Real} + in_{\rm Imag}$ (blue) and $n_2 = n_{\rm Real} - in_{\rm Imag}$ (red), respectively.

reflectionless phenomenon. In Secs. III B and III C, we show the switching of the direction and the distribution of energy of unidirectional reflectionlessness. In Sec. III D, we focus on the unidirectional invisibility. Finally, our conclusions are summarized in Sec. IV.

II. MODEL AND THEORY

A. OTOWN model

In this paper, we design a $\mathcal{PT}$-symmetric periodic structure using OTOWN. An optical structure with N unit cells is shown in Fig. 1, in which each waveguide segment is composed of two subsegments with equivalent length d but different refractive indices, i.e., $n(x) = n^*(-x)$, forming a $\mathcal{PT}$-symmetric structure. To be more specific, the refractive index for each waveguide subsegment is defined as $n_1 = n_{\rm Real} + in_{\rm Imag}$ and $n_2 = n_{\rm Real} - in_{\rm Imag}$. The upper and lower waveguides are of equal length in our model. We defined $n_{\rm Real} = 2$ and $n_{\rm Imag} = 0.2$ for three reasons. First, materials with the refractive index of approximately 2.0 are relatively cheaper and richer, which offers experimental convenience. Second, doping in quantum dots or modulating the density of a material can be used to tune the value of $n_{\rm Imag}$. Structures with smaller values of $n_{\rm Imag}$ are easier to fabricate. Third, this simplifies our calculations. The generalized eigenfunction method [25,26] can be used to calculate the transmissivity, reflectivity, and photonic localization in the optical system.

B. Theory and formulation

We regard the waveguides between two nodes in the OTOWN model as a unit cell. The transfer matrix of the unit cell is $\mathcal{PT}$ symmetric and is given by

$$M_1 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} 1/t^* & r_{R-1}/t \\ -r_{L-1}/t & 1/t \end{pmatrix}, \quad (1)$$

where t is the transmission, and r_{L-1} and r_{R-1} are the reflection coefficients from the left and right sides of the cell, respectively. The OTOWN model can be considered a system with N unit cells. Because all unit cells have the same transfer matrix, the transfer matrix M_N of the N unit-cell system can be written as the Nth power of the unimodular matrix M_1, i.e., M_1^N [30]. Using the Chebyshev identity, one can rewrite M_N as follows [30,31]:

$$M_N = M_1^N = \begin{pmatrix} m_{11}U_{N-1} - U_{N-2} & m_{12}U_{N-1} \\ m_{21}U_{N-1} & m_{22}U_{N-1} - U_{N-2} \end{pmatrix}, \quad (2)$$

$$U_N = \frac{\sin{(N+1)\phi}}{\sin\phi}, \quad (3)$$

where ϕ is the Bloch phase, which is given by the two eigenvalues of M_1 (λ_1 and λ_2) [30]:

$$\cos\phi = \tfrac{1}{2}(\lambda_1 + \lambda_2) = {\rm Re}(1/t). \quad (4)$$

Therefore,

$$M_N = \begin{pmatrix} \frac{1}{t_N^*} & \frac{r_{R-N}}{t_N} \\ \frac{-r_{L-N}}{t_N} & \frac{1}{t_N} \end{pmatrix} = \begin{pmatrix} \frac{1}{t^*}\frac{\sin N\phi}{\sin\phi} - \frac{\sin{(N-1)\phi}}{\sin\phi} & \frac{r_{R-1}}{t}\frac{\sin N\phi}{\sin\phi} \\ \frac{-r_{L-1}}{t}\frac{\sin N\phi}{\sin\phi} & \frac{1}{t}\frac{\sin N\phi}{\sin\phi} - \frac{\sin{(N-1)\phi}}{\sin\phi} \end{pmatrix}, \quad (5)$$

where t_N is the transmission, and r_{L-N} and r_{R-N} are the reflection coefficients from the left and right sides of the entire system, respectively. These can be written as follows:

$$\begin{cases} t_N = \dfrac{t\sin\phi}{\sin N\phi - t\sin{(N-1)\phi}}, \\ r_{L-N} = \dfrac{r_{L-1}\sin N\phi}{\sin N\phi - t\sin{(N-1)\phi}}, \\ r_{R-N} = \dfrac{r_{R-1}\sin N\phi}{\sin N\phi - t\sin{(N-1)\phi}}. \end{cases} \quad (6)$$

Equation (6) clearly shows that t_N, r_{L-N}, and r_{R-N} are related to t, r_{L-1}, r_{R-1}, ϕ, and N. In systems with unit cells that have identical structure, t, r_{L-1}, r_{R-1}, and ϕ are only dependent on frequency. Therefore, the transmission and reflection curves should oscillate as functions of the number of unit cells N and frequency. From this, we can deduce the condition for reflectionlessness in this system.

III. RESULTS AND DISCUSSION

A. Unidirectional and bidirectional reflectionlessness

The forward and backward traveling waves in the $\mathcal{PT}$-symmetric waveguide network shown in Fig. 1 can be described using the scattering matrix $S(\omega)$:

$$\begin{pmatrix} E_f^+ \\ E_b^- \end{pmatrix} = S(\omega)\begin{pmatrix} E_f^- \\ E_b^+ \end{pmatrix}, \quad S(\omega) = \begin{pmatrix} t_N & r_{R-N} \\ r_{L-N} & t_N \end{pmatrix}. \quad (7)$$

The $\mathcal{PT}$-symmetric nature of the opticals system leads to

$$\begin{pmatrix} E_f^{-*} \\ E_b^{+*} \end{pmatrix} = S(\omega)\begin{pmatrix} E_f^{+*} \\ E_b^{-*} \end{pmatrix}. \quad (8)$$

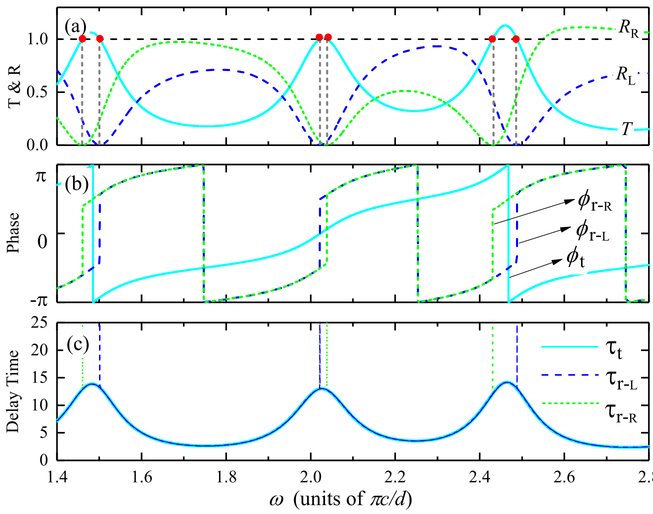

FIG. 2. (a) Transmittance and reflectance from the left and right in the unit-cell system. The red points are the EPs. (b), (c) Corresponding phase and delay time, respectively.

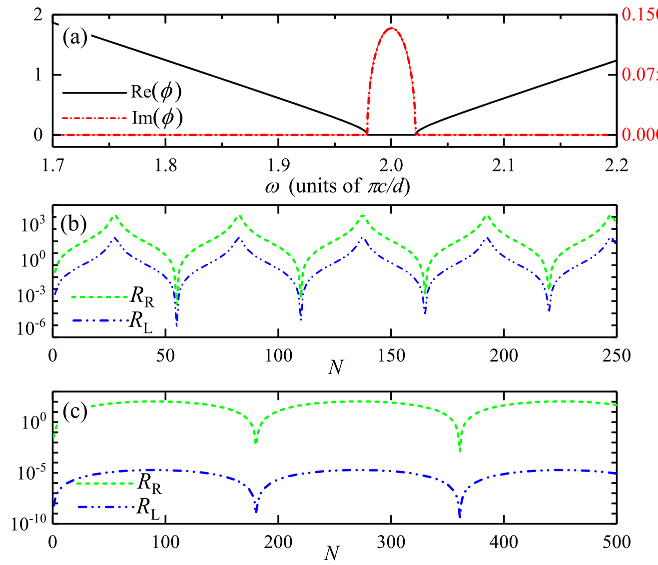

FIG. 3. (a) The real and imaginary parts of ϕ. (b), (c) R_{L-N} and R_{R-N} vs N when $\omega = 2.02350\pi c/d$ and $2.02180\pi c/d$, respectively.

One finds $S^*(\omega) = S^{-1}(\omega)$ when Eqs. (7) and (8) are compared. From this relation, one can conclude that $r_{L-N}r^*_{R-N} = 1 - |t_N|^2$, which implies [20]

$$\sqrt{R_{L-N}R_{R-N}} = |T_N - 1|. \tag{9}$$

One implication of this relationship is that the EPs of the $\mathcal{PT}$-symmetric opticals system are a sufficient condition for reflectionlessness. At the EPs of the system, we have perfect transmission ($T_N = 1$), and either $R_{L-N} = 0$ or $R_{R-N} = 0$ in order to satisfy Eq. (9). Therefore, the $\mathcal{PT}$-symmetric system is either bidirectionally or unidirectionally reflectionless. Figure 2(a) shows the transmittance and reflectance in the unit-cell system, which were calculated using the generalized eigenfunction method. The results show that the left or right side reflectance is equal to zero at these EPs [marked with red dots in Fig. 2(a)]. One should note that the N unit-cell system is reflectionless if the reflectance R_{L-1} or R_{R-1} of the unit-cell system is zero, according to Eq. (6).

The underlying physical mechanism of reflectionlessness is complete destructive interference inside the $\mathcal{PT}$-symmetric structure [15,21,32,33]. Destructive interference at the EPs causes the amplitude of the reflected wave to vanish, and the corresponding delay time diverges as the phase becomes uncertain [34], thereby forming a phase singularity [29]. Accordingly, reflectionlessness can be judged from the divergence of the corresponding delay time. The delay time is defined as $\tau_{r,t} = d\Phi_{r,t}/d\omega$, where $\Phi_{r,t}$ is the phase of the transmission or reflection coefficient [19]. Figure 2(b) shows the phase of the transmission and reflection coefficients versus incident wave frequency. At every EP, there is an abrupt phase change for waves reflected from the left or right side, which is an indication of the divergence of the delay time for reflected waves, as shown in Fig. 2(c). The amplitude of the corresponding reflected wave is zero, as shown in Fig. 2(a).

Regarding our $\mathcal{PT}$-symmetric OTOWN system, the system is unidirectional reflectionless from the left and right sides when $r_{L-1}\sin N\phi = 0$ and $r_{R-1}\sin N\phi = 0$, respectively, according to Eq. (6). Intuitively, reflectionlessness in our model is strongly affected by the number of unit cells N. Furthermore, Eq. (6) permits us to extract $\sin N\phi = 0$ as a sufficient condition for bidirectional reflectionlessness in $\mathcal{PT}$-symmetric periodic structures. This sufficient condition will be satisfied periodically as N increases as long as the Bloch phase ϕ is a real number. ϕ from our model is shown in Fig. 3(a), although it is not necessarily a real number as shown in the figure. Fortunately, Eq. (4) indicates $\phi = x$ or $m\pi + yi$, where x and y are real numbers and m is an integer. In both cases, $\sin N\phi = 0$ if ϕ is purely real. Therefore, bidirectional reflectionlessness occurs only if the imaginary part of the Bloch phase is zero. Obviously, given the characteristics of a sinusoidal function, bidirectional reflectionless points appear periodically as N increases and satisfies $N = k\pi/\phi$, where k is any positive integer.

To verify this theory, we calculated the reflectance from the left and right sides, and transmittance in the $\mathcal{PT}$-symmetric periodic structure as N increases from 1 to 250 when $\omega = 2.02350\pi c/d$. The numerical results are shown in Fig. 3(b). As shown in Fig. 3(a) $\phi = 0.057$ when $\omega = 2.02350\pi c/d$. The optical system should exhibit bidirectional reflectionlessness when $N = k\pi/\phi = 55k$ according to our theory. In Fig. 3(b), $R_{L-N} < 10^{-5}$ and $R_{R-N} < 10^{-3}$ when $N = 55k(55, 110, 165)$, thus the $\mathcal{PT}$-symmetric periodic structure is bidirectionally reflectionless. In addition, when $55k - 5 < N < 55k + 5$, R_{R-N} increases rapidly while $R_{L-N} << 1$. These points can be regarded as unidirectional reflectionless from the left side.

In the above, we analyzed reflectionlessness as a function of frequency and N. In short, there are two cases where system is reflectionless.

(1) The frequency of the incident wave coincides with the EPs of the unit-cell system.

(2) $N = k\pi/\phi$.

We can refer to the first case as the frequency condition and the second as the unit-cell number condition. The $\mathcal{PT}$-symmetric periodic system is reflectionless in both cases.

Predictably, when the two conditions are satisfied at the same frequency, reflection from the left or right side will become exceptionally small and the optical system exhibits ideal reflectionlessness. For example, $\omega = 2.02180\pi c/d$ corresponds to one of the EPs of the unit-cell system as shown in Fig. 2(a), where $N = 180k$ according to our method. We calculated reflectance from the left and right sides in the $\mathcal{PT}$-symmetric periodic structure as N was increased from 1 to 500 when $\omega = 2.02180\pi c/d$. The results in Fig. 3(c) show that $R_{L-N} < 10^{-5}$ for all periods when $1 < N < 500$. In particular, R_{L-N} achieved the order of magnitude of the negative ninth power at the periodic points $N = 180k(180, 360)$, which could be considered ideal unidirectional reflectionlessness.

B. Directional switching of reflectionlessness

The direction of unidirectional reflectionlessness can be controlled, which is essential for several key applications in photonic circuits, e.g., designing optical integrated circuits, reducing the size of optical systems, and developing reconfigurable optical components [35,36]. In a $\mathcal{PT}$-symmetric optical system, multiple interference inside the structure suppresses reflection. The direction of reflectionlessness can be controlled by choosing the appropriate phase of waves reflected from waveguides with loss and gain [15,33]. This can be implemented in our $\mathcal{PT}$-symmetric OTOWN system. The phase of the reflected waves could be regulated by tuning the spacing between waveguides with loss and gain. Specifically, we separate the waveguides with gain from those with loss. Waveguides with loss and gain in a unit cell form a ring and are connected by a passive waveguide with length L and refractive index $n = 1$, as shown in Fig. 4(a). When length L is changed, complex interference between waves reflected from the waveguides with loss and gain inside the system changes constantly. Complete destructive interference of left or right reflected waves will occur for specific values of L. Figure 4(b) shows that waves reflected from the left or right sides of the $\mathcal{PT}$-symmetric system are approximately periodic functions of L when $\omega = 1.45300\pi c/d$. Unidirectional reflectionlessness from the left or right side occurs periodically as L increases from zero to $3.5d$, where the period is $0.68823d$. More interestingly, the period of unidirectional reflectionlessness is $\lambda_n/2 = c\pi/\omega n = 0.68823d$, where λ_n is the incident wavelength in the passive waveguide. This result is in reasonable agreement with interference theory. In the case of left side reflectionlessness, a wave that is incident from the left is first reflected by a waveguide with loss, which then passes through the passive waveguide and is subsequently reflected from a waveguide with gain. The phase difference between waves reflected from the waveguides with loss and gain will increase by 2π as L is increased by integer multiples of $\lambda_n/2$. The same can be said for right side reflectionlessness. Therefore, the left reflectance and right reflectance of the $\mathcal{PT}$-symmetric system are periodic functions of L.

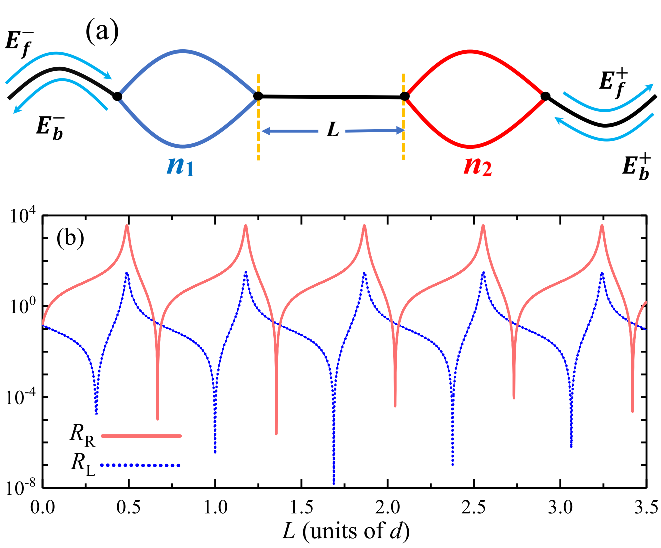

FIG. 4. (a) Schematic waveguide network consisting of a loss ring, a gain ring, and a passive waveguide of length L. The length of each loss or gain waveguide is d. (b) Left and right reflectances vs L when $\omega = 1.45300\pi c/d$. The reflectance curves are periodic functions of L.

One should note that the directionality of reflectionlessness can also be switched by tuning the refractive index of the passive waveguide.

C. Intensity of photonic localization

Energy changes could reflect the underlying mechanism of physical phenomena. Here we provide an example of the unit-cell system of our model and discuss how energy changes when reflectionless light propagates from the left or right side of the structure. The energy carried by waves in $\mathcal{PT}$-symmetric waveguide networks can be expressed in terms of the intensity of photonic localization [27]. Figure 5 shows the intensity of photonic localization near the incident frequency corresponding to the left or right reflectionless points, respectively. The results show that the $\mathcal{PT}$-symmetric system exhibits unitary transmission at the unidirectionally reflectionless frequency, regardless of whether incident light initially passes through a waveguide with loss or gain. In Fig. 5(a), incident light with frequency near $\omega = 1.50120\pi c/d$ first

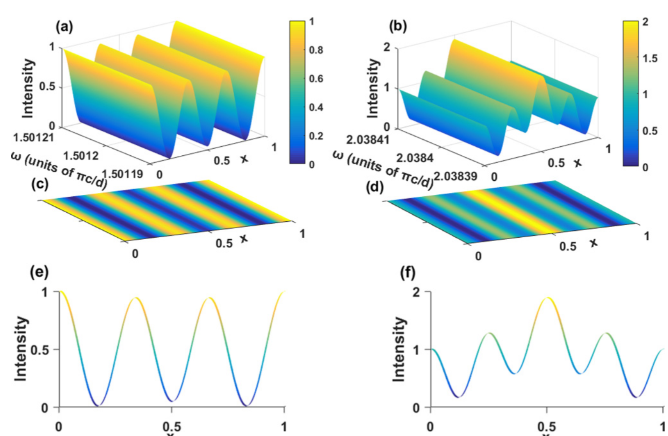

FIG. 5. Intensity distribution of photonic localization near the reflectionless points. (a), (c), (e) Three-, two-, and one-dimensional distributions for the light propagating from the left of the system, respectively. (b), (d), (f) Corresponding distributions for light propagating from the right.

passes through the loss portion from the left side, and the intensity of photonic localization decreases to 0.046. The intensity then gradually increases to 1 in the gain portion. In Fig. 5(b), right incident light with frequency near $\omega = 2.03840\pi c/d$ first passes through the gain portion and the intensity increases to 1.890. The intensity gradually decreases to 1 in the loss portion. The spatially symmetric intensity of photonic localization indicates that energy absorbed in the loss waveguide is completely reproduced in the gain waveguide at the mirrored position, leading to unitary transmission and reflectionlessness. However, it is worth noting that the incident intensities in the waveguides with gain and loss are not monotonically functions, as shown in Figs. 5(e) and 5(f). Gain and attenuation of the incident light do not originate from media with gain or loss in the $\mathcal{PT}$-symmetric system. Instead, this results from complex interference inside the $\mathcal{PT}$-symmetric structure, although energy in the system is still conserved due to the odd symmetry of imaginary modulation in our $\mathcal{PT}$-symmetric OTOWN system.

D. Unidirectional invisibility

A $\mathcal{PT}$-symmetric optical system can exhibit unidirectional or bidirectional reflectionlessness, thus it has the potential of appearing invisible in appropriate conditions [13]. However, unidirectional reflectionlessness is not equivalent to unidirectional invisibility in general. If there is a phase difference when light passes through a homogeneous background medium and a $\mathcal{PT}$-symmetric medium with equal lengths, the observer can still sense the existence of the $\mathcal{PT}$-symmetric medium from simple time-of-flight measurements. The phase of the transmission coefficient ($\Phi_{t(N)}$) in the $\mathcal{PT}$-symmetric optical must have just the right value to produce invisibility. $\Phi_{t(N)}$ is the phase difference if no phase change occurs after light passes through a homogeneous background medium with one unit cell, which could be satisfied by adjusting the length of the unit cell.

The transmission coefficient in the N unit-cell system is given by Eq. (6), and one can transform t_N into the following form:

$$t_N = \left[\cos N\phi + i\mathrm{Im}(1/t)\frac{\sin N\phi}{\sin\phi}\right]^{-1}. \tag{10}$$

Thus, the phase $\Phi_{t(N)}$ of the transmission coefficient in the N unit-cell system can be expressed using the following formula:

$$\tan\Phi_{t(N)} = -\frac{\mathrm{Im}(1/t)}{\sin\phi}\tan N\phi, \tag{11}$$

where the transmission coefficient t of the unit-cell system and ϕ are invariant at a given frequency when the system consists of identical unit cells. As a result, $\tan\Phi_{t(N)}$ is a periodic function of the number of unit cells N and has the same period as $\tan N\phi$. $\Phi_{t(N)} = N\phi = 2k\pi$ at points where the phase difference is zero, where k is an integer. Thus, the $\mathcal{PT}$-symmetric system can be regarded as unidirectionally invisible if and only if $N = 2k\pi/\phi$. As an example, Fig. 6 shows the relationship between $\Phi_{t(N)}$ and N when $\omega = 2.0384\pi c/d$.

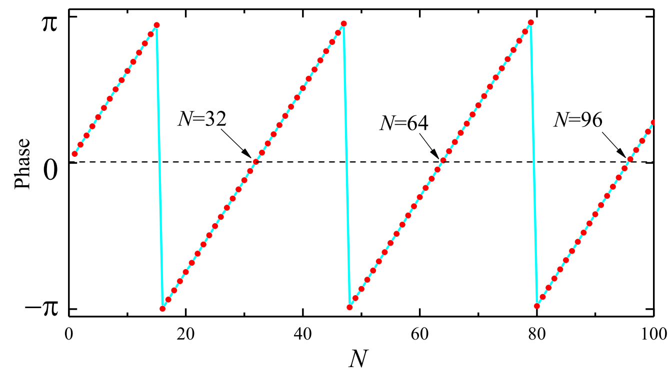

FIG. 6. The phase of transmitted waves propagating through the $\mathcal{PT}$-symmetric optical system vs the unit-cell number N at the frequency of $\omega = 2.0384\pi c/d$.

As analyzed in Sec. III A, we calculate $\pi/\phi = 16$. Therefore, the $\mathcal{PT}$-symmetric periodic system should exhibit unidirectional invisibility when N is an even multiple of 16 (for example, $32, 64, 96, \ldots$). For odd multiples (for example, $16, 48, 80, \ldots$), the optical system is not invisible, although reflectionlessness still occurs, and neither phenomenon occurs for other values of N.

IV. CONCLUSION

In conclusion, we investigated reflectionlessness in a $\mathcal{PT}$-symmetric periodic structure using the OTOWN model. The analytical results show that reflectionlessness can occur in a $\mathcal{PT}$-symmetric system when the incident wave frequency coincides with EPs of the unit-cell system or when the number of unit cells $N = k\pi/\phi$. In addition, the underlying physical mechanism is explained in terms of destructive interference inside the $\mathcal{PT}$-symmetric structure. Our results show that the orientation of unidirectional reflectionlessness can be controlled by tuning the spacing between waveguides with loss and gain, which explains the nonmonotonic behavior of energy changes when unidirectional reflectionless light propagates through this optical structure. Finally, we show that $\mathcal{PT}$-symmetric periodic systems should exhibit unidirectional invisibility when $N = 2k\pi/\phi$. Our results show that the concept of $\mathcal{PT}$ symmetry can be exploited in waveguide networks. Achieving reflectionless propagation in waveguide-cavity systems is important for integrated optical chips. Unidirectional invisibility in optics also has remarkable significance for cloaking physics. We believe that our work will have great potential in the immediate future.

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