Preservation of quantum correlation between nitrogen-vacancy-center ensembles by squeezed-reservoir engineering

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We propose a theoretical scheme to investigate the dynamics of quantum correlation between two nitrogen-vacancy-center ensembles (NVEs) coupled to a common superconducting coplanar waveguide resonator, driven by a broadband microwave squeezed field working as a squeezed-vacuum reservoir. Based on the reduced master equation for NVEs by the adiabatical elimination method in the bad-resonator limit, our results reveal quantum correlation preservation due to the interplay between the nonclassical feature of the squeezing reservoirs and the original dissipation mechanisms. The required operations are very close to the capabilities of current superconducting circuit-QED techniques. Our work may open interesting perspectives for devising active decoherence-immune quantum information devices.

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I. INTRODUCTION

Recently, there has been remarkable progress in engineering the hybrid system consisting of nitrogen-vacancy-center spin ensembles (NVEs) and a superconducting resonator, where both the spin ensembles and the resonator are modeled as interacting harmonic oscillators [1–10]. These systems combine the high scalability and controllability of superconducting circuits with the long coherence times of the NVEs. Experimentally, strong coupling between a spin ensemble and a superconducting resonator has been demonstrated in the linear or Gaussian regime [4–6]. One attractive example is that coherent coupling between two macroscopically separated NVEs and the transverse ensemble-ensemble coupling have been experimentally demonstrated via virtual photons [10].

From the point of view of quantum information processing (QIP), it remains an interesting future challenge, both theoretically and experimentally, to preserve steady-state quantum correlation [11] between two or more separated NVEs. As we know, generation and preservation of entanglement are basic ingredients in scalable QIP demanding preexisting entangled states, either at short distances or at large separations, such as quantum cryptography [12,13], quantum communication [14], and quantum computation [15–17]. However, with the growth of system complexity, the environmental dissipation unavoidably becomes more and more detrimental to quantum coherence, which makes preservation or protection of entanglement more difficult in the long-time limit.

On the other hand, various strategies proposed previously have attempted to counteract the harmful influence from the system-environment coupling [18–27], among which the quantum technologies of reservoir engineering [22–27] have been of great interest due to their unique vantage in the coherence control. Especially, squeezed-vacuum reservoir engineering has been widely used in various quantum information tasks, such as high-efficiency measurement [28,29], entanglement distribution [30], unique resonance fluorescence spectroscopy [31,32], and squeezing or cooling of the mechanical resonator [33,34].

Inspired by these advances, we propose in this work a potential experimental scheme using squeezed-reservoir engineering to preserve quantum correlation between two NVEs coupled to a common superconducting coplanar waveguide resonator (CPWR), driven by a broadband microwave squeezed field working as a squeezed-vacuum reservoir. We show that the delicate interplay between the squeezing reservoir and the dissipative mechanisms could stabilize these two NVEs into a steady state with quantum correlation, whose magnitudes are essentially determined by the squeezing features of the broadband driving field as well as other key parameters, such as detunings and NVE-CPWR coupling strengths. Based on the reduced master equation for NVEs using the adiabatic elimination method [35] in the bad-resonator limit, we first investigate the dynamics of quantum correlation between the two NVEs characterized by both logarithmic negativity [36,37] and Gaussian quantum discord (GQD) [38,39]. Additionally, the influence of system parameters, such as the squeezing degree, the detuning, and the coupling strength, on the performance of the scheme is also investigated. This could help us analyze the optimal parameter condition for obtaining the maximal steady-state quantum correlations. Our

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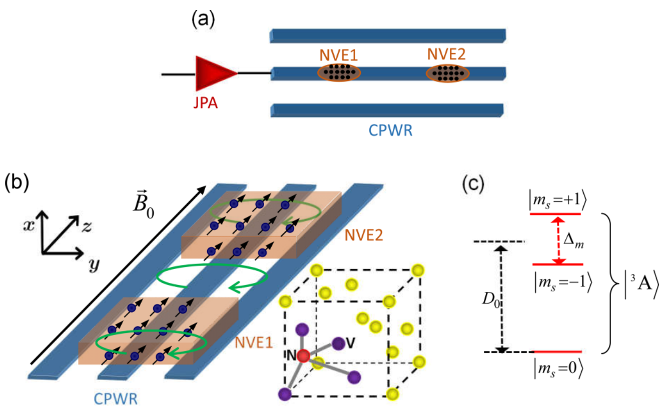

FIG. 1. (a) Proposed setup with two separated NVEs coupled to the same CPWR driven by a microwave squeezed field, which is generated by a flux-pumped Josephson parametric amplifier (JPA). (b) The cubic diagram of this hybrid system. Here, a static magnetic field $\vec{B}_0$ parallel to the z axis is applied to the NVE to lift the degenerated levels $|\pm 1\rangle$ of the ground state $|^3A\rangle$. (c) The ground state of the NV center is a spin-triplet state with three levels $|m_s = 0, \pm 1\rangle$. The level splitting width between $|\pm 1\rangle$ induced by $\vec{B}_0$ is $\Delta_m = g_s\mu_B B_0/\hbar$, with $g_s \simeq 2$ and μ_B being the Bohr magneton.

scheme includes three important features. First, different from traditional quantum-state engineering or dynamical control of the system [40–43], our method does not require active control of the system and accurate control of the evolution time. Second, a robust steady-state quantum correlation can be maintained in the long-time limit such that quantum correlation is available at any time, regardless of the initial state of the system. Third, our scheme can be extended to more spin ensembles and our work provides a building block for preserving quantum correlation among multi-NVEs using the reservoir-engineering approach.

The remainder of this paper is organized as follows. In Sec. II we introduce the model of our system and derive the reduced master equation for NVEs using the adiabatic elimination method. In Sec. III we preserve the quantum correlation between NVEs by squeezed-reservoir engineering. We discuss and summarize our results in Sec. IV.

II. MODEL AND HAMILTONIAN

As illustrated in Fig. 1(a), the system under consideration is composed of two spatially separated NVEs bonded individually onto a CPWR. The Hamiltonian of the whole system can be written as $H_t = H_c + H_n + H_{cn}$, where $H_c = \omega_c c^\dagger c$ represents the Hamiltonian of a CPWR in units of $\hbar = 1$, with c ($c^\dagger$) being the annihilation (creation) operator of the full-wave CPWR mode and ω_c being the eigenfrequency. $H_n = \sum_{j=1}^{2} \omega_{\rm eg}^j S_j^z/2$ is the Hamiltonian of two NVEs, with $\omega_{\rm eg}^j$ being the eigenfrequency of the jth NVE and $j = 1$ or 2, where $S_j^\mu = \sum_{i=1}^{N_0} \sigma_{i,j}^\mu$ ($\mu = z, \pm$) is the collective spin operator for NVE [containing N_0 nitrogen-vacancy (NV) centers] with the spin operators $\sigma_i^z = |-1\rangle_i\langle -1| - |0\rangle_i\langle 0|$, $\sigma_i^+ = |-1\rangle_i\langle 0|$, and $\sigma_i^- = |0\rangle_i\langle -1|$ of the ith NV center [10]. Through the collective magnetic-dipole coupling, the NVE-CPWR interaction Hamiltonian is given by $H_{cn} = \sum_{j=1}^{2} g_j(S_j^+ c + S_j^- c^\dagger)$, with g_j being the single NV vacuum Rabi frequency of the jth NVE.

Assuming that the spatial dimension of the NVE is smaller than the mode wavelength of the CPWR, we consider that all the NV spins in NVEs interact symmetrically with the CPWR field with the same coupling strength g_j. Using the Holstein-Primakoff transformation [44], we can map the spin operators into the bosonic operators with the forms $\sum_{i=1}^{N_0} \sigma_{i,j}^+ = a_j^\dagger\sqrt{N_0 - a_j^\dagger a_j} \simeq \sqrt{N_0}a_j^\dagger$, $\sum_{i=1}^{N_0} \sigma_{i,j}^- = a_j\sqrt{N_0 - a_j^\dagger a_j} \simeq \sqrt{N_0}a_j$, and $\sum_{i=1}^{N_0} \sigma_{i,j}^z = a_j^\dagger a_j - N_0/2$, where the bosonic operators a_j and $a_j^\dagger$ obey the standard commutator $[a_j, a_j^\dagger] \simeq 1$ in the limit of weak excitation $\langle a_j^\dagger a_j\rangle \ll N_0$. Based on these transformations, the total Hamiltonian of the system is

$$\begin{aligned} H &= H_0 + H_1, \\ H_0 &= \omega_c c^\dagger c + \sum\nolimits_j \omega_{\rm eg}^j a_j^\dagger a_j, \\ H_1 &= \sum\nolimits_j G_j(a_j^\dagger c + a_j c^\dagger), \end{aligned} \tag{1}$$

where H_0 is the free Hamiltonian and H_1 is the interaction Hamiltonian with $G_j = \sqrt{N_0}g_j$ being the collective coupling strength between the jth NVE and the CPWR.

To obtain the stationary quantum correlation between the spin ensembles, the CPWR is driven by a broadband microwave squeezed field generated by a flux-pumped Josephson parametric amplifier (JPA) [29,45] with squeezing degree r around the central frequency ω_s, where the broadband microwave squeezed field acts as a squeezed-vacuum reservoir. Thus, the dynamics of the whole system is governed by the Born-Markovian master equation

$$\dot{E}(t) = -i[H, E(t)] + \mathcal{L}_n E(t) + \mathcal{L}_c E(t), \tag{2}$$

where $E(t)$ represents the density matrix of the total system. $\mathcal{L}_n\cdot = \sum_j \gamma_j[(\bar{n}_j + 1)\check{\mathcal{D}}_{a_j,a_j^\dagger}\cdot + \bar{n}_j\check{\mathcal{D}}_{a_j^\dagger,a_j}\cdot]$ and $\mathcal{L}_c\cdot = \kappa[(N + 1)\check{\mathcal{D}}_{c,c^\dagger}\cdot + N\check{\mathcal{D}}_{c^\dagger,c}\cdot - (Me^{2i\omega_s t}\check{\mathcal{D}}_{c,c}\cdot + \text{H.c.})]$ represent the dissipators for the NVEs caused by their independent finite-temperature reservoirs and the dissipators of the CPWR caused by the squeezed-vacuum reservoir, respectively. γ_j and κ represent the damping rates of the jth NVE and the CPWR, respectively. $\check{\mathcal{D}}_{m,n}\cdot = 2m\cdot n - nm\cdot - \cdot nm$ and $\bar{n}_j = 1/(e^{\omega_{\rm eg}^j/k_B T_j} - 1)$ denotes the mean thermal bath boson number at the environment temperature T_j with k_B being the Boltzmann constant. $M = \cosh r \sinh r$ and $N = \sinh^2 r$ describe the strength of the two-photon correlation and the mean photon number of the broadband squeezed field [46], respectively.

Assuming that $\omega_s = \omega_c$ and transforming the density matrix $E(t)$ into $\bar{E}(t) = U_s U_c E(t) U_c^\dagger U_s^\dagger$ by the canonical transformation $U_c = e^{iH_0 t}$ and the squeezing transformation $U_s = e^{r(c^2 - c^{\dagger 2})/2}$, Eq. (2) becomes [47]

$$d\bar{E}(t)/dt = -i[\bar{H}_1(t), \bar{E}(t)] + (\mathcal{L}_n + \mathcal{L}_k)\bar{E}(t), \tag{3}$$

where $\bar{H}_1(t) = X(t)c^\dagger + \text{H.c.}$ with $X(t) = \sum_j G_j(\sqrt{N}e^{i\Delta_j t}a_j^\dagger + \sqrt{N+1}e^{-i\Delta_j t}a_j)$ and $\Delta_j = \omega_{\rm eg}^j - \omega_c$ being the detuning between the jth NVE and the CPWR. $\mathcal{L}_k\bar{E} = \kappa\check{\mathcal{D}}_{c,c^\dagger}\bar{E}$ describes the dissipation of the CPWR. Besides, Eq. (3) implies that the nonclassical effect of the squeezed-vacuum reservoir is effectively transferred to the NVE-CPWR system through the interaction Hamiltonian $\bar{H}_1(t)$ and the CPWR mode is damped only by the vacuum field.

In the bad-resonator limit ($\kappa \gg G_j \gg \gamma_j$), $\bar{E}$ is approximately expressed by $\bar{E} \simeq \mathrm{Tr}_c[\bar{E}] \otimes (|0\rangle\langle 0|)_c$ because the CPWR field governed by the dissipator $\mathcal{L}_k$ could rapidly reach the steady state $(|0\rangle\langle 0|)_c$. Therefore, the degree of freedom of the CPWR can be adiabatically eliminated and a reduced master equation satisfied by the NVEs can be obtained. To this end, we first make the transformation $\breve{E}(t) = e^{-\mathcal{L}_k t}\bar{E}(t)$ in a dissipation picture, then Eq. (3) can be written as $d\breve{E}(t)/dt = [\mathcal{L}_n + \breve{\mathcal{L}}_1(t)]\breve{E}(t)$, with $\breve{\mathcal{L}}_1(t)\cdot = -ie^{-\mathcal{L}_k t}[\bar{H}_1(t), \cdot]e^{\mathcal{L}_k t}$. Then tracing out the variables of the CPWR as $\breve{\rho}(t) = \mathrm{Tr}_c[\breve{E}(t)]$ under the Born-Markovian approximation, we have

$$\frac{d\breve{\rho}(t)}{dt} = \mathrm{Tr}_c \int_0^\infty d\tau \breve{\mathcal{L}}_1(t)\breve{\mathcal{L}}_1(t-\tau)\breve{\rho}(t)(|0\rangle\langle 0|)_c + \mathcal{L}_n\breve{\rho}(t). \tag{4}$$

Here $\breve{\mathcal{L}}_1(t)\cdot = -i[X^\dagger(t)\mathcal{A}_-(t) + X(t)\mathcal{A}_+(t) - \mathrm{H.c.}]\cdot$, with $\mathcal{A}_+(t)\cdot = e^{-\mathcal{L}_k t}(c^\dagger\cdot)e^{\mathcal{L}_k t}$ and $\mathcal{A}_-(t)\cdot = e^{-\mathcal{L}_k t}(c\cdot)e^{\mathcal{L}_k t}$. Making a time derivative to $\mathcal{A}_\pm(t)$, we obtain $\mathcal{A}_+(t) = e^{\kappa t}(c^\dagger\cdot) + (e^{-\kappa t} - e^{\kappa t})(\cdot c^\dagger)$ and $\mathcal{A}_-(t) = e^{-\kappa t}(c\cdot)$, respectively, and derive the nonzero correlation function of the CPWR fields as $\langle \mathcal{A}_-(t)\mathcal{A}_+(t')\rangle = \langle \mathcal{A}_+(t)\mathcal{A}_+^\dagger(t')\rangle = \langle \mathcal{A}_-^\dagger(t)\mathcal{A}_+^\dagger(t')\rangle = \langle \mathcal{A}_+^\dagger(t)\mathcal{A}_+(t')\rangle = e^{-\kappa(t-t')}$, with $\langle\cdot\rangle = \mathrm{Tr}_c[\cdot|0\rangle\langle 0|_c]$ and $t' = t - \tau$. Then we rewrite Eq. (4) as

$$\begin{aligned} d\breve{\rho}(t)/dt = \int_0^\infty & e^{-\kappa\tau}[X(t)\breve{\rho}(t)X^\dagger(t-\tau) \\ & - X^\dagger(t)X(t-\tau)\breve{\rho}(t) + \mathrm{H.c.}]d\tau + \mathcal{L}_n\breve{\rho}(t). \end{aligned} \tag{5}$$

Returning to the Schrödinger picture with $\rho(t) = \exp(-iH_I t)\breve{\rho}(t)\exp(iH_I t)$ and $H_I = \sum_j \omega_{eg}^j a_j^\dagger a_j$, the reduced master equation satisfied by the NVEs has the following form:

$$\begin{aligned} \dot{\rho}(t) = & -i[H_I, \rho(t)] + L_n\rho(t) + \sum_{j,k=1,2} G_j G_k\{(N+1) \\ & \times [(T_k + T_j^*)a_j\rho(t)a_k^\dagger - T_j^* a_k^\dagger a_j\rho(t) - T_k\rho(t)a_k^\dagger a_j] \\ & + N[(T_k^* + T_j)a_j^\dagger\rho(t)a_k - T_j a_k a_j^\dagger\rho(t) - T_k^*\rho(t)a_k a_j^\dagger]\} \\ & + \sum_{j,k=1,2} G_j G_k\{M e^{2i\omega_c t}[(T_k^* + T_j^*)a_j\rho(t)a_k \\ & - T_j^* a_k a_j\rho(t) - T_k^*\rho(t)a_k a_j] + \mathrm{H.c.}\}, \end{aligned} \tag{6}$$

where $T_k = 1/(\kappa + \Delta_k i)$. One can find that Eq. (6) is traceless, and the last two terms in the right-hand side of Eq. (6) reflect all the dynamical squeezed effects of the CPWR on the two NVEs. Equation (6) consists of two aspects: it induces the direct dissipation on each individual NVE ($j = k$) and also gives rise to incoherent interaction between these two NVEs through the exchange of virtual bosons ($j \neq k$). Based on these processes, a stable quantum correlation shared by the two NVEs could be preserved.

III. STABLE QUANTUM CORRELATION BETWEEN TWO NVEs

In this section, we investigate quantum correlation between two NVEs characterized by both logarithmic negativity [36,37] and GQD [38,39] through solving the reduced master equation of Eq. (6). Based on Eq. (6), we can obtain a covariance matrix regarding the two NVEs.

In general, the states of the NVEs can be fully characterized by the covariance matrix in a 2×2 block form:

$$V = \begin{pmatrix} A & B \\ B^T & A' \end{pmatrix},$$

where $A(A')$ and B represent the local properties of the NVEs and the nonlocal correlation between them, respectively. The covariance matrix V can be estimated via the homodyne measurements on the amplitude quadratures X_j and P_j in experiments, and the elements of the covariance matrix V have the form of $V_{ij} = \langle \Delta\zeta_i\Delta\zeta_j + \Delta\zeta_j\Delta\zeta_i\rangle/2$, with $\vec{\zeta} = (X_1, P_1, X_2, P_2)$, $\Delta\zeta_j = \zeta_j - \langle\zeta_j\rangle$, $X_j = (a_j + a_j^\dagger)/\sqrt{2}$, and $P_j = (a_j - a_j^\dagger)/\sqrt{2}i$. Additionally, the commutation relations $[\zeta_i, \zeta_j] = iW_{ij}$ could be satisfied with

$$W = \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix} \text{ and } C = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

which determine the symplectic structure of the system with the symplectic eigenvalues $\lambda = (\lambda_1, \lambda_2)$ of the matrix $iW\cdot V$. If we make a partial transposition [48] on the covariance matrix V as $\breve{V} = \Gamma\cdot V\cdot\Gamma$, with $\Gamma = \mathrm{diag}(1, 1, 1, -1)$, the Peres-Horodecki criterion [49,50] could judge if the state is separable or entangled, corresponding, respectively, to the conditions $\breve{\lambda}_i \geqslant 1/2$ or $\breve{\lambda}_i < 1/2$. Therefore, quantum entanglement between two NVEs measured by the logarithmic negativity can be quantified as

$$E_N = \max\{0, -\log_2[2\min(\breve{\lambda}_1, \breve{\lambda}_2)]\}. \tag{7}$$

In our work, the NVEs are initially prepared in the two-mode squeezed state $\exp[r'(a_1a_2 - a_1^\dagger a_2^\dagger)]|00\rangle$, with $|0\rangle$ being the vacuum state of a single NVE [51–53], and the time evolution of such a state under the government of Eq. (1) keeps the Gaussianity. Therefore we could use the GQD to evaluate quantum correlation between spin ensembles. The total amount of correlation for a bipartite system is quantified by quantum mutual information $I(\rho_{12}) = S(\rho_1) + S(\rho_2) - S(\rho_{12})$, with $S(\rho) = -\mathrm{Tr}(\rho\log_2\rho)$ being the von Neumann entropy of the density matrix ρ and $\rho_{1(2)}$ being the reduced density matrix of ρ_{12} by tracing out subsystem 2 (1). In addition, the amount of one-way classical correlation extractable from a Gaussian measurement is $C(\rho_{12}) = S(\rho_1) - \inf_{V_1} S(\rho_{1|V_1})$, where V_1 is the covariance matrix by performing the measurement on subsystem 2. Therefore, the GQD (measure of Gaussian quantum correlation) is defined as the difference between two ways representing mutual information $D(\rho_{12}) = I(\rho_{12}) - C(\rho_{12})$. The explicit expression about the GQD can be expressed as [38,39]

$$D(V) = g(\sqrt{I_2}) + g(\sqrt{\delta}) - g(\sqrt{\eta_+}) - g(\sqrt{\eta_-}), \tag{8}$$

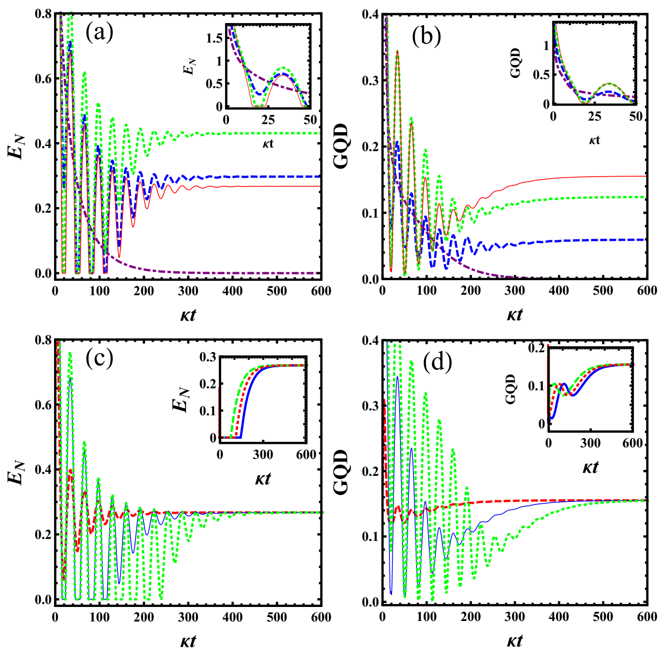

FIG. 2. Simulated dynamics of quantum correlation E_N (a) and GQD (b), under different squeezing degrees with $r = 0$ (dotted-dashed line), 0.3 (dashed line), 0.6 (dotted line), and 1 (solid line), respectively, where $r' = 1$. The dynamics of quantum correlation E_N (c) and GQD (d), under different initial two-mode squeezed states, with $r' = 0.2$ (dashed line), 1 (solid line), and 2 (dotted line), respectively, where $r = 1$. Insets in panels (c) and (d) show the dynamics of quantum correlation E_N and GQD, under different initial thermal states with the thermal boson number $n'_1 = n'_2 = n'_0 = 1$ (dashed line), 2 (dotted line), and 4 (solid line), respectively, where $r = 1$. Other parameters are $\Delta_0 = -0.1$, $\bar{n}_0 = 0$, $\gamma_0 = 0.01$, and $G_0 = 0.3$, respectively. The parameters are in units of $\kappa = 1$.

with the function $g(m) = (m + 1/2)\ln(m + 1/2) - (m - 1/2)\ln(m - 1/2)$ and

$$\delta = \begin{cases} \left[\frac{2|I_3| + \sqrt{4I_3^2 + (4I_2 - 1)(4I_4 - I_1)}}{4I_2 - 1}\right]^2 & \text{if } \frac{4(I_4 - I_1 I_2)^2}{(4I_2 + 1)(4I_4 + I_1)I_3^2} \leqslant 1, \\ \frac{I_4 + I_1 I_2 - I_3^2 - \sqrt{(I_4 + I_1 I_2 - I_3^2)^2 - 4I_1 I_2 I_4}}{2I_2} & \text{otherwise}. \end{cases} \quad (9)$$

Here $I_1 = \det A$, $I_2 = \det A'$, $I_3 = \det B$, and $I_4 = \det V$ represent the symplectic invariants derived from the covariance matrix V and $\eta_{\pm} = \sqrt{\Lambda \pm \sqrt{\Lambda^2 - 4I_4}}/\sqrt{2}$ are the symplectic eigenvalues of V, with $\Lambda = I_1 + I_2 + 2I_3$. It is now believed that the GQD characterizes the quantumness of correlations for the Gaussian state more generally than quantum entanglement because E_N cannot exhaust the nonclassicality in the correlations [54].

We first study the simple case where the two NVEs have identical parameters with $\Delta_1 = \Delta_2 = \Delta_0$, $\bar{n}_1 = \bar{n}_2 = \bar{n}_0$, $\gamma_1 = \gamma_2 = \gamma_0$, and $G_1 = G_2 = G_0$. Figures 2(a) and 2(b) describe the dynamics of quantum correlation characterized by both E_N and GQD under different values of squeezing degree r. We can find that stationary quantum correlations (both E_N and GQD) cannot be preserved when the squeezing parameter $r = 0$, which corresponds to the case of a vacuum reservoir. Once the driving field with nonzero squeezing degree is switched on, a stable quantum correlation (both E_N and GQD) is stimulated asymptotically for a wide range of r in the long-time limit. It implies that the formation of steady-state quantum correlation between two NVEs is determined by the features of the squeezing driving field. It also indicates that this nonclassical squeezing character of the broadband driving field can be transferred into the NVEs via the CPWR field playing the role of a quantum bus. The physics behind the preservation of quantum correlation is that the nonunitary dynamics resulting from the balance between the squeezing driving and the original dissipation mechanisms can stabilize the NVEs into a desired stationary quantum correlation. In addition, we consider the dynamics of both quantum correlations E_N and GQD under different initial states, such as the two-mode squeezed states and thermal states, as shown in Figs. 2(c) and 2(d), respectively. We find that the system has a unique steady state because all the quantum correlations converge to an identical line in the long-time limit. It demonstrates that the preservation of quantum correlation in our work does not depend on the initial state of the system. Besides, we also study the time evolution of quantum correlation (both E_N and GQD) under different detunings Δ_0. It can be seen from Fig. 3 that steady-state quantum correlations [denoted by $E_N(\infty)$ and GQD(∞)] between NVEs exist inside a broad parameter regime $\Delta_0 \in [-0.1, 0.1]$, and the maximum $E_N(\infty)$ and GQD(∞) could be obtained when $\Delta_0 = 0$. Once the values of Δ_0 are outside the approximative regime $[-0.1, 0.1]$, both E_N and GQD will quickly decay to zero in the long-time limit, even in the presence of a squeezing driving field with high squeezing degrees. So to carry out our scheme efficiently, we require to regulate the detunings and limit them inside an accurate parameter regime.

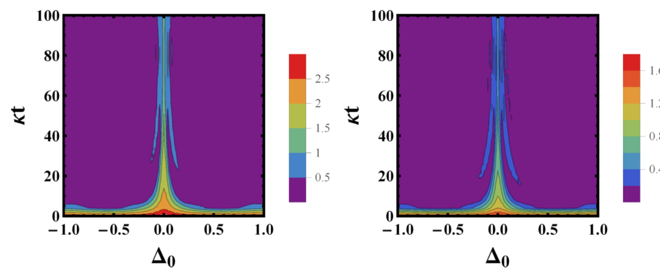

FIG. 3. Simulated dynamics of quantum correlations E_N (left panel) and GQD (right panel) versus the detuning Δ_0 and time t. Other parameters are $r = 1$, $r' = 1$, $\bar{n}_0 = 0$, $\gamma_0 = 0.01$, and $G_0 = 0.3$, respectively. The parameters are in units of $\kappa = 1$.

To get a clear picture of how other key parameters influence the steady-state quantum correlation, we plot in Figs. 4(a)–4(d) the stationary quantum correlations $E_N(\infty)$ and GQD(∞) under different system parameters r, $\bar{n}_0$, γ_0, and G_0, respectively. Figure 4(a) demonstrates again the importance of the squeezing feature of the driving field in the preservation of quantum correlation. An optimal squeezing degree r exists for maintaining the maximum steady-state quantum correlation. Figure 4(b) shows that the stable quantum correlation can also be maintained at a finite environmental temperature $\bar{n}_0$, and $E_N(\infty)$ decays faster to zero than GQD(∞) with the growth of $\bar{n}_0$. The point $E_N(\infty) = 0$

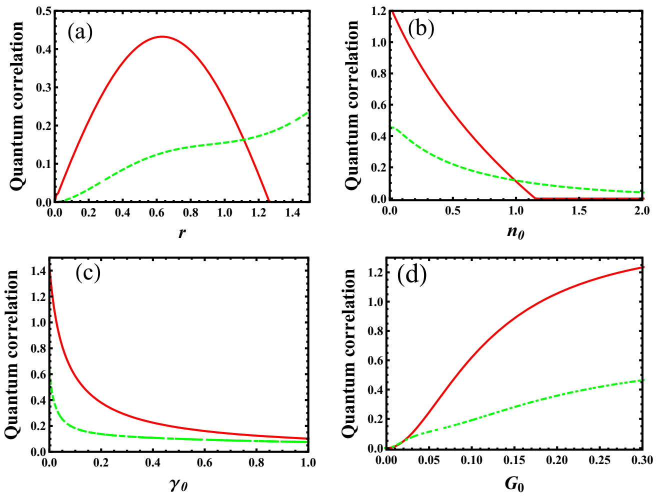

FIG. 4. Simulated steady-state quantum correlations $E_N(\infty)$ (red solid line) and GQD(∞) (green dashed line) under different parameters. (a) $\bar{n}_0 = 0$, $\gamma_0 = 0.01$, $G_0 = 0.3$, and $\Delta_0 = -0.1$; (b) $r = 1$, $\gamma_0 = 0.01$, $G_0 = 0.3$, and $\Delta_0 = 0$; (c) $r = 1$, $\bar{n}_0 = 0$, $G_0 = 0.3$, and $\Delta_0 = 0$; (d) $r = 1$, $\bar{n}_0 = 0$, $\gamma_0 = 0.01$, and $\Delta_0 = 0$. The parameters are in units of $\kappa = 1$.

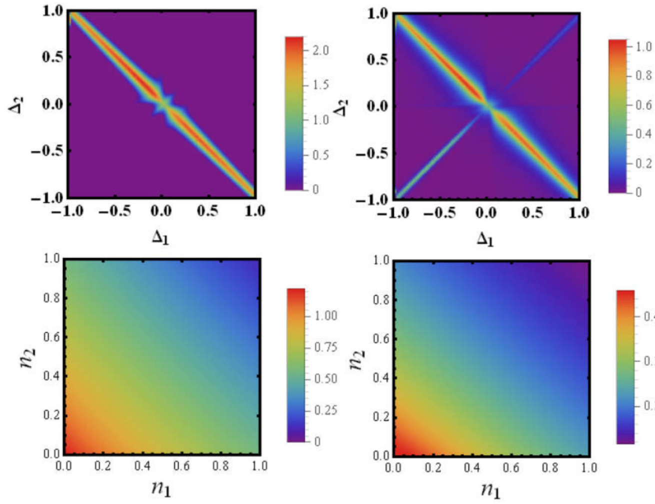

FIG. 5. Simulated steady-state quantum correlations $E_N(\infty)$ (left panels) and GQD(∞) (right panels) in different parameter spaces $\{\Delta_1, \Delta_2\}$ and $\{n_1, n_2\}$. Other common parameters are $r = 1$, $G_1 = G_2 = 0.3$, and $\gamma_1 = \gamma_2 = 0.01$. In addition, $\bar{n}_1 = \bar{n}_2 = 0$ (in the upper panels) and $\Delta_1 = \Delta_2 = 0$ (in the lower panels). The parameters are in units of $\kappa = 1$.

corresponds to the reservoir temperature in the order of magnitude of millikelvins, which fulfills the experimental condition [10]. Figure 4(c)–4(d) shows that the stable quantum correlation decreases (increases) with the growth of the decay rate γ_0 (coupling strength G_0). The reason behind this phenomenon is that a larger decay rate γ_0 induces a faster decay of the spin ensemble into the ground state and thus destroys quantum correlation more seriously. In contrast, a larger coupling strength G_0 could exploit and transfer the squeezing features of the broadband driving field to the spin ensembles more effectively and then make a larger amount of stable quantum correlation between the NVEs.

In the above discussion, two NVEs are assumed to share the same parameter values. However, these system parameters in a realistic situation may be unequal due to parameter fluctuations or stochastic errors. Next we survey the effect of these imperfect factors on the quantum correlation, as plotted in Figs. 5 and 6. In the upper panels of Fig. 5, the region with respect to maximal stable quantum correlation $E_N(\infty)$ primarily locates along the off-diagonal line $|\Delta_1 + \Delta_2| \simeq 0$, in the plane $\{\Delta_1, \Delta_2\}$. In addition, a smaller value of $|\Delta_1 + \Delta_2|$ induces a larger amount of stable quantum correlation $E_N(\infty)$. For GQD(∞), except for the off-diagonal line $|\Delta_1 + \Delta_2| \simeq 0$ in the plane $\{\Delta_1, \Delta_2\}$, there is another diagonal line $\Delta_1 \simeq \Delta_2$ in the plane $\{\Delta_1, \Delta_2\}$ where stable quantum correlation GQD(∞) with minor value also exists. Additionally, we also study the behavior of $E_N(\infty)$ and GQD(∞) in the parameter space $\{n_1, n_2\}$ (see lower panels of Fig. 5). We find that the two reservoir parameters $\bar{n}_1$ and $\bar{n}_2$ influence the stable quantum correlation ina similar manner. Also, the values of $E_N(\infty)$ decrease faster than those of GQD(∞) with the growth of the reservoir temperature.

The upper panels of Fig. 6 show the behavior of $E_N(\infty)$ and GQD(∞) versus the parameters γ_1 and γ_2. We find that $E_N(\infty)$ has large values primarily located in a series of ellipselike regions in the plane $\{\gamma_1, \gamma_2\}$. Compared with $E_N(\infty)$, the parameter regime where the stable quantum correlation GQD(∞) has large values shrinks. Furthermore, the steady characters of $E_N(\infty)$ and GQD(∞) in the parameter space $\{G_1, G_2\}$ are also investigated in the lower panels of Fig. 6, where both $E_N(\infty)$ and GQD(∞) are distributed symmetrically about the coupling strength G_2 when G_1 is fixed in the plane $\{G_1, G_2\}$. All the results in Fig. 6 indicate that quantum correlations between the NVEs can also be manipulated by the decay rates of the NVEs and the NVE-CPWR

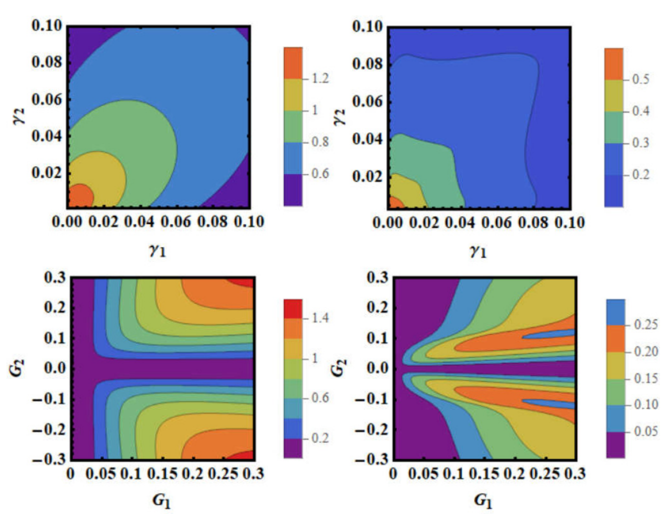

FIG. 6. Simulated steady-state quantum correlations $E_N(\infty)$ (left panels) and GQD(∞) (right panels) in different parameter spaces $\{\gamma_1, \gamma_2\}$ and $\{G_1, G_2\}$. Other common parameters are $r = 1$, $\bar{n}_1 = \bar{n}_2 = 0$, and $\Delta_1 = \Delta_2 = 0$. In addition, $G_1 = G_2 = 0.3$ (in the upper panels), and $\gamma_1 = \gamma_2 = 0.01$ (in the lower panels). The parameters are in units of $\kappa = 1$.

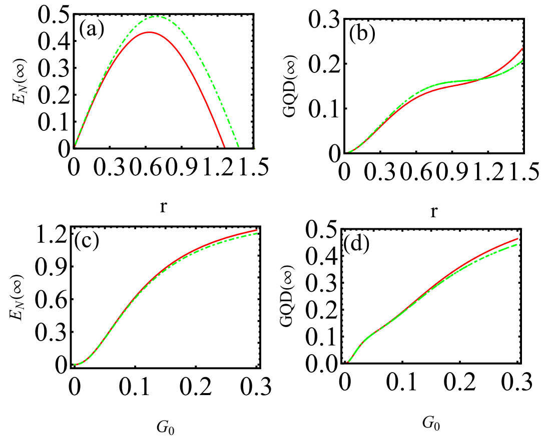

FIG. 7. (a, b) Dependence of steady-state quantum correlations $E_N(\infty)$ and GQD(∞) on the squeezing degree r, where $G_0 = 0.3$ and $\Delta_0 = -0.1$. (c, d) Dependence of steady-state quantum correlations $E_N(\infty)$ and GQD(∞) on the coupling strength G_0, where $r = 1$ and $\Delta_0 = 0$. The red solid line and the green dashed line denote the quantum correlation calculated with and without the adiabatic elimination method, respectively. Other parameters are $\bar{n}_0 = 0$ and $\gamma_0 = 0.01$. The parameters are in units of $\kappa = 1$.

coupling strength. Reducing the NVEs' decay rates and the mean thermal boson number $\bar{n}_i$ may help us to efficiently maintain higher amount of quantum correlation.

In order to verify the validity of the adiabatic elimination method employed in the derivation of Eq. (6), we plot in Fig. 7 the relation between stable quantum correlation [both $E_N(\infty)$ and GQD(∞)] and the squeezing parameter r (coupling strength G_0) with and without the adiabatic elimination of the CPWR mode. Slight differences can be found between the two methods, where the adiabatic elimination works well for small squeezing degree r and almost all of the value of the coupling strength G_0. In other regimes, the steady-state quantum correlations with and without the adiabatic elimination keep the same quantitative tendency with slight differences. Since the parameters used in our scheme are consistent with the conditions of the adiabatic elimination method, the results obtained in the present work are valid.

IV. DISCUSSION AND CONCLUSION

We survey the relevant experimental parameters for the feasibility of our scheme. In a recent experiment about two NVEs coupled to a CPWR [10], the full-wave frequency of the CPWR is $\omega_c/2\pi = 2.7491$ GHz, and the distance between NVEs is approximately 5 mm. The NVE-CPWR coupling strengths are $G_1/2\pi = 7.5 \pm 0.1$ MHz and $G_2/2\pi = 5.6 \pm 0.1$ MHz. The decay rates of the NVEs are $\gamma_1/2\pi = 2.45 \pm 0.18$ MHz and $\gamma_2/2\pi = 2.28 \pm 0.16$ MHz, respectively. In addition, the NV centers have long coherence time and excellent quantum controllability, e.g., an electron-spin relaxation time T_1 of 6 ms at room temperature [55] and even 28–265 s at lower temperatures [56]. Besides, using a spin-echo sequence makes the electron spin decouple from its local environment, and thus the dephasing time T_2 can be greatly prolonged. Based on this spin-echo technique, the dephasing time T_2 of a NVE with a natural abundance of ^{13}C can reach 0.6 ms at room temperature [57]. In addition, the electron-spin relaxation time T_1 for a NVE could reach up to 10 s at low temperature under an appropriately chosen magnetic field [58]. In our work, we assume that $\kappa : G_j : \gamma_j = 1 : 0.3 : 0.01$ to fulfill the condition of the bad-resonator limit ($\kappa \gg G_j \gg \gamma_j$). Note that the microwave squeezed field with a squeezing bandwidth up to tens of megahertz has been reported in an experiment [59] via a JPA, which is much larger than the linewidth of a typical microwave resonator with hundreds of kilohertz.

For the present complex systems, experimentally detecting quantum correlations between the NVEs is tough work because the standard methods, such as full state tomography, are impractical [60]. To verify the quantum correlation, we could resort to the well-developed experimental techniques in the field of cold-atom ensembles, such as the conventional method of "collective quantum nondemolition measurement" [61–64]. We could pass a "verifying" light pulse (the measurement sequence aimed at the verification of the entanglement, as in Refs. [61–63]) through the two spin ensembles. Using two verifying pulses independently, we may achieve measurements on the statistical variance of the collective spin components for the two spin ensembles. Alternatively, we could also combine the method of joint measurement with the technique of real-time feedback to verify entanglement between the ensembles. A joint measurement of the population of NVEs in excited states is engineered by measuring the frequency shift of the CPWR mode through the homodyne detection of the probe light reflected from the CPWR, as in Ref. [65].

In summary, we have proposed a practical scheme to maintain stable quantum correlation between two NVEs placed in a CPWR, which has a squeezed-vacuum reservoir resulting from a broadband squeezing driving field. Using a delicate interplay of the squeezing reservoir with the unavoidable dissipative mechanisms to stabilize the NVEs into a steady state regarding quantum correlation, we have found that the obtained stable quantum correlation of two NVEs is mainly determined by the squeezing feature of the broadband squeezing driving field and other key parameters, such as the detunings and NVE-CPWR coupling strengths. Our study has also provided a way to extract the optimal experimental parameters for maximal steady-state quantum correlation between the NVEs through the technology of squeezed-reservoir engineering. Moreover, straightforward extension of our idea to more NVEs is possible. Therefore, the present study may open interesting perspectives for devising an active decoherence-immune quantum information processor.

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APPENDIX

In the Appendix, we present the solution to the matrix elements of the covariance matrix V. Defining the vector

$$R^{(10)}(t) = [V_{11}(t), V_{22}(t), V_{33}(t), V_{44}(t), V_{12}(t), V_{13}(t), V_{14}(t), V_{23}(t), V_{24}(t), V_{34}(t)]^T,$$

the time-evolution equation of $R^{(10)}(t)$ can be obtained from Eq. (6) as

$$\dot{R}^{(10)}(t) = L^{(10)}R^{(10)}(t) + C_0 + NC_1 + M(C_2 e^{2i\omega_c t} + \text{c.c.}), \tag{A1}$$

where

$$L^{(10)} = \begin{pmatrix} -Q_1 & 0 & 0 & 0 & Z_1 & J_2 & K_2 & 0 & 0 & 0 \\ 0 & -Q_1 & 0 & 0 & -Z_1 & 0 & 0 & -K_2 & J_2 & 0 \\ 0 & 0 & -Q_2 & 0 & 0 & J_1 & 0 & K_1 & 0 & Z_2 \\ 0 & 0 & 0 & -Q_2 & 0 & 0 & -K_1 & 0 & J_1 & -Z_2 \\ -Z_1/2 & Z_1/2 & 0 & 0 & -Q_1 & -K_2/2 & J_2/2 & J_2/2 & K_2/2 & 0 \\ J_1/2 & 0 & J_2/2 & 0 & K_1/2 & -S & Z_2/2 & Z_1/2 & 0 & K_2/2 \\ -K_1/2 & 0 & 0 & K_2/2 & J_1/2 & -Z_2/2 & -S & 0 & Z_1/2 & J_2/2 \\ 0 & K_1/2 & -K_2/2 & 0 & J_1/2 & -Z_1/2 & 0 & -S & Z_2/2 & J_2/2 \\ 0 & J_1/2 & 0 & J_2/2 & -K_1/2 & 0 & -Z_1/2 & -Z_2/2 & -S & -K_2/2 \\ 0 & 0 & -Z_2/2 & Z_2/2 & 0 & -K_1/2 & J_1/2 & J_1/2 & K_1/2 & -Q_2 \end{pmatrix},$$

$$C_0 = (W_1, W_1, W_2, W_2, 0, p_\alpha/2, p_\beta/2, -p_\beta/2, p_\alpha/2, 0)^T,$$
$$C_1 = (P_1, P_1, P_2, P_2, 0, p_\alpha, p_\beta, -p_\beta, p_\alpha, 0)^T,$$
$$C_2 = (-Y_1, Y_1, -Y_2, Y_2, -iY_1, -Y_0, -iY_0, -iY_0, Y_0, -iY_2)^T.$$

Here,

$$J_i = -G_1G_2(T_i + T_i^*), K_i = iG_1G_2(T_i - T_i^*),$$
$$P_i = G_iG_i(T_i + T_i^*), Q_i = P_i + 2\gamma_i,$$
$$W_i = P_i/2 + \phi_i, S = (Q_1 + Q_2)/2,$$
$$Y_i = G_iG_iT_i^*, Y_0 = G_1G_2(T_1^* + T_2^*)/2,$$
$$Z_i = 2\omega_{eg}^i + iG_iG_i(T_i - T_i^*),$$
$$p_\alpha = -(J_1 + J_2)/2, p_\beta = (K_1 - K_2)/2,$$
$$\phi_i = (2\bar{n}_i + 1)\gamma_i \quad (i = 1, 2).$$

Therefore, the dynamical solution of Eq. (A1) can be simply written as

$$R^{(10)}(t) = O\exp(Ft)O^{-1}R^{(10)}(0) - OF^{-1}[I - \exp(Ft)]O^{-1}(C_0 + NC_1) - M\{O(F - 2i\omega_c I)^{-1}[I\exp(2i\omega_c t) - \exp(Ft)] \times O^{-1}C_2 + \text{c.c.}\}, \tag{A2}$$

with $F = O^{-1}L^{(10)}O$ and I being the unit matrix. Furthermore, the steady-state solutions of Eq. (A2) asymptotically approach

$$R^{(10)}(\infty) = -[L^{(10)}]^{-1}(C_0 + NC_1) - M[(L^{(10)} - 2i\omega_c I)^{-1}C_2\exp(2i\omega_c t) + \text{c.c.}]. \tag{A3}$$

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