Reply to "Comment on 'Inverse Doppler shift and control field as coherence generators for the stability in superluminal light' "

Fazal Ghafoor,[1,*] Bakht Amin Bacha,[2] and Salman Khan[1]

[1]*Department of Physics, COMSATS University Islamabad, Chak Shahzad, Islamabad 44000, Pakistan*

[2]*Department of Physics, University of Malakand, Chakdara, KPK, Pakistan*

(Received 1 April 2019; published 26 August 2019)

In the presence of inverse Doppler shift, we verify the superluminal effect in a weak probe field through the medium of our atomic system using classes of the atoms of high average velocity. Additional analysis of analytical results for the advancement of Gaussian pulse in time is presented. The pulse advancement through the single transparency window under the weak control field increases. The pulse advancement through the central (each of the sides) window of the medium under the strong control field increases (decreases) when its intensity is gradually increased.

DOI: 10.1103/PhysRevA.100.027801

This is a Reply to the Comment [1] on our paper [2]. We agree with some points raised in Ref. [1], which mainly refer to different typographical errors and the one we identified in the part on numerical simulation of analytical results in Ref. [2]. However, there are some claims in Ref. [1] with which we disagree and provide some further analysis to correctly explain the claims of [2]. We begin from the relation of susceptibility [2] given by

$$\chi = \frac{2N|\wp_{ac}|^2\widetilde{\rho}_{ac}}{\epsilon_0\hbar\Omega_p}, \tag{1}$$

with

$$\widetilde{\rho}_{ac} = -i\Omega_p Ł^{-1}(\Delta)\sum_{j=1}^{4}\mathbb{k}_j(\Delta)/8,$$

where N is the atom number density of the medium and the Einstein A coefficient is given by $A = |\wp_{ac}|^2\omega_{ac}^3/3\pi\epsilon_0\hbar c^3$ [3], as pointed out in the comment [1]. The forms of other parameters in Eq. (1) are given in the Appendix of [2]. Judicially, we do not agree with the comment along with the other relevant discussion that the incorrect value of $\omega_{ac} = 10^3\Gamma$ was used for the sodium D_2 line in Ref. [2] which, according to the authors of the Comment, reveals weak atomic density for the medium. This frequency that appeared in Fig. 2, as compared to its correct wavelength in the nanometer range in the same figure, is a typographical error, as evidenced from our estimation of the prefactor of the susceptibility (or unit of the relevant parameters) shown in Ref. [2] and in what follows.

The prefactor $\frac{N\wp_{ac}^2}{\epsilon_0\hbar}$ of Eq. (1), where N is the atom's number density and $\wp_{ac}$ the dipole moment developed by absorption of the probe field of frequency ω_{ac}, becomes $2\pi \times 10$ MHz for a cold temperature medium of the sodium D_1 (D_2) line which is also equal to the corresponding radiative decay rate γ. This factor could be associated with the linewidth of probe resonance for the D_1 (D_2) line of wavelength 508.33 THz (508.86 THz) [4]. However, here we consider a warm temperature medium wherein the gain in the probe transmission is subjected to the Doppler effect when a lighter buffer gas is also added to the vapor cell of the medium (Sodium D_1 line) with a suitable pressure. Elements of the family of inert gases, for example, Xe or He, could be used as the buffer gas for our medium. The linewidth of the probe field resonance becomes much larger than γ in such a situation. Moreover, coherence preserving velocity changing collisions (CPVCs) between the atoms of medium and the atoms of the buffer gas [5] could be produced under suitable pressure. In such a situation, we expect a rather inverse Doppler shift in the gain of the probe field than the traditional Doppler shift. According to the experiment of Ref. [6], we consider the linewidth $2\pi \times 20$ MHz of the probe resonance in case of Xe buffer gas as the unit of the relevant quantities of the system. For notation clarity we represent it by Γ. As an additional example, we consider the linewidth $2\pi \times 35$ MHz for the probe field resonance when He is added in place of Xe to the vapor cell as the buffer gas. To this end, we show a quantitative comparison of the advancement of the probe pulse in time for the two cases in our analytical nonlinear input-output theory. We further employ the condition $\Omega_p = 0.01\Gamma \ll \Omega_{1,2,3}$ to keep transmission of the probe field through the medium in a linear regime.

We revise the graphical results of [2] using a suitable linewidth for the probe field absorption in the presence of the buffer gas in the vapor cell of the medium. They are retraced in Figs. 1–3. It is important to mention here that these corrections in the graphical results leaves the qualitative behavior unchanged and only affect them quantitatively. We found that the gain in the probe field is less than 1 [see Figs. 1(a), 1(c), and 3(a)], which defines a stability in the probe transmission, since instability kicks in around the value 3.

We agree with the claim of [1] that there is no superluminal effect in the Gaussian probe field. However, our

*rishteen@yahoo.com

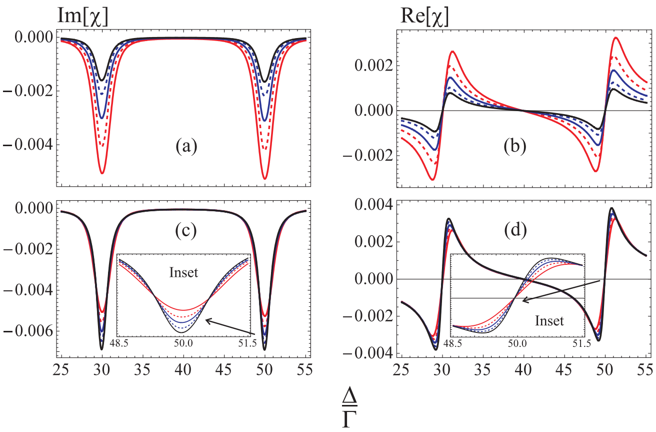

FIG. 1. Im[χ], Re[χ] vs $\frac{\Delta}{\Gamma}$ ($\Gamma = 2\pi \times 20$ MHz for the probe linewidth when Xe is loaded as the buffer gas in the vapor cell of the medium) using $\Omega_1 = 2.55\Gamma$, $\Omega_2 = 1.5\Gamma$, $\Omega_3 = 2\Gamma$, $\delta_2 = 50\Gamma$, $\delta_1 = 30\Gamma$, and $\delta_3 = 0$ with the atom's radiative decay rates $\Gamma_{ad} = \Gamma_{ac} = \Gamma_{bd} = \Gamma_{bc} = 2.05\Gamma$ and collisional dephasing rates $\Gamma_{cd} = \Gamma_{ab} = 0.5\Gamma$. In (a) and (b) [(c) and (d)] we use the class of average atomic velocity in the unit of m/s for $v = 3\Gamma/k$ ($v = 10\Gamma/k$) where $k = 2\pi/\lambda$ with $\lambda = 589.6$ nm in case of the Sodium D_1 line [4] and $V_D = 0$ (0) (red), 4Γ (2Γ) (red dashed), 6Γ (3Γ) (blue), 9Γ (4Γ) (blue dashed), 12Γ (5Γ) (black). In both cases, the class of the atom's velocity is kept fixed. The insets in (c) and (d) are, respectively, the gain and dispersion around $\frac{\Delta}{\Gamma} = 50$.

present investigations show that this is not an absolute reality and is partially valid only for $\Delta_p = 40\gamma$, as with this choice of Δ_p, $-N_g$ approximately tends to zero [see Figs. 2(c) and 3(c)]. Below, we show that changing the choice of Δ_p gradually, does result in superluminality in the Gaussian probe field, since in this case $-N_g$ also increases gradually.

The complex wave number of the probe field expanded via Taylor series in terms of the group index of the gain medium

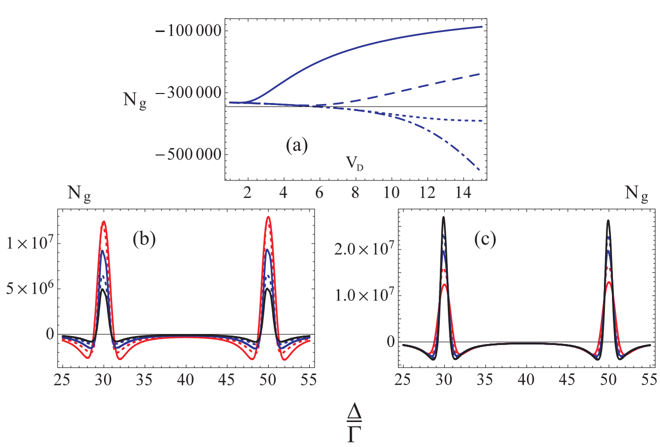

FIG. 2. N_g vs $\frac{\Delta}{\Gamma}$ with Γ of the same probe linewidth as we considered in Fig. 1. The V_D and average v for (b) and (c) are chosen from Fig. 2, correspondingly. However, in (a) vs V_D with classes of the atoms velocity v in m/s of $3\Gamma/k$ (blue solid), $10\Gamma/k$ (blue dashed), $20\Gamma/k$ (blue dotted), and $30\Gamma/k$ (blue dash-dotted).

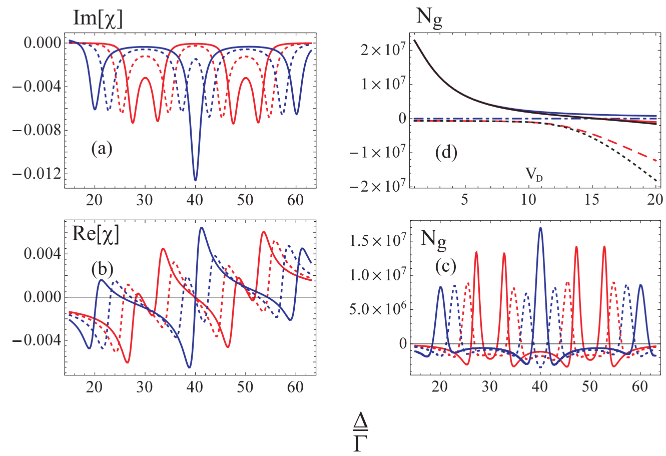

FIG. 3. In (a) Im[χ], in (b) Re[χ], and in (c) N_g vs $\frac{\Delta}{\Gamma}$ of the same probe linewidth as we used in Fig. 1. The other parameters are selected as $\Omega_1 = 2.5\Gamma$, $\Omega_2 = 4.3\Gamma$, and $\Omega_3 = 6\Gamma$ (red), 10Γ (red dashed), 15Γ (blue dashed), 20.5Γ (blue). The other parameters are chosen from Fig. 2. In (d) N_g vs V_D with $\Omega_3 = 6\Gamma$ (red) for average velocity $v = 10\Gamma/k$ m/s and $\Delta = 40\Gamma, 30\Gamma$ (blue, blue-dot-dashed), $v = 30\Gamma/k$ m/s and $\Delta = 40\Gamma, 30\Gamma$ (red, red-dashed) and for average v of $v = 10^3\Gamma/k$ m/s and $\Delta = 40\Gamma, 30\Gamma$ (black, black dotted). The other parameters used are similar to Fig. 1

is rewritten as [2]

$$k(\omega) = \omega c^{-1} N_g^{(0)} + \frac{c^{-1}}{2!}(\omega - \omega_0)^2 \frac{\partial N_g}{\partial \omega}\bigg|_{\omega\to\omega_0} + \frac{c^{-1}}{3!}(\omega - \omega_0)^3 \frac{\partial^2 N_g}{\partial \omega^2}\bigg|_{\omega\to\omega_0} + \cdots$$
$$= \mathbb{A}_a + \mathbb{A}_b\omega + \mathbb{A}_c\omega^2 + \mathbb{A}_d\omega^3 + \cdots, \qquad (2)$$

with $\mathbb{A}_a = \frac{\omega_o^3 N_2}{2c} - \frac{\omega_o^4 N_3}{6c} \ldots$, $\mathbb{A}_b = \frac{N_o}{c} - \frac{\omega_o N_1}{c} + \frac{\omega_o^2 N_2}{2c} - \frac{\omega_o^3 N_3}{6c} \ldots$, $\mathbb{A}_c = \frac{N_1}{c} - \frac{\omega_o N_2}{c} + \frac{\omega_o^2 N_3}{2c} \ldots$, and $\mathbb{A}_d| = \frac{N_2}{2c} + \frac{\omega_o N_3}{2c} \ldots$, where $N_g^{(o)}(\omega)|_{\omega\to\omega_0} = N_o$, $\frac{d}{d\omega}N_g(\omega)|_{\omega\to\omega_0} = N_1$, $\frac{d^2}{d\omega^2}N_g(\omega)|_{\omega\to\omega_0} = N_2$, and $\frac{d^3}{d\omega^3}N_g(\omega)|_{\omega\to\omega_0} = N_3$. Using the convolution theorem, the output pulse becomes [2]

$$S_{\text{out}}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S_{\text{in}}(\omega) H(\omega) e^{i\omega t} d\omega. \qquad (3)$$

Note that with the aid of Eq. (2), the transfer function $H(\omega)$, and the input Gaussian pulse $S_{\text{in}}(\omega)$ as of [2], the exponential function in the integrand of Eq. (3) becomes a polynomial of order 3 in ω, which constitutes the third part of the integrand. The first two parts of the expression are Gaussian integrals which can be solved straightaway whereas the third part is solved through Taylor expansion up to the term involving ω^3. The higher orders of this expansion are neglected due to their negligible contribution. The integral is evaluated analytically to obtain

$$S_{\text{out}}(t) = \mathcal{M}\sqrt{\frac{\pi}{f_2}}\bigg[\exp\left(\frac{f_1^2}{4f_2}\right) + \frac{f_2 f_3 (6f_1 + f_2^2)}{8(f_2)^{5/2}} \exp\left(\frac{f_1^2}{4f_2}\right)\bigg], \qquad (4)$$

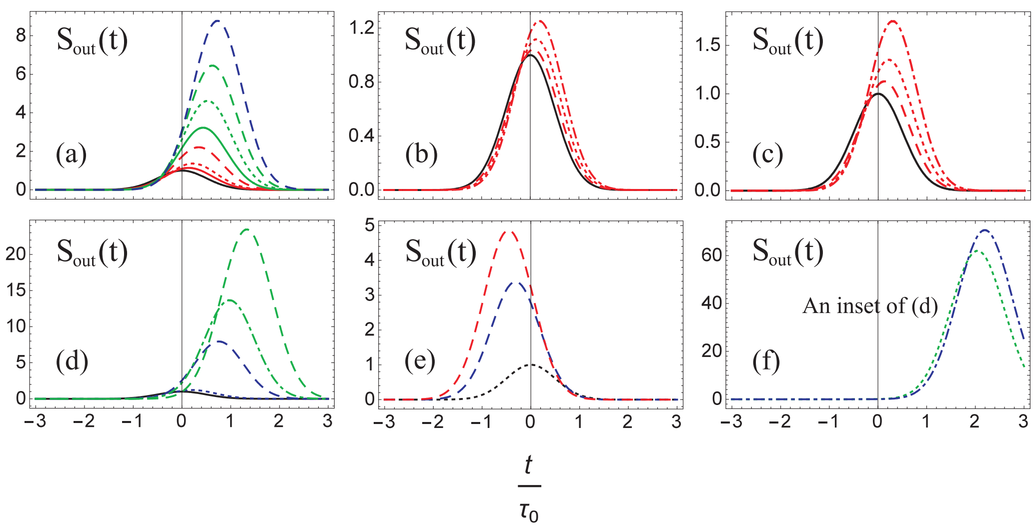

FIG. 4. The input (black solid line) and output intensity $S_{\text{out}}(t)$ of a Gaussian probe pulse vs the normalized time t/τ_0 is shown with $\xi = 0.001$, $\tau_0 = 1\ \mu$s, $L = 0.03$ m. In (a) we use the class of the average atom's velocity $v = 3\Gamma/k$ m/s with the probe detuning Δ at 45.40Γ (the inner red-solid line), 45.30Γ (red dotted), 34.44Γ (red dashed), 34.43Γ (green solid), 34.42Γ (green dotted), 34.41Γ (green dashed), and 34.40Γ (blue dashed). The other parameters, in this case, are kept similar as we considered in Fig. 1. In (b) [(c)] we use $\Delta = 45.4\Gamma$ whereas the class of the atoms velocity is gradually varied as $v = 2.9\Gamma/k$ m/s (red dot-dashed) $v = 2.8\Gamma/k$ m/s (red dotted), $2.75\Gamma/k$ m/s (red dashed) when the probe resonance linewidth $\Gamma = 2\pi \times 20$ MHz in case of the Xe buffer gas ($\Gamma = 2\pi \times 35$ MHz in case of the He buffer gas) is considered. The probe pulse in the central (left side) window at $\Delta = 39\Gamma$ ($\Delta = 28\Gamma$) with $\Omega_3 = 6\Gamma$ is shown in (d) as indicated by the gray-dotted line [in (f) as an inset of (d) as shown by the blue-dashed line]. Similarly, with $\Omega_3 = 10\Gamma$, the probe field in the central (left side) window is shown in (d) as represented by the gray-dashed (green-dashed) line and with $\Omega_3 = 15\Gamma$ in (d) as represented by the blue dot-dashed line [in (f) as the inset of (d) as designated by the gray dot-dashed line]. However, the transmission with $\Delta = 40\Gamma$, $\Omega_3 = 20.5\Gamma$, $v = 3\Gamma/k$ m/s, and $L = 0.09$ m ($L = 0.07$ m) is shown in (e), as shown by the red-dashed (blue-dashed) line. The other parameters in (d), (e) and (f) are, respectively, the same as used in Fig. 3. In all the cases we consider $V_D = 6\Gamma$.

where $\mathcal{M} = \frac{\tau_o}{2\sqrt{\pi}} \exp[-(\omega_o + \epsilon)^2 \frac{\tau_o^2}{4} - i\mathbb{A}_a L]$, $f_1 = 2(\omega_o + \xi)\frac{\tau_o^2}{4} + i\mathbb{A}_b L + it$, $f_2 = i\mathbb{A}_c L - \frac{\tau_o}{4}$, and $f_3 = -i\mathbb{A}_d L$.

In Fig. 4(a), we show time advancement of the probe pulse through the medium in the weak control field regime using suitable parameters of the system. The dispersion (negative group index), as shown in Fig. 1(d) [Fig. 2(c)] at the detuning $\Delta = 40\Gamma$ of the probe field is mild. Consistent with Fig. 5 of Ref. [2], there is no appreciable superluminality at this value. In this case, the reference and the advanced Gaussian pulse match at the output of the medium.

Beyond this review, we provide an additional analysis of our input-output theory to show the superluminal effect of the Gaussian probe pulse through the medium [2]. A gradual advancement in the pulse in time is seen when the detuning Δ deviates successively from 40Γ, as described in Fig. 4(a). Here, we note that the longer the time of the pulse advancement, the more asymmetric amplification and broadening appear in the probe pulse when it is retrieved from the medium. To show the influence of the inverse Doppler shift on the superluminality of the Gaussian probe pulse through the medium, the behavior is further shown in Fig. 4(b) using various classes of high average velocity of the atoms in the medium. Here, we note that the time advancement in the probe transmission is measurably enhanced with the linewidth Γ of $2\pi \times 20$ MHz at the probe resonance [6] when the class of the atom's velocity is increased successively while using Xe as buffer gas with a suitable pressure in the vapor cell. It is interesting to note here that when He with the linewidth Γ of $2\pi \times 35$ MHz for the probe resonance is selected as the buffer gas in the medium [6] rather than the Xe, the time advancement in the probe pulse is increased, however, at a cost of the relatively large pulse distortion, as shown in Fig. 4(c). The behavior of the pulse distortion with its long time advancement is in agreement with the ones of Fig. 4(a). We note that a similar dependence of the linewidth of the probe resonance on the advancement of the probe pulse through the medium is established in all the cases of this study.

Intuitively, the closer the two gain lines in a gain spectrum, the sharper the anomalous dispersion is expected in the transparency window between the two lines and vice versa. We notice that the anomalous dispersion at and around the probe detuning $\Delta = 40\Gamma$ (or $\Delta = 30\Gamma$, for example) which is flattened (sharpened) for coupling of the relatively weak control field becomes sharpened (flattened) when the coupling strength increases [see Fig. 3(b)]. As a result, the negative group index increases (decreases) as shown in Fig. 3(c). In this case, the time advancement of the probe pulse through the left side (central) window becomes, respectively, less (more) advanced for the Rabi frequency Ω_3 as 6Γ, 10Γ, and 15Γ, as shown in Fig. 4(d) and as an inset in Fig. 4(f) for optimal advancement of the probe pulse through both the windows. This behavior indicates that time advancement of the probe pulse through the central window increases in the strong control field regime compensated by simultaneous decrease in the one through the left side and in the one through the right side window of the medium, since as of the left window the superluminal behavior of the probe pulse through the right window appears identical.

As reported in Ref. [2] and retraced in Figs. 3(a) and 3(b), the transparency window at $\Delta = 40\Gamma$ collapses to create a gain line due to the merging effect of the adjacent two gain lines using $\Omega_3 = 20.5\Gamma$. As a result, quantum constructive interference exhibits in the spectrum converting the anomalous dispersion into normal. In this case, the pulse retrieved is rather with a time delay than the time of an advancement [see Fig. 4(e)] where the time delay is long due to high normal dispersion [for the high normal dispersion, see Fig. 3(b)]. The amplification and broadening in the pulse retrieved can, however, be minimized by increasing the length L of the gain medium [compare red-dotted and blue-dashed lines in Fig. 4(e)].

In the experiment of Wang *et al.* [7], small superluminality was observed due to small anomalous dispersion at the probe transparency through the window of the medium of atomic Cesium where the pulse retrieved from the vapor cell was almost lossless and symmetric (see Fig. 4 of [8]). A closed observation of this figure when zoomed indicates a relatively short time for advancement of the back tail of the probe pulse as compared to the advancement of the front tail. In the present study, the pulse distortion for sufficiently small anomalous dispersion is also negligible. The additional coherence effect by the control field and Doppler narrowing increases the anomalous dispersion of the probe pulse through the single and the left side windows of the multiple transparency windows under their respective conditions of the control field. As a result, the time advancement of the probe pulse increases due to the steeped anomalous dispersion, however, with an asymmetric amplification and broadening.

In general, the phenomenon of the time advancement in the probe pulse through a gain medium is counterintuitive as compared to the time delay in electromagnetically induced transparency (EIT) [9]. In the former case, the dispersion of a probe field where the gain is absent is anomalous, whereas it is normal through an EIT medium when absorption of the probe pulse is absent. On the other side, in the presence of the gain, the dispersion is normal and it is anomalous when absorption of the probe pulse through an EIT medium exhibits. The probe pulse is amplified and broadened due to a lossless gain medium of a sharp anomalous dispersion whereas it is compressed and broadened due to a lossless EIT medium of a sharp normal dispersion. The former superluminal behavior of a probe pulse through a gain medium holds true for the present system as well.

In conclusion, we have reevaluated the graphical and numerical results of Ref. [2] using an appropriate linewidth for the resonance of the probe field through the medium of our system when a lighter buffer gas with a suitable pressure is also added to the vapor cell of the medium. We have concluded that these changes in our analytical and numerical results leaves the qualitative behavior unchanged and only affect them quantitatively. As a result, the discussions on the results presented in Ref. [2] remain the same. We have further verified the superluminal effect quantitatively in a Gaussian probe pulse with coherence due to the inverse Doppler broadening and control field using additional analysis of the nonlinear input-output theory. To this end, the quantitative analysis in terms of the pulse advancement in time and qualitative analysis in terms of the group index in the probe field frequency is in excellent agreement for the transmission of the probe field. The additional analysis supports our claims reported in Ref. [2].

We are grateful to the anonymous reviewer whose constructive comments and useful suggestions enabled us to present this Reply in a readable form. F.G. acknowledges financial support from Higher Education Commission, Islamabad, through Grant No. HEC/NRPU/20-2475.

[1] B. Macke and B. Segard, Phys. Rev. A **99**, 047801 (2019).
[2] F. Ghafoor, B. A. Bacha, and S. Khan, Phys. Rev. A **91**, 053807 (2015).
[3] F. Ghafoor and R. G. Nazmitdinov, J. Phys. B: At. Mol. Opt. Phys. **49**, 175502 (2016).
[4] G. S. Agarwal and S. Dasgupta, Phys. Rev. A **70**, 023802 (2004).
[5] See, for example, G. Alzetta, A. Gozzini, L. Moi, and G. Orriols, Nuovo Cimento B **36**, 5 (1976).
[6] See, for example, L. J. Rothberg and N. Bloembergen, Phys. Rev. A **30**, 2327 (1984).
[7] L. J. Wang, A. Kuzmich, and A. Dogariu, Nature (London) **406**, 277 (2000).
[8] A. Dogariu, A. Kuzmich, and L. J. Wang, Phys. Rev. A **63**, 053806 (2001).
[9] Y.-F. Chen, Y.-M. Kao, W.-H. Lin, and I. A. Yu, Phys. Rev. A **74**, 063807 (2006).