Green's-function formulation for studying the backaction cooling of a levitated nanosphere in an arbitrary structure

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In this paper, we present a formulation based on the Green's function to study the backaction cooling of a levitated nanosphere in an arbitrary structure. This formulation has enabled us to study the dynamical backaction effect and the possibility of cooling in the absence of a cavity or when the nanosphere is trapped outside a cavity and excites a continuum of electromagnetic modes. We also investigate the roles of the Stokes and anti-Stokes processes, separately, and show that the anti-Stokes process is not necessarily the sole cooling agent. This is in sheer contrast with the intracavity cooling scenario.

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I. INTRODUCTION

The past decade has witnessed a growing interest in optomechanical systems based on levitated nanoparticles [1–6]. This surge of interest is attributable to the fact that levitated nanoparticles do not need mechanical support in ultrahigh vacuum and thus actualize having mechanical resonance with a high quality factor [1,2]. Numerous attempts have been made to cool the center of mass motion of a levitated nanoparticle down to microkelvin temperatures by applying active optical feedback [7–10], or by using dynamical backaction which is sometimes referred to as the passive cooling [1,2]. Ground-state cooling that might be thus achieved is of much importance because it can enhance the sensitivity of force measurements [11–15], facilitate the observation of quantum behavior on macroscopic scale, and generate nonclassical light-matter states [1–3]. To passively cool the motion of submicron particles, high finesse cavities are usually needed [1,2]. In such a case, an external tweezer can be employed to trap the nanoparticle inside the cavity [2], or two driven modes of the cavity can be used to trap and cool the motion of the nanoparticle at the same time. This latter scheme is referred to as the self-trapping scheme [1,16,17]. Recently, it has been shown that use of the coupled cavities is advantageous to cool the motion of nanoparticles in the resolved sideband regime [18], and can provide ground state cooling in the unresolved sideband regime [19]. Furthermore, cooling of charged nanoparticles using a Paul trap has been recently reported which thwarts instabilities that might occur in optical traps at very low pressures [20,21].

Dynamics of cavity-assisted optomechanical systems are conventionally studied by Hamiltonian formulation [1,16], which is more undemanding when the nanoparticle interacts predominantly with only a small number of discrete resonance modes within a high-quality cavity. Studying the dynamics of the nanoparticle motion in the presence of a continuum of electromagnetic modes in a cavityless structure or outside a cavity becomes burdensome if one wishes to apply the conventional Hamiltonian formulation. In such a case, it would be more beneficial to employ the Green's-function formulation to study the dynamics of the trapped nanoparticle motion because the Green's function encompasses the whole spectrum of eigenmodes supported by the electromagnetic structure in which the nanoparticle motion is to be cooled down. It is worth noting that studying the static backaction effects on the time-averaged force exerted upon Rayleigh particles has been recently reported by using the Green's-function formulation [22].

In this paper, we present a formulation based on the Green's-function to study the dynamical effects of the electromagnetic backaction on the center-of-mass motion of a levitated nanosphere. This formulation enables us to study cooling of a levitated nanosphere in a cavityless structure or outside a cavity. We apply the proposed formulation to study cooling of a levitated nanosphere at the center of a spherical reflector using a monochromatic incident electromagnetic field. The spherical reflector can be either an ideal mirror or a high-quality cavity. We show that the optical cooling rate depends strongly on the radius of the spherical reflector, and cooling rates of up to 1 MHz are conceivable if the spherical reflector forms a high-quality cavity. In sheer contrast to the case of intracavity cooling in which the anti-Stokes process is always responsible for the cooling, we show that under certain circumstances the Stokes process might cool the nanosphere's motion.

The organization of this paper is as follows: In Sec. II the mathematical formulation to model the dynamical backaction effect based on the Green's function is presented and the cooling rate expressions are also derived. In Sec. III, as a numerical example, we investigate the cooling of a levitated nanosphere at the center of a spherical reflector. The conclusions are made in Sec. IV.

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II. FORMULATION

To investigate the dynamics of the nanosphere's motion around its equilibrium position in the presence of a monochromatic incident electromagnetic field, we consider the position of the nanosphere as

$$\boldsymbol{r}'(t) = \boldsymbol{r}_p + \delta\boldsymbol{r}(t), \tag{1}$$

where $\boldsymbol{r}_p$ is the equilibrium position of the nanosphere, and $\delta\boldsymbol{r}(t)$ is the small fluctuation of the nanosphere's position around $\boldsymbol{r}_p$.

The electric field at an arbitrary point $\boldsymbol{r}$ when the nanosphere is at $\boldsymbol{r}'$ can be expressed as

$$\boldsymbol{\mathcal{E}}(\boldsymbol{r}, \boldsymbol{r}'; t) = \tfrac{1}{2}\boldsymbol{E}(\boldsymbol{r}, \boldsymbol{r}'; t)e^{-i\omega_L t} + \text{c.c.}, \tag{2}$$

where ω_L is the frequency of the incident electromagnetic field, and $\boldsymbol{E}(\boldsymbol{r}, \boldsymbol{r}'; t)$ is the complex amplitude of the electric field, given by

$$\boldsymbol{E}(\boldsymbol{r}, \boldsymbol{r}'; t) = \boldsymbol{E}_0(\boldsymbol{r}) + \int \overleftrightarrow{\boldsymbol{G}}_s(\boldsymbol{r}, \boldsymbol{r}'; t - t') \cdot \boldsymbol{p}(\boldsymbol{r}'; t')e^{i\omega_L(t-t')}dt'. \tag{3}$$

Here, $\boldsymbol{E}_0$ is the electric field in the absence of the nanosphere. The second term in the above equation is the scattered field due to the radiation of the nanosphere which is referred to as the backaction field. It should be noted that $\overleftrightarrow{\boldsymbol{G}}_s$ is the scattering Green's function of the structure, and $\boldsymbol{p}$ is the complex amplitude of the nanosphere's equivalent electric dipole moment which can be obtained from

$$\boldsymbol{p}(\boldsymbol{r}'; t) = \alpha\boldsymbol{E}(\boldsymbol{r}', \boldsymbol{r}'; t), \tag{4}$$

where α is the free space polarizability of the nanosphere, given by

$$\alpha = 4\pi\epsilon_0 R_p^3 \frac{\epsilon_p - 1}{\epsilon_p + 2} \tag{5}$$

in which R_p and ϵ_p are the radius and relative permittivity of the nanosphere, respectively. It is worth noting that the validity of Eqs. (4) and (5) is subject to two approximations: first, we neglect the impact of the nanosphere's motion on the free space polarizability of the nanosphere and second, the dispersion of the nanosphere polarizability should be insignificant. The former holds true because the optical time scale is much shorter than the mechanical time scale and thus the mechanical motion appears as nearly frozen to the optical field. The latter remains valid because the radiation reaction is usually negligible. [22].

Then, by linearizing the above expressions with respect to $\delta\boldsymbol{r}$, it can be easily shown that

$$\boldsymbol{E}(\boldsymbol{r}, \boldsymbol{r}'; t) \simeq \bar{\boldsymbol{E}}(\boldsymbol{r}, \boldsymbol{r}_p) + \delta\boldsymbol{E}(\boldsymbol{r}, \boldsymbol{r}'; t), \tag{6}$$

where $\bar{\boldsymbol{E}}(\boldsymbol{r}, \boldsymbol{r}_p)$ is the averaged electric field seen by the nanosphere, given by

$$\bar{\boldsymbol{E}}(\boldsymbol{r}, \boldsymbol{r}_p) = \boldsymbol{E}_0(\boldsymbol{r}) + \overleftrightarrow{\boldsymbol{G}}_s(\boldsymbol{r}, \boldsymbol{r}_p; \omega_L) \cdot \boldsymbol{p}(\boldsymbol{r}_p) \tag{7}$$

and $\delta\boldsymbol{E}(\boldsymbol{r}, \boldsymbol{r}'; t)$ is the linearized fluctuation of the electric field due to the motion of the nanosphere, which can be written as

$$\delta\boldsymbol{E}(\boldsymbol{r}, \boldsymbol{r}'; t) = \frac{1}{2\pi}\int \delta\boldsymbol{r}(\omega) \cdot \overleftrightarrow{\boldsymbol{\mathcal{M}}}(\boldsymbol{r}, \boldsymbol{r}_p; \omega_L, \omega)e^{-i\omega t}d\omega. \tag{8}$$

In this expression,

$$\begin{aligned}\mathcal{M}_{ij}(\boldsymbol{r}, \boldsymbol{r}_p; \omega_L, \omega) = {} & \partial_i' G_{s_{jk}}(\boldsymbol{r}, \boldsymbol{r}_p; \omega_L + \omega)p_k(\boldsymbol{r}_p) \\ & + G_{s_{jk}}(\boldsymbol{r}, \boldsymbol{r}_p; \omega_L + \omega)\alpha_{\text{eff}_{kl}}(\boldsymbol{r}_p; \omega_L + \omega) \\ & \times [\partial_i E_{0_l}(\boldsymbol{r}_p) + \partial_i G_{s_{lm}}(\boldsymbol{r}_p, \boldsymbol{r}_p; \omega_L)p_m(\boldsymbol{r}_p) \\ & + \partial_i' G_{s_{lm}}(\boldsymbol{r}_p, \boldsymbol{r}_p; \omega_L + \omega)p_m(\boldsymbol{r}_p)]\end{aligned} \tag{9}$$

is the optomechanical transfer function of the system, and $\boldsymbol{p}(\boldsymbol{r}_p) = \overleftrightarrow{\boldsymbol{\alpha}}_{\text{eff}}(\boldsymbol{r}_p; \omega_L) \cdot \boldsymbol{E}_0(\boldsymbol{r}_p)$ is the averaged electric dipole moment in which

$$\overleftrightarrow{\boldsymbol{\alpha}}_{\text{eff}}(\boldsymbol{r}_p; \omega) = \alpha[\overleftrightarrow{\boldsymbol{I}} - \alpha\overleftrightarrow{\boldsymbol{G}}_s(\boldsymbol{r}_p, \boldsymbol{r}_p; \omega)]^{-1} \tag{10}$$

is the effective polarizability of the nanosphere [22]. Now the backaction has an insignificant effect on trapping of the nanosphere, i.e., $\bar{\boldsymbol{E}}(\boldsymbol{r}, \boldsymbol{r}_p) \simeq \boldsymbol{E}_0(\boldsymbol{r})$, when $||\alpha\overleftrightarrow{\boldsymbol{G}}_s|| \ll 1$, and $\boldsymbol{E}_0$ is sufficiently confined. In such a case, the optomechanical transfer function can be further simplified to

$$\begin{aligned}\mathcal{M}_{ij}(\boldsymbol{r}, \boldsymbol{r}_p; \omega_L, \omega) \simeq {} & \alpha\partial_i' G_{s_{jk}}(\boldsymbol{r}, \boldsymbol{r}_p; \omega_L + \omega)E_{0_k}(\boldsymbol{r}_p) \\ & + \alpha G_{s_{jk}}(\boldsymbol{r}, \boldsymbol{r}_p; \omega_L + \omega)\partial_i E_{0_k}(\boldsymbol{r}_p).\end{aligned} \tag{11}$$

It should be noted that even though we can neglect the impact of the scattering Green's function on the trapping of the nanosphere, we cannot necessarily neglect its significant effect on the optomechanical transfer function which can play a significant role in cooling the nanosphere's motion.

The mechanical motion of the nanosphere around its equilibrium position is governed by

$$m\frac{d^2\boldsymbol{r}'}{dt^2} = \boldsymbol{F}_{em} - m\Gamma_m\frac{d\boldsymbol{r}'}{dt} + \boldsymbol{\xi}(t), \tag{12}$$

where

$$\boldsymbol{F}_{em} = \frac{1}{2}\text{Re}\left[\alpha\sum_k E_k\nabla E_k^*\right] \tag{13}$$

is the electromagnetic force exerted upon the nanosphere under the rotating wave approximation [23]. Here, m is the mass of the nanosphere, and Γ_m is the mechanical damping rate. Furthermore, $\boldsymbol{\xi}(t)$ stands for the thermal noise whose correlation is given by

$$\langle\xi_i(t)\xi_j(t')\rangle = 2m\Gamma_m k_B T\delta_{ij}\delta(t - t'), \tag{14}$$

where δ_{ij} is the Kronecker delta function, k_B is the Boltzmann constant, and T is the ambient temperature [18]. Then, the linearized equation of motion of the nanosphere can be written as

$$\begin{aligned}m\frac{d^2}{dt^2}\delta\boldsymbol{r} = {} & \frac{1}{2}(\delta\boldsymbol{r} \cdot \nabla)\text{Re}\left[\alpha\sum_k \bar{E}_k\nabla\bar{E}_k^*\right] \\ & + \frac{1}{2}\text{Re}\left[\alpha\sum_k(\delta E_k\nabla\bar{E}_k^* + \bar{E}_k\nabla\delta E_k^*)\right] \\ & - m\Gamma_m\frac{d}{dt}\delta\boldsymbol{r} + \boldsymbol{\xi}\end{aligned} \tag{15}$$

and thereby the mechanical response of the system is given by

$$[\boldsymbol{\chi}^{-1}]_{ij} = m\big(\omega_{m_{ij}}^2 - \omega^2\delta_{ij} - i\omega\Gamma_m\delta_{ij}\big) - \tfrac{1}{4}\alpha[\mathcal{M}_{jk}(\boldsymbol{r}_p,\boldsymbol{r}_p;\omega_L,\omega)\partial_i\bar{E}_k^* + \mathcal{M}_{jk}^*(\boldsymbol{r}_p,\boldsymbol{r}_p;\omega_L,-\omega)\partial_i\bar{E}_k] - \tfrac{1}{4}\alpha[\bar{E}_k^*\partial_i\mathcal{M}_{jk}(\boldsymbol{r}_p,\boldsymbol{r}_p;\omega_L,\omega) + \bar{E}_k\partial_i\mathcal{M}_{jk}^*(\boldsymbol{r}_p,\boldsymbol{r}_p;\omega_L,-\omega)], \tag{16}$$

where

$$\omega_{m_{ij}} = \left[\frac{1}{2m}\mathrm{Re}\left[\alpha\partial_j\sum_k \bar{E}_k\partial_i\bar{E}_k^*\right]\right]^{1/2} \tag{17}$$

is the tensor of the mechanical frequency. In the weak-coupling regime, when the mechanical response of the system can be approximated by a Lorentzian line shape, the optical cooling rate tensor can be defined as

$$\Gamma_{\mathrm{opt}_{ij}} = \Gamma_{\mathrm{S}_{ij}} + \Gamma_{\mathrm{A}_{ij}} \tag{18}$$

in which

$$\Gamma_{\mathrm{S}_{ij}} = \frac{\alpha}{4m\omega_{m_{ii}}}\mathrm{Im}\big[\bar{E}_k\partial_i\mathcal{M}_{jk}^*(\boldsymbol{r}_p,\boldsymbol{r}_p;\omega_L,-\omega_{m_{ii}}) + \mathcal{M}_{jk}^*(\boldsymbol{r}_p,\boldsymbol{r}_p;\omega_L,-\omega_{m_{ii}})\partial_i\bar{E}_k\big] \tag{19}$$

and

$$\Gamma_{\mathrm{A}_{ij}} = \frac{\alpha}{4m\omega_{m_{ii}}}\mathrm{Im}\big[\mathcal{M}_{jk}(\boldsymbol{r}_p,\boldsymbol{r}_p;\omega_L,\omega_{m_{ii}})\partial_i\bar{E}_k^* + \bar{E}_k^*\partial_i\mathcal{M}_{jk}(\boldsymbol{r}_p,\boldsymbol{r}_p;\omega_L,\omega_{m_{ii}})\big] \tag{20}$$

are the rates of the Stokes and anti-Stokes processes, respectively.

First, we apply the proposed formulation to investigate the cooling of a levitated nanosphere in a high-quality cavity. We assume that the nanosphere is trapped by an external tweezer at $\boldsymbol{r}_p$, and is cooled by a discrete resonant mode of the cavity. Thus, the electric field inside the cavity can be approximated by a single resonant mode $\boldsymbol{E}_0 = E_{in}\boldsymbol{u}(\boldsymbol{r})$, where E_{in} is the amplitude of the mode, and $\boldsymbol{u}(\boldsymbol{r})$ is its normalized electric-field profile. The scattering Green's function can then be approximated by

$$G_s(\boldsymbol{r},\boldsymbol{r}';\omega) = \frac{i\omega}{2\epsilon_0 V_m}\frac{\boldsymbol{u}(\boldsymbol{r})\boldsymbol{u}^*(\boldsymbol{r}')}{\frac{\kappa}{2} + i(\omega_c - \omega)}, \tag{21}$$

where ω_c and κ are the resonance frequency and decay rate of the resonant mode, respectively, and V_m is its mode volume [22]. Hence, the Stokes and anti-Stokes rates are given by

$$\Gamma_{\mathrm{S}_{ij}} = \frac{-\Omega}{\left(\frac{\kappa}{2}\right)^2 + \left(\Delta - \omega_{m_{ii}}\right)^2}\partial_i\Psi|_{\boldsymbol{r}_p}\partial_j\Psi|_{\boldsymbol{r}_p}, \tag{22}$$

$$\Gamma_{\mathrm{A}_{ij}} = \frac{\Omega}{\left(\frac{\kappa}{2}\right)^2 + \left(\Delta + \omega_{m_{ii}}\right)^2}\partial_i\Psi|_{\boldsymbol{r}_p}\partial_j\Psi|_{\boldsymbol{r}_p} \tag{23}$$

in which $\Delta = \omega_L - \omega_c$ is the detuning, $\Psi = |\boldsymbol{u}(\boldsymbol{r})|^2$ is the intensity profile of the resonant mode, and $\Omega = \frac{\alpha^2|E_{in}|^2\omega_L\kappa}{16m\omega_{m_{ii}}\epsilon_0 V_m}$. As expected, Eqs. (22) and (23) show that in the conventional intracavity cooling scheme the Stokes and anti-Stokes processes will indeed heat and cool the nanosphere's motion, respectively. However, the complexity of Eqs. (19) and (20) begs the question of whether it is generally true that the heating and cooling are caused by the Stokes and anti-Stokes processes; respectively, particularly when there is a continuum of electromagnetic modes at play. In the following, we present cases in which the cooling of the nanosphere's motion is caused by the Stokes process.

III. NUMERICAL EXAMPLES

In this section, we apply the proposed formulation to study the cooling of a levitated nanosphere at the center of a spherical reflector. A fused silica nanosphere whose radius is $R_p = 100$ nm, and has a relative permittivity of $\epsilon_p = 2.1$ with the mass density of $\rho_m = 2201\ \mathrm{kg\,m^{-3}}$ is trapped by an optical tweezer. The optical tweezer is an x-polarized Gaussian beam which propagates along the y direction, has a beam waist of $w_0 = 2\lambda_0$, and is of intensity I_0. The wavelength of the Gaussian beam is $\lambda_0 = 1064$ nm, and its intensity is $I_0/w_0^2 = 2\ \mathrm{W}\,\mu\mathrm{m}^{-4}$ which provides the trapping frequency of 352 kHz. In accordance with Fig. 1, the trapping position is at the center of either a single spherical mirror of radius R_0 or two concentric spherical mirrors whose radii are R_0 and $R_0 + d$. In both scenarios, the central angle of the apparatus is $\theta_0 = \pi/3$. It is worth noting that the reflectivity of the second scenario is frequency selective because two concentric spherical reflectors establish a resonator. This proves beneficial to the cooling process even though the nanosphere is trapped outside of the established cavity and thus does not follow the dynamics of a nanosphere trapped inside a Fabry-Perot resonator whose Stokes and anti-Stokes rates are given by Eqs. (22) and (23), respectively.

In Fig. 1, the impact of the spherical reflector on the trapping of the nanosphere is negligible, and the particle will be trapped at the center of the Gaussian beam, i.e., $\boldsymbol{r}_p = 0$. After some algebraic manipulations, it can be shown that in the weak-coupling regime, the Stokes and anti-Stokes rates can be simplified to

$$\Gamma_{\mathrm{S}_{\mathrm{ij}}} = \frac{1}{4m\omega_{m_{ii}}}|\alpha|^2|E_{0_x}|^2\mathrm{Im}\big[\partial_i\partial_j' G_{s_{xx}}^*(\omega_L - \omega_{m_{ii}})\big] \tag{24}$$

and

$$\Gamma_{\mathrm{A}_{\mathrm{ij}}} = \frac{1}{4m\omega_{m_{ii}}}|\alpha|^2|E_{0_x}|^2\mathrm{Im}\big[\partial_i\partial_j' G_{s_{xx}}(\omega_L + \omega_{m_{ii}})\big] \tag{25}$$

respectively. Now, using the physical optics approximation, it can be easily shown that

$$\partial_i\partial_j' G_{s_{xx}}(\omega) \simeq \frac{i\omega^5}{8\pi\epsilon_0 c^5}\rho(\omega)e^{2i\omega R/c} \times (0.126\delta_{xx} + 0.181\delta_{yy} + 0.419\delta_{zz}) \tag{26}$$

for $\theta_0 = \pi/3$. In the above expression, $\rho(\omega)$ is the reflection coefficient of the structure. It is clear that $\rho(\omega) = -1$ when the structure is made of a single spherical mirror, and $\rho(\omega) = \rho_0[1 - \exp(\frac{2i\omega d}{c})]/[1 - \rho_0^2\exp(\frac{2i\omega d}{c})]$ when it is made of two concentric spherical reflectors. In this latter expression, ρ_0 is the reflection coefficient of each reflector, and d is the difference between the radii of the inner and outer spherical reflectors. According to Eq. (26) it can be figured out that the cooling rate tensor is diagonal, and its diagonal entries have the same profile. Hence, in the following we just investigate the cooling rates along the z direction, i.e., Γ_{zz}.

Figure 2(a) shows the optical cooling rate when the structure is made of a single spherical mirror in accordance with Fig. 1(a). In this figure the radius of the spherical mirror is $R = N\lambda_0 + \delta R$ where N is an integer and δR determines the phase of the scattering Green's function. The cooling rate at $\delta R = 0.125\lambda_0$ can be as high as 3.7 kHz for $N = 10^5$. In Fig. 2(b) the rates of the Stokes and anti-Stokes processes are plotted separately. This figure shows that the anti-Stokes process cools the nanosphere's motion only when $0.125\lambda_0 < \delta R < 0.375\lambda_0$. Otherwise, and in sheer contrast to what is expected, the Stokes process is responsible for the cooling. It is worth noting that the cooling rate grows by increasing N, and can reach 2.3 MHz when $N \simeq \lambda_m/2\lambda_0$ in which $\lambda_m = 2\pi c/\omega_m$ is the mechanical wavelength. However, this is not feasible because the spherical mirror would be extremely large for such a scenario.

Figure 3 shows the optical cooling rate when the structure is made of two concentric reflectors in accordance with Fig. 1(b). In this figure, $\rho_0 = 0.99996$ and $d = 1$ cm which provides a high-frequency selective reflection around $\omega_0/2\pi = 300$ THz, and $\Delta = \omega - \omega_0$ is the detuning of the drive frequency from ω_0. This figure shows that the nanosphere's motion can be cooled at both negative and positive frequency detuning depending on the value of δR.

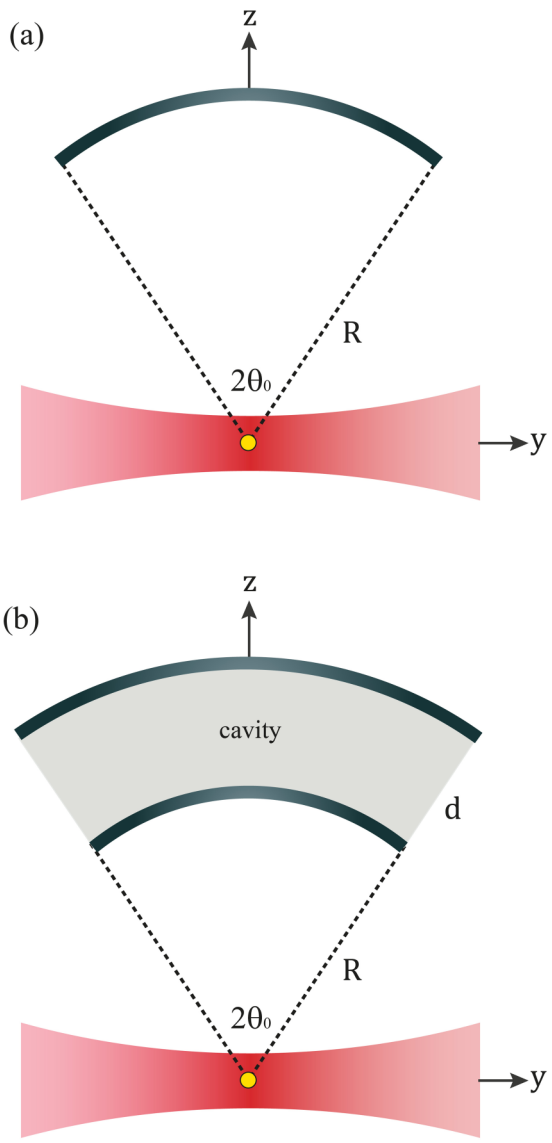

FIG. 1. Schematic of the proposed system. An optical tweezer made of a Gaussian beam traps a nanosphere at the center of (a) a single spherical mirror, (b) two concentric spherical reflectors.

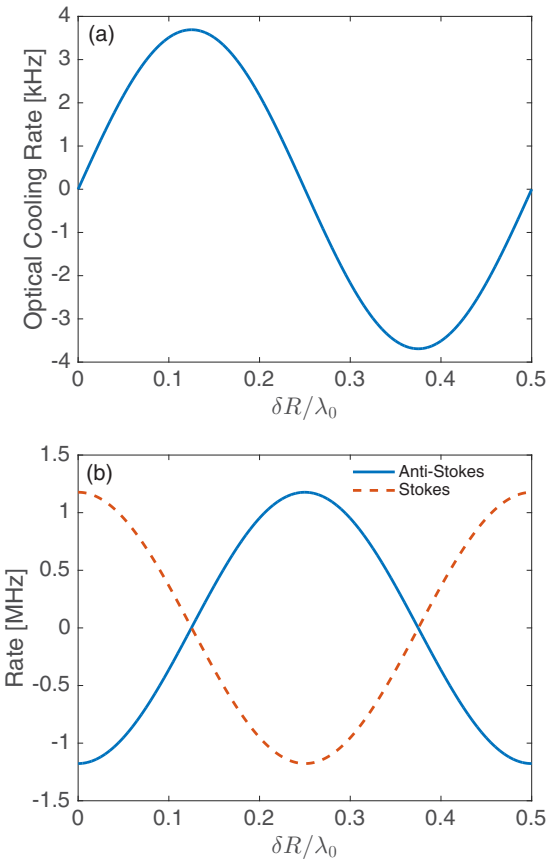

FIG. 2. (a) The optical cooling rate, and (b) the Stokes and anti-Stokes rates vs δR for the case of spherical mirror for $N = 10^5$.

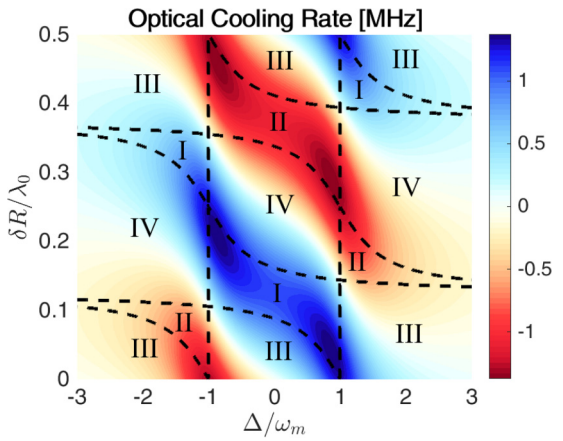

FIG. 3. The optical cooling rate for the case of two concentric spherical reflectors. This figure is separated into different regions: (I) both Stokes and anti-Stokes processes are cooling; (II) both Stokes and anti-Stokes processes are heating (III); Stokes is heating and anti-Stokes is cooling; (IV) Stokes is cooling and anti-Stokes is heating.

Furthermore, the contributions of the Stokes and anti-Stokes processes in cooling or heating of the nanosphere's motion are looked into and four different regions are distinguished. Interestingly, we can find conditions under which both the Stokes and anti-Stokes processes cool the nanosphere's motion, simultaneously, and it is worth noting that the optimum cooling lies in this regime.

IV. CONCLUSION

In conclusion, we have presented a formulation for studying the dynamical backaction cooling of a levitated nanosphere based on the Green's function of the structure. This formulation enables us to study the center of mass cooling of a levitated nanosphere in a cavityless structure, or in the extra-cavity scenario. We have shown that this formulation reaches the same results as the Hamiltonian formulation for the case of the intracavity cooling. We have also shown that the anti-Stokes process is not necessarily always the sole cooling agent whenever the Green's function of the structure can no longer be approximated by retaining only one eigenmode of the system, i.e., when the structure is cavityless or is excited from outside of the cavity. In the latter case, the red detuning of the structure is no longer a must.

[1] D. E. Chang, C. Regal, S. Papp, D. Wilson, J. Ye, O. Painter, H. J. Kimble, and P. Zoller, Cavity opto-mechanics using an optically levitated nanosphere, Proc. Natl. Acad. Sci. USA **107**, 1005 (2010).

[2] O. Romero-Isart, M. L. Juan, R. Quidant, and J. I. Cirac, Toward quantum superposition of living organisms, New J. Phys. **12**, 033015 (2010).

[3] O. Romero-Isart, A. C. Pflanzer, M. L. Juan, R. Quidant, N. Kiesel, M. Aspelmeyer, and J. I. Cirac, Optically levitating dielectrics in the quantum regime: Theory and protocols, Phys. Rev. A **83**, 013803 (2011).

[4] N. Kiesel, F. Blaser, U. Delić, D. Grass, R. Kaltenbaek, and M. Aspelmeyer, Cavity cooling of an optically levitated submicron particle, Proc. Natl. Acad. Sci. USA **110**, 14180 (2013).

[5] L. P. Neukirch and A. N. Vamivakas, Nano-optomechanics with optically levitated nanoparticles, Contemp. Phys. **56**, 48 (2015).

[6] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, Rev. Mod. Phys. **86**, 1391 (2014).

[7] J. Gieseler, B. Deutsch, R. Quidant, and L. Novotny, Subkelvin Parametric Feedback Cooling of a Laser-Trapped Nanoparticle, Phys. Rev. Lett. **109**, 103603 (2012).

[8] T. Li, M. G. Raizen, and S. Kheifets, Millikelvin cooling of an optically trapped microsphere in vacuum, Nat. Phys. **7**, 527 (2011).

[9] P. Mestres, J. Berthelot, M. Spasenović, J. Gieseler, L. Novotny, and R. Quidant, Cooling and manipulation of a levitated nanoparticle with an optical fiber trap, Appl. Phys. Lett. **107**, 151102 (2015).

[10] J. Vovrosh, M. Rashid, D. Hempston, J. Bateman, M. Paternostro, and H. Ulbricht, Parametric feedback cooling of levitated optomechanics in a parabolic mirror trap, J. Opt. Soc. Am. B **34**, 1421 (2017).

[11] A. A. Geraci, S. B. Papp, and J. Kitching, Short-Range Force Detection Using Optically Cooled Levitated Microspheres, Phys. Rev. Lett. **105**, 101101 (2010).

[12] G. Ranjit, M. Cunningham, K. Casey, and A. A. Geraci, Zeptonewton force sensing with nanospheres in an optical lattice, Phys. Rev. A **93**, 053801 (2016).

[13] A. Arvanitaki and A. A. Geraci, Detecting High-Frequency Gravitational Waves with Optically Levitated Sensors, Phys. Rev. Lett. **110**, 071105 (2013).

[14] J. Gieseler, L. Novotny, and R. Quidant, Thermal nonlinearities in a nanomechanical oscillator, Nat. Phys. **9**, 806 (2013).

[15] V. Jain, J. Gieseler, C. Moritz, C. Dellago, R. Quidant, and L. Novotny, Direct Measurement of Photon Recoil from a Levitated Nanoparticle, Phys. Rev. Lett. **116**, 243601 (2016).

[16] G. A Pender, P. F. Barker, F. Marquardt, J. Millen, and T. S. Monteiro, Optomechanical cooling of levitated spheres with doubly resonant fields, Phys. Rev. A **85**, 021802(R) (2012).

[17] T. Monteiro, J. Millen, G. Pender, F. Marquardt, D. Chang, and P. Barker, Dynamics of levitated nanospheres: Towards the strong coupling regime, New J. Phys. **15**, 015001 (2013).

[18] M. A. Abbassi and K. Mehrany, Employing coupled cavities to increase the cooling rate of a levitated nanosphere in the resolved sideband regime, J. Opt. Soc. Am. B **35**, 1563 (2018).

[19] J.-S. Feng, L. Tan, H.-Q. Gu, and W.-M. Liu, Auxiliary-cavity-assisted ground-state cooling of an optically levitated nanosphere in the unresolved-sideband regime, Phys. Rev. A **96**, 063818 (2017).

[20] J. Millen, P. Z. G. Fonseca, T. Mavrogordatos, T. S. Monteiro, and P. F. Barker, Cavity Cooling a Single Charged Levitated Nanosphere, Phys. Rev. Lett. **114**, 123602 (2015).

[21] P. Z. G. Fonseca, E. B. Aranas, J. Millen, T. S. Monteiro, and P. F. Barker, Nonlinear Dynamics and Strong Cavity Cooling of Levitated Nanoparticles, Phys. Rev. Lett. **117**, 173602 (2016).

[22] M. A. Abbassi and K. Mehrany, Inclusion of the backaction term in the total optical force exerted upon rayleigh particles in nonresonant structures, Phys. Rev. A **98**, 013806 (2018).

[23] L. Novotny and B. Hecht, *Principles of Nano-optics* (Cambridge University Press, Cambridge, UK, 2012).