Bose polaron in spherical trap potentials: Spatial structure and quantum depletion

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We investigate how the presence of a localized impurity in a Bose-Einstein condensate of trapped cold atoms that interact with each other weakly and repulsively affects the profile of the condensed and excited components at zero temperature. By solving the Gross-Pitaevskii and Bogoliubov–de Gennes equations, we find that an impurity-boson contact attraction (repulsion) causes both components to change in spatial structure in such a way as to be enhanced (suppressed) around the impurity, while slightly declining (growing) in a region far from the impurity. Such behavior of the quantum depletion of the condensate can be understood by decomposing the impurity-induced change in the profile of the excited component with respect to the radial and azimuthal quantum numbers. The significant role of the centrifugal potential and the hole excitation level is thus clarified.

I. INTRODUCTION

Polarons are quasiparticles conceptually well established in solid-state physics. For instance, a conducting electron in an ionic crystal moves together with the locally induced polarization to form a polaron that has an energy spectrum modified from that of an electron in vacuum [1–3]. Such a medium-modified electron serves as an essential building block for more complex many-body phenomena, e.g., high-T_c superconductivity [4–6]. Recently, cold atomic gas experiments have offered various types of atomic polarons, i.e., impurities (minority atoms) that are immersed either in a trapped Bose gas with Bose-Einstein condensation (BEC) or in a degenerate Fermi gas and eventually dressed by a virtual cloud of the corresponding particle-hole excitations, called Bose polarons [7–13] and Fermi polarons [14–18], respectively. At sufficiently low temperatures, the interaction between the impurity and medium atoms is characterized by a low-energy s-wave scattering length, which in turn is tunable by external magnetic fields as predicted by the Fano-Feshbach theory [19]. The polaron energy and spectral strength have been measured by, e.g., radio-frequency spectroscopy [11,12,14,15], which utilizes the radio absorption probability of impurity atoms in two different hyperfine states that interact with medium atoms only weakly and relatively strongly.

The above experiments have also energized theoretical investigations, which include various issues of atomic polarons in zero-temperature media: quasiparticle properties of Bose polarons [20–30] and Fermi polarons [31–44], the self-localization of impurities in quasi-one-dimensional BEC media [45–49], polaronic spectral changes from a weak- to

a strong-coupling regime for attractive interactions [50–54], few-body physics around the unitarity limit [55–59], dynamics of the polaron formation [59–62], and open quantum dynamics [63–65]. In addition to these "conventional" atomic polarons, more exotic ones have been proposed, e.g., p-wave polarons [66], dipolar-type polarons [67,68], angulons [69–72], and Bose polarons near the transition temperature [73,74]. Moreover, thermal evolution of Fermi polarons has been observed in a recent experiment [75] and studied in theoretical investigations [76,77], and critical properties of Bose polarons have been experimentally observed [78]. Whereas most of the theoretical studies mentioned above assume that the system is spatially uniform, Bose polarons in rotationally symmetric trap potentials [79,80] that are used in experiments have been studied theoretically to figure out the ground-state properties at a given total angular momentum, a conserved quantity in the system considered here. Note that the total momentum is the corresponding quantity in translationally symmetric uniform systems.

However, theoretical studies of Bose polarons have yet to reveal the detailed structures of the condensate and fluctuations in the presence of impurity, especially in three-dimensional trapped systems. This is because the Bogoliubov approximation has been used basically within a theoretical framework that assumes that the presence of the impurity does not change the spatial profile of the condensate or the number of condensed bosons. The Bogoliubov approximation is supposed to be valid if the fraction of noncondensed bosons that are caused by all interactions assumed in the system is negligibly small (much less than one). There are some estimates of this condition with respect to the boson-boson and the impurity-boson interaction strengths [81,82]. In general, a repulsive interaction among bosons disturbs them in their condensing into a coherent state by causing fluctuations to generate a finite fraction of noncondensed bosons even at zero temperature, a phenomenon referred to as the quantum depletion of the condensate [19,83,84]. In superfluid ^{4}He, for

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instance, the fraction is no less than about 80–90 % [85], while in cold atomic condensates it is less than 10% because a very weak repulsive interaction can be naturally realized [86]; it has been recently observed in an experiment [87] in a box trap. Aside from the validity argument of the approximations used in theoretical studies of Bose polarons, it is interesting to find a missing piece, namely, to examine the open question of how the impurity gives local feedback to the medium bosons: The impurity inevitably induces a local deformation of the condensate and local quantum fluctuations. Our motivation is to examine such local modifications caused by an impurity localized in a trap potential. In the present paper, therefore, we consider a single Bose polaron in spherically symmetric trap potentials and figure out details of the spatial structure of the condensate and Bogoliubov excitations by allowing for changes induced around the impurity by the impurity-boson interaction.

This paper is organized as follows. In Sec. II we set up our system, write the effective Hamiltonian in detail, and derive a set of coupled equations using some reasonable approximations. In Sec. III we show numerical results obtained by solving the equations derived in Sec. II. In Sec. IV we summarize.

II. FORMULATION

In this section we write a set of equations that describe the spatial structure of an impurity and the condensed and excited components of a weakly interacting Bose gas in trap potentials.

A. Effective Hamiltonian

We consider the zero-temperature system of a single atomic impurity immersed in a dilute atomic Bose gas. The impurity and the gas are trapped in the confinement potentials $V_I(\mathbf{r})$ and $V_B(\mathbf{r})$. Bosons are assumed to interact with each other weakly and repulsively, while the impurity-boson interaction is assumed to be tunable between positive and negative values using the Fano-Feshbach resonance. Such a system can be described by the low-energy effective Hamiltonian $\mathcal{H} = H_{imp} + H_B + H_{int}$ that is composed of the trapped single impurity, the trapped boson gas, and the impurity-boson interaction, respectively, i.e.,

$$H_{imp} = \frac{\hat{\mathbf{p}}^2}{2m_I} + \frac{m_I \omega_I^2}{2}\hat{\mathbf{q}}^2, \tag{1}$$

$$H_B = \int_{\mathbf{r}} \hat{\phi}^\dagger(\mathbf{r})\Big[h_B + \frac{g_{BB}}{2}\hat{\phi}^\dagger(\mathbf{r})\hat{\phi}(\mathbf{r}) - \mu\Big]\hat{\phi}(\mathbf{r}), \tag{2}$$

$$H_{int} = g_{IB}\int_{\mathbf{r}} \hat{\phi}^\dagger(\mathbf{r})\delta^{(3)}(\hat{\mathbf{q}} - \mathbf{r})\hat{\phi}(\mathbf{r}). \tag{3}$$

Here $h_B = -\frac{\hbar^2 \nabla^2}{2m_B} + V_B(\mathbf{r})$ and we have used the first (second) quantized form for the impurity (bosons). We have introduced the abbreviated notation for the spatial integral $\int_{\mathbf{r}} = \int d^3\mathbf{r}$. For the boson-boson and boson-impurity interactions, the effective coupling constants and *s*-wave scattering lengths are

related by

$$g_{BB} = \frac{4\pi\hbar^2}{m_B}a_{BB}, \tag{4}$$

$$g_{IB} = \frac{2\pi\hbar^2}{m_{red}}a_{IB}, \tag{5}$$

where $m_{red} = m_B m_I/(m_B + m_I)$.

We aim to figure out the ground-state properties of this system at zero temperature. In the formulation we first take the expectation of $\mathcal{H}$ with respect to a normalized impurity state $|imp\rangle$ that is yet to be determined. The resultant expression reads

$$\mathcal{H}_B = \langle\mathcal{H}\rangle_{imp}$$
$$= \langle H_{imp}\rangle_{imp} + H_B + g_{IB}\int_{\mathbf{r}} |\psi(\mathbf{r})|^2 \hat{\phi}^\dagger(\mathbf{r})\hat{\phi}(\mathbf{r}), \tag{6}$$

where $\psi(\mathbf{r}) = \langle\mathbf{r}|imp\rangle$ is the wave function of the impurity and $\langle H_{imp}\rangle_{imp} = \int_{\mathbf{r}} \psi^*(\mathbf{r})[-\frac{\hbar^2\nabla^2}{2m_I} + V_I(\mathbf{r})]\psi(\mathbf{r})$. The $\mathcal{H}_B$ defined above represents an effective Hamiltonian for bosons in the presence of an additional potential due to the impurity wave function.

Now we expand the boson field operator into the condensate and its fluctuations as $\hat{\phi}(\mathbf{r}) = \phi(\mathbf{r}) + \hat{\varphi}(\mathbf{r})$, where $\phi(\mathbf{r}) := \langle\hat{\phi}(\mathbf{r})\rangle$, with $\langle\cdots\rangle$ defined as the expectation value by the vacuum of bosons in the presence of an impurity,[1] corresponding to the ground state of the unperturbed Hamiltonian to be specified below. This implies $\langle\hat{\varphi}(\mathbf{r})\rangle = 0$. Expanding $\mathcal{H}_B$ with respect to the fluctuations, we obtain

$$\mathcal{H}^{(0)} = \langle H_{imp}\rangle_{imp} + \int_{\mathbf{r}} \phi^*\Big[h_B + g_{IB}|\psi|^2 + \frac{g_{BB}}{2}|\phi|^2 - \mu\Big]\phi, \tag{7}$$

$$\mathcal{H}^{(1)} = \int_{\mathbf{r}} \hat{\varphi}^\dagger(h_B + g_{IB}|\psi|^2 + g_{BB}|\phi|^2 - \mu)\phi + \text{H.c.}, \tag{8}$$

$$\mathcal{H}^{(2)} = \frac{1}{2}\int_{\mathbf{r}} \begin{pmatrix}\hat{\varphi}^\dagger & \hat{\varphi}\end{pmatrix}\begin{pmatrix}\mathcal{L} & \mathcal{M} \\ \mathcal{M}^* & \mathcal{L}^*\end{pmatrix}\begin{pmatrix}\hat{\varphi} \\ \hat{\varphi}^\dagger\end{pmatrix}, \tag{9}$$

$$\mathcal{H}^{(3)} = g_{BB}\int_{\mathbf{r}}(\phi\hat{\varphi}^\dagger\hat{\varphi}^\dagger\hat{\varphi} + \phi^*\hat{\varphi}^\dagger\hat{\varphi}\hat{\varphi}), \tag{10}$$

$$\mathcal{H}^{(4)} = \frac{g_{BB}}{2}\int_{\mathbf{r}} \hat{\varphi}^\dagger\hat{\varphi}^\dagger\hat{\varphi}\hat{\varphi}, \tag{11}$$

where $\mathcal{L} = h_B + g_{IB}|\psi|^2 + 2g_{BB}|\phi|^2 - \mu$ and $\mathcal{M} = g_{BB}\phi^2$.

Let us now choose $\mathcal{H}^{(1)} + \mathcal{H}^{(2)}$ as the unperturbed Hamiltonian. Then we find $\mathcal{H}^{(1)} = 0$ from $i\hbar\partial_t\langle\hat{\varphi}\rangle = \langle[\hat{\varphi}, \mathcal{H}^{(1)} + \mathcal{H}^{(2)}]\rangle = 0$ in the interaction picture, which leads to the Gross-Pitaevskii equation [88]

$$(h_B + g_{IB}|\psi|^2 + g_{BB}|\phi|^2 - \mu)\phi = 0. \tag{12}$$

This does not include fluctuation effects explicitly, but only through the impurity's wave function (15). In fact, Eq. (12) is equivalent to the stationary condition from $\mathcal{H}^{(0)}$. When higher-order fluctuation effects from $\mathcal{H}^{(3)}$ and $\mathcal{H}^{(4)}$ are negligible due to sufficiently weak interactions, i.e., $\mathcal{H}^{(3)}$ and $\mathcal{H}^{(4)}$

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[1]The definition of the expectation depends on the choice of the unperturbed Hamiltonian and the approximation scheme.

terms in the Hamiltonian can be neglected, the Hamiltonian approximately reads

$$\mathcal{H}_\mathrm{B} \simeq \mathcal{H}^{(0)} + \mathcal{H}^{(2)}. \tag{13}$$

The ground and excited states for the above approximate Hamiltonian can be determined solely from the diagonalization of $\mathcal{H}^{(2)}$ once the condensation profile is obtained from (12). In the Bogoliubov representation $\hat{\varphi}(\mathbf{r}) = \sum_i [u_i(\mathbf{r})\hat{\alpha}_i + v_i^*(\mathbf{r})\hat{\alpha}_i^\dagger]$, where the operators $\hat{\alpha}_i$ satisfy the canonical commutation relation $[\hat{\alpha}_i, \hat{\alpha}_j^\dagger] = \delta_{ij}$; we thus obtain the Bogoliubov–de Gennes (BdG) equations [89,90]

$$\begin{pmatrix} \mathcal{L} & \mathcal{M} \\ -\mathcal{M}^* & -\mathcal{L} \end{pmatrix} \begin{pmatrix} u_i \\ v_i \end{pmatrix} = E_i \begin{pmatrix} u_i \\ v_i \end{pmatrix}, \tag{14}$$

where E_i is the eigenenergy of $\mathcal{H}^{(2)}$, i.e., the boson excitation energy. We remark that these excited bosons can be interpreted as phonons whose bilinear coupling with the impurity is similar to the Fröhlich-type electron-phonon coupling in polar semiconductors. The variational condition with respect to the impurity state $\delta\langle\mathcal{H}_\mathrm{B}\rangle/\delta\psi^* = 0$ leads to another equation

$$(h_\mathrm{I} + g_\mathrm{IB}|\phi|^2 + g_\mathrm{IB}\langle\hat{\varphi}^\dagger\hat{\varphi}\rangle)\psi = 0, \tag{15}$$

where the expectation is taken with respect to the bosonic ground state of $\mathcal{H}^{(2)}$, i.e., the Fock vacuum of $\hat{\alpha}_i$.

Putting Eqs. (12), (14), and (15) together, we obtain a set of equations to be solved simultaneously. We note that the chemical potential μ will be determined from

$$N_\mathrm{B} = \int_\mathbf{r} \langle\hat{\phi}(\mathbf{r})^\dagger\hat{\phi}(\mathbf{r})\rangle := N_0 + N_\mathrm{ex}, \tag{16}$$

where N_0 is the number of the bosons in the condensate and N_ex is the number of the bosons in the excited states. At zero temperature, N_ex represents a depletion of the condensate [86], because all the bosons are in the condensate for a noninteracting impurity-free system. The depletion of the condensate can be described by

$$N_\mathrm{ex} = \sum_i \int_\mathbf{r} |v_i(\mathbf{r})|^2, \tag{17}$$

under the condition that the total number of bosons N_B is kept constant by μ.

B. Spherical trap potentials and further approximations

Hereafter, for theoretical simplicity, we assume that the impurity and the gas are trapped in the respective spherical harmonic-oscillator potentials whose centers coincide: $V_\mathrm{B}(\mathbf{r}) = \frac{m_\mathrm{B}\omega_\mathrm{B}^2}{2}\mathbf{r}^2$ and $V_\mathrm{I}(\mathbf{r}) = \frac{m_\mathrm{I}\omega_\mathrm{I}^2}{2}\mathbf{r}^2$. This theoretical setup is motivated by experiments [12] in which the optical and magneto-optical traps are well described by the harmonic-oscillator potentials.

In this work we focus on the quantum depletion of the condensate in the presence of an impurity. For simplicity, we ignore the dynamical kickback to the impurity from the medium Bose gas and consider a situation where the impurity is strongly bounded by the spherical harmonic trap potential, i.e., $\hbar\omega_\ell \gg |g_\mathrm{IB}\bar{n}_\mathrm{B}|$, with $\bar{n}_\mathrm{B}$ the average boson density $\bar{n}_\mathrm{B} = N_\mathrm{B}/(\frac{4\pi}{3}d_\mathrm{B}^3)$, where $d_\mathrm{B} = \sqrt{\hbar/m_\mathrm{B}\omega_\mathrm{B}}$. In this situation, because of the wide energy gap between the ground and other excited

states, the wave function of the impurity remains in the ground state. Moreover, the impurity wave function is shrunk by the potential and experiences the kickback from the condensate as a constant potential which affects only the energy shift; hence, the kickback term from the condensate in Eq. (15) can be ignored. Therefore, we have

$$\psi(\mathbf{r}) \simeq \left(\frac{\pi\hbar}{m_\mathrm{I}\omega_\mathrm{I}}\right)^{-3/4} \exp\left(-\frac{m_\mathrm{I}\omega_\mathrm{I}}{2\hbar}r^2\right). \tag{18}$$

We note that this condition is inequivalent to the heavy mass limit of the impurity $m_\mathrm{I} \gg m_\mathrm{B}$ because the wave function of the impurity is shrunk under this condition but the excitation gap remains unchanged. Since the wave function of the impurity is isotropic, the ground state of the condensate also has an isotropic form

$$\phi(\mathbf{r}) = \sqrt{\frac{N_0}{4\pi}}\Phi(r) \tag{19}$$

and the BdG eigenfunctions can be assumed to have a separable form

$$\begin{Bmatrix} u_{n_r\ell m}(\mathbf{r}) \\ v_{n_r\ell m}(\mathbf{r}) \end{Bmatrix} = \begin{Bmatrix} \mathcal{U}_{n_r\ell}(r) \\ \mathcal{V}_{n_r\ell}(r) \end{Bmatrix} Y_{\ell m}(\theta_1, \theta_2), \tag{20}$$

where (n_r, ℓ, m) denote the radial, azimuthal, and magnetic quantum number, respectively. Finally, the depletion of the condensate is characterized by

$$N_\mathrm{ex} = \sum_{n_r,\ell} \int_\mathbf{r} n_{\mathrm{ex},n_r\ell}(r), \tag{21}$$

$$n_{\mathrm{ex},n_r\ell}(r) = \frac{2\ell + 1}{4\pi}|\mathcal{V}_{n_r\ell}(r)|^2. \tag{22}$$

III. NUMERICAL RESULTS AND DISCUSSION

Let us now present numerical results for the quantum depletion. To this end, we consider a situation in which a ^{40}K Fermi impurity is immersed in an ^{87}Rb Bose condensate, i.e., $m_\mathrm{I}/m_\mathrm{B} \simeq 0.460$. We set the total number of the bosons as $N_\mathrm{B} = 10^5$ and the ratio of the strength of the trap potentials as $\omega_\mathrm{I}/\omega_\mathrm{B} = 5$ with $\omega_\mathrm{B} = 20 \times 2\pi$ Hz. For the boson-boson and boson-impurity interactions, we take $1/a_\mathrm{BB}\bar{n}_\mathrm{B}^{1/3} = 100$ and $1/a_\mathrm{IB}\bar{n}_\mathrm{B}^{1/3} = \pm 10$, with

$$\bar{n}_\mathrm{B} = N_\mathrm{B} \bigg/ \left(\frac{4\pi}{3}d_\mathrm{B}^3\right) = 1.70 \times 10^{15} \text{ cm}^{-3}, \tag{23}$$

$$d_\mathrm{B} = \sqrt{\frac{\hbar}{m_\mathrm{B}\omega_\mathrm{B}}} = 2.41 \times 10^{-4} \text{ cm}. \tag{24}$$

Throughout the numerical calculations we keep the number of total bosons N_B fixed by tuning the chemical potential.

A. Condensate and depletion in real space

We first consider the spatial dependence of the condensate and depletion. Figure 1 shows the radial profile of the order parameter in the absence of the impurity $\Phi^{(0)}(r)$ and the change in the order parameter due to the presence of the impurity, defined by

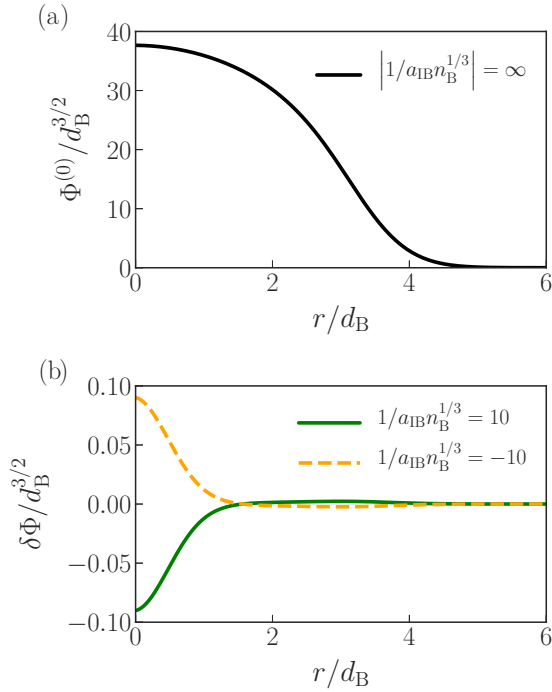$$\delta\Phi(r) := \Phi(r) - \Phi^{(0)}(r). \tag{25}$$

FIG. 1. (a) Radial profile of the order parameter $\Phi(r)$ in the absence of an impurity and (b) difference of the radial profile of the order parameter in the presence of an impurity from the result depicted in (a).

FIG. 2. Same as Fig. 1 but for the density of depletion $\sum_{n_r,\ell} n_{\text{ex},n_r\ell}(r)$.

Hereafter, we use the superscript (0), which denotes the absence of the impurity. Note that the impurity is localized at the center of the trap potential.[2] When the boson-impurity interaction is repulsive $g_{\text{IB}} > 0$ (attractive $g_{\text{IB}} < 0$), therefore, the condensate seems to sense an additional small bump (dip) at the center and eventually is slightly repelled from (pulled toward) the impurity as shown in Fig. 1.

Correspondingly, Fig. 2 exhibits the radial profile of the density of the quantum depletion, $\sum_{n_r,\ell} n_{\text{ex},n_r\ell}(r) := n_{\text{ex}}(r)$, in the absence of the impurity and the change in the density of the quantum depletion due to the presence of the impurity,

$$\delta n_{\text{ex}}(r) := \sum_{n_r,\ell} \left[n_{\text{ex},n_r\ell}(r) - n_{\text{ex},n_r\ell}^{(0)}(r) \right]. \tag{26}$$

We find that the profiles of the order parameter and the quantum depletion are similar. This result reflects the fact that the quantum depletion at a given position arises from zero-range boson-boson repulsion and thus follows the local density of the condensate. The impurity nearby induces a decrease (increase) in the number of atoms in the condensate by 2.38×10^{-3} (2.40×10^{-3}) and hence leads to a decrease (increase) of the quantum depletion. We remark that in a region far from the impurity, i.e., beyond $r/d_{\text{B}} \sim 1.7$, the change in the condensate and depletion due to the presence

of the impurity is tiny but opposite in sign to that around the impurity.

Finally, we remark on the details of numerical calculations. As pointed out in Ref. [86], the convergence of the sum in Eq. (21) is significantly slow. We truncate the terms whose energy exceeds $1000/\hbar\omega_{\text{B}}$. Even with this truncation, we can reproduce 90% of the depletion obtained in the semiclassical approximation [86], which implies that our approach is quantitatively reasonable.

B. Depletion vs quantum numbers

The n_r dependence of the quantum depletion of the condensate is shown for $\ell = 0, 1, 2$ in Fig. 3. The figure indicates a qualitative difference in the behaviors between $\ell = 0$ and $\ell = 1, 2$: When $g_{\text{IB}} > 0$, the quantum depletion is always suppressed by $\ell = 0$ excitations with any n_r, while it is suppressed (enhanced) by high-lying (low-lying) excitations of $\ell \neq 0$. When $g_{\text{IB}} < 0$, the quantum depletion behaves oppositely to when $g_{\text{IB}} > 0$.

To understand such behavior we illustrate the BdG eigenfunctions $\mathcal{V}_{n_r\ell}(r)$ which correspond to hole excitations in the absence of the impurity and differences in the quantum depletion at each state,

$$\delta n_{\text{ex},n_r\ell}(r) := n_{\text{ex},n_r\ell}(r) - n_{\text{ex},n_r\ell}^{(0)}(r), \tag{27}$$

due to the presence of the impurity in Fig. 4. We can see that for given ℓ and n_r, the corresponding change in the density of depletion due to the presence of the impurity is controlled by where the bottom of the dip of the BdG eigenfunction $\mathcal{V}_{n_r\ell}(r)$ is located.

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[2]We do not discuss here the self-localization of impurities [45,46]. The weak boson-impurity coupling constant used here keeps the system from a possible Bose collapse expected for strong attractive couplings in trapped systems [47,48].
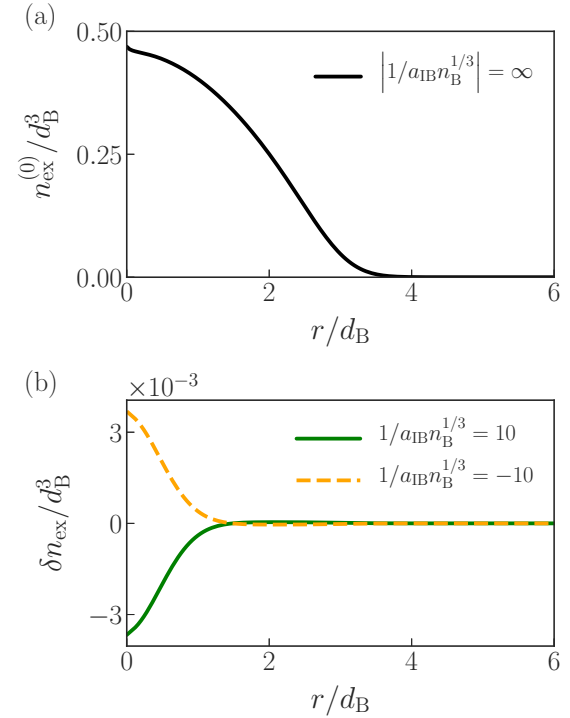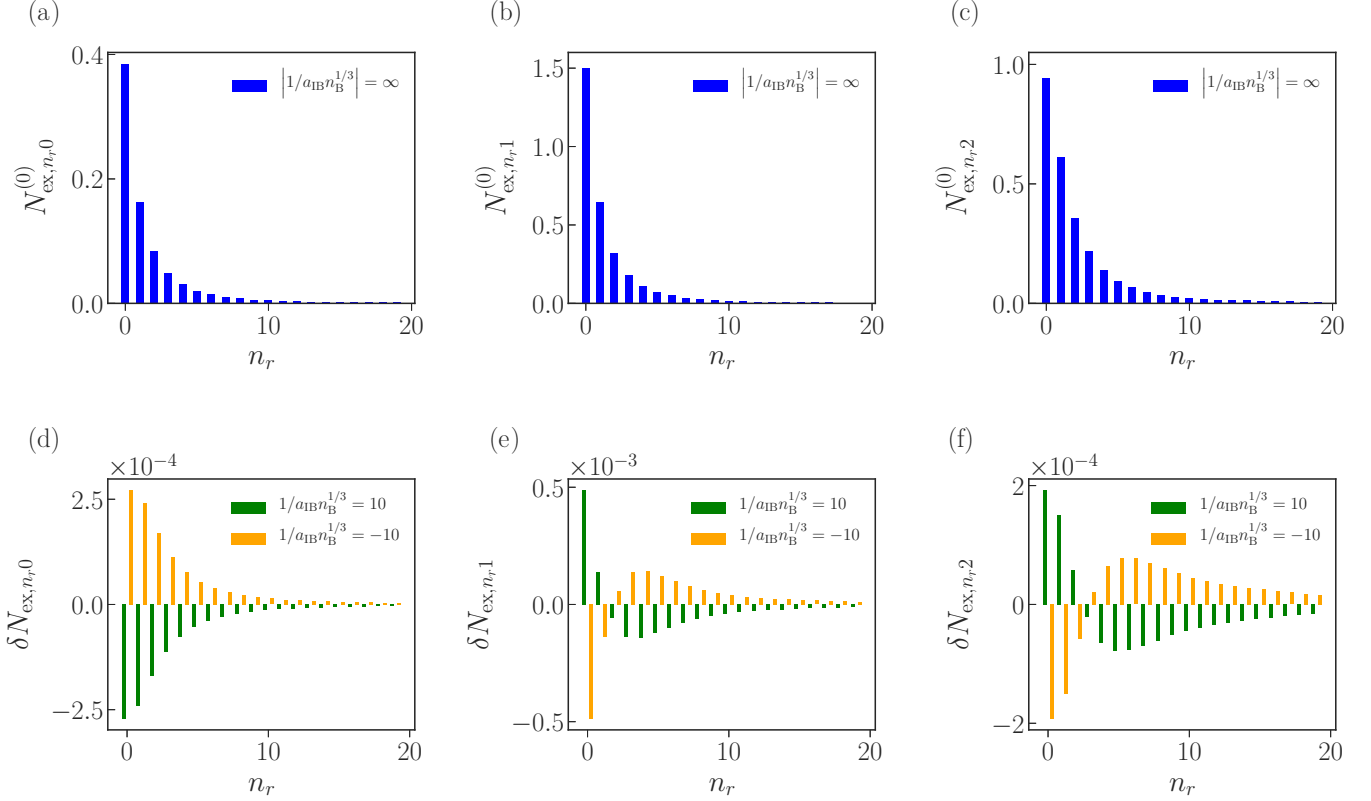
FIG. 3. Amount of quantum depletion $N_{\text{ex},n_r\ell} = \int_{\mathbf{r}} n_{\text{ex},n_r\ell}(r)$ obtained as a function of n_r for (a) and (d) $\ell = 0$, (b) and (e) $\ell = 1$, and (c) and (f) $\ell = 2$. The impurity-free cases are depicted in (a)–(c), while the impurity-induced changes are in (d)–(f).

It should be noted that the behavior of the hole excitation function $\mathcal{V}_{n_r,\ell}(r)$ is determined mainly from the effective potential $V_{\text{eff},\ell}(r) := V_B(r) + \frac{\hbar^2 \ell(\ell+1)}{2r^2} + g_{\text{IB}} \frac{\psi^2(r)}{4\pi} + g_B N_0 \frac{\Phi^2(r)}{2\pi} - \mu$ in the diagonal part of Eq. (14), $\mathcal{L}_\ell = -\frac{\hbar^2}{2m_B}(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}) + V_{\text{eff},\ell}(r)$. We show in Fig. 5 the difference of the effective potential in the presence of an impurity from that in its absence: $\delta V_{\text{eff},\ell}(r) := V_{\text{eff},\ell}(r) - V_{\text{eff},\ell}^{(0)}$.

Figure 5 shows that the $\delta V_{\text{eff},\ell}(r)$ directly reflects the difference of the radial profile of the condensate [see Fig. 1(b)] and changes the sign at $r/d_B \sim 1.7$. When the peak of $\mathcal{V}_{n_r,\ell}(r)$ comes near this point, the sign of the quantum depletion changes. In the case of $\ell = 0$ in which no centrifugal potential occurs, the dip bottom of $\mathcal{V}_{n_r\ell}(r)$ appears at the center of the effective potential, as shown in Fig. 4, and the presence of the impurity gives a small isotropic change to the potential around the center, which thus leads to negative or positive $\delta n_{\text{ex},n_r0}(r)$ therein. On the other hand, when $\ell \neq 0$, the presence of the centrifugal potential allows the dip bottom of $\mathcal{V}_{n_r\ell\neq0}(r)$ to move away from $r = 0$ and, for sufficiently small n_r (corresponding to a sufficiently high hole excitation level $-E_i$), to enter the regime where the impurity-induced change in the density of the condensate is opposite in sign, while $\mathcal{V}_{n_r\ell\neq0}(0) = 0$ is required for regularization. Consequently, the change in the depletion for $\ell \neq 0$ appears inevitably away from the center but within the influence of the impurity's wave function, which can be summarized in terms of real-space densities at $z = 0$ in Figs. 6 and 7. For these reasons, the

depletion of the condensate depends strongly on the excitation energy as well as the ℓ value.

We note that the depletion for $\ell = 0$ is relatively small in comparison with that for $\ell = 1, 2$. Since the integration (21) includes a Jacobian proportional to r^2, the contributions from the amplitude of $\mathcal{V}_{n_r,\ell=0}(r)$ dipped around $r = 0$ are suppressed, while those from the amplitudes of $\mathcal{V}_{n_r,\ell\neq0}(r)$ around the dip bottoms remain unsuppressed. This is why the depletion for $\ell = 0$ is smaller than $\ell = 1, 2$. This situation is the same as the problems of quantum mechanics in harmonic oscillators.

IV. SUMMARY

We have studied the quantum depletion of the condensate in the trapped Bose-polaron system and found that the net change of the depletion, $\delta N_{\text{ex}} = \sum_{n_r,\ell} \int_{\mathbf{r}} \delta n_{\text{ex},n_r\ell}(r)$, induced by the impurity can be negative (positive) for the repulsive (attractive) impurity-boson interaction. These qualitative results can be understood as follows. (i) We have solved a set of coupled equations for the condensate, the impurity's wave function, and Bogoliubov excitations. (ii) Among them, the first two are apparently coupled with each other, but the condensate alone is affected by the other because we have ignored the kickback effect on the impurity by assuming that the impurity is tightly bounded in the trap. (iii) Then the resultant condensate, together with the impurity's wave
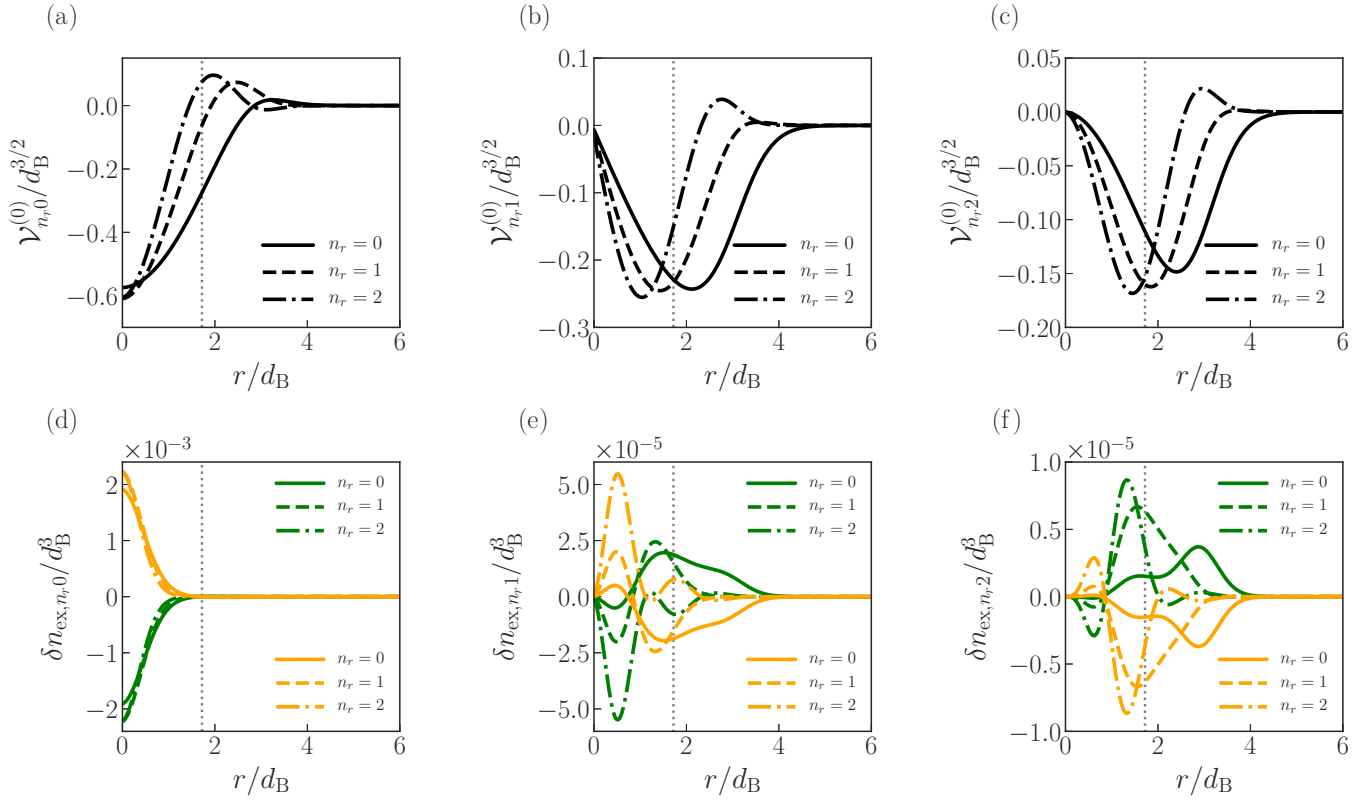
FIG. 4. (a)–(c) Radial profile of the BdG eigenfunctions $\mathcal{V}_{n_r \ell}(r)$ in the absence of the impurity and (d)–(f) the corresponding change in the density of the quantum depletion due to the presence of the impurity for (a) and (d) $\ell = 0$, (b) and (e) $\ell = 1$, and (c) and (f) $\ell = 2$. In (d)–(f) the green (dark gray) and the orange (light gray) lines denote $1/a_{\mathrm{IB}} n_{\mathrm{B}}^{1/3} = +10$ and -10, respectively. The vertical dotted lines are drawn in the same way as in Fig. 5.

function, gives a small dip (bump) to the effective potential in the BdG equation (see Fig. 5), which determines the spectra of excitations, i.e., the depletion.

The negative change of the depletion that we have found in the case of the repulsive impurity-boson interaction in this study seems counterintuitive, since in general the impurity-boson interaction, whether repulsive or attractive, disturbs the condensation in uniform systems. Thus, we consider this result as a specific feature in trapped systems. Moreover, this result does not depend on the shape of trap potentials and can be observed, for example, in a box trap potential.
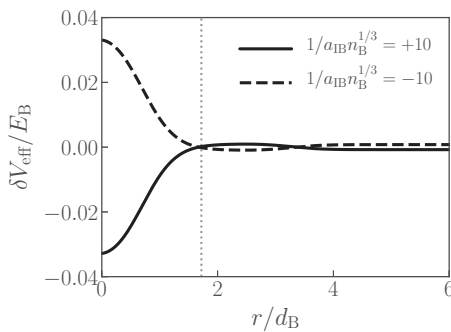

FIG. 5. Difference of the radial profile of the effective potential nondimensionalized by $E_{\mathrm{B}} = \frac{\hbar \omega_{\mathrm{B}}}{2}$. The vertical dotted line denotes the point where the sign of δV_{eff} changes.
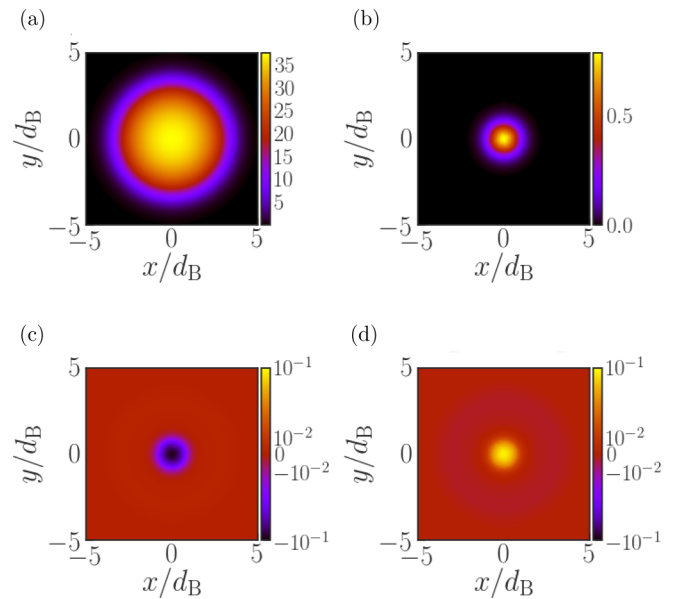

FIG. 6. Real-space profiles at $z = 0$ of (a) the order parameter $\phi_{\mathrm{B}}(x, y, z = 0)$ in the absence of the impurity ($\phi^{(0)}/d_{\mathrm{B}}^{3/2}$), (b) the impurity's wave function $\psi_{\mathrm{I}}(x, y, z = 0)$ ($\psi/d_{\mathrm{B}}^{3/2}$), and the differences of the order parameter $\delta\phi_{\mathrm{B}}(x, y, z = 0)$ in the presence of the impurity with (c) repulsive [$\delta\phi/d_{\mathrm{B}}^{3/2}(1/a_{\mathrm{IB}} n_{\mathrm{B}}^{1/3} = +10)$] and (d) attractive [$\delta\phi/d_{\mathrm{B}}^{3/2}(1/a_{\mathrm{IB}} n_{\mathrm{B}}^{1/3} = -10)$] boson-impurity interactions.
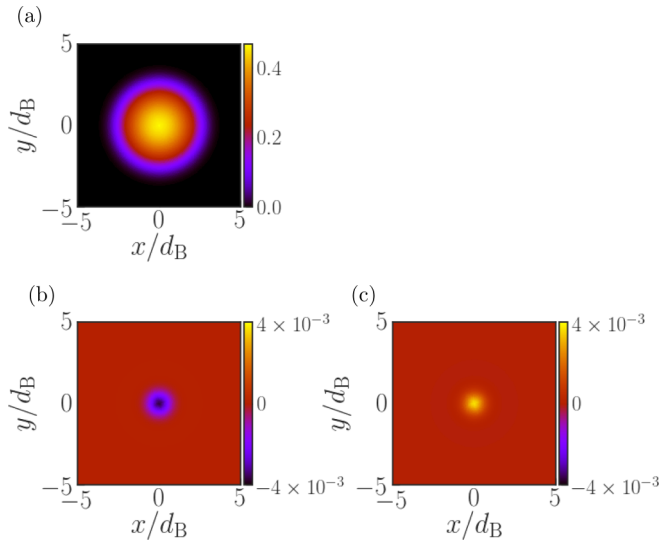
FIG. 7. Real-space profiles at $z = 0$ of (a) the density of depletion in the absence of the impurity $n_{ex}(x, y, z = 0)$ $(n_{ex}^{(0)}/d_B^3)$ and the differences of the density of depletion $\delta n_{ex}(x, y, z = 0)$ in the presence of the impurity with (b) repulsive $[\delta n_{ex}/d_B^3(1/a_{IB}n_B^{1/3} = +10)]$ and (c) attractive $[\delta n_{ex}/d_B^3(1/a_{IB}n_B^{1/3} = -10)]$ boson-impurity interactions.

Finally, we comment briefly on possible experimental observations of the results found in this study. Although the single-impurity-induced change in the amount of depletion is minuscule, i.e., $|\delta N_{ex}| \sim 10^{-3}$ for the weak coupling considered here, we expect that such a change in the amount of depletion is detectable for a dilute impurity gas even in weak coupling and more easily in strong coupling. For instance, *in situ* experiments [91–96] could be used to obtain images of bosonic excitations, while the photoabsorption spectroscopy by lasers [97,98] could select the azimuthal quantum number ℓ in the excitation number $N_{ex,n_r\ell}$. Nevertheless, these experimental observations seem challenging because of the smallness of the depletion. To enhance the impurity-induced change in the quantum depletion, we proposed a method that uses bosonic impurities. With this setting, the amplitude of the impurity wave function is increased; therefore, the impurity-induced change in the amount of quantum depletion can be enhanced. For studies of such situations, the framework of our theory can be progressively extended to deal with the system with many impurities, incorporating the many-body effects of impurities, e.g., the effective impurity-impurity interactions mediated by medium fluctuations [18,82]. The present method can be further developed in other directions of research to look into, e.g., zero-mode physics [99–101], few-body physics, and self-localization.

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