Finite-duration interaction quench in dilute attractively interacting Fermi gases: Emergence of preformed pairs

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We investigate the nonequilibrium behavior of dilute, attractively interacting Fermi gases subjected to finite-duration ramps of their internal interaction strength. We identify three dynamical regimes as a function of ramp duration using a time-dependent version of the Bardeen-Cooper-Schrieffer theory of superconductivity to model these systems. For short ramp durations, these systems become nonsuperconducting; however, fermions with opposite momenta remain paired albeit with reduced amplitudes, and the associated pair amplitude distribution is nonthermal. In this first regime, the disappearance of superconductivity is due to the loss of phase coherence between pairs. By contrast, for intermediate ramp durations, superconductivity survives but the magnitude of the order parameter is reduced and presents long-lived oscillations. Finally, for long ramp durations, phase coherence is almost fully retained during the finite-duration interaction quench, and the steady state is characterized by a thermal-like pair amplitude distribution. Using this approach, one can therefore dynamically tune the coherence between pairs to control the magnitude of the superconducting order parameter and even engineer a nonequilibrium state made of preformed pairs.

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I. INTRODUCTION

The properties of quantum materials are extremely sensitive to external stimuli. In these systems, the interactions associated with, for example, the spin, charge, lattice, and orbital degrees of freedom are often similar in magnitude with the electronic kinetic energy. The delicate balance between competing states can therefore be readily altered via external perturbations leading to the emergence of novel properties. Taking advantage of this distinctive characteristic of quantum materials and building on the tremendous technical progress achieved in the last decade, scientists can now dynamically engineer complex states and follow their nonequilibrium evolution. Phase transitions were photoinduced in strongly interacting solid-state compounds using ultrafast optical pulses [1–4], and similar achievements were also reported in ultracold atomic systems using time-dependent electromagnetic fields [5,6]. For example, in a striped-order cuprate, a Josephson plasmon, a hallmark of the superconducting state, was activated above the critical temperature by the application of midinfrared femtosecond pulses [7]. While these results are truly remarkable, the mechanisms underlying the nonequilibrium dynamics of strongly correlated matter are still being investigated.

Identifying the processes governing the evolution of order parameters when interactions are tuned over time remains an open question. As order parameters are global quantities often made up from sums of local or quasilocal (in position or momentum space) expectation values, one would like to understand how the time-dependent behavior of these different local components conspires to control the dynamics of the global order parameter. Fermionic systems described within the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity constitute an interesting example as in these the collective order parameter is built out of individual Cooper pair states labeled by their internal momentum. A similar situation also arises in magnets where the magnetization is a sum over all local spins.

Going back to our first example, superconductivity, in this case understanding the dynamics arising from the subtle interplay between the BCS collective mode and its constituting elements following a sudden quench of the pairing strength has been the focus of various works [8–20]. The renewed interest for this problem has been triggered, in part, by the possibility in dilute fermionic gases cooled below degeneracy to control the interaction strength using Feshbach resonances [5]. For the case of a sudden quench, using both analytical and numerical methods, three different dynamical regimes were unveiled in a space spanned by the ratio of the equilibrium superconducting order parameters of the initial to final states. When this ratio is sufficiently small, the order parameter oscillates without damping, while for intermediate ratios, it is damped, exhibits decaying oscillations and saturates at an asymptotic value. In contrast, for larger ratios, the order parameter is overdamped following the sudden interaction quench and ultimately superconductivity is destroyed.

Experimentally, quenches are typically realized within a finite ramp duration. This can be achieved, for example, by exciting phononic modes in solids [21] or in cold gases by ramping a magnetic field. We study in this article the dynamics of the BCS order parameter when the interaction between fermions is changed in time following a finite-duration schedule amenable to cold atom setups. Within BCS theory, we focus on the dynamical region where a sudden change of the interaction strength would have obliterated the superconducting order parameter. In contrast to the sudden quench case, we find, as shown in Fig. 1, that for such quenches three different dynamical regimes emerge as a function of the ramp

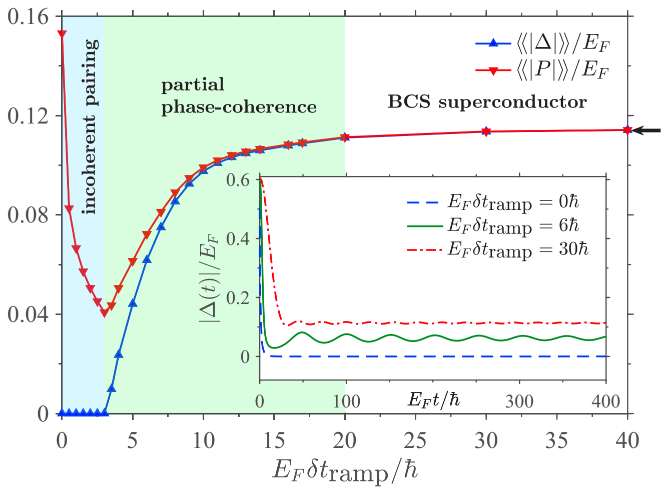

FIG. 1. Time-averaged value of the superconducting order parameter $\langle\langle|\Delta|\rangle\rangle$ and of the sum over the magnitude of the pair amplitudes $\langle\langle|P|\rangle\rangle$ as a function of ramp duration. The interaction strength is ramped down from $1/(k_F a) = -0.1072$ ($|\Delta_{0,i}| = 0.60E_F$) to $1/(k_F a) = -1.3493$ ($|\Delta_{0,f}| = 0.11E_F$), and the time-average is taken between $100\hbar/E_F$ and $400\hbar/E_F$. These two quantities signal three different dynamical regimes (highlighted by shaded regions). For short quenches, the system is characterized by preformed pairs (incoherent pairing state). For intermediate quench durations, superconductivity is maintained but with only partial phase coherence; while, for longer ramp durations, phase coherence is mostly unaffected and the order parameter asymptotes to $\Delta_{0,f}$ (value marked by the arrow). The boundary between the partial phase coherence and BCS superconductor regimes is located at $|\langle\langle|\Delta|\rangle\rangle - \langle\langle|P|\rangle\rangle| \sim 10^{-4}E_F$. Inset: Evolution of the order parameter as a function of time for three different ramp durations.

duration. In particular, for short ramp durations, a state made of incoherent preformed pairs, the so-called phase-disordered superconductor [22], is stabilized. In the remainder of this article, we detail further the three regimes and the subtle mechanism responsible for their dynamics.

II. TIME-DEPENDENT BCS THEORY

We consider a situation applicable to both solid-state systems and cold atom gases: a three-dimensional gas made of two species of fermions described by the BCS s-wave Hamiltonian

$$H_{\mathrm{BCS}} = \sum_{\mathbf{k},\sigma=\{1,2\}} \epsilon_{\mathbf{k}}\, n_{\mathbf{k},\sigma} + \sum_{\mathbf{k}} [\Delta\, c^{\dagger}_{\mathbf{k},1} c^{\dagger}_{-\mathbf{k},2} + \mathrm{H.c.}], \quad (1)$$

where $c^{(\dagger)}_{\mathbf{k},\sigma}$ are the fermionic annihilation (creation) operators, $n_{\mathbf{k},\sigma}$ is the particle number operator of momentum $\mathbf{k}$ and species $\sigma = \{1, 2\}$, and $\epsilon_{\mathbf{k}} = \hbar^2\mathbf{k}^2/(2m)$ is the single-particle dispersion. The superconducting order parameter enters this Hamiltonian as

$$\Delta = \frac{g}{V}\sum_{\mathbf{k}} P_{\mathbf{k}}, \quad (2)$$

with V the system volume and g the interaction strength. Here, the expectation value $P_{\mathbf{k}} = \langle c_{-\mathbf{k},2}c_{\mathbf{k},1}\rangle$ relates to individual Cooper pairs. Additionally, to assess the individual pairing strength, we introduce a second quantity corresponding to the sum over the magnitude of the momentum-dependent pair amplitudes

$$P = \frac{g}{V}\sum_{\mathbf{k}} |P_{\mathbf{k}}|. \quad (3)$$

For ultracold gases, the strength of the interaction between the fermions of two different hyperfine states can be tuned via Feshbach resonances [5], and at sufficiently low temperatures the s-wave scattering is the dominant contribution. In this situation, the interaction can be parametrized by a single parameter, the s-wave scattering length a, via $\frac{1}{k_F a} = \frac{8\pi E_F}{g k_F^3} + \frac{2}{\pi}\sqrt{\frac{E_C}{E_F}}$ with k_F and E_F the Fermi momentum and energy, respectively, and E_C a suitably chosen energy cutoff.

The equilibrium phase diagram for a system described by this Hamiltonian has been thoroughly studied (see [23] and references therein). For $\frac{1}{k_F a} < 0$, the interaction is attractive and below the critical temperature, T_c, this system arranges into a superfluid of Cooper pairs. In this situation, the value of the superconducting order parameter decreases with increasing temperature as the thermal generation of single-particle excitations leads to the breaking of Cooper pairs.

Here, we consider finite-duration interaction changes of the parameter $\frac{1}{k_F a} < 0$ using the schedule

$$\frac{1}{k_F a(t)} = \frac{1}{k_F a(t_f)} + h(t, t_i, t_f)\left[1 - \sin^2\left(\frac{\pi}{2}\frac{t - t_i}{\delta t_{\mathrm{ramp}}}\right)\right],$$

where $h(t, t_i, t_f) = \Theta(t_f - t)[1/(k_F a(t_i)) - 1/(k_F a(t_f))]$, $\delta t_{\mathrm{ramp}} = t_f - t_i$ with t_i and t_f the times at which the interaction ramp begins and ends, and Θ is the Heaviside function. $\delta t_{\mathrm{ramp}} \to 0$ corresponds to a sudden interaction quench while $\delta t_{\mathrm{ramp}} \to \infty$ would correspond to an adiabatic interaction change. The sinusoidal nature of the ramp protocol allows for a smooth (differentiable) change of the interaction strength, thereby avoiding additional excitations at the beginning and end of the ramp. We focus on the situation where the interaction strength is ramped down such that the initial equilibrium value of the superconducting order parameter, $\Delta_{0,i}$, is larger than the final equilibrium value, $\Delta_{0,f}$.

To understand the dynamics of the order parameter, we obtain a set of coupled differential equations connecting the superconducting order parameter to the expectation values of individual pairs and atom densities:

$$\begin{aligned}\hbar\frac{\partial}{\partial t}\langle c_{-\mathbf{k},2}c_{\mathbf{k},1}\rangle &= i\{-2\epsilon_{\mathbf{k}}\langle c_{-\mathbf{k},2}c_{\mathbf{k},1}\rangle + \Delta(\langle n_{\mathbf{k},1}\rangle \\ &\quad + \langle n_{-\mathbf{k},2}\rangle - 1)\},\\ \hbar\frac{\partial}{\partial t}\langle n_{\mathbf{k},1}\rangle &= -2\,\mathrm{Im}\{\Delta^*\langle c_{-\mathbf{k},2}c_{\mathbf{k},1}\rangle\},\\ \hbar\frac{\partial}{\partial t}\langle n_{-\mathbf{k},2}\rangle &= -2\,\mathrm{Im}\{\Delta^*\langle c_{-\mathbf{k},2}c_{\mathbf{k},1}\rangle\}. \end{aligned} \quad (4)$$

Solving numerically this set of equations together with the self-consistency condition for Δ, Eq. (2), we compare and contrast the nonequilibrium evolution due to a sudden interaction change, $\delta t_{\mathrm{ramp}} \to 0$, to ones due to longer ramp durations.

III. EMERGENCE OF THREE DISTINCT DYNAMICAL REGIMES

The main results are summarized in Fig. 1 comparing the time-averaged superconducting order parameter $\langle\langle|\Delta|\rangle\rangle$ and the time-averaged sum over the magnitude of the pair amplitudes $\langle\langle|P|\rangle\rangle$. Here, the time average of a time-dependent quantity $f(t)$ is given by

$$\langle\langle f(t)\rangle\rangle = \frac{1}{\delta t}\int_{t_0}^{t_0+\delta t} dt\, f(t), \tag{5}$$

where $E_F t_0 = 100\hbar$ is the beginning of the averaging window, chosen well after the end of the ramp, and $E_F \delta t = 300\hbar$ its duration, taken suitably long to average over several oscillations in the postramp state.

For a sudden interaction change and for ramp times up to $3\hbar/E_{\mathrm{F}}$, the superconducting order parameter $\langle\langle|\Delta|\rangle\rangle$ is found to average to zero whereas, for slower interaction ramps, this order parameter retains a finite value. The precise ramp duration at which the crossover occurs depends on the interaction strength and the cutoff. In contrast, we find that the time-averaged sum over the magnitude of the pair amplitudes $\langle\langle|P|\rangle\rangle$ remains finite for all ramp times. This important difference in behavior between these two quantities signals that the evolution of the relative phase between individual pairs plays a crucial role in the dynamics. As the amplitude of pairs is reduced but remains finite, the destruction of superconductivity for short ramp durations is associated with the loss of phase coherence between pairs. Therefore, phase unlocking is the main mechanism responsible for the suppression of superconductivity.

This is in stark contrast to the finite-temperature equilibrium scenario where superconductivity is suppressed by an increase in thermal fluctuations resulting in pair breaking. Interestingly, this result implies that stabilizing a state made of preformed pairs is possible via a fast ramp. Within the scope of the BCS model, this state is long-lived. However, in experimental realizations, various mechanisms, such as electron-phonon coupling in solids, could likely affect the long-time stability of this state such that it would only remain stable up to intermediate timescales.

Unexpectedly, $\langle\langle|P|\rangle\rangle$, the sum over the magnitude of the pair amplitudes, is nonmonotonous as a function of ramp duration: it first decreases with increasing ramp durations and then increases again. For a sudden quench, this quantity has a large value due to the freezing of the initial state which is then projected onto the new Cooper pairs. Within this new basis, this frozen state contains excited quasiparticle pairs which contribute to the sum over the magnitude of pair amplitudes. As these quasiparticle pairs are not coherent, their contributions to the total order parameter dephase after a short time resulting in the suppression of the superconducting order parameter. For short but finite ramp durations, the same mechanism persists until the sum over the magnitude of the pair amplitudes reaches a minimum. For longer ramp durations, this quantity rises again and the dephasing becomes less important such that the value of the superconducting order parameter is finite at longer times.

In the following, we analyze carefully the behavior of the excitations responsible for the emergence of the

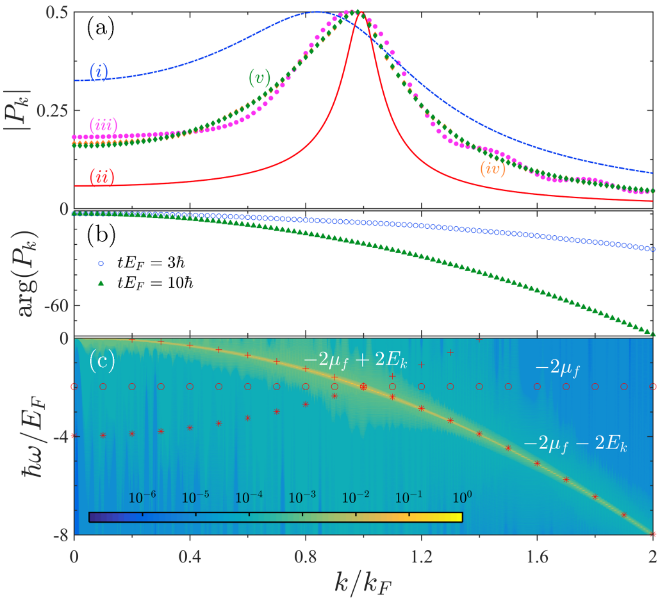

FIG. 2. Sudden quench of the interaction strength from $1/(k_{\mathrm{F}}a) = -0.1072$ to $1/(k_{\mathrm{F}}a) = -1.3493$. (a) Distribution of the magnitude of the pair amplitude as a function of momentum: (i, blue) ground state at $1/(k_{\mathrm{F}}a) = -0.1072$; (ii, red) ground state at $1/(k_{\mathrm{F}}a) = -1.3493$; (iii, pink) snapshot at $3\hbar/E_{\mathrm{F}}$; (iv, orange) snapshot at $10\hbar/E_{\mathrm{F}}$; and (v, green) time-average between $100\hbar/E_{\mathrm{F}}$ and $400\hbar/E_{\mathrm{F}}$. This distribution is already hardly distinguishable form its steady-state configuration at $10\hbar/E_{\mathrm{F}}$. It is nonthermal and signals the presence of preformed pairs. Note that lines (iv) and (v) are on top of each other. (b) Phase of the pair amplitude as a function of momentum. Rapid phase unlocking is responsible for the destruction of superconductivity. Note: The legend is in descending order. (c) Fourier transform of the momentum-dependent pair amplitude $|\mathcal{F}[\mathrm{Im}[P_k(t)]]|$. The sudden quench generates quasiparticle pair excitations along the parabolic line $\pm 2E_{\mathbf{k}} - 2\mu_{\mathrm{f}}$ (marked by the red crosses and stars, respectively; red circles mark the coherent evolution at $-2\mu_{\mathrm{f}}$).

nonequilibrium states detailed above. Useful information can be obtained by analyzing the momentum distribution of the Cooper pair amplitudes. As displayed in Figs. 2(a), 3(a), and 4(a), initially the distribution of pair amplitudes follows the zero temperature and interaction-dependent expression $P_{\mathbf{k}} = \frac{1}{2}\sqrt{1 - \xi_{\mathbf{k}}^2/E_{\mathbf{k}}^2}$ with $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu_{\mathrm{i}}$ where μ_{i} is the chemical potential at the initial interaction strength. For $\frac{1}{k_{\mathrm{F}}a} = -0.1072$, this distribution (see curve (i)) has a maximum close to the Fermi momentum and drops down for larger momentum values. In contrast, the distribution corresponding to the final ground state of an adiabatic quench to $\frac{1}{k_{\mathrm{F}}a} = -1.3493$ [see curve (ii)] is strongly peaked around the Fermi momentum and is much lower in value than the initial distribution.

To understand the time evolution of the distribution of pair amplitudes, we consider both snapshots of $|P_{\mathbf{k}}|$ at particular times and values denoted by $\langle\langle|P_{\mathbf{k}}|\rangle\rangle$ that are time-averaged between $100\hbar/E_{\mathrm{F}}$ and $400\hbar/E_{\mathrm{F}}$. As shown in Fig. 2, for a sudden ramp, the magnitudes of the pair amplitudes settle quickly as the snapshot distribution at $E_{\mathrm{F}}t = 3\hbar$ already agrees approximately with the one obtained via time-averaging. However, even at long times, this distribution takes much larger values

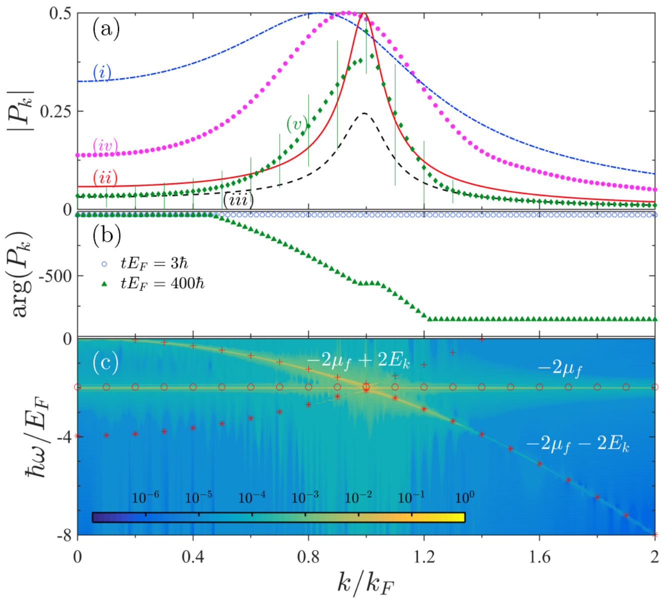

FIG. 3. Slow quench performed in $E_\mathrm{F}\delta t_\mathrm{ramp} = 6\hbar$ (same interaction strengths as in Fig. 2). (a) Distribution of the magnitude of the pair amplitude as a function of momentum: (i, blue) and (ii, red) same as in Fig. 2; (iii, black dashed) thermal distribution at $1/(k_\mathrm{F}a) = -1.3493$, $T = 0.89T_c$ is chosen such that $\langle\langle|\Delta|\rangle\rangle = \Delta(T)$; (iv, pink) snapshot at $3\hbar/E_\mathrm{F}$; (v, green) time-average between $100\hbar/E_\mathrm{F}$ and $400\hbar/E_\mathrm{F}$. The time-averaged distribution is nonthermal, and $|P_k|$ exhibit strong oscillations represented, together with (v), by vertical bars (peak-to-peak amplitude of the oscillations). (b) Phase of the pair amplitude as a function of momentum. Note: The legend is in descending order. (c) Fourier transform of the momentum-dependent pair amplitude $|\mathcal{F}[\mathrm{Im}[P_\mathbf{k}(t)]]|$. While this quench generates quasiparticle pair excitations, all $P_\mathbf{k}$ signals have a strong in-phase component at $-2\mu_\mathrm{f}$.

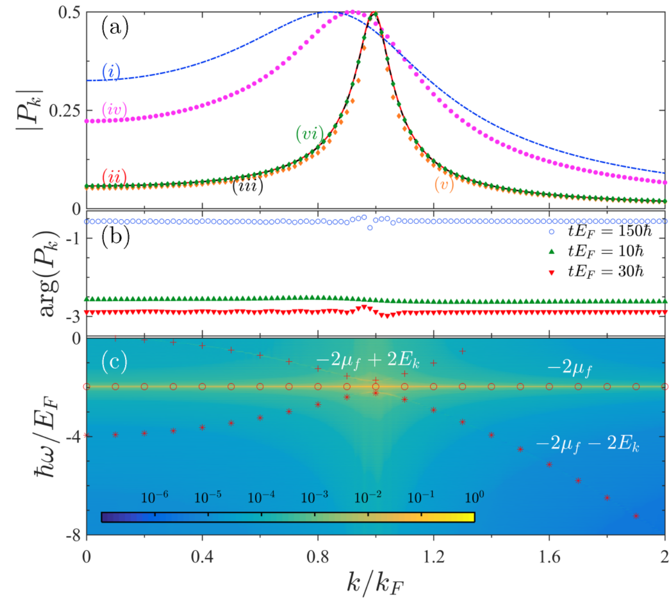

FIG. 4. Slow quench performed in $E_\mathrm{F}\delta t_\mathrm{ramp} = 30\hbar$ (same interaction strengths as in Fig. 2). (a) Distribution of the magnitude of the pair amplitude as a function of momentum: (i, blue) and (ii, red) same as in Fig. 2; (iii, black dashed) thermal distribution at $1/(k_\mathrm{F}a) = -1.3493$, $T = 0.35T_c$ is chosen such that $\langle\langle|\Delta|\rangle\rangle = \Delta(T)$; (iv, pink) snapshot at $10\hbar/E_\mathrm{F}$; (v, orange) snapshot at $30\hbar/E_\mathrm{F}$; (vi, green) time-average between $100\hbar/E_\mathrm{F}$ and $400\hbar/E_\mathrm{F}$. The steady-state distribution is thermal. Note that lines (ii), (iii), (v), (vi) are on top of each other. (b) Phase of the pair amplitude as a function of momentum. Phase coherence is only slightly lost near k_F. Note: The legend is in descending order. (c) Fourier transform of the momentum-dependent pair amplitude $|\mathcal{F}[\mathrm{Im}[P_\mathbf{k}(t)]]|$. Quasiparticle pairs are only generated near k_F (red crosses and stars). P_k are dominated by the coherent phase evolution at $-2\mu_\mathrm{f}$.

than the ones expected in equilibrium at the final interaction strength $\frac{1}{k_\mathrm{F}a} = -1.3493$. This result explains the large finite value of the sum over the magnitude of the pair amplitudes presented in Fig. 1 signaling that Cooper pairs survive through the quench (even though with a smaller amplitude than in the initial state).

As shown in the central panel of Fig. 2, for the sudden ramp, each pair rapidly acquires a particular phase proportional to $2E_\mathbf{k}$ leading to complete dephasing such that already at $E_\mathrm{F}t = 10\hbar$ the superconducting order parameter is totally obliterated.

The evolution of the phases can be understood via the Fourier transform of the pair amplitudes, $|\mathcal{F}[\mathrm{Im}[P_\mathbf{k}(t)]]|$, and as $\mathrm{Im}[P_\mathbf{k}(t)]$ and $\mathrm{Re}[P_\mathbf{k}(t)]$ provide the same information about the phase evolution, we only consider the former without loss of generality. From this quantity, we see that the sudden quench generates quasiparticle pairs at $-2\mu_\mathrm{f} \pm 2E_\mathbf{k}$ (see lower panel of Fig. 2) with μ_f the chemical potential at the final interaction strength. These quasiparticle pairs are at the origin of the parabolic distribution of the phases (central panel). This result indicates that the system dynamically organizes into a nonthermal state made of preformed but dephased Cooper pairs.

The pair amplitude distributions are also nontrivially affected when the interaction strength is slowly ramped down. For the ramp duration $E_\mathrm{F}\delta t_\mathrm{ramp} = 6\hbar$, both the snapshot and time-averaged distributions are clearly finite and nonthermal. Only the small and large momentum tails of the time-averaged distribution agree with the thermal equilibrium distribution at $\frac{1}{k_\mathrm{F}a} = -1.3493$. The temperature T used in Fig. 3 is found by solving the finite-temperature gap equation [24] assuming that $\langle\langle|\Delta|\rangle\rangle = \Delta(T)$.

As we see in the lower panel of Fig. 3, the $E_\mathrm{F}\delta t_\mathrm{ramp} = 6\hbar$ ramp creates fewer quasiparticle pairs at $-2\mu_\mathrm{f} \pm 2E_\mathbf{k}$ and all $P_\mathbf{k}$ signals have a strong component at $-2\mu_\mathrm{f}$. At short times compared to the ramp duration, the phase remains fully locked, then as the evolution goes on, in the momentum interval where most of the quasiparticle pairs are generated, each Cooper pair begins accumulating a particular phase. This process leads to a partial loss of phase coherence, but, as shown in Fig. 1, the Cooper pairs are still sufficiently synchronized for superconductivity to survive at the considered times.

Finally, for $E_\mathrm{F}\delta t_\mathrm{ramp} \geqslant 20\hbar$, we find that the dynamics enters a different regime as the distribution of pair amplitudes becomes thermal. As one sees from Fig. 4, for $E_\mathrm{F}\delta t_\mathrm{ramp} = 30\hbar$ the distribution obtained at the end of the ramp strongly resembles the one expected for a superconducting system in equilibrium at $T = 0.35T_c$ for an interaction strength of $\frac{1}{k_F a} = -1.3493$. For this ramp schedule, quasiparticle pairs are solely generated in a small momentum region around k_F (see lower panel of Fig. 4). The phase coherence remains for the most part undisturbed by the interaction ramps. As illustrated in Fig. 4, during the ramp the phase starts to ripple around k_F, the region where quasiparticle pairs are generated, but phase locking is for the most part maintained throughout the system.

IV. CONCLUSION

To summarize, we analyze the nonequilibrium dynamics of dilute attractively interacting Fermi gases described within BCS theory when their interaction strength is ramped down within a finite duration. We identify three different dynamical regimes and, in particular, we demonstrate the dynamical creation of a steady state of preformed pairs without global phase coherence. The insights gained from this study will likely pave the way to employ slow quenches to create other steady states with novel properties absent in thermal equilibrium.

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