Z_1^3-dependent range contributions*

J. C. Ashley and R. H. Ritchie
Health Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

Werner Brandt
Department of Physics, New York University, New York, New York 10003
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Previously derived comprehensive formulas for the Z_1^3 contributions to the ranges of charged particles in matter are amended to apply at high, nonrelativistic particle velocities.

In recent publications[1-3] formulas were derived for the Z_1^3-dependent contributions to the stopping of charged particles and to their ranges in matter. It is the purpose of this Addendum to correct a statement in the text of Ref. 3 (last sentence of first paragraph on p. 2406) to the effect that a small discrepancy between our range differences of π^+ and π^- mesons in nuclear emulsions and the numerical results reported by Jackson and McCarthy[4] stem from relativistic corrections not included in our theory.

The small discrepancies (of the order of $\leq 15\%$) displayed in Fig. 3 of Ref. 3 near $E_{\pi^+} = 5$ MeV between the curves labeled Eqs. (22) and (23), and the cross marks representing numerical results from Ref. 4, come mainly from the slow decrease of $\kappa(b, x_c)$ with increasing x_c, relative to the mean constant κ_0 with the value $\kappa_0 = 0.32$ appropriate for $b = 1.8$ employed in calculating the curves. It is important for the application to the ranges of energetic particles to incorporate this slow x_c dependence of κ into Eqs. (22) and (23) of Ref. 3 in a simple but accurate manner. The small (by comparison) relativistic contributions at the highest x values can be estimated from the formula given in Ref. 4. We return to the integration of Eq. (17) of Ref. 3 and write κ in the form

$$\kappa(b, x_c) = \kappa_0 + [\kappa(b, x_c) - \kappa_0] \simeq \kappa_0 + a x_c^\gamma . \qquad (1)$$

The small x_c-dependent correction is approximated well over the range of $x > 1$ tabulated in Table I of Ref. 3 by $a x_c^\gamma$, with constants $a < 0$ and $\gamma \simeq 0.7$. On inserting Eq. (1) of this paper into Eq. (17) of Ref. 3, one retrieves Eqs. (22) and (23) of Ref. 3, where now, κ_0 is replaced by a mean value $\bar{\kappa}(x_c)$ given by

$$\bar{\kappa}(x_c) = \kappa_0 + a \int^{x_c} \frac{x'^\gamma dx'}{x' S_0(x')} \Bigg/ \int^{x_c} \frac{dx'}{x' S_0(x')}, \qquad (2)$$

S_0 being the stopping power. We calculate the correction to κ_0 in Eq. (2) with the approximation $S_0 = \eta k^{1/\eta} E_1^{1-1/\eta}$, which corresponds to the relation $R_0 = (E_1/k)^{1/\eta}$, where E_1 is the particle energy, R_0 the particle range, η the range-energy index, and k a constant. Equation (2) with $\gamma = 0.7$ and $\eta = 0.59$ (Table II in Ref. 3) becomes

$$\bar{\kappa}(x_c) = \kappa_0 + \frac{1-\eta}{1-\eta(1-\gamma)} a x_c^\gamma \simeq \frac{\kappa_0 + \kappa(b, x_c)}{2}. \qquad (3)$$

The function $\kappa(b, x)$ is tabulated in Table I of Ref. 3. On replacing κ_0 by the mean value $\bar{\kappa}(x_c)$ as given in Eq. (3) in this paper, the range formulas Eqs. (22) and (23) in Ref. 3 yield range differences for π^+ and π^- mesons in nuclear emulsions in agreement with the computed values given in Ref. 4, and in reasonable agreement with present experimental data.

In conclusion, the range formulas Eqs. (22) and (23) in Ref. 3 with κ_0 replaced by $\frac{1}{2}[\kappa_0 + \kappa(b, x_c)]$ apply to the same wide domain of particle velocities as the stopping-power formula given in Eq. (11) of Ref. 3.

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