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Theory of the thermomagnetic force*

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A theory of the thermomagnetic force in dilute polyatomic gases is presented which, like that for the thermomagnetic torque (Scott effect), depends on a Burnett transport coefficient. The theory is shown to be in good agreement with experiments in NO and N_2.

Recently Larchez and Adair[1] and Taboada[2,3] have observed a magnetic-field-induced change in the radiometer force which a polyatomic gas exerts normal to a disk in the presence of a thermal gradient. In this paper we propose an explanation which, like the Scott effect,[4] depends on a Burnett transport coefficient. The effect therefore requires a nonuniform temperature gradient.

A kinetic theory of the thermomagnetic force was presented by Hess.[5] His theory utilized a direct coupling between the force and $\vec{\nabla}T$. This type of coupling is forbidden because ordinary transport coefficients always couple quantities which have different time-reversal properties.[6] Since $\vec{F}$ and T are both even under time reversal there can be no such coupling (in the bulk).

We assume that the experimental apparatus[1-3] is axially symmetric about the z axis. We must assume that one of the surfaces is not planar in order for the temperature field to possess a nonuniform gradient (i.e., nonzero $\vec{\nabla}\vec{\nabla}T$). This conclusion is consistent with more recent experiments,[2,3] but in disagreement with Hess's theory.

The force normal to the disk is related to the stress tensor by

$$F_z = \tau_{zz} \ . \tag{1}$$

In the presence of a nonzero $\vec{\nabla}\vec{\nabla}T$ the stress tensor is related to the temperature field via a Burnett coefficient[7]

$$\overleftrightarrow{\tau} = \vec{\mathrm{B}} : \vec{\nabla}\vec{\nabla}T \,, \tag{2}$$

where the derivative is evaluated on the disk.

The force on the disk can therefore be written as

$$F = [B_{zzzz} - \tfrac{1}{2}(B_{zzxx} + B_{zzyy})]\frac{\partial^2 T}{\partial z^2} + 2B_{zzxy}\frac{\partial^2 T}{\partial x\,\partial y} + 2B_{zzzx}\frac{\partial^2 T}{\partial z\,\partial x} + 2B_{zzzy}\frac{\partial^2 T}{\partial z\,\partial y} \,, \tag{3}$$

where we have used the fact that the temperature field is an axially symmetric solution of Laplace's equation. Because of the symmetry of the problem we have

$$T(x, y, z) = T(-x, y, z,) = T(x, -y, z). \tag{4}$$

Therefore we have

$$\frac{\partial^2 T(x, y, z)}{\partial x\,\partial y} = -\frac{\partial^2 T(-x, y, z)}{\partial x\,\partial y} = -\frac{\partial^2 T(x, -y, z)}{\partial x\,\partial y} \,, \tag{5a}$$

$$\frac{\partial^2 T(x, y, z)}{\partial x\,\partial y} = \frac{\partial^2 T(y, x, z)}{\partial z\,\partial y} = -\frac{\partial^2 T(x, -y, z)}{\partial z\,\partial y} \ . \tag{5b}$$

Thus, if the disk is axially symmetric, the average over the disk of each cross derivative vanishes. Therefore they will not contribute to the total force.

Thus averaging Eq. (3) over the disk yields

$$F = [B_{zzzz} - \tfrac{1}{2}(B_{zzxx} + B_{zzyy})]\frac{\partial^2 T}{\partial z^2} \ . \tag{6}$$

The Burnett coefficient for a polyatomic gas in the presence of a magnetic field has been calculated by several authors.[7,8] In the notation of Ref. 7 we have for diamagnetic molecules

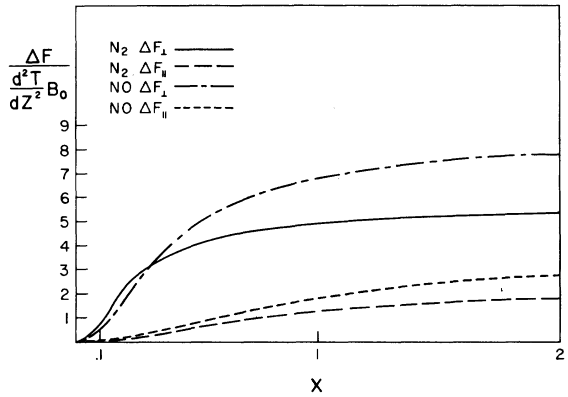

FIG. 1. Plots of $\Delta F_\perp/(\partial^2 T/\partial z^2 B_0)$ and $\Delta F_\parallel/(\partial^2 T/\partial z^2 B_0)$ [cf. Eq. (7)] vs $x = 0.0000707H/P$ using collision parameters appropriate for N_2 and vs $x = \bar{x} = 0.0174H/P$ using collision parameters appropriate for NO.

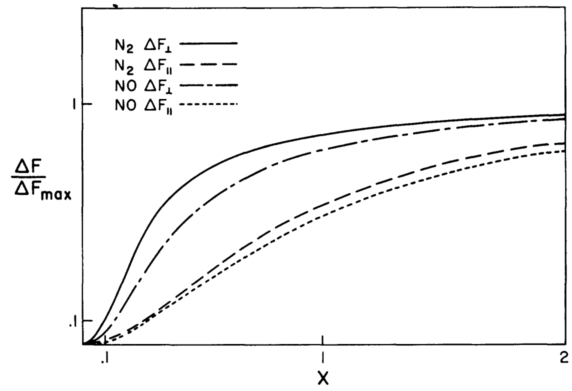

FIG. 2. Plots of $\Delta F_\perp$ and $\Delta F_\parallel$ normalized to their maximum value vs $x = 0.0000707H/P$ using collision parameters appropriate for N_2 and vs $x = \bar{x} = 0.0174H/P$ using collision parameters appropriate for NO.

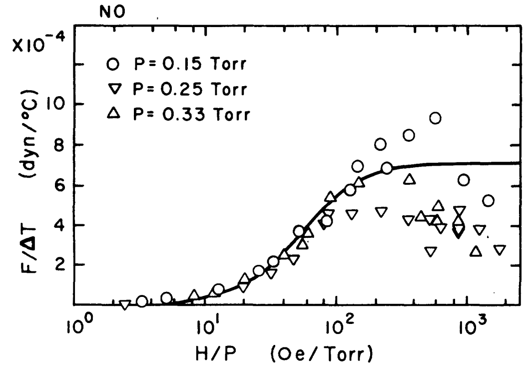

FIG. 4. Comparison of ΔF normalized to the experimental extrapolated maximum value with experimental results of Larchez and Adair (Ref. 1) in NO.

$$\frac{\Delta F_\perp}{(\partial^2 T/\partial z^2)B_0} = -\tfrac{3}{4}c_0 f(2h_s/\Gamma_{22})$$

$$- (\tfrac{3}{4}f_0 + \tfrac{5}{6}d_0) f(2h_s/\Gamma) - \tfrac{7}{12}d_0 f(h_s/\Gamma)\,,$$

$$\frac{\Delta F_\parallel}{(\partial^2 T/\partial z^2)B_0} = -\tfrac{1}{3}d_0[\tfrac{5}{2}f(h_s/\Gamma) + f(2h_s/\Gamma)]\,, \tag{7}$$

where $f(x) = x^2/(1+x^2)$. $\Delta F_\perp (\Delta F_\parallel)$ denotes the change in the force on the disk when the magnetic field is perpendicular (parallel) to the temperature gradient. h_s is the Larmor frequency of the molecules, and Γ and Γ_{22} are collision frequencies which depend on the details of the interaction between two molecules. B_0 denotes the field-free value of the Burnett coefficient.

For nitrogen,[7] $c_0 = 6.85\times10^{-3}$, $d_0 = 1.86\times10^{-3}$, and $f_0 = -2.84\times10^{-3}$. Both $\Delta F_\perp$ and $\Delta F_\parallel$ for nitrogen are plotted vs $x = h_s/\Gamma = 0.0000707H/P$ in Fig. 1. In Fig. 2, $\Delta F_\perp$ and $\Delta F_\parallel$ normalized to their maximum values are plotted to illustrate the difference in their field dependences. In Fig. 3, Taboada's experiments[2] are compared with our calculations of $\Delta F_\parallel$. Since the actual magnitude of the second temperature gradient is not known, we normalize all the data to extrapolated maximum values.

For paramagnetic molecules we must average Eq. (7) over a distribution of magnetic moments.[7] For nitric oxide, however, to a good approximation, we can assume two low-lying states, each with its magnetic moment, populated in the ratio

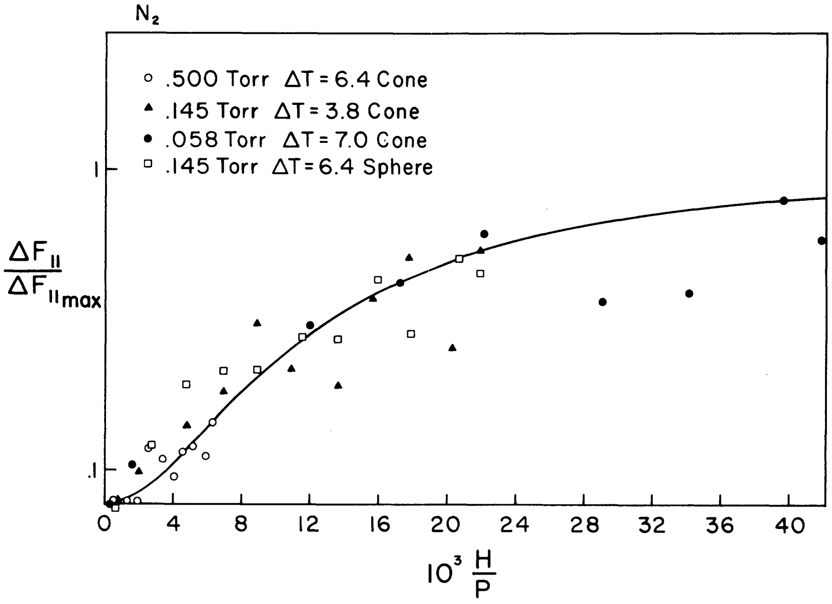

FIG. 3. Comparison of $\Delta F_\parallel$ normalized to its maximum value with experimental results of Taboada (Ref. 2) at several different pressures and geometries for N_2.

of their Boltzmann factors. Then we have[9] at room temperature

$$\langle f(x)\rangle \approx 0.64f(\bar{x})+0.36f(0.74\bar{x}), \qquad (8)$$

where $\bar{x}=0.0174H/P$.

Using[7] $c_0=11.64\times10^{-3}$, $d_0-2.99\times10^{-3}$, and $f_0=-6.04\times10^{-3}$ and Eq. (8), we have plotted $F_\perp$ and $F_\parallel$ for NO vs $\bar{x}$ in Fig. 1. In Fig. 2, $F_\perp$ and $F_\parallel$ normalized to their maximum values are plotted versus $\bar{x}$. In Fig. 4, $\Delta F_\parallel$ for NO is compared with the experiments in Ref. 1. The size of the effect has again been scaled to best fit the data.

In conclusion we observe that the field dependence of the force effect is in agreement with the experiments. The theory has been used by Taboada to obtain good agreement with his more recent experiments.[3]

To obtain the size of the effect we need the magnitude of $\partial^2T/\partial z^2$ evaluated on the disk. This involves the solution of Laplace's equation for complicated boundary conditions and is beyond the scope of this paper.

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