Singularities in the Galitskii-Feynman T matrix

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The Galitskii-Feynman T matrix, which sums the ladder diagrams for both particle-particle and hole-hole scattering in a many-fermion system, is shown to have a singularity at negative values of the c.m. energy of the interacting pair. The singularity appears as a first-order pole in the T matrix, the position of which is a function of both the density of the system and the total c.m. momentum of the pair, and corresponds to the formation of bound pairs in the medium. The singularity structure exists in two distinct Fermi systems (liquid He^3 and nuclear matter), and detailed numerical results are presented using a potential appropriate to liquid He^3. The singularity in this case is shown to be present at all densities above a certain critical density which is lower than the physical density.

I. INTRODUCTION

In any many-body theory calculation which ultimately has its origins in perturbation theory, one has to decide from the outset whether to work in a time-dependent or a time-independent formalism. Translated into the language of diagrammatic perturbation theory the choice made implies, respectively, the use of time-ordered or non-time-ordered diagrams. The first formalism leads to the linked cluster perturbation series of Goldstone,[1] and the associated time-ordered diagrams are known as Goldstone diagrams. The usual starting point for the time-independent theory is the Green's-function formalism of Galitskii and Migdal,[2] which grew from the relativistic quantum field theory techniques initiated principally by Feynman, and the associated non-time-ordered diagrams are usually known as Feynman diagrams. Both formalisms provide equally valid frameworks for calculations and in the context of calculations of the ground-state energy of dilute Fermi systems, the connections between, and the relative advantages of the two methods have been discussed by one of the present authors.[3]

For virtually all physical systems of interest, a straightforward use of perturbation theory in either formalism in powers of the potential is fruitless because convergence would demand a prohibitively large number of terms. Progress is then only made by a rearrangement of the series, in which a partial summation is made of a suitable (and normally infinite) subset of the diagrams. The potential is thereby eliminated in favor of the function which performs the summation, and a reordered perturbation series is obtained in powers of the new function, in which hopefully only a few terms will suffice for convergence. Perhaps the best-known example of this procedure is the summation of the so-called "ladder diagrams" which sum the repeated two-body interactions between a pair of fermions imbedded in the many-body background, to obtain a T matrix. In the time-dependent formalism this procedure leads to the Bethe-Goldstone[4] equation, and in the time-independent formalism to the analog of the (ladder approximation to the) Bethe-Salpeter[5] equation for the respective T matrices appropriate to each method.

This particular procedure of summing the ladder diagrams is (as is well known) necessary for systems wherein the interparticle potential contains strong short-range repulsion, as is the case for nuclear matter and liquid He^3. In particular, following the original work of Brueckner and Gammel[6] on the liquid He^3 system, several extensive calculations[7-9] have been performed, all within the context of the Brueckner-Bethe-Goldstone formalism; a similar situation prevails for nuclear matter and other many-fermion systems. The overall motivation for the present work derives from the fact that the time-independent formalism has received little attention, and we have accordingly embarked on a series of calculations, for these systems, using it. In this context it is natural to initially focus attention upon the T matrix which sums the two-body scattering series (the ladder diagrams) inside the medium, using non-relativistic time-independent perturbation theory, and which we shall henceforth refer to as the Galitskii-Feynman (GF) T matrix. The relationship between the GF T matrix and the corresponding quantity in the Bethe-Goldstone (BG) formalism has been documented elsewhere, and, in particular, calculations have been performed using both for the model case of purely repulsive potentials, including the extreme hard-core limit.[3] The restriction to purely repulsive potentials was necessary to eliminate any complications associated with the possibility of Cooper-pair[10] formation, leading to a superconducting state of lower energy than the normal ground-state. On the other hand, when dealing with real many-fermion systems, one has to handle potentials containing both

short-range repulsion and long-range attraction, and accordingly this work presents some results on the GF T matrix for real physical systems. In particular we present detailed numerical results for a potential appropriate to liquid He^3, and we show that the GF T matrix contains a singularity in the bound-state region. Insofar as the singularity will have consequences for later calculations of physical observables, particular emphasis is placed on its properties and behavior as the relevant parameters are changed.

Our over-all philosophy is twofold, namely, that the detailed results for a physical system like liquid He^3 are interesting in their own right, and secondly that a knowledge of the behavior of the bound-state singularity is necessary for a full calculation of the thermodynamic functions of the system, within the framework of the GF formalism that we outline. In addition, the intermediate states in the diagrams that are summed to give the GF T matrix consist of an indefinite number of pairs of either particles outside or holes inside the Fermi sea. In this way one sums compactly a considerably larger class of the analogous Goldstone diagrams than by solving the Bethe-Goldstone equation for the corresponding T matrix, which sums only repeated scatterings between two particles outside the Fermi sea. The numerical investigation of pair states in a many-body medium is of interest in its own right, and we feel that a comparison between results in the GF and BG formalisms can be used to shed further light on this topic. In particular, the singularity structure found for the GF T matrix with our liquid He^3 potential appears to be absent for the corresponding BG T matrix.

In the next section we outline the formalism of the GF T matrix and its connection to the proper self-energy, from which the self-consistent single-particle energy spectrum and the thermodynamic functions can be obtained. Details of the potentials used for the numerical study and the numerical techniques themselves are presented in Sec. III. In Sec. IV we give the results of the calculations focussing particularly on the singularity structure and its behavior as a function of the total momentum of the scattering pair and the density of the medium in which they are imbedded. The results are summarized in Sec. V and possible consequences of the singularity structure are examined, together with a discussion of the merits of the GF formalism in these cases.

II. T-MATRIX FORMALISM

For scattering via a two-body potential V, the GF T matrix is readily shown[3] to be the solution to an integral equation which is the analog of the Bethe-Salpeter equation[5] in the ladder approximation. In terms of the incoming and outgoing momenta of the interacting pair, $\vec{p}_1, \vec{p}_2$ and $\vec{p}_1', \vec{p}_2'$, respectively, we define relative and c.m. momenta as

$$\vec{p} = \tfrac{1}{2}(\vec{p}_1 - \vec{p}_2), \quad \vec{p}' = \tfrac{1}{2}(\vec{p}_1' - \vec{p}_2'),$$
$$\vec{P} = \tfrac{1}{2}(\vec{p}_1 + \vec{p}_2) = \tfrac{1}{2}(\vec{p}_1' + \vec{p}_2'),$$

and we similarly define $2P_0$ to be the total energy of the pair. We further let s/M be the relative energy of the pair in the c.m. frame,[11]

$$s = M(2P_0 - \vec{P}^2/M) = P_0 - \vec{P}^2,$$

since $\vec{P}^2/M$ is the energy carried by the c.m. itself, where M is the mass of each (identical) fermion. In this representation, the fully off-shell GF T matrix has the form $T(\vec{p}, \vec{p}'; s, \vec{P}) = \langle \vec{p} \mid T(s, \vec{P}) \mid \vec{p}' \rangle$, and written in operator notation in the c.m. frame of the interacting pair, the GF T matrix equation is[3]

$$T(s, \vec{P}) = u - u[Q(\vec{P}) g_0(s) - \overline{Q}(\vec{P}) g_0^{\dagger}(s)]\, T(s, \vec{P}), \tag{1}$$

where $u = M\hbar^{-2}V = \frac{1}{2}V$, and where operator multiplication is defined in the relative momentum representation as

$$\langle \vec{p} \mid AB \mid \vec{p}' \rangle = (2\pi)^{-3} \int d\vec{k} \langle \vec{p} \mid A \mid \vec{k} \rangle \langle \vec{k} \mid B \mid \vec{p}' \rangle .$$

The operator Q ($\overline{Q}$) is equal to unity if both particles are outside (inside) the Fermi sea, and zero otherwise; and $g_0(s) = (\frac{1}{2}H_0 - s - i\eta)^{-1}$ is the free two-body propagator, where H_0 is the relative kinetic energy operator of the pair and η is a positive infinitesimal in the scattering region ($s > 0$) and zero elsewhere. If the operator $\overline{Q}$ in Eq. (1) is replaced by zero, the corresponding form of the Bethe-Goldstone equation is obtained. The propagators g_0 and $g_0^{\dagger}$ associated with the two-particle and two-hole states, respectively, define the corresponding boundary conditions at infinity to be outgoing and incoming waves, as is required by the general formalism of the one-body Feynman Green's function.

With the GF T matrix so defined, one can associate a generalized fully off-shell wave function operator $X(s, \vec{P})$ and a two-body propagator in the medium $g(s, \vec{P})$ by the definitions

$$T(s, \vec{P}) = uX(s, \vec{P}); \quad g(s, \vec{P}) = X(s, \vec{P}) g_0(s). \tag{2}$$

Using these definitions in Eq. (1), it is trivial to show that g has the expected form

$$g^{-1}(s, \vec{P}) = \tfrac{1}{2}H_0 - s + \{Q(\vec{P}) - \overline{Q}(\vec{P})\} u \equiv L(\vec{P}) - s, \tag{3}$$

with the appropriate factors of $\pm i\eta$ inserted on in-

version to take account of the required boundary conditions. It is obvious that a knowledge of any one of T, X, or g suffices to trivially determine the other two. It is useful to introduce the dual sets of wave functions $|\phi_\nu(\vec{P})\rangle$ and $\langle\psi_\nu(\vec{P})|$ which are the right and left eigenfunctions for the operator L defined in Eq. (3),

$$L|\phi_\nu(\vec{P})\rangle = s_\nu(\vec{P})|\phi_\nu(\vec{P})\rangle,$$
$$L^\dagger|\psi_\nu(\vec{P})\rangle = s_\nu^*(\vec{P})|\psi_\nu(\vec{P})\rangle, \tag{4}$$

with the appropriate boundary conditions determined by Eq. (1), and with the eigenvalue $s_\nu(\vec{P})$. The index ν includes both a continuous scattering spectrum (labeled $\nu \to \vec{k}$, $s_\nu \to k^2$) and any possible discrete boundstates (labeled $\nu \to i$, $s_\nu \to s_i < 0$, $i = 0, 1, 2, \ldots$). The dual set of functions form a bi-orthogonal set of eigenfunctions, in terms of which g has the spectral representation

$$g(s,\vec{P}) \sim \sum_\nu \frac{|\phi_\nu(\vec{P})\rangle\langle\psi_\nu(\vec{P})|}{s_\nu(\vec{P}) - s}, \tag{5a}$$

or in the momentum representation

$$g(\vec{p},\vec{p}\,';s,\vec{P}) \sim \int \frac{d\vec{k}}{(2\pi)^3}\frac{\phi_{\vec{k}}(\vec{p},\vec{P})\psi_{\vec{k}}^*(\vec{p}\,',\vec{P})}{k^2 - s} + \sum_i \frac{\phi_i(\vec{p},\vec{P})\psi_i^*(\vec{p}\,',\vec{P})}{s_i(\vec{P}) - s}, \tag{5b}$$

where the symbol ~ indicates equality when the relevant factors of $\pm i\eta$ have been inserted as demanded by the boundary conditions, and after any non-completeness of the eigenfunctions due to any pathological behavior of the potential as discussed in Ref. (12), is taken into account. In the usual case of a non-singular potential, the completeness relation

$$\sum_\nu |\phi_\nu(\vec{P})\rangle\langle\psi_\nu(\vec{P})| = 1$$

holds true. These bi-orthogonal sets of wave functions are thus seen to diagonalize g, and the T matrix itself is easily expressed in terms of them. The scattering eigenfunctions $|\phi_{\vec{k}}(\vec{P})\rangle$ are themselves easily related to the half-off-shell value of the generalized wave function $X(s,\vec{P})$. From Eqs. (1) and (2), we find

$$(L(\vec{P}) - s)X(s,\vec{P}) = g_0^{-1},$$

with appropriate boundary conditions, and comparison with Eq. (4) shows

$$\langle\vec{p}\,|X(k^2,\vec{P})|\vec{k}\rangle = \langle\vec{p}\,|\phi_{\vec{k}}(\vec{P})\rangle \equiv \phi_{\vec{k}}(\vec{p},\vec{P}).$$

As can be seen from Eqs. (5) together with the easily derived relation

$$g_0^{-1}gg_0^{-1} = g_0^{-1} - (Q - \overline{Q})T,$$

the bound-state eigenfunctions lead to poles in both g and T at values $s = s_i(\vec{P})$, the bound-state energy eigenvalues, the residues at which are easily calculated and seen to be factorizable in $\vec{p}$ and $\vec{p}\,'$ in the momentum representation. Using either time-reversal invariance or Eq. (1) directly, the T matrix is symmetric in this representation;

$$T(\vec{p},\vec{p}\,';s,\vec{P}) = T(\vec{p}\,',\vec{p};s,\vec{P}),$$

and hence the residues at $s = s_i(\vec{P})$ must also be symmetric factorizable functions of $\vec{p}$ and $\vec{p}\,'$.

The GF T matrix is not amenable to direct physical observation, but within the ladder approximation it leads simply and directly[13] to the proper self-energy function $\Sigma^*(\vec{p},p_0)$ from which one can obtain the self-consistent single-particle energy spectrum as the solution to the equation[13]

$$p_0 = \vec{p}^2 + \Sigma^*(\vec{p},p_0).$$

The solution $p_0 = \epsilon(\vec{p}) + i\gamma(\vec{p})$ is in general complex, and $\gamma(\vec{p})$ is a direct measure of the quasiparticle lifetime. From the spectrum, one then easily makes contact with the thermodynamic functions.

For these reasons, we have chosen to study the GF T matrix for some potentials of physical interest, and, in particular, for a potential appropriate to liquid He^3, as detailed in Sec. III. The starting point for the calculations is the GF T matrix Eq. (1). Unlike the case of free scattering, the GF T matrix depends on the vector $\vec{P}$, and in order to be able to make a decoupled partial-wave expansion, it is necessary and usual to make the approximation of replacing Q and $\overline{Q}$ by their angle-averaged values. In this approximation Eq. (1) can be rewritten

$$T(\vec{p},\vec{p}\,';s,\vec{P}) = \sum_l (2l+1)T_l(p,p';s,P)P_l(\hat{p}\cdot\hat{p}\,'),$$

$$T_l(p,p';s,P) = u_l(p,p') - \int_0^\infty \frac{k^2dk}{2\pi^2}u_l(p,k) \times \left(\frac{Q(k,P)}{k^2 - s - i\eta} - \frac{\overline{Q}(k,P)}{k^2 - s + i\eta}\right) \times T_l(k,p';s,P), \tag{6}$$

$$Q(k,P) = \tfrac{1}{2}\int_{-1}^{1} d\mu\, Q(\vec{k},\vec{P}),$$

$$Q(\vec{k},\vec{P}) = \theta(|\vec{P} + \vec{k}| - k_F)\theta(|\vec{P} - \vec{k}| - k_F),$$

where $\mu = \hat{k}\cdot\hat{P}$, and $\overline{Q}$ is defined similarly under the replacement $\theta(x) \to 1 - \theta(x)$, where $\theta(x)$ is the unit-step function. The function T_l is also an implicit function of the Fermi momentum k_F, defined in terms of the particle density n for identical spin-$\frac{1}{2}$ fermions as $k_F = (3\pi^2 n)^{1/3}$.

We have examined both Eq. (6) and the BG equation, which can be obtained from it under the replacement $\overline{Q} \to 0$, for several two-body potentials

of interest, both in the scattering region ($s>0$) and in the bound-state region ($s<0$). Details of the potentials used and the numerical techniques employed are given in Sec. III.

III. DETAILS OF THE CALCULATIONS

Most of the detailed calculations reported in this work concern S-wave results for the modified Frost-Musulin (MFM) potential,[14] applicable to liquid He^3,

$$V(r) = -\epsilon[1 + c(1 - r_0/r)]\exp[c(1 - r/r_0)]\,,$$
$$\epsilon = 12.54\,^{\circ}\mathrm{K}, \quad c = 8.01, \quad r_0 = 2.98\ \text{Å}, \tag{7}$$

with the value $\hbar^2/2M = 8.0425\,^{\circ}\mathrm{K}\ \text{Å}^2$. We have also investigated both the higher partial waves for this potential, and the isotriplet S-wave soft-core Reid potential[15] appropriate to nuclear-matter calculations. Neither of these potentials supports any bound state, although both would need only a small increase in strength to do so.

The integral Eq. (6) is solved numerically by first discretizing it using an N-point quadrature formula, and then inverting the resulting matrix equation. In the bound-state region the equation is nonsingular, the T-matrix is real, and this procedure is applied directly to an $N\times N$ matrix. In the scattering region, the integral equation is singular and we have preferred to solve for the (real) K matrix obtained from Eq. (6) by letting $\eta \to 0$ and regarding the resulting integral as of principal-value type. In this case we subtract from the integral a term (which is identically zero) which has the numerator evaluated at the point $k = +s^{1/2}$, thereby replacing the principal-value condition by a smooth integrand.[16] The real and imaginary parts of the T matrix can then be found from the K matrix by using a generalized unitarity relation. For the numerical results reported below the integration variable has been mapped on to the interval $(-1, 1)$ suitable for Gauss quadrature by the transformation $k = \tan\frac{1}{4}\pi(1+x)$. The numerical techniques were first checked for the case of free scattering obtained from Eq. (6) by letting $Q \to 1$, $\overline{Q} \to 0$, and in this limit $T_l \to t_l$, the free two particle t-matrix. In particular the fully on-shell t matrix can be parametrized in terms of the phase shifts $\delta_l(p)$, and we found that for the MFM potential the functions $\delta_l(p)$ are smooth and extremely stable against changes in N for $N>48$. In the calculations reported in Sec. IV we use $N=96$.

IV. RESULTS

Using the MFM potential initially an investigation of the GF T matrix was then undertaken. The T matrix was found to be a smooth continuous function of all of its arguments in the scattering region, but exhibited singular behavior in the bound-state region for certain values of the arguments, and it is this behavior which is reported.

In Fig. 1 is plotted the S-wave GF T matrix at zero c.m. momentum for a typical value of k_F, over a range of negative c.m. energies, for several different values of the diagonal relative momentum p. Near $s=0$ the T matrix is negative, and as s is decreased the T matrix goes through infinity and becomes positive. That the singularity is a first-order pole in s is confirmed by obtaining the residue, and for a given value of P and k_F, the position of the pole is independent of the relative incoming and outgoing momenta. This behavior is typical for a wide range of values of the parameters P and k_F.

The position s_0 of the singularity can thus be obtained as a function of the two parameters P and k_F, as displayed in Fig. 2. For a given value of P, there exists a critical density or equivalently a critical Fermi momentum, k_{F_c}, below which the singularity disappears. The function $k_{F_c}(P)$ is smooth and monotonic, with a value at $P=0$ of $0.7146\ \text{Å}^{-1}$. By contrast the density of real liquid He^3 at zero temperature corresponds to a value $k_F = 0.79\ \text{Å}^{-1}$. The surface $s_0(P, k_F)$ appears to be very rich in detail. In particular the behavior at $P=0$ is very striking, where s_0 appears to attain a set of discrete values only, as a function of k_F, as can be seen clearly from Fig. 3. Moreover, as one moves away from the $P=0$ plane, for a general $k_F[> k_{F_c}(0)]$, at each of the discrete values s_0 there

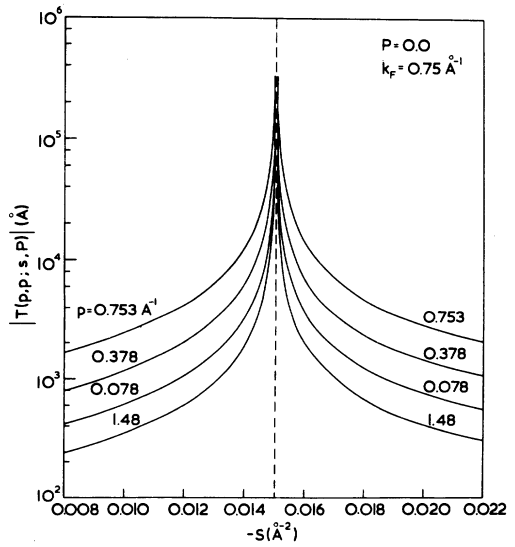

FIG. 1. S-wave GF T matrix for the MFM potential as a function of s, for four values of the diagonal relative momentum.

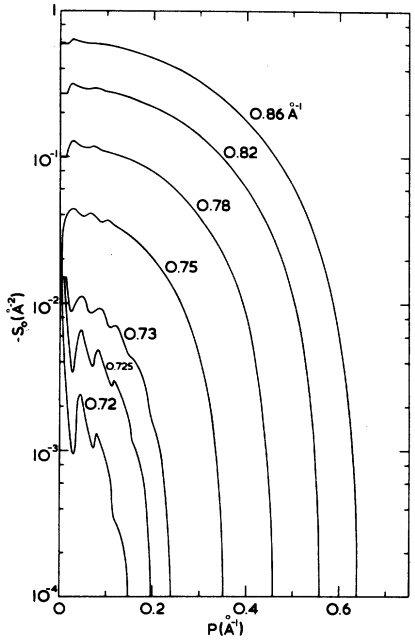

FIG. 2. Position of the singularity s_0 in the S-wave GF T matrix for the MFM potential as a function of the c.m. momentum P, for seven values of k_F.

exists a small horizontal "ledge," where s_0 is reckoned to be plotted vertically. The width of the ledge appears to be a periodic function of k_F. At a value of k_F which corresponds to a point where s_0 has just flipped to a new discrete value the shelf-width vanishes, and as k_F is further increased the width first increases to a maximum and then decreases to zero at that value of k_F which marks the next flip to a new value of s_0. From Fig. 2 one also observes that the s_0-vs-P curves exhibit oscillatory behavior which dies away rapidly with increasing k_F. The extrema however appear to occur at values of P which are independent of k_F. We also have some preliminary results which indicate that the pole whose behavior is plotted in Fig. 2, may not be the only pole present for the S-wave MFM potential. However any other pole certainly only appears at considerably higher values of k_F, and hence probably outside the density region of interest for liquid He^3.

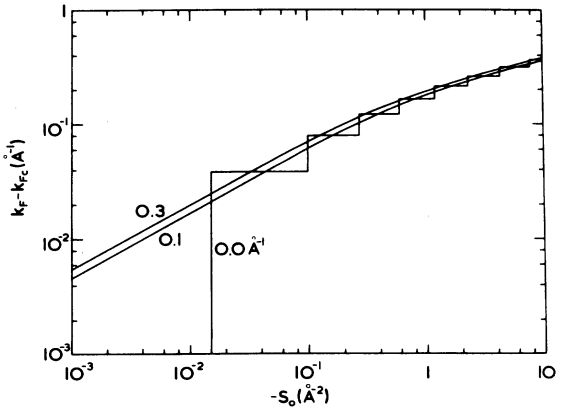

FIG. 3. Fermi momentum k_F vs the position of the singularity s_0 in the S-wave GF T matrix for the MFM potential, for three values of the c.m. momentum $P = 0.0$, 0.1, and 0.3 $Å^{-1}$, for which the critical values k_{F_c} are, respectively, 0.7146, 0.7158, and 0.7375 $Å^{-1}$.

The singularity structure detailed above does not seem to be present in the higher partial waves for the MFM potential, nor does it occur in the S-wave BG T matrix equation. We have however examined the GF T matrix for other potentials, and in particular have located a similar behavior in the isotriplet S-wave soft-core Reid internucleon potential.[15] In this case however, the critical value k_{F_c} at zero c.m. momentum of the pair occurs around 4.1 F^{-1}, which is considerably higher than the physical value of 1.35 F^{-1} appropriate to nuclear matter.

The positive identification of the pole structure plotted in Fig. 2 with a bound state can be made by reference to the remarks in Sec. II (and by analogy with free two-particle potential scattering) and the numerical observations that: (a) the residue has the sign expected for an S-wave potential, (b) the residue is factorizable in the sense that

$$(s - s_0)T(p,p';s,P) \xrightarrow[s\to s_0]{} -f(p,P)f(p',P),$$

and (c) the pole s_0 moves when only the strength of the potential is varied. Also, in the extreme high-density limit ($k_F \to \infty$) for $P = 0$, it is readily seen that the GF T matrix approaches the negative of the free-scattering t matrix for a potential $-V$. Insofar as the negative of the MFM potential does support bound-states, it is perhaps not surprising that the bound-state structure persists at finite densities.

The results reported here are summarized in Sec. V and possible consequences are indicated.

V. DISCUSSION AND SUMMARY

The numerical results of Sec. IV indicate that for the MFM potential, the GF T matrix (which describes the interaction to all orders in the potential between either a pair of particles outside or a pair of holes inside the Fermi sea, propagating freely in intermediate states except for restrictions due to the exclusion principle) has a singularity in the lowest partial wave at densities of the order of and lower than the physical density of real liquid He^3. This is seen to be caused by a bound state of the pair in the medium which forms

above a certain critical density, which itself depends on the total momentum of the pair. The corresponding Bethe-Goldstone T matrix which allows for only interactions between two particles outside the Fermi sea has no such singularity for the same potential. A similar pole structure was observed in the GF T matrix appropriate to the soft-core Reid internucleon potential in the S-wave isotriplet channel although only at densities appreciably higher than ordinary nuclear matter. Both the GF and BG T matrices can be used as the starting points for calculations of the properties of a many-body system, and the former has summed more terms of the complete perturbation series. This is generally "a good thing", although it is certainly not obvious that this necessarily results in an improved value of the ground-state energy, or of any other quantities. What is true is that in principle either T matrix can be employed to calculate the higher-order cluster terms. The GF T matrix has the added attraction of treating particles and holes symmetrically.

In a different context a generalized GF T matrix has been examined by Emery.[17] The T matrix examined by him is identical with ours except that the free two-body propagator $g_0(s)$ in Eq. (1) is modified to a self-consistent Hartree-Fock propagator in which each member of the pair travels in its own Hartree-Fock field. With this modification, Emery was able to show that this version of the GF T matrix is singular if and only if there exists an energy gap in the energy spectrum of the type directly associated with the BCS theory of superconductivity.[18] Emery[19] has also shown that the criterion for the existence of a singularity in the corresponding self-consistent Hartree-Fock version of the BG T matrix appears to be quite distinct from the criterion for the existence of an energy gap. These results are intuitively reasonable since in contrast to the usual Brueckner-Bethe-Goldstone approach, the Galitskii formalism and the BCS theory both treat particles and holes in a symmetric manner. In this sense one might also expect that the simple GF formalism used by us could provide a better description of the normal state than the usual BG approach insofar as it may contain the essential physics and detail of a more general theory. Within a different theoretical framework Goldberg and Puff[20] also discuss the generalized (in the Hartree-Fock sense) GF bound-state wave functions analogous to our eigenfunctions $|\phi_i(\vec{P})\rangle$ discussed in Sec. II. They show that the BCS gap function $\Delta(k)$ is given by

$$\Delta(k) \propto \int d\vec{q}\, u(\vec{k}-\vec{q})\langle \vec{q}\,|\,\phi_0(0)\rangle$$

when they employ the correct BCS form of the fermion distribution function in place of our step-function form, and the self-consistent Hartree-Fock two-body propagator in place of our propagator $g_0(s)$. It is worth noting that in our case this relation would imply a very direct link between the gap function and the T matrix, since the right-hand side of the above expression is trivially related to the T matrix.

We are presently investigating the potential dependence of the singularity structure in a systematic way and are extending the present results to finite temperatures using a temperature-dependent formalism. Results of these investigations and a calculation leading to the thermodynamic functions for a system wherein the two-body interaction leads to a pole structure of the sort discussed here, will be published elsewhere.

In summary, it is our belief that the simple GF formalism outlined in this work forms a simple starting-point for real calculations, and one which already has the possibility of containing the important physical elements of pairing in zeroth order. It is probably too early to fully answer the most interesting question of whether the pole structure in the T matrix observed by us is of merely mathematical interest, or whether it will really herald the onset of some physical phenomenon—in the sense that we have discussed above and that one knows, for example, that the Cooper pairs responsible for superconductivity manifest themselves in a similar fashion.

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