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Atoms in moderately strong electromagnetic fields: General method and its application to the two-level atom

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The desirability of considering "dressed" rather than "bare" states of atoms in strong electromagnetic fields has been noted by others. It is suggested here that such a treatment is conveniently carried out by performing a unitary transformation of the Hamiltonian so that the "dressed" states in the new basis are precisely the "bare" states of the usual basis. A perturbative method for constructing such a transformation, closely related to a procedure described by Heitler, is proposed. The method is applied through second order to a two-level atom model, and it is verified that the transformation makes the dressed ground atomic state stable, and the excited state unstable with the proper decay probability; furthermore, the energies of the two states are renormalized by the correct amounts. The method is then compared with the "nonperturbative" momentum-translation approximation, and shown to contain already in first order the entire operator content of that approximation, but to yield different transition probabilities. Finally, the transformation method is applied through second order to the standard scalar-field model and shown to produce the usual dressing transformation, mass renormalization, and induced nucleon-nucleon potential of that model, a result which tends to substantiate the hypothesis that the transformation produces the correct dressed states of a physical system.

I. INTRODUCTION

The availability of lasers of steadily increasing power has led, for both fundamental and practical reasons, to much interest in the study of atoms and molecules in intense electromagnetic fields.[1-22] Theoretical considerations of this problem have taken the form both of evaluation of higher-order terms in perturbation theory[2,4,9,11-13,17-20] and attempts to develop nonperturbative approximation techniques.[1,3,5-8,10,14-16,22] Calculations have been carried out both in semiclassical[1,3,5-10,12-14,16,18,20,22] and full quantum-electrodynamical[2,4,11,15,17] models. One feature common to the majority of these studies, though, is that they attempt to calculate by some method or other the transition probabilities induced by the electromagnetic interaction between those states of the atom which are determined in the *absence* of the interaction.

However, it is well known from experience with quantum field theories that interactions between fields serve not only to induce transitions but also to alter the properties of particles of each field, even in the absence of particles of any of the other fields—e.g., in quantum electrodynamics renormalizations of the electron's mass and gyromagnetic ratio occur even in the absence of "physical" (cf. below) photons.[23] Furthermore, the state representing a "physical" electron—i.e., the electron as actually observed in the laboratory—can be thought of within the context of a perturbation-theory calculation as consisting of the "bare" electron, obtained in the theory of the noninteracting electron field, "dressed" by a cloud of virtual particles consisting of both "bare" photons and "bare" electron-positron pairs.

In the same way, an atom described by a Schrödinger equation containing interparticle potentials is a bare atom which, when it interacts with the electromagnetic field, is dressed by that field. Only states of the dressed atom are observable, and therefore calculations of transition probabilities should properly be made for dressed-atom states rather than bare-atom states. The desirability of formulating the theory in terms of dressed- rather than bare-atom states has been urged even for weak-field situations.[24] It seems reasonable to suppose that use of the dressed- rather than bare-atom states becomes increasingly important for the case of an atom interacting with increasingly strong electromagnetic fields.[17,18]

A major problem in carrying out such a formulation is the difficulty of properly identifying the dressed states of the system. This is a nontrivial matter even for the states of the dressed atom in the absence of any photons, and it becomes more challenging still for the case of a dressed atom in a strong electromagnetic (photon) field. A large part of the difficulty could be eliminated by simply transforming the Hamiltonian for the interacting atom and electromagnetic field to a new basis in such a way that the bare states of the old (usual) basis become the dressed states of the new basis. Identification of the physical states of the system then becomes a straightforward matter, and the computational problem is primarily that of deter-

mining the transformed Hamiltonian.

An explicit perturbative procedure for calculating the required unitary transformation and the transformed Hamiltonian, closely related to one previously described by Heitler,[25] is proposed here. The basic idea motivating the particular choice given for the unitary transformation is that it maps the eigenstates of the total Hamiltonian onto the eigenstates of the noninteracting Hamiltonian in a natural way so that in the new basis the ground-atomic-state vector of the transformed Hamiltonian is precisely the bare-ground-state vector of the original basis. Because of the "naturalness" of the transformation, it is then conjectured that in the transformed basis the excited states of the physical dressed atom and the states of the dressed atom with "physical" photons present are also represented by the appropriate bare-state vectors of the original basis. Some support for this conjecture is obtained by applying the described procedure through second order to a model consisting of a two-level atom interacting with a denumerable discrete set of photon modes, with both rotating and counter-rotating terms present in each mode, and with one mode resonant with the (bare) atomic-transition frequency. It is found that the hypothesis that the dressed-state vectors for the transformed basis are the bare-state vectors of the original basis leads to the following conclusions:

(i) In the absence of photons the ground state is stable, and the excited state is unstable with the expected decay rate.

(ii) Both states undergo the expected energy renormalizations in second order.

(iii) Both states experience second-order energy shifts (in opposite directions) in the presence of an electromagnetic field, the shift due to a particular mode being proportional to the number of photons present in that mode.

Conclusions (i) and (ii) indicate that in the transformed basis the physical atomic states are correctly represented by the bare-state vectors, at least through second order in the interaction.

A description of the procedure for calculating the unitary transformation of the Hamiltonian and a comparison with Heitler's method are given in Sec. II, and application of the procedure to a two-level atom is given in Sec. III. The paper concludes with a discussion section in which the method described here is compared with another well-known technique of atomic physics and is also applied to a standard particle-theory model.

II. UNITARY TRANSFORMATION

Let H_0 be the Hamiltonian for the noninteracting atom and electromagnetic field, and λH_I the interaction Hamiltonian. The total Hamiltonian H is then given by

$$H = H_0 + \lambda H_I . \tag{1}$$

Assume (by putting the system in a box if necessary, with an appropriate limit to be taken later) that both H and H_0 have purely discrete spectra, with

$$H|n\rangle = \epsilon_n |n\rangle . \tag{2}$$

Let $|n\rangle^{(0)}$ be an orthonormal enumeration of the eigenstates of H_0 chosen so that

$$|n\rangle \underset{\lambda \to 0}{\longrightarrow} |n\rangle^{(0)} ,$$

and define the unitary transformation U by

$$U|n\rangle = |n\rangle^{(0)} . \tag{3}$$

(If H_0 has degenerate eigenvalues U is not uniquely defined by these requirements. We shall later make it unique by a requirement of "simplicity.") Equations (2) and (3) imply

$$UHU^{-1}|n\rangle^{(0)} = \epsilon_n |n\rangle^{(0)} , \tag{4}$$

so that UHU^{-1} and H_0 commute. Put

$$UHU^{-1} = H_0 + D, \tag{5}$$

where

$$[H_0, D] = 0, \tag{6}$$

$$\lim_{\lambda \to 0} U = I \quad \text{(identity)}, \tag{7}$$

$$\lim_{\lambda \to 0} D = 0 . \tag{8}$$

We attempt a perturbative solution of Eqs. (5)–(8) by assuming[26]

$$D = \sum_{n=1}^{\infty} \lambda^n D_n \tag{9}$$

with

$$[H_0, D_n] = 0, \tag{10}$$

and

$$U = e^{iK} \tag{11}$$

with

$$K = \sum_{n=1}^{\infty} \lambda^n K_n = K^{\dagger} . \tag{12}$$

Substituting these expressions into Eq. (5), expanding the exponentials, and equating coefficients of like powers of λ (which hereafter will be set equal to 1) on the two sides of the equation leads to the infinite set of equations,

$$D_1 = i[K_1, H_0] + H_I, \tag{13}$$

$$D_2 = i[K_2, H_0] + (i^2/2!)[K_1, [K_1, H_0]] + i[K_1, H_I], \tag{14}$$

$$D_3 = i[K_3, H_0] + (i^2/2!)[K_1, [K_2, H_0]] + (i^2/2!)[K_2, [K_1, H_0]] + (i^3/3!)[K_1, [K_1, [K_1, H_0]]] + i[K_2, H_I] + (i^2/2!)[K_1, [K_1, H_I]], \quad (15)$$

$$\vdots$$

$$D_n = \sum_{m=1}^{n} \frac{i^m}{m!} \sum_{\substack{j_1 \\ (j_1+\cdots+j_m=n)}} \cdots \sum_{j_m} [K_{j_1}, [\ldots, [K_{j_m}, H_0]\ldots]] + \sum_{m=1}^{n-1} \frac{i^m}{m!} \sum_{\substack{j_1 \\ (j_1+\cdots+j_m=n-1)}} \cdots \sum_{j_m} [K_{j_1}, [\ldots, [K_{j_m}, H_I]\ldots]], \quad (16)$$

$$\vdots$$

Equation (13) is now to be used for the simultaneous determination of D_1 and K_1, Eq. (14) for the simultaneous determination of D_2 and K_2, etc. These determinations will be made as follows. By Eq. (10), D_1 must commute with H_0. By an earlier assumption, H_I can be expressed as a discrete sum of products of creation and annihilation operators, or as a discrete sum of products of creation and annihilation operators and of atomic-state raising and lowering operators, for the two components of the system. The sum of those terms in H_I which individually commute with H_0 will constitute D_1; the remainder of the terms in H_I must be canceled by the commutator $i[K_1, H_0]$, and K_1 will be chosen to be the *simplest* operator for which the commutator can produce the desired cancellation. In practice this means K_1 will be i times the sum of the *non*commuting terms for H_I, with an appropriate energy denominator inserted in each term. With this choice for K_1, the second and third terms on the right-hand side of the expression for D_2 are determined. D_2 is now taken to be the part of these two terms which commutes with H_0, and K_2 to be the simplest operator such that $i[K_2, H_0]$ cancels the remainder—i.e., K_2 is i times the *non*commuting part of $(i^2/2!)[K_1, [K_1, H_0]] + i[K_1, H_I]$, with an appropriate energy denominator inserted in each term. The process is continued to higher-order equations in the obvious way. It is easy to see that this construction process automatically makes each K_n Hermitian.

Note that a procedure has been introduced for eliminating the ambiguity in the definition of U by making the most obvious—and the "minimal"—choice for K_n in each order of perturbation theory. An additional point in the definition of the transformation which should be considered is the indicated manner of initially dividing the total Hamiltonian into H_0 and H_I, a matter obviously influencing the form of the final result. It is assumed here that electromagnetic interactions are treated in the Coulomb gauge, a gauge having the best group-theoretical basis[27] and the fewest mathematical anomalies (indefinite Hilbert-space metric, etc.). In the Coulomb gauge the electromagnetic interaction is composed of two parts, an instantaneous Coulomb interaction between the charge densities of the charged fields in question and an interaction of the transverse part of the current vector of each charged field with the transverse electromagnetic field. The kinetic energy (including free electromagnetic field energy) and Coulomb interaction parts of the total Hamiltonian comprise an operator which is mathematically reasonably well defined and which leads to no dressing phenomena in the normal sense; this part of the Hamiltonian is chosen as H_0. The transverse-current–transverse-electromagnetic-field part of the Hamiltonian, however, leads to dressing phenomena and to such well-known pathological behavior as divergences, the necessity for scaling coupling constants in the infinite-volume limit, etc.; this part of the Hamiltonian is chosen as H_I. More generally, one can say that H_0 consists of those terms of the Hamiltonian describing kinetic energies of, and instantaneous potential interactions between, fields, while H_I consists of those terms containing direct field-field interactions described in coordinate space by expressions involving products of three or more field operators evaluated at the same space-time point. This prescription is adequate to determine H_0 and H_I for most theories of current interest, and will be seen to lead to reasonable results—at least through second order—in the two examples treated here.

Heitler[25] has described a canonical transformation whose purpose is identical to that of the transformation given above, to obtain a new Hamiltonian which generates the dynamics of the dressed particles of the system. His description is formulated in the interaction picture, and in terms of our notation can be stated as follows: A time-dependent unitary operator $U(t)$ and a time-dependent Hermitian operator $D(t)$ are to be determined by the equation

$$D(t) = U^{-1}(t) H_I(t) U(t) - i U^{-1}(t) \dot{U}(t), \quad (5')$$

where, as usual,

$$H_I(t) \equiv e^{iH_0 t} H_I e^{-iH_0 t},$$

and where the dot above the last $U(t)$ indicates time differentiation. $D(t)$ is to satisfy the requirement that it commute with H_0 and $U(t)$ the requirement that it have no nonzero matrix elements between degenerate eigenstates of H_0 except those

"diagonal with respect to all variables"—i.e., connecting a given eigenstate of H_0 to itself. It is assumed that if these conditions are satisfied then $D(t)$ will be the operator generating the interaction-picture time development of the dressed physical states.

To obtain a solution of Eq. (5′) Heitler expands $D(t)$ and $U(t)$ in perturbation series,

$$D=\sum_{n=1}^{\infty}\lambda^n D_n, \tag{9′}$$

$$U=1+\sum_{n=1}^{\infty}\lambda^n U_n. \tag{11′}$$

Since $U(t)$ itself rather than an exponentiated anti-Hermitian operator (as in the transformation presented here) is being expanded, unitarity is no longer automatic, and therefore a set of subsidiary conditions

$$\begin{aligned} &U_1+U_1^\dagger=0,\\ &U_2+U_2^\dagger=U_1^2,\\ &U_3+U_3^\dagger=U_1U_2+U_2U_1-U_1^3,\\ &\cdots \end{aligned} \tag{12′}$$

must be added to the above perturbation expansion of $U(t)$ to insure unitarity. When Eqs. (9′) and (11′) are substituted into Eq. (5′) and coefficients of like powers of λ set equal to zero, one obtains the infinite sequence of equations

$$D_1(t)=H_I(t)-i\dot{U}_1(t), \tag{13′}$$

$$D_2(t)=H_I(t)U_1(t)-U_1(t)D_1(t)-i\dot{U}_2(t), \tag{14′}$$

$$D_3(t)=H_I(t)U_2(t)-U_1(t)D_2(t)-U_2(t)D_1(t)-i\dot{U}_3(t),$$

$$\cdots . \tag{15′}$$

According to Heitler, these equations are to be solved for the $U_n(t)$ and $D_n(t)$, subject to the above-stated restrictions [the $D_n(t)$ commute with H_0, the $U_n(t)$ satisfy the unitarity conditions (12′) and have no nonzero matrix elements between degenerate eigenstates except those diagonal in all variables].

It is obvious that Heitler's transformation is very close in both intent and effect to the transformation described here. However, Heitler's prescription for the unique determination of the U_n—that they have no nonvanishing matrix elements between degenerate eigenstates of H_0 except those diagonal in all variables—cannot be applied in general because it is inconsistent with the unitarity conditions (12′) in many cases of interest. For example, if two distinct degenerate eigenstates of H_0 are both connected via H_I to one or more H_0 eigenstates with which they are nondegenerate, as in the case of a hydrogen atom interacting with the electromagnetic field, then it is seen that U_1^2 will in general have a nonzero matrix element between these two distinct degenerate eigenstates; and by the second of the unitarity conditions (12′), U_2 must also have such a nonzero matrix element. This is contrary to Heitler's condition for the determination of U_2. (For comparison it should be observed that the method described here for constructing the K_n insures that K will have no nonzero matrix elements between distinct degenerate eigenstates of H_0 but makes no assertions about U in this regard.)

If one were to alter Heitler's criterion for the determination of the U_n by requiring that they have no nonzero matrix elements between distinct degenerate eigenstates of H_0 except those required by unitarity, his transformation definition would become self-consistent in general and probably equivalent to the one given here. Since the criterion in this form is awkward, and the unitarity conditions in any case a nuisance, we shall avoid these problems altogether by utilizing the transformation as described in Eqs. (9)–(16) and the discussion immediately following. We shall end our consideration of Heitler's method with two final comments regarding the relationship between his treatment and the present one: (i) Heitler states that when the Hamiltonian contains a Coulomb interaction term then this term should be a part of H_I, while it has been argued here that such a term induces no dressing phenomena and is properly a part of H_0. Apart from the question of principle involved there is also a somewhat practical matter here, for Heitler's procedure would necessitate the description of all physical states, including bound atomic states, in terms of kinetic-energy eigenstates—an inconvenient procedure whose physical significance is not clear. (ii) Heitler makes no application of his method (though he states the desirability of doing so) to the case of principal interest here, the interaction of an atom with the electromagnetic field.

We now turn our attention to this case of interest.

III. APPLICATION TO A TWO-LEVEL ATOM

The two-level atom has been a popular model for use in testing various calculational techniques[28] because its simplicity eliminates both the tediousness of calculation and the complexity of interpretation of results encountered in more realistic models. A two-level atom is (for obvious reasons) conveniently described by using a set of three operators $R_\pm$, R_3 having angular momentum commutation relations

$$[R_+, R_-]=2R_3, \quad [R_3, R_\pm]=\pm R_\pm, \tag{17}$$

with the requirement that R_3 have eigenvalues $\pm\frac{1}{2}$. In terms of these operators and a set of creation and annihilation operators $a_k^\dagger$ and a_k for the photon modes, one may write the Hamiltonian to be studied as

$$H=\omega_0 R_3+\sum_k \omega_k a_k^\dagger a_k+\sum_k g_k(a_k+a_k^\dagger)(R_-+R_+), \tag{18}$$

where ω_0 is the bare-atom transition energy and the ω_k are the bare-photon energies ($\hbar=1$). It will be assumed that one photon mode, the lth, has a bare energy equal to the bare-atom transition energy,

$$\omega_l=\omega_0. \tag{19}$$

The third term on the right in (18) arises from a product of three field operators evaluated at the same space-time point and is, of course, H_I. Note that it contains counter-rotating wave terms for all modes. Note also that the dipole approximation is not used in (18). It is a straightforward matter to verify that in realistic situations, such as bound-bound transitions in hydrogen, one has

$$\sum_k |g_k|^2<\infty, \tag{20}$$

provided the mode coupling constants are calculated exactly rather than in the dipole approximation; the validity of (20) will thus also be assumed in the present calculation. The existence of two polarization states for the photon has been ignored, since it adds nothing of interest to the calculation for this simple model.

We shall now use the method described in Sec. II to calculate K_1, D_1, K_2, and D_2. The determination of D_1 is obvious, since precisely two terms in H_I commute with H_0:

$$D_1=g_l(R_+a_l+R_-a_l^\dagger). \tag{21}$$

Then by inserting appropriate energy denominators in the remainder of the terms in H_I and multiplying by i one obtains

$$K_1=i\sum_{k\neq l}\frac{g_k}{\omega_k-\omega_0}(a_kR_+-a_k^\dagger R_-)+i\sum_k\frac{g_k}{\omega_k+\omega_0}(a_kR_--a_k^\dagger R_+). \tag{22}$$

From the explicit form (22) for K_1 one now calculates

$$\begin{aligned}\frac{i^2}{2!}[K_1,[K_1,H_0]]+i[K_1,H_I]=&\,2R_3\Big(\sum_{k\neq l}\frac{g_kg_l}{\omega_0-\omega_k}(a_k^\dagger a_l+a_l^\dagger a_k)+\sum_k\frac{g_kg_l}{\omega_0+\omega_k}(a_k^\dagger a_l^\dagger+a_ka_l)\Big)\\&+R_3\Big(\sum_{k\neq l}\sum_{j\neq l}\frac{g_kg_j}{\omega_0-\omega_k}(a_k^\dagger a_j+a_j^\dagger a_k)+\sum_{k\neq l}\sum_j\frac{g_kg_j}{\omega_0-\omega_k}(a_k^\dagger a_j^\dagger+a_ka_j)\\&+\sum_k\sum_{j\neq l}\frac{g_kg_j}{\omega_0+\omega_k}(a_k^\dagger a_j^\dagger+a_ka_j)+\sum_k\sum_j\frac{g_kg_j}{\omega_0+\omega_k}(a_k^\dagger a_j+a_j^\dagger a_k)\Big)\\&-R_+R_-\sum_{k\neq l}\frac{g_k^2}{\omega_k-\omega_0}-R_-R_+\sum_k\frac{g_k^2}{\omega_k+\omega_0}.\end{aligned} \tag{23}$$

According to the prescription of Sec. II, the sum of the terms commuting with H_0 makes up D_2:

$$D_2=-\Big(\sum_{k\neq l}\frac{g_k^2}{\omega_k-\omega_0}\Big)R_+R_--\Big(\sum_k\frac{g_k^2}{\omega_k+\omega_0}\Big)R_-R_++2R_3\Big(-\sum_{k\neq l}\frac{g_k^2}{\omega_k-\omega_0}a_k^\dagger a_k+\sum_k\frac{g_k^2}{\omega_k+\omega_0}a_k^\dagger a_k\Big), \tag{24}$$

and the remaining terms in (23) when multiplied by i and supplied with appropriate energy denominators become K_2:

$$\begin{aligned}K_2=&\,R_3\Big(i\sum_{k\neq l}\frac{g_kg_l}{(\omega_k-\omega_0)(\omega_k-\omega_l)}(a_k^\dagger a_l-a_l^\dagger a_k)+i\sum_k\frac{g_kg_l}{(\omega_k+\omega_0)(\omega_k+\omega_l)}(a_ka_l-a_k^\dagger a_l^\dagger)\Big)\\&+R_3\Big(i\sum_{k\neq l}\sum_{j\neq k}\frac{g_kg_j}{(\omega_k-\omega_0)(\omega_k-\omega_j)}(a_k^\dagger a_j-a_j^\dagger a_k)+i\sum_{k\neq l}\sum_j\frac{g_kg_j}{(\omega_k-\omega_0)(\omega_k+\omega_j)}(a_k^\dagger a_j^\dagger-a_ka_j)\\&+i\sum_k\sum_j\frac{g_kg_j}{(\omega_k+\omega_0)(\omega_k+\omega_j)}(a_ka_j-a_k^\dagger a_j^\dagger)+i\sum_k\sum_{j\neq k}\frac{g_kg_j}{(\omega_k+\omega_0)(\omega_j-\omega_k)}(a_k^\dagger a_j-a_j^\dagger a_k)\Big).\end{aligned} \tag{25}$$

The important quantity for dynamics is the Hamiltonian $H'=H_0+D$ in the new basis, which to second order is now seen to be

$$\begin{aligned}H'\approx H_0+D_1+D_2=&\,\omega_0R_3+\sum_k\omega_ka_k^\dagger a_k+g_l(R_+a_l+R_-a_l^\dagger)-\Big(\sum_{k\neq l}\frac{g_k^2}{\omega_k-\omega_0}\Big)R_+R_--\Big(\sum_k\frac{g_k^2}{\omega_k+\omega_0}\Big)R_-R_+\\&+2R_3\Big(-\sum_{k\neq l}\frac{g_k^2}{\omega_k-\omega_0}a_k^\dagger a_k+\sum_k\frac{g_k^2}{\omega_k+\omega_0}a_k^\dagger a_k\Big).\end{aligned} \tag{26}$$

According to the ideas of Sec. II, the bare states of the original basis will be assumed to represent the dressed states of the new basis, and their time evolution will be governed (to second order) by (26). Let $|1\rangle$ and $|2\rangle$ denote the product of the bare-atom ground and excited states with the photon vacuum in the original basis. Then $|1\rangle, |2\rangle, a_k^\dagger|1\rangle, a_k^\dagger|2\rangle, \ldots$, should represent the *physical* ground atomic state, excited atomic state, ground atomic state with one photon present in mode k, excited atomic state with one photon present in mode $k, \ldots$, in the basis in which H' is the Hamiltonian. Let us note the reasonableness of this interpretation by observing the following points:

(a) $|1\rangle$ is *stable* under H', with no "virtual" non-energy-conserving transitions induced. However, $|2\rangle$ is unstable, as one would expect, with a nonzero probability of decaying to $a_l^\dagger|1\rangle$. The transition matrix element between $|2\rangle$ and $a_l^\dagger|1\rangle$ is g_l, to second order. This agrees with the lowest-order transition matrix element $\langle 1|a_l H_I|2\rangle$ of the usual theory.

(b) Under H' the ground atomic state $|1\rangle$ has an energy shifted from the bare-atom value by the amount $-\sum_k g_k^2/(\omega_k+\omega_0)$, and the excited atomic state has its energy shifted from the bare-atom value by the amount $-\sum_{k\neq l} g_k^2/(\omega_k-\omega_0)$. These energy renormalizations are of a well-known form,[28] but are here finite [by (20)] because the dipole approximation has not been used. In addition, the excited state $|2\rangle$ has an energy "spread" due to its instability; it is, in fact, a linear combination of two eigenstates of H' having energies

$$E_\pm = \tfrac{1}{2}(\omega_0+\alpha_+ +\alpha_- - g_l^2/2\omega_0) \pm[g_l^2+\tfrac{1}{4}(-\alpha_+ +\alpha_- + g_l^2/2\omega_0)^2]^{1/2}, \quad (27)$$

where for convenience the abbreviations

$$\alpha_+ = -\sum_{k\neq l}\frac{g_k^2}{\omega_k-\omega_0}, \quad \alpha_- = -\sum_k \frac{g_k^2}{\omega_k+\omega_0} \quad (28)$$

have been introduced. The energy spread in $|2\rangle$ is then

$$\Delta E = [4g_l^2+(-\alpha_+ +\alpha_- + g_l^2/2\omega_0)^2]^{1/2}. \quad (29)$$

In addition, one notes that D_2 contains a term yielding the known result[29,30] that the ground- and excited-state energies of the physical atom are shifted (in this case in opposite directions) by the presence of an electromagnetic field, with the part of the shift due to a given mode of the field being proportional to the number of photons present in that mode. It is the existence of such terms that suggests the importance of considering dressed rather than bare states in the case of atoms in relatively strong electromagnetic fields. This term is also responsible for the qualification ("moderately strong") on field strength in the title of this paper, since when the number of photons present in the field is sufficiently great this "second-order" energy shift can become large; in this case higher-order corrections to H', which have not been explicitly calculated here for this model, are required for an accurate description of the problem.

The implications of (26) discussed under points (a) and (b) above tend to justify the supposition that under the transformation described in Sec. II the Hamiltonian is transformed to a new basis in which the physical atomic states—with or without photons present—are properly represented by the bare states of the original basis, at least through second order. The possibility that this might be true for higher orders as well remains a question for future investigations, both theoretical and experimental.

In concluding the discussion of this model, two additional comments need to be made. The first is that the absence of two-photon-creation and two-photon-annihilation terms in D_2 is a peculiarity of the two-level-atom model used; such terms are present in the case of a general atom, to be discussed in a later paper. The second comment is that Hamiltonians of the form (18) are frequently used to describe a *system* of identical two-level atoms,[31] in which case R_3 has eigenvalues $-J, -J+1, \ldots, J$ for some integer or half integer $J>\frac{1}{2}$. For this case D_1, K_1, and D_2 retain exactly the forms given here, but K_2 acquires additional terms containing the operators R_+^2 and R_-^2 (which are zero for the case of a single two-level atom).

IV. DISCUSSION

The method described in Sec. II is perturbative in nature, and consequently suffers from the usual shortcomings of perturbation expansions—primarily the difficulty of calculating higher-order terms. Considerable work has been devoted recently to calculations of transition rates of atoms in intense electromagnetic fields by so-called "nonperturbative" approximations, and particularly by use of the "momentum-translation approximation" (whose validity has, however, recently been questioned—see below).[5-10,15,21,22] It is therefore of some interest to compare the method described here with the momentum-translation approximation, to which one may expect it to be somewhat related because both deal with unitary transformations (but in opposite directions) between the states of H_0 and H.

The momentum-translation approximation appears in both semiclassical (external electromag-

netic field[5-10]) and quantum-mechanical (quantized electromagnetic field[15]) versions, and in both cases is usually concerned with an electromagnetic wave of a single nonresonant frequency interacting with the atom. In fact, a criterion usually stated for the validity of the method[6,8] is

$$\omega/\omega_0 \ll 1, \tag{30}$$

where ω is the electromagnetic field frequency and ω_0 a typical atomic transition frequency. A comparison between the present method and the semiclassical version of the momentum-translation approximation will be given here. The comparison with the quantum-mechanical version is easily carried out and leads to similar conclusions.

Suppose, then, that the system to be considered consists of an atom with bare Hamiltonian H_0 with eigenstates $|i\rangle$,

$$H_0|i\rangle = E_i|i\rangle, \tag{31}$$

and that the interaction Hamiltonian is ($\hbar = c = 1$)

$$H_I = -(e/m)\vec{A}\cdot\vec{p}, \tag{32}$$

where $\vec{A}$ is the vector potential for an external electromagnetic field in the Coulomb-Lorentz gauge ($A_0 = 0$, $\vec{\nabla}\cdot\vec{A} = 0$). One can then formally apply the methods of Sec. II to calculate

$$K_1 = i\sum_{\substack{i,j\\(E_i\neq E_j)}}\sum |i\rangle \frac{\langle i|[-(e/m)\vec{A}\cdot\vec{p}]|j\rangle}{E_j - E_i}\langle j|, \tag{33}$$

$$D_1 = \sum_{\substack{i,j\\(E_i = E_j)}}\sum |i\rangle\langle i|[-(e/m)\vec{A}\cdot\vec{p}]|j\rangle\langle j|. \tag{34}$$

One follows the standard technique of replacing $(i/m)\vec{p}$ in (33) by $[\vec{x}, H_0]$ to obtain

$$K_1 = -e\sum_{\substack{i,j\\(E_i\neq E_j)}}\sum |i\rangle\langle i|\vec{x}\cdot\vec{A}|j\rangle\langle j| - e\sum_{\substack{i,j\\(E_i\neq E_j)}}\sum |i\rangle \frac{\langle i|[H_0,\vec{A}]\cdot\vec{x}|j\rangle}{E_j - E_i}\langle j|. \tag{35}$$

Now one of the basic assumptions of the momentum-translation approximation is precisely that the coefficients in the second summation in (35) can be neglected compared to those in the first summation.[6,8] A second basic assumption of the usual form of the approximation, implicit both in the inequality (30) and in all applications of the method, is that matrix elements of H_I between eigenstates of H_0 with the same energy are negligible. It follows that to the accuracy appropriate to the momentum-translation approximation

$$D_1 \approx 0, \tag{36}$$

$$K_1 \approx -e\vec{x}\cdot\vec{A}, \tag{37}$$

and therefore the transformation $U^\dagger$ from the eigenstates of H_0 to those of H is, to lowest order in the interaction,

$$U^\dagger \approx e^{-iK_1} = e^{ie\vec{x}\cdot\vec{A}}. \tag{38}$$

The right-hand side of (38) is precisely the unitary transformation of the momentum-translation approximation, and (36) shows that in the limit in which this transformation is valid the correction to the Hamiltonian of corresponding order, D_1, is negligible. Thus, from a formal standpoint, the lowest-order terms obtained by the method of Sec. II already encompass the entirety of the usual operator structure of the momentum-translation approximation.

However, from the standpoint of physical interpretation the techniques and predictions of the momentum-translation approximation and those of Sec. II are quite different. According to the viewpoint adopted in the momentum translation approximation, e^{-iK_1} generates transition probabilities (between states of the bare atom) whose calculation is the primary goal of the method. According to the viewpoint adopted in this paper, e^{iK_1} serves only to dress the states of the atom[32]; transitions between these dressed states are then induced only by the D_n (to which, however, K_1 contributes for $n \geq 2$). Transition probabilities calculated by the two methods will therefore bear little resemblance to each other in general.

One is thus led by this comparison to two interesting conclusions about the momentum-translation approximation: (i) Far from being "nonperturbative," it can in fact be thought of (though not originally derived this way) as a rather restrictive case of a first-order perturbation result, and (ii) the "transition matrix elements" calculated in this approximation may have little relation to multiphoton atomic transition probabilities as observed in the laboratory.[33] Correspondingly, it seems possible that it is not necessary to be overly apologetic concerning the perturbative nature of the method of Sec. II of the present paper.

It should be commented that application of the method of Sec. II through second order to a realistic Hamiltonian for a three-dimensional atom or molecule is straightforward and has been carried out, though the results naturally are of a much more complex form than those for the two-level atom treated in Sec. III. The calculation for this general case will be presented in a subsequent paper.

Finally, the obvious remark will be made that the method of Sec. II is applicable to a much wider array of quantum systems than just that of an atom or molecule interacting with the electromagnetic

field; it can be used in any problem involving the interaction of two or more quantum fields. For instance, for the well-known scalar-field model[34] with

$$H_0 = m_0 \int d\vec{p}\, \psi^\dagger(\vec{p})\psi(\vec{p}) + \int d\vec{k}\, \omega(\vec{k}) a^\dagger(\vec{k}) a(\vec{k}), \tag{39}$$

$$H_I = \frac{\lambda}{(2\pi)^{3/2}} \int d\vec{p} \int \frac{d\vec{k} f(\vec{k}^2)}{[2\omega(\vec{k})]^{1/2}} \psi^\dagger(\vec{p}+\vec{k})\psi(\vec{p})[a(\vec{k}) + a^\dagger(-\vec{k})], \tag{40}$$

and the standard (anti) commutation relations for the fields, one obtains by the methods of Sec. II

$$D_1 = 0, \tag{41}$$

$$K_1 = \frac{i\lambda}{(2\pi)^{3/2}} \int d\vec{p} \int \frac{d\vec{k} f(\vec{k}^2)}{[2\omega^3(\vec{k})]^{1/2}} \psi^\dagger(\vec{p}+\vec{k})\psi(\vec{p})[a(\vec{k}) - a^\dagger(-\vec{k})], \tag{42}$$

$$D_2 = -\frac{\lambda^2}{(2\pi)^3} \int \frac{d\vec{k}\,|f(\vec{k}^2)|^2}{2\omega^2(\vec{k})} \int d\vec{p}\, \psi^\dagger(\vec{p})\psi(\vec{p}) + \frac{\lambda^2}{(2\pi)^3} \int d\vec{p} \int d\vec{q} \int \frac{d\vec{k}\,|f(\vec{k}^2)|^2}{2\omega^2(\vec{k})} \psi^\dagger(\vec{p}+\vec{k})\psi^\dagger(\vec{q})\psi(\vec{p})\psi(\vec{q}+\vec{k}). \tag{43}$$

One immediately sees that e^{-iK_1} is the dressing operator,[35,36] the first term in D_2 the mass-renormalization operator, and the second term in D_2 the induced nucleon-nucleon potential term of the "canonical" scalar-field theory. These results suggest the possibility that the method of Sec. II may lead to the correct formulation of the dynamics of the physical dressed states for a variety of physical system.

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[35]The direction of the transformation described in this paper is opposite to that used in the conventional scalar-field theory, since it is somewhat simpler to have the original bare field operators represent the dressed particles in the transformed basis. Thus our D_2 represents the transformed Hamiltonian in terms of bare operators, while that in the conventional scalar-field formalism presents the transformed Hamiltonian in terms of dressed operators. However, the formal appearance and physical predictions in the two cases are precisely the same.

[36]It should be noted, however, that in the usual treatment of the scalar field model the transformation e^{-iK_1} is interpreted as the *total* dressing operator, while in the method of Sec. II there are higher-order corrections K_n—and also D_n—which must be included in a complete calculation.