Multiphoton cyclotron resonance absorption of laser radiation*

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The laser heating of plasma electrons in a uniform static magnetic field via the inverse bremsstrahlung process is considered. A kinetic equation is derived, and the change in kinetic energy of the electrons is calculated. For laser radiation propagating parallel to the magnetic field and incident on cold electrons, it is found that multiphoton processes are dominant when the laser frequency is near the electron cyclotron frequency. The multiphoton absorption coefficient is found to decrease as the laser frequency approaches the electron cyclotron frequency. The presence of the magnetic field reduces the intensity threshold above which multiphoton absorption occurs.

I. INTRODUCTION

There has been recent interest in using CO_2 laser radiation to heat plasma in θ-pinch or solenoidal magnetic fields.[1-3] Since the laser frequency is much greater than the electron cyclotron frequency in these experiments, the magnetic field probably has little effect on the rate of absorption of laser energy by the electrons, but has a major effect on particle confinement and gas breakdown thresholds.[3] However, a resonance condition, where the laser frequency is equal to the electron cyclotron frequency, may be approached by increasing the magnetic field strength or by using longer-wavelength lasers. Intense submillimeter lasers are becoming available,[4] and it is important to consider the cyclotron resonance absorption of this radiation.

The inverse bremsstrahlung process is believed to play a role in the heating of a plasma by laser radiation.[5] During this process, a plasma electron gains energy from the electromagnetic field of the laser beam by absorbing laser photons during a collision with a nucleus. We consider here the inverse bremsstrahlung absorption of laser radiation, and include the effects of a strong magnetic field.

The laser beam is treated as a classical plane electromagnetic wave in the dipole approximation. The plasma electrons are described by the solution to the Schrödinger equation for an electron in the laser field and a uniform static magnetic field. The scattering of electrons by nuclei, which are assumed to be uncorrelated and fixed in space, is treated using first-order perturbation theory.[6-9] The transition probabilities are used to write a kinetic equation for the electrons. For the case of laser radiation propagating parallel to the magnetic field and incident on cold electrons, the absorption coefficient is found to decrease as the laser frequency approaches the electron cyclotron frequency.

II. SOLUTION TO THE SCHRÖDINGER EQUATION

We let the magnetic field be in the $+z$ direction. The spatial dependence of the electromagnetic field of the laser beam is neglected (dipole approximation). The vector potential of the laser and magnetic fields is taken to be

$$\vec{A}(y,t) = \vec{A}(t) - By\,\vec{e}_x ,$$

where B is the magnetic field strength and $\vec{A}(t)$ represents the field of the laser beam. The solution to the time-dependent Schrödinger equation is[10]

$$\psi = \exp(-iE_n t/\hbar)\exp(i\vec{p}\cdot\vec{x}/\hbar) \times \exp\left(-(i/2m\hbar)\int^t R(t')dt'\right)\bar{H}_n(\xi) , \qquad (1)$$

where $E_n = \hbar\omega_c(n+\frac{1}{2})$, $n = 0, 1, 2, \ldots$,

$$\vec{p} = (p_x, Q(t), p_z),$$

$$R(t) = |\vec{p} - (e/c)\vec{A}(t)|^2 - [p_x - G(t)]^2 ,$$

$$\bar{H}_n(\xi) = (m\omega_c/\pi\hbar)^{1/4}(2^n n!)^{-1/2} e^{-\xi^2/2} H_n(\xi) ,$$

$$\xi = (m\omega_c/\hbar)^{1/2} y - (m\omega_c\hbar)^{-1/2}[p_x - G(t)] .$$

Here $\omega_c = |e|B/mc$ is the cyclotron frequency, p_x and p_z are constants of the motion, n is the Landau level, and $\bar{H}_n(\xi)$ is the harmonic-oscillator wave function. The real functions of time $G(t)$ and $Q(t)$ are determined by the equation[10]

$$G(t) + iQ(t) = (e\omega_c/c)\int^t dt' [A_y(t') - iA_x(t')] \times \exp[i\omega_c(t-t')] , \qquad (2)$$

where $A_x(t)$ and $A_y(t)$ are components of $\vec{A}(t)$.

For later use, we consider here some properties of the wave function (1). The orthonormality relation is

$$\int d^3x\,\psi_2^*\psi_1 = (2\pi\hbar)^2\delta(p_{2x} - p_{1x})\delta(p_{2z} - p_{1z})\delta_{n_2,n_1} ,$$

where unit normalization volume is assumed. The average value of the energy of the electron is

$$\epsilon = \int d^3x\, \psi^*(2m)^{-1}\left|\frac{\hbar}{i}\vec{\nabla} - \frac{e}{c}\vec{A}(y,t)\right|^2 \psi$$
$$= E_n + (2m)^{-1}\left[\left(G - \frac{e}{c}A_x\right)^2 + \left(Q - \frac{e}{c}A_y\right)^2 + \left(p_z - \frac{e}{c}A_z\right)^2\right]. \quad (3)$$

Since Eq. (3) does not contain p_x, the system is degenerate with respect to p_x. The degeneracy is given by

$$(2\pi\hbar)^{-1}\int_{-\infty}^{\infty} dp_x\, \psi^*\psi = m\omega_c/2\pi\hbar\,, \quad (4)$$

which is the same degeneracy that would occur if only the magnetic field were present.[11]

III. TRANSITION PROBABILITY

Treating the nuclear potential $V(\vec{x})$ as a perturbation, the probability amplitude for the transition from state 1 with quantum numbers p_{1x}, p_{1z}, n_1 to state 2 with quantum numbers p_{2x}, p_{2z}, n_2 is

$$a(1\to 2) = -(i/\hbar)\int\int_{-T/2}^{T/2} d^3x\, dt\, \psi_2^* V(\vec{x})\psi_1\,. \quad (5)$$

We write the Coulomb potential in the form

$$V(\vec{x}) = -4\pi Z e^2\hbar^2 \sum_{\vec{q}} q^{-2}\exp[i\vec{q}\cdot(\vec{x}+\vec{x}_\alpha)/\hbar]\,, \quad (6)$$

where $\vec{x}_\alpha$ is the position of the nucleus. Substituting Eqs. (1) and (6) into Eq. (5) and performing the integration over x and z, we obtain:

$$a(1\to 2) = 2iZe^2(2\pi\hbar)^3 \sum_{\vec{q}} q^{-2}\exp(i\vec{q}\cdot\vec{x}_\alpha/\hbar)\delta(p_{2x}-p_{1x}-q_x)\,\delta(p_{2z}-p_{1z}-q_z)\int_{-T/2}^{T/2} dt\exp[i(E_{n_2}-E_{n_1})t/\hbar]$$
$$\times\exp\left(\frac{i}{2m\hbar}\int^t dt'(R_2-R_1)\right)I(t)\,, \quad (7)$$

where

$$\xi_i = (m\omega_c/\hbar)^{1/2}y - (m\omega_c\hbar)^{-1/2}[p_{ix} - G(t)]\,,$$
$$i = 1 \text{ or } 2$$
$$I(t) = \int_{-\infty}^{\infty} dy\exp(iq_y y/\hbar)\bar{H}_{n_2}(\xi_2)\bar{H}_{n_1}(\xi_1)\,. \quad (8)$$

Integrals similar to the one in Eq. (8) have been considered by several authors.[12] The result is

$$I(t) = H(n_1,n_2)J(n_1,n_2,\rho)$$
$$\times\exp[iq_y(p_{1x}+p_{2x}-2G)/2m\hbar\omega_c]\,,$$

where

$$H(n_1,n_2) = (-1)^{n_2}e^{i(n_1-n_2)\phi}\,,$$
$$J(n_1,n_2,\rho) = (n_1!\,n_2!)^{-1/2}e^{-\rho/2}\rho^{(n_1+n_2)/2}$$
$$\times {}_2F_0(-n_1,-n_2;-1/\rho)\,, \quad (9)$$

$$\rho = (q_x^2+q_y^2)/2m\hbar\omega_c\,, \quad (10)$$
$$\phi = \tan^{-1}(q_y/q_x)\,,$$

and ${}_2F_0(-n_1,-n_2;-1/\rho)$ is the hypergeometric function. The integration over t in Eq. (7) may be performed after expanding the factors that are periodic in time in the Fourier series

$$\sum_{s=-\infty}^{\infty} F_s(\vec{q})e^{-is\omega t} = \exp\left[\frac{iq_x}{m\hbar}\int^t dt'\left(G-\frac{e}{c}A_x\right)\right.$$
$$\left.-\frac{iq_y G}{m\hbar\omega_c} - \frac{ieq_z}{mc\hbar}\int^t dt' A_z\right]. \quad (11)$$

Then Eq. (7) may be written

$$a(1\to 2) = 2iZe^2(2\pi\hbar)^4 \sum_{\vec{q}} q^{-2}\exp(i\vec{q}\cdot\vec{x}_\alpha/\hbar)\delta(p_{2x}-p_{1x}-q_x)\delta(p_{2z}-p_{1z}-q_z)H(n_1,n_2)J(n_1,n_2,\rho)$$
$$\times\exp\left(\frac{iq_y(p_{2x}+p_{1x})}{2m\hbar\omega_c}\right)\sum_{s=-\infty}^{\infty} F_s(\vec{q})\delta(E_{n_2}+p_{2z}^2/2m - E_{n_1} - p_{1z}^2/2m - s\hbar\omega)\,. \quad (12)$$

Equation (12) is now squared to obtain the transition probability per unit time. We assume that the nuclei are randomly distributed in space. Then the sums over the positions of the uncorrelated nuclei are[13]

$$\sum_\alpha\sum_\beta \exp[i(\vec{q}\cdot\vec{x}_\alpha - \vec{q}'\cdot\vec{x}_\beta)/\hbar] = N_i\,\delta_{\vec{q},\vec{q}'}\,,$$

where N_i is the ion density. The transition probability per unit time, summed over the nuclei, is

$$|a(1\to 2)|^2/T = \sum_{s=-\infty}^{\infty} T(s, 1\to 2) ,$$

$$T(s, 1\to 2) = 4Z^2e^4N_i(2\pi\hbar)^3 \sum_{q_y} q_0^{-4} |J(n_1, n_2, \rho_0)|^2 |F_s(\vec{q}_0)|^2 \delta(E_{n_2} + p_{2z}^2/2m - E_{n_1} - p_{1z}^2/2m - s\hbar\omega) , \quad (13)$$

where $\vec{q}_0 = (p_{2x} - p_{1x}, q_y, p_{2z} - p_{1z})$ and ρ_0 is given by Eq. (10) with $\vec{q}_0$ instead of $\vec{q}$. From the δ function of Eq. (13), we see that transitions are induced between Landau levels n_1 and n_2, with the absorption ($s > 0$) or emission ($s < 0$) of $|s|$ photons.

IV. KINETIC EQUATION

The change in $N_e(2)$, the number of electrons in state 2, may be written schematically as in Fig. 1, Eq. (14), where the second sum is over the quantum numbers of state 1. As in Ref. 9, we convert the schematic equation (14) to a mathematical equation by substituting the transition probability (13). For example, the second term in Eq. (14) becomes

$$T(-s, 1\to 2) N_e(1)[1 - N_e(2)],$$

where $N_e(1)$ is the square of the matrix element of the fermion destruction operator and $[1 - N_e(2)]$ is the square of the matrix element of the fermion creation operator. These factors appear in the transition probability when the electrons are treated using second-quantized theory rather than the first-quantized theory used in Sec. III.[14] From Eq. (13), it may be shown that $T(s, 1\to 2) = T(-s, 2\to 1)$. Thus Eq. (14) may be written

$$\frac{\partial N_e(2)}{\partial t} = \sum_{\substack{s=-\infty \\ s\neq 0}}^{\infty} \sum_{1} T(s, 1\to 2)[N_e(1) - N_e(2)] . \quad (15)$$

We now take the classical limit of Eq. (15) by letting

$$\hbar \to 0 \quad (16)$$

and

$$n \to \infty , \quad (17)$$

such that

$$\hbar\omega_c(n + \tfrac{1}{2}) \to \tfrac{1}{2}mv_\perp^2 . \quad (18)$$

Taking into account the degeneracy in p_x given by Eq. (4), the sum over the quantum numbers of state 1 is

$$\sum_1 \equiv \frac{m\omega_c}{2\pi\hbar} \sum_{n=0}^{\infty} \sum_{p_z} .$$

Letting the sums over n and p_z become integrals and using $dn = (m/\hbar\omega_c)\, v_\perp\, dv_\perp$, we obtain in the classical limit

$$\sum_1 \to \left(\frac{m}{2\pi\hbar}\right)^3 \int d^3v .$$

A Maxwellian distribution is assumed for the electrons. The classical limit of Eq. (15) is

$$\frac{\partial f_e(\vec{v}_2)}{\partial t} = 4Z^2e^4N_i N_e m^3 (m/2\pi kT)^{3/2} \int d^3v_1 \,[\exp(-mv_1^2/2kT) - \exp(-mv_2^2/2kT)]$$
$$\times \sum_{q_y} \{|J(n_1, n_2, \rho_0)|^2\}_{\text{C.L.}} q_0^{-4} \sum_{\substack{s=-\infty \\ s\neq 0}}^{\infty} |F_s(\vec{q}_0)|^2 \delta(\Omega - s\hbar\omega) , \quad (19)$$

where $f_e(\vec{v})$ is the electron distribution function, the subscript C.L. indicates that the classical limits (16)–(18) are to be taken in the brackets, and the change in kinetic energy of the electrons is

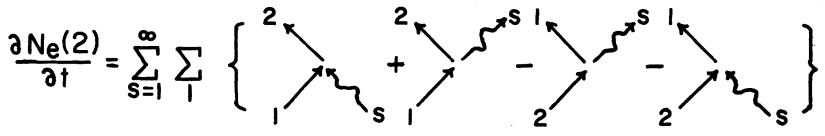
$$\Omega = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 .$$

Equation (19) is the kinetic equation for the electrons.

V. HEATING RATE

The sums and integrals in the kinetic equation (19) are in general difficult to evaluate. However, Eq. (19) takes a simpler form for the special case of circularly polarized laser radiation propagating parallel to the magnetic field. Then the Fourier coefficients F_s are Bessel functions.

We assume the right-hand circularly polarized plane wave



(14)

FIG. 1. Change in $N_e(2)$, the number of electrons in state 2.

$$\vec{A}(t) = (cE_0/\omega)(\vec{e}_x \cos\omega t + \vec{e}_y \sin\omega t) .$$

From Eq. (2), $G(t) = -eE_0\omega_c(\cos\omega t)/\omega(\omega - \omega_c)$. Using Eq. (11), the square of the Fourier coefficient is found to be

$$|F_s(\vec{q}_0)|^2 = |J_s(\lambda/\hbar\omega)|^2 , \tag{20}$$

where J_s is the Bessel function and

$$\lambda = eE_0 q_{0\perp}/m(\omega - \omega_c) . \tag{21}$$

The field parameter λ depends on the laser field strength E_0, the laser frequency ω, and the electron cyclotron frequency ω_c. The case $\omega_c \ll \omega$ is essentially the problem considered in a previous paper.[9] We consider here only the case $\omega_c \sim \omega$. Then $\lambda \gg \hbar\omega$ and the argument of the Bessel function is large.

For large values of argument, the Bessel function is small except when the order s is equal to the argument. The sum over s in Eq. (19) may be written approximately

$$\sum_{\substack{s=-\infty \\ s\neq 0}}^{\infty} |J_s(\lambda/\hbar\omega)|^2 \delta(\Omega - s\hbar\omega) = \tfrac{1}{2}[\delta(\Omega - \lambda) + \delta(\Omega + \lambda)] .$$

The factor $\frac{1}{2}$ may be verified by integrating both sides of the equation over Ω. The kinetic equation (19) becomes

$$\frac{\partial f_e(\vec{v}_2)}{\partial t} = 2Z^2 e^4 N_i N_e m^3 (m/2\pi kT)^{3/2} \exp(-mv_2^2/2kT) \times \int d^3v_1 \sum_{q_y} \{|J(n_1, n_2, \rho_0)|^2\}_{\text{C.L.}} q_0^{-4} \times [(e^{\lambda/kT} - 1)\delta(\Omega - \lambda) + (e^{-\lambda/kT} - 1)\delta(\Omega + \lambda)] . \tag{22}$$

The first δ function corresponds to the absorption and the second to the emission of $\lambda/\hbar\omega$ photons. Since $\lambda \gg \hbar\omega$, only multiphoton processes are significant. We assume that the electron temperature is low. Then $kT \ll \lambda$ and the emission term in Eq. (22) is negligible compared to the absorption term. Equation (22) becomes

$$\frac{\partial f_e(\vec{v}_2)}{\partial t} = 2Z^2 e^4 N_i N_e m^3 (m/2\pi kT)^{3/2} \times \int d^3v_1 \sum_{q_y} \{|J(n_1, n_2, \rho_0)|^2\}_{\text{C.L.}} q_0^{-4} \times \exp(-mv_1^2/2kT)\delta(\Omega - \lambda). \tag{23}$$

In the limit of low temperature, the Maxwellian distribution $(m/2\pi kT)^{3/2}\exp(-mv^2/2kT)$ reduces to the δ function $\delta(\vec{v})$. Using this δ function to evaluate the integration over velocity, Eq. (23) becomes

$$\frac{\partial f_e(\vec{v}_2)}{\partial t} = 2Z^2 e^4 N_i N_e m^3 \sum_{q_y} \{|J(0, n_2, \rho_2)|^2\}_{\text{C.L.}} \times q_2^{-4}\delta(mv_2^2/2 - eE_0 q_{2\perp}/m(\omega - \omega_c)), \tag{24}$$

where $\vec{q}_2 = (p_{2x}, q_y, p_{2z})$ and ρ_2 is given by Eq. (10) with $\vec{q}_2$ instead of $\vec{q}$. Using ${}_2F_0(0, -n_2; -1/\rho_2) = 1$ and Eq. (9), we write

$$|J(0, n_2, \rho_2)|^2 = (n_2!)^{-1} e^{-\rho_2} \rho_2^{n_2} . \tag{25}$$

In the classical limits $n_2 \to \infty$ and $\rho_2 \to \infty$, it may be shown that $|J(0, n_2, \rho_2)|^2$ is nonzero only for $n_2 = \rho_2$. From Eq. (18) and the definition $\rho_2 = (p_{2x}^2 + q_y^2)/2m\hbar\omega_c$, this implies that the only terms in the sum over q_y which contribute to Eq. (24) are those for which $m^2v_{2y}^2 = q_y^2$. There are two such terms corresponding to $q_y = \pm mv_{2y}$. Using Stirling's approximation for the factorial in Eq. (25), we find $|J(0, n_2, \rho_2)|^2 \cong (2\pi)^{-1/2}$ when $n_2 = \rho_2$. Substituting

$$\sum_{q_y} \{|J(0, n_2, \rho_2)|^2\}_{\text{C.L.}} = 2(2\pi)^{-1/2} \sum_{\rho_2} \delta_{n_2, \rho_2} \cong \sum_{q_y^2} \delta_{q_y^2, m^2v_{2y}^2}$$

into Eq. (24), we obtain

$$\frac{\partial f_e(\vec{v}_2)}{\partial t} = 2Z^2 e^4 N_i N_e m^{-1} v_2^{-4} \delta\left(\frac{mv_2^2}{2} - \frac{eE_0 v_{2\perp}}{\omega - \omega_c}\right) . \tag{26}$$

The change in average kinetic energy of the electrons is found by substituting Eq. (26) into

$$\frac{d\langle\epsilon\rangle}{dt} = \int d^3v_2 \frac{mv_2^2}{2} \frac{\partial f_e(\vec{v}_2)}{\partial t} . \tag{27}$$

Choosing the coordinate axes so that $v_{2\perp} = v_2 \sin\theta_2$, we write the δ function appearing in Eq. (26) in the form[15]

$$\delta\left(\frac{mv_2^2}{2} - \frac{eE_0 v_2 \sin\theta_2}{\omega - \omega_c}\right) = \frac{|\omega - \omega_c|}{eE_0 \sin\theta_2}\left[\delta(v_2) + \delta\left(v_2 - \frac{2eE_0 \sin\theta_2}{m(\omega - \omega_c)}\right)\right] .$$

Substituting Eq. (26) into Eq. (27), the change in average kinetic energy of the electrons may be written in the form

$$\frac{d\langle\epsilon\rangle}{dt} = \epsilon_0 \nu_{\text{eff}} ,$$

where ϵ_0 is the oscillatory energy of the electron

found from Eq. (3),

$$\epsilon_0 = e^2 E_0^2 / 2m(\omega - \omega_c)^2 ,$$

and the effective collision frequency ν_{eff} is

$$\nu_{\text{eff}} = 8\pi^2 N_i N_e Z^2 e m |\omega - \omega_c|^3 / E_0^3 . \quad (28)$$

The effective collision frequency is proportional to $\epsilon_0^{-3/2}$. The absorption coefficient given by $I^{-1}(d\langle\epsilon\rangle/dt)$, where I is the laser intensity, is proportional to $|\omega - \omega_c|$. Thus the absorption coefficient decreases as the laser frequency approaches the electron cyclotron frequency. This is because the oscillatory energy of the electrons is high, and the frequency of collisions between high-energy electrons and nuclei is small.

We note that Eq. (28) does not contain a Coulomb logarithm term which appears in other classical[16] and quantum-mechanical[7] calculations. The Coulomb logarithm normally enters because an integral over impact parameter is divergent and must be cut off. However, the sums and integrals leading to Eq. (28) are convergent, and so no cutoff need be assumed.

From Eq. (26), the number of photons absorbed by a cold electron during a collision with a nucleus is

$$S = \frac{eE_0 v_\perp / (\omega - \omega_c)}{\hbar\omega} , \quad (29)$$

where $v_\perp$ is the velocity of the electron perpendicular to the laser beam. Using Eqs. (20) and (13), it may be shown that the transition probability is zero when $v_\perp = 0$. Most photons are absorbed by electrons whose paths are nearly perpendicular to the direction of propagation of the laser radiation. Setting $\frac{1}{2}mv_\perp^2 \cong S\hbar\omega$ in Eq. (29), we find

$$S \cong \frac{[2e^2 E_0^2 / m(\omega - \omega_c)^2]}{\hbar\omega} . \quad (30)$$

Thus the number of photons absorbed is approximately equal to $\epsilon_0/\hbar\omega$, the ratio of electron oscillatory energy to photon energy. Upon comparison with a previous calculation[9] in which no magnetic field was assumed, we find that Eq. (30) is larger by a factor of $(1 - \omega_c/\omega)^{-2}$. Thus the presence of the magnetic field reduces the laser intensity threshold above which multiphoton processes occur by a factor of $(1 - \omega_c/\omega)^2$.

The above analysis of multiphoton cyclotron resonance absorption is valid for oscillatory energy ϵ_0 greater than the photon energy $\hbar\omega$ but less than the electron rest energy mc^2. This corresponds to laser intensities of

$$5 \times 10^2 \omega(\omega - \omega_c)^2 \ll I \ll 4 \times 10^{11}(\omega - \omega_c)^2 ,$$

where ω and ω_c are in units of 10^{12} Hz, and I is in units of W/cm^2. Absorption far from resonance ($\omega_c \ll \omega$) is essentially the problem considered in a previous paper.[9]

It has been shown that the inverse bremsstrahlung absorption coefficient decreases as the laser frequency ω approaches the cyclotron frequency ω_c. Thus inverse bremsstrahlung absorption may be insignificant compared to other effects, such as the cyclotron damping which occurs when $\omega - \omega_c \cong k V_{\text{th}}$. Investigations, which take into account the strong field of intense laser radiation, of other resonant processes such as cyclotron damping are needed.

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