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Differential cross sections for the elastic scattering of electrons from atomic hydrogen. II. Medium energies

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Differential cross sections for the elastic scattering of electrons from atomic and molecular hydrogen have been measured at 30, 50, 100, and 200 eV. The results for atomic hydrogen are compared with various theoretical calculations based on both low-energy and improved high-energy approximations. The ratios of the total cross sections for elastic scattering of electrons from atomic and molecular hydrogen are also given for incident energies of 9.4, 12.0, 20, 30, 50, 100, and 200 eV.

I. INTRODUCTION

The scattering of electrons by atomic hydrogen is the simplest atomic scattering process and therefore provides a convenient testing ground for atomic-collision theories. At low energies, where only a few inelastic channels are open, the close-coupling calculations are considered to be reliable. This view is reinforced by the excellent agreement of the close-coupling results with the experimental differential cross sections for the elastic scattering of low-energy electrons from atomic hydrogen presented by us in a previous paper,[1] hereafter referred to as I. However, as the energy of the incident electron is increased the number of open inelastic channels becomes too large for close-coupling calculations to be practicable.

Several schemes have been proposed to cope with the difficulty. One approach depends on modifying the low-energy close-coupling methods to artificially allow for the continuum of states. This can be done by introducing a set of pseudostates into the close-coupling expansions[2] or by introducing second-order potentials.[3] Other approaches depend on employing improved high-energy approximations. One of these, the Glauber approximation, has been widely used in recent years.[4-9] This has given considerably better agreement than the Born approximation with inelastic e-H cross sections in the range 30–200 eV.[6] A variation of the Glauber approximation based on angle trajectories has been used by Chen *et al.*,[10,11] whereas Byron and Joachain[12] have recently developed an approach based on the eikonal-Born series. Sinfailam and Chen[13] and Hutton and Roberts[14] have investigated the application of the Faddeev-Watson multiple scattering expansion (FWMSE) to atomic problems. This is a high-energy iterative expansion of the Faddeev equations. Another recent development is the method of extrapolation of the scattering amplitudes in the complex energy plane.[15]

Obviously experimental data are needed in the intermediate- and high-energy range for a critical evaluation of these theoretical approaches. However, the number of elastic electron–atomic-hydrogen scattering experiments has been quite limited, primarily owing to the difficulty of producing atomic hydrogen and distinguishing electons elastically scattered by the target atoms from those scattered by the background gas and surrounding apparatus. The only available data in this energy range are earlier work by one of us[16] which was subject to systematic errors at forward angles.

In this paper we report the measurement of differential cross sections for the elastic scattering of electrons from atomic hydrogen over the angular range 15°–135° at incident energies of 30, 50, 100, and 200 eV, thus covering the intermediate- to high-energy range. This supplements the low-energy data taken at 9.4, 12.0, and 20 eV recently reported by us in I and the 50-eV data reported earlier[17] in an abbreviated form. Our measured differential cross sections are also compared with those calculated using a variety of the above medium- and high-energy formalisms.

II. EXPERIMENTAL TECHNIQUE AND APPARATUS

The apparatus shown schematically in Fig. 1 has been described in I and only a brief description will be given here. A beam of electrons from an electron gun intersected a modulated beam of hydrogen atoms produced in a tungsten furnace. The energies of the scattered electrons were analyzed with a retarding-potential analyzer (RPA) and the electrons were detected in a channel electron multiplier. The output of the multiplier was recorded in two scalers, one gated on in phase with the modulated hydrogen beam and

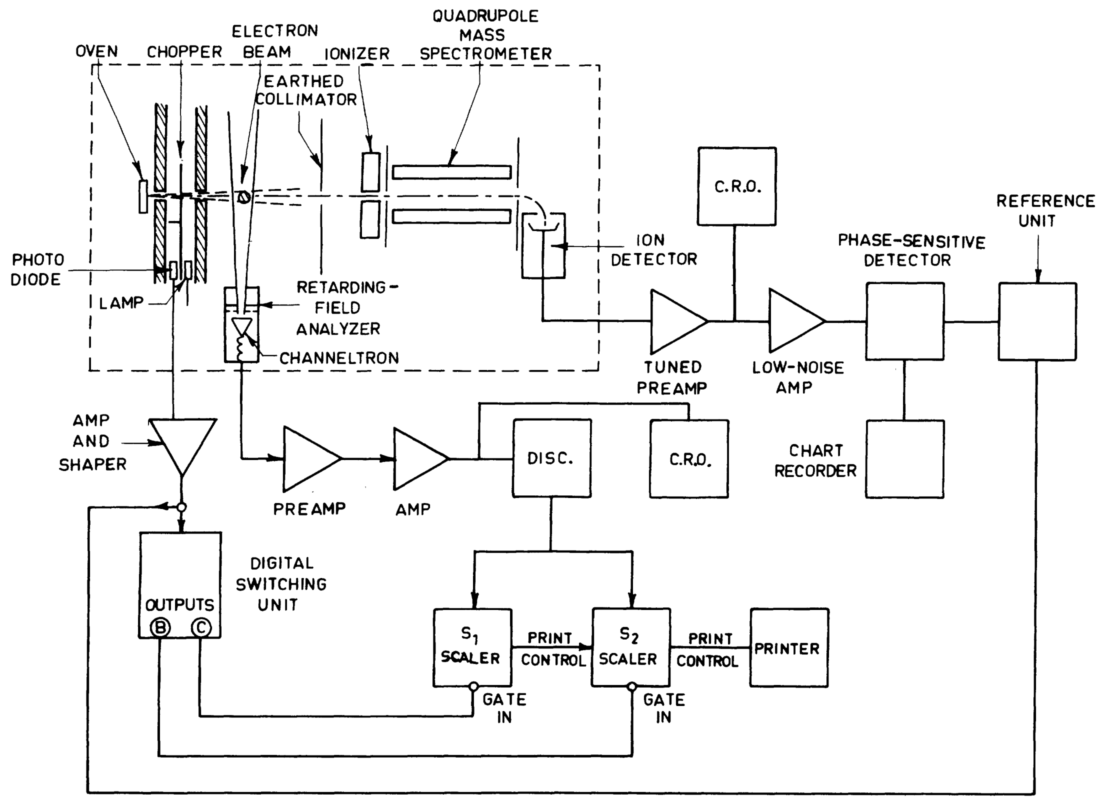

FIG. 1. Schematic diagram of the apparatus.

the other, counting background events, gated on in antiphase with the atomic-hydrogen beam. As described in detail in I, the ratio $R=\sigma_H(\theta)/\sigma_{H_2}(\theta)$, the ratio of cross sections for the elastic scattering of electrons by atomic and molecular hydrogen, was then measured at each energy and each scattering angle. The differential cross section for elastic scattering of atomic hydrogen was then determined by forming the product of $\sigma_{H_2}(\theta)$ and R, $\sigma_{H_2}(\theta)$ being measured in a separate experiment.

At high incident electron energies, photons produced in the collision region were also detected in the channel electron multiplier, since the retarding-potential analyzer was a line-of-sight instrument. Photons from the hydrogen beam resulted in an in-phase signal which of course was not related to the elastically scattered electron signal. The photon component was determined by setting the retarding potential on the RPA at approximately twice the incident electron energy. The correction for photon signal was quite small, of the order of 1% of the elastic signal at forward angles at all energies. The maximum correction was 20% at the largest scattering angle and the highest incident energy (200 eV) employed in the present investigation. Inelastically scattered electrons resulting in the excitation of electronic transitions were discriminated against by adjusting the potential on the retarding-potential analyzer.

Care was taken to minimize all stray electric and magnetic fields, and the areas bounding the interaction region were coated with colloidal graphite. The angular divergence of the electron beam at all energies was less than 0.03 rads. As in I, the angular resolution was 3° and the error in symmetry about 0° less than $\pm\frac{1}{2}°$. This was checked over the whole energy range by measuring the elastic scattering of electrons by argon, this cross section showing a considerable amount of sharp structure.

III. RESULTS AND DISCUSSION

A. Molecular hydrogen

Unlike the situation at low energies[1] no reliable absolute differential elastic e-H_2 cross sections exist for the energies under discussion. Trajmar *et al.*[18] measured differential cross sections from 10° to 80° for electron energies up to 81.6 eV. At 20 eV and below, these results were normalized using the total e-H_2 elastic scattering data of Golden *et al.*[19] Above 20 eV, however, they normalized their data by using theoretical results

TABLE I. Comparison of the measured ratios of elastic e-H_2 scattering cross sections at various incident energies with the ratios obtained using the normalization procedures discussed in the text.

E_1 (eV)	E_2 (eV)	θ (deg)	$\sigma_{E_1}(\theta)/\sigma_{E_2}(\theta)$ Measured	$\sigma_{E_1}(\theta)/\sigma_{E_2}(\theta)$ Using normalization
20	100	50	8.2 ± 2	8.0
50	100	60	2.5 ± 0.6	2.8
100	200	50	3.5 ± 0.9	3.2
20	30	40	1.5 ± 0.35	1.5

calculated with the aid of the Born-plus-polarization approximation.

The present angular distributions were normalized at 30 and 50 eV by using the absolute values of Trajmar *et al.* at 60°. Because the specific energies used by these workers did not correspond with those used in the present measurements, it was necessary to interpolate their data. At impact energies of 100 and 200 eV the present data were normalized at 60° to the theoretical results of Khare and Moiseiwitsch,[20] whose calculated angular distributions agree very well with the present data at these energies.

It was, however, possible to experimentally relate the angular distributions taken at different energies by measuring the ratios of cross sections at various energies but at a given angle. It is well known that this type of measurement is difficult to perform accurately using the modulated-crossed-beam technique, uncertainties being introduced by changes in the electron-beam shape and by possible changes in the transmission of the

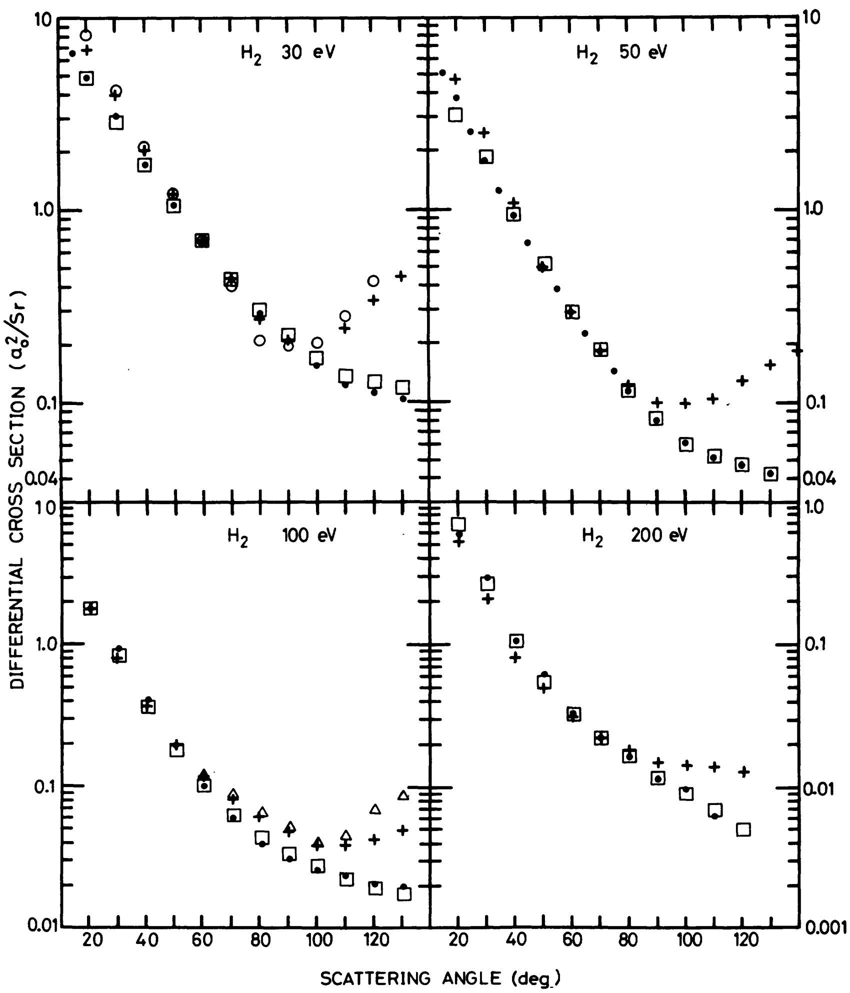

FIG. 2. Differential cross section for the elastic scattering of electrons from molecular hydrogen at 30, 50, 100, and 200 eV. The data of Williams (Ref. 21) (□), Webb (Ref. 22) (+), Bullard and Massey (Ref. 24) (○), and McMillen and Hughes (Ref. 23) (△) were generally normalized to the present data (●) at 60°. The procedure used for absolute normalization is discussed in the text.

TABLE II. Ratios $R = \sigma_H(\theta)/\sigma_{H_2}(\theta)$ of electrons elastically scattered from atomic hydrogen to those elastically scattered from molecular hydrogen at different angles for various incident energies.

Scattering angle (deg)	R 30 eV	50 eV	100 eV	200 eV
15	0.53± 0.02	0.54± 0.01	0.51± 0.02	
20	0.49± 0.02	0.57± 0.015	0.61± 0.02	0.74± 0.03
30	0.52± 0.02	0.60± 0.02	0.64± 0.02	0.75± 0.02
40	0.56± 0.02	0.65± 0.02	0.69± 0.04	0.81± 0.05
45		0.67± 0.02		
50	0.60± 0.02	0.72± 0.03	0.83± 0.04	0.75± 0.03
60	0.65± 0.03	0.77± 0.03	0.80± 0.06	0.81± 0.05
70	0.75± 0.03	0.87± 0.04	0.86± 0.07	0.73± 0.04
80	0.84± 0.03	0.94± 0.06	0.85± 0.08	0.70± 0.04
90	0.90± 0.04	1.01± 0.06	0.77± 0.05	0.65± 0.06
100	0.925± 0.05	0.95± 0.05	0.65± 0.06	0.51± 0.05
110	0.96± 0.06	1.00± 0.05	0.63± 0.05	0.63± 0.10
120	0.95± 0.07	0.91± 0.05	0.58± 0.06	
130			0.52± 0.04	
135	1.0 ± 0.08	0.80± 0.06		

electron spectrometer with energy. These effects were minimized by careful design of the RPA and measurement of beam profiles. The resulting ratios, given in Table I, are in good agreement with those obtained from the cross sections formed using the above normalization procedures.

Figure 2 shows the present angular distributions compared with those obtained by previous workers. Their results have all been normalized to the present results to provide the best visual fit. For all energies from 30 to 200 eV there is good agreement with the results of Williams[21] but poor agreement at large scattering angles with the earlier results of Webb,[22] Hughes and McMillen,[23] and Bullard and Massey.[24] The statistical errors in the present data range from 1% at forward angles to 4% at the most backward angles.

Since the energy resolution of the detection system used in the present experiments was approximately 1 eV, excitation of the first few vibrational levels as well as rotational levels was included in the measurements. Calculations[18] have shown that the total cross section for the sum of all vibrational states should be less than 1% of the total elastic cross section. In addition, it has been shown[25] that the shapes of the differential cross sections for excitation of the first two vibrational states are similar to the elastic differential cross sections. We therefore believe that the contributions of vibrational states produce negligible errors in the shape of the cross sections. This is supported by the excellent agreement of our low-energy results[1] with those of Trajmar

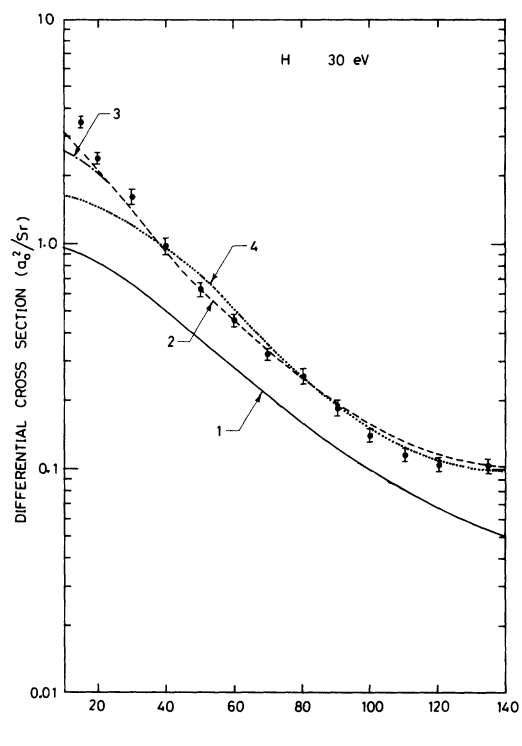

FIG. 3. Differential cross section for the elastic scattering of electrons from atomic hydrogen at an incident energy of 30 eV compared with the Born approximation (1), the close-coupling calculation of Scott (Ref. 27) (2), the close-coupling calculation of Burke *et al.* (Ref. 26) (3), and the variational calculation of Geltman (Ref. 28) (4).

et al.,[18] the latter results being measured with energy resolution sufficient to distinguish between vibrational states.

B. Atomic hydrogen

The ratios R of electrons elastically scattered from atomic hydrogen to those elastically scattered from molecular hydrogen at different angles and for incident energies of 30, 50, 100, and 200 eV are shown in Table II. The quoted errors of one standard deviation arise from statistical considerations as discussed in I. Figures 3–6 show the differential cross sections for the elastic scattering of electrons from atomic hydrogen obtained by forming the product $R\sigma_{H_2}(\theta)$. The errors shown in the figures include the errors in R and the errors in the relative cross section $\sigma_{H_2}(\theta)$, but do not include any errors which may arise in the normalization of $\sigma_{H_2}(\theta)$ as discussed above. This error, estimated to be less than 40%, does not affect the shape of the angular distribution. Nevertheless it must be emphasized that the final error

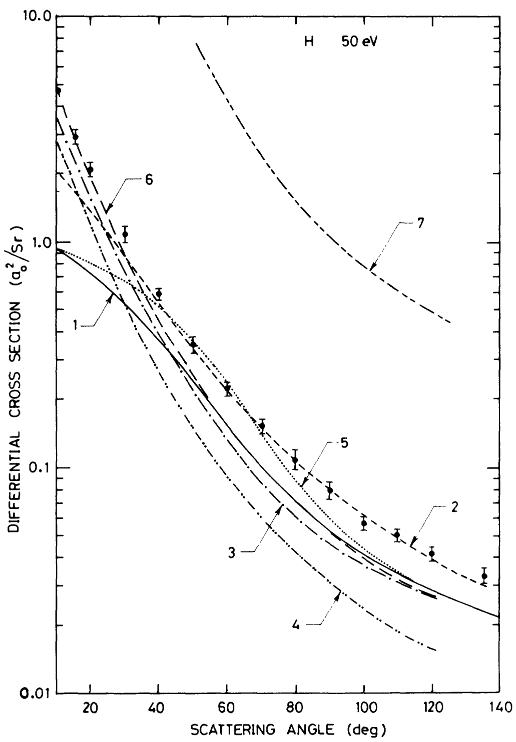

FIG. 4. Differential cross section for the elastic scattering of electrons from atomic hydrogen at 50 eV. The present results (●) are compared with the Born approximation (1), the close-coupling approximation (Ref. 27) (2), the Glauber (angle) approximation (Ref. 11) (3), the straight-line Glauber approximation (Ref. 11) (4), a variational calculation (Ref. 28) (5), a second-order potential calculation (Ref. 29) (6), and a calculation based on the FWMSE (Ref. 13) (7).

in the absolute value of the cross section for the elastic scattering of electrons by atomic hydrogen is primarily determined by the error in the corresponding molecular cross section.

Figure 3 shows the present experimental results at 30 eV compared with the close-coupling calculations of Burke *et al.*[26] and Scott[27] as well as with the variational calculation of Geltman.[28] Since these calculations are essentially low-energy results, normally only considered reliable up to the $n = 2$ or 3 thresholds, good agreement with the experimental results is not to be expected. Nevertheless, the close-coupling calculations are in surprisingly good agreement with the data over nearly the whole of the angular range. This is particularly so for the results of Scott, which were calculated using the reactance matrix elements of Burke *et al.* supplemented by Born-approximation values for higher partial waves. The effect of adding the extra l values is to raise the cross section at forward angles. Also shown for comparison is the Born-approximation result, which, as expected, does not provide an adequate description of the data at such low energies.

At the higher energy of 50 eV (Fig. 4) the shape predicted by the variational calculation is in much worse agreement with the data, and the close-coupling calculations begin to deviate significantly from the data for angles below 40°. The recent second-order potential calculation of Winters *et al.*[29] gives a cross section which fits the data at forward angles but drops off too rapidly with increasing angle.

The high-energy approximations are, in general, also in disagreement with the data. The Born approximation predicts an angular distribution which is far too low at forward angles. The (straight-line) Glauber approximation gives a somewhat better description of the shape of the cross section especially at forward angles, but is too low by approximately a factor of 2 at backward angles. The cross section predicted by the FWMSE is too high by an order of magnitude. The best of the high-energy approximations is obviously the Glauber approximation with angle trajectories.

At 100 eV (Fig. 5) the Born approximation gives a much better description of the data, although it again predicts a shape that is too low at forward angles. The straight-line Glauber approximation gives a somewhat better description of the data, but not as good as that given by the Glauber approximation with angle trajectories. The recent calculation of Byron and Joachain[12] based on the eikonal-Born series also gives a good fit to the shape of the angular distribution. This calculation predicts greater scattering at forward angles

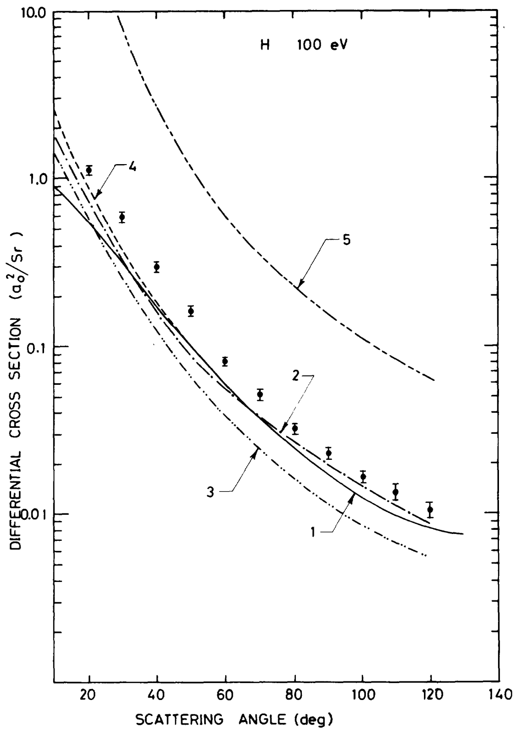

FIG. 5. Differential cross section for the elastic scattering of 100-eV electrons from atomic hydrogen. The experimental points (●) are compared with the Born approximation (1), the Glauber approximation with angle trajectories (Ref. 11) (2), the straight-line Glauber approximation (Ref. 11) (3), the eikonal-Born series result (Ref. 12) (4), and the FWMSE (Ref. 13) (5).

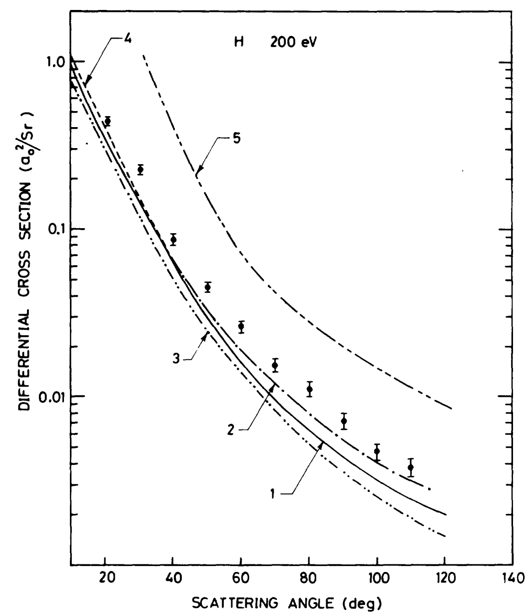

FIG. 6. Differential cross section for the elastic scattering of 200-eV electrons from atomic hydrogen. The experimental points (●) are compared with the Born approximation (1), the Glauber approximation with angle trajectories (Refs. 10 and 11) (2), the straight-line Glauber approximation (Refs. 10 and 11) (3), the eikonal-Born series result (Ref. 12) (4), and the FWMSE (Ref. 13) (5).

compared with the Born approximation. Not shown in the figure is the result of the second-order potential formalism of Bransden *et al.*,[3] which for angles greater than 20° does not differ significantly from the Glauber (straight-line) approximation. A more recent second-order potential calculation by Winters *et al.*[30] gives an angular distribution essentially in agreement with the eikonal-Born-series result for angles greater than 20°. Although all these calculated cross sections fall a little below the experimental points, the error in normalization must be taken into account. Once again the FWMSE calculation[13] predicts a cross section too high by nearly an order of magnitude.

Although it is generally regarded that the Born approximation should be valid for electron-hydrogen scattering at 200 eV and above, some doubt has recently been cast on this low-energy bound for the range of validity of the Born approximation by Damburg,[31] Geltman and Burke,[32] and Chen.[33] In particular, Damburg showed that channel coupling, assumed small in the Born approximation, may be significant into the kilovolt region. This may account for the 20% discrepancy in cross sections[31,32] between close-coupling calculations and experiment for excitation of the 2*s* and 2*p* states near threshold, since the experimental cross sections are normalized to the Born approximation at 200 eV. The reasons given by Chen are based on the results of calculations using the FWMSE, which suggest that the Born approximation may not be valid until at least the kilovolt region. However, the present results, shown in Fig. 6, support the Born approximation rather than the FWMSE, since the latter calculations are too high by a factor of approximately 3, while the Born calculations are too low by only of the order of (20–40)%. This latter discrepancy is not too serious in view of the normalization errors. It is interesting to note that Byron and Joachain[34] recently also gave arguments in favor of the Born approach and pointed out that great care must be taken in applying the FWMSE method in atomic physics.

Also shown in Fig. 6 are the calculated differential cross sections using the Glauber approximation[10,11] for both straight-line and angle trajectories, and the calculations of Byron and Joachain[12] based on the eikonal-Born series. The best fit to the data is given by the calculation

TABLE III. Ratios of the total cross sections for the elastic scattering of electrons from atomic and molecular hydrogen. The total elastic e-H cross sections in the last column are obtained by using the absolute e-H_2 cross sections of Golden *et al.* (Ref. 19).

E (eV)	$\sigma(H)/\sigma(H_2)$	$\sigma(H)$ (a_0^2)
9.4	0.71 ± 0.05	24.6 ± 1.9
12.0	0.66 ± 0.03	19.1 ± 1.0
20.0	0.63 ± 0.02	
30.0	0.60 ± 0.02	
50.0	0.62 ± 0.02	
100	0.62 ± 0.02	
200	0.72 ± 0.03	

based on the Glauber approximation with angle trajectories. Not shown in the figure are the second-order potential calculations of Bransden *et al.*[3] and Withers *et al.*[30] which for angles greater than 15–20° follow closely the Glauber (straight-line) and eikonal-Born-series calculations, respectively.

It is interesting to note that neither the high- nor the low-energy approximations give an adequate description of the data over the whole angular range in this intermediate-energy region. Below 50 eV, however, the low-energy approximations appear to give more satisfactory results.

The ratios of the total elastic e-H to the total elastic e-H_2 scattering cross sections are listed in Table III. For the sake of completeness the total cross sections obtained from the differential cross sections reported in I have been included in Table III. Absolute total cross sections for the elastic scattering of electrons by atomic hydrogen were obtained at 9.4 and 12.0 eV by using the absolute molecular-hydrogen data of Golden *et al.*[19] At the higher energies no absolute molecular-hydrogen data exist. The importance of obtaining accurate absolute cross sections for the elastic scattering of electrons by molecular hydrogen cannot be overemphasized. Both the differential and total elastic electron–atomic-hydrogen cross sections depend on these data.

ACKNOWLEDGMENT

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