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Radiative corrections to relativistic magnetic-dipole decays of hydrogenlike ions

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The radiative correction to the decay $2S_{1/2} \to 1S_{1/2} + 1\gamma$ in hydrogenlike atoms is calculated to relative order α. All terms of order $\alpha \ln\alpha$ and α are found to cancel. The cancellation of the order-α terms does not occur in other relativistic $M1$ transitions in hydrogenlike atoms.

I. INTRODUCTION

The $M1$ decay $2S_{1/2} \to 1S_{1/2} + 1\gamma$ in hydrogenlike ions and the related decay $2\,{}^3S \to 1\,{}^1S + 1\gamma$ in heliumlike ions have been intensively studied, both theoretically[1] and experimentally,[2] in recent years. There remains a small discrepancy between theory and experiment in the decay rate of heliumlike Ar^{+16} and Cl^{+15}. The previous calculations[1] give a leading matrix element of order $(Z\alpha)^4$ for both the hydrogenlike and heliumlike decays, and various corrections of higher order in $Z\alpha$ have been calculated[3] or estimated.[1] In this note, we present the results of a calculation of the radiative correction to the hydrogenic decay $2S_{1/2} \to 1S_{1/2} + 1\gamma$, and of other $M1$ decays of hydrogenlike atoms. The detailed calculations will be presented elsewhere. A calculation of radiative corrections to a decay rate does not appear to have been carried out previously in detail, although order-of-magnitude estimates exist.[4]

The radiative correction to the decay matrix element, to relative order α, is given by the sum of graphs in Figs. 1(a)–1(e). We omit graphs in which the emitted photon emerges from a closed electron loop, as those graphs are higher order in $Z\alpha$. In these graphs the double lines represent electron propagators, or represent external electron lines, each including the effect of the nuclear Coulomb potential V. Experience in the calculation of the Lamb shift[5] suggests that it is inadvisable to expand such propagators in powers of V, and indeed it is easy to see that such an expansion would give an infinite series of terms, all of which are the same order in $Z\alpha$. Instead, we have followed the method of Erickson and Yennie,[5] retaining the Coulomb potential in the propagators whenever possible, and expanding instead in powers of the external Coulomb field $E_i = -\partial V/\partial x_i$. This does give a series of increasing powers in $Z\alpha$, and we have calculated the leading term in this expansion, whose relative order is α. In FS, it was suggested that terms of relative order $\alpha \ln Z\alpha$ might occur in the answer. Although such terms do occur in individual graphs such as 1(c) and 1(d), they cancel when all graphs are added.[6] This cancellation has been checked by a different calculation along the lines used by Fried and Yennie[7] to calculate the Lamb shift. However, the latter method is not easily extended to obtain the terms of relative order α. The latter terms also cancel for $M1$ transitions of the form $nS_{1/2} \to mS_{1/2} + 1\gamma$ $(n \neq m)$, but do not cancel for transitions such as $nP_{1/2} \to mP_{1/2} + 1\gamma$ $(n \neq m)$, or $nS_{1/2} \to nS_{1/2} + 1\gamma$.

II. CALCULATION

The calculation of the terms of relative order α, with no additional powers of $Z\alpha$, is simplified by several observations. One is that the "vacuum

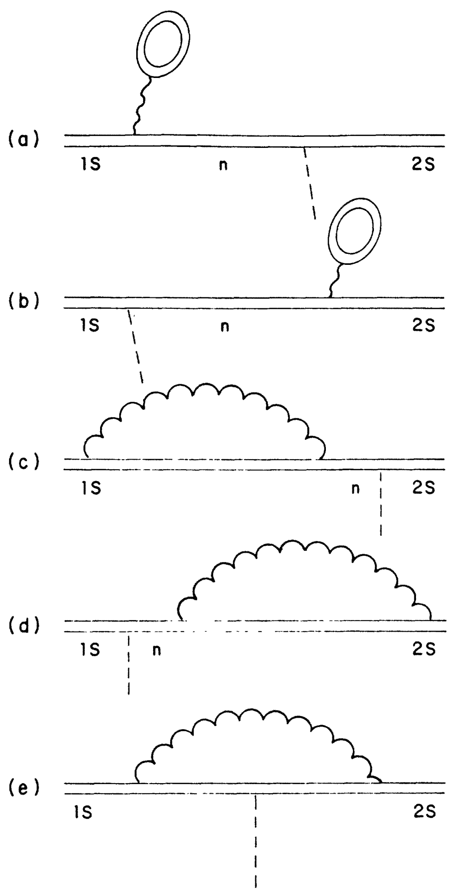

FIG. 1. (a) and (b) Vacuum polarization graph contributing to the decay matrix element. (c) and (d) Electron self-energy graph contributing to the decay matrix element. (e) Vertex graph contributing to the decay matrix element.

polarization" graphs 1(a) and 1(b) do not contribute to this order. This will be shown below. A second observation is that the calculation of the vertex graph, 1(e), can be reduced to the calculation of the mass renormalized self-energy operator Σ^{el}, in an external potential, through the following identity[8]:

$$\epsilon_\mu \Lambda_\mu(\pi, q) = \lim_{u\to 0} \frac{d}{du}\left[\Sigma^{el}(\pi_\alpha - ue\epsilon_\alpha e^{iq\cdot x})\right], \tag{2.1}$$

where the inner parentheses on the right-hand side indicates functional dependence. Here $\pi_\alpha \equiv p_\alpha - eV_\alpha$, ϵ_μ is the photon polarization, Λ_μ is the proper vertex operator of diagram 1(e), and Σ^{el} is the electron self-energy operator of Fig. 2. The value of this identity is that it expresses Λ_α in terms of Σ^{el}, where the latter is evaluated for the total "external" potential $V_\alpha + u\epsilon_\alpha e^{iq\cdot x}$. Since we must evaluate Σ^{el} for the external field V_α in order to calculate the self-energy graphs 1(c) and 1(d), little extra work is needed to obtain Λ_α, and several cancellations between graphs can be seen without detailed evaluation.

The sum of the graphs 1(a)–1(d) gives the expression

$$M^\Sigma = \frac{e}{\sqrt{2\omega_q}} \langle f | \Sigma_{tot} \frac{1}{\not\pi - m} \not\epsilon e^{iq\cdot x} + \not\epsilon e^{iq\cdot x} \frac{1}{\not\pi - m} \Sigma_{tot} | i\rangle , \tag{2.2}$$

where $\Sigma_{tot} = \Sigma^{el} + \Sigma^{\gamma}$ is the total self-energy operator in the Coulomb field, originating from Fig. 2. The initial and final states satisfy

$$(\not\pi - m)|i\rangle = 0, \quad (\not\pi - m)|f\rangle = 0,$$

where $\not A = -\beta\vec\alpha\cdot\vec A + \beta A_0$ for any four-vector A. ω_q is the photon energy, and ϵ_μ its polarization vector, which we choose to have no time component.

We write

$$\Sigma^{el} = \Sigma^{el}_{LS} + (\not\pi - m)\Sigma^{el}_R + \Sigma^{el}_L(\not\pi - m). \tag{2.3}$$

Therefore Σ^{el}_{LS} is the part of the electron self-energy operator contributing to the energy shift of bound states. However, the other terms in Σ^{el} will contribute to the matrix element we consider here

$$M^\Sigma = \langle f | \left((\Sigma^{el}_{LS} + \Sigma^{\gamma}) \frac{1}{\not\pi - m} \frac{e\not\epsilon}{\sqrt{2\omega_q}} e^{iq\cdot x} + \frac{e\not\epsilon}{\sqrt{2\omega_q}} e^{iq\cdot x} \frac{1}{\not\pi - m} (\Sigma^{el}_{LS} + \Sigma^{\gamma}) \right) |i\rangle + \langle f | \left(\Sigma^{el}_L \frac{e\not\epsilon}{\sqrt{2\omega_q}} e^{iq\cdot x} + \frac{e\not\epsilon}{\sqrt{2\omega_q}} e^{iq\cdot x} \Sigma^{el}_R \right) |i\rangle . \tag{2.4}$$

In writing Eq. (2.4), we have used a naive cancellation of $(\not\pi - m)$ against $(\not\pi - m)^{-1}$. This procedure is questionable because Eq. (2.2) implicitly involves intermediate states for which $\not\pi - m = 0$. It is known that a similar procedure must be carried out with some care to get the correct cancellation of wave function and vertex renormalizations in scattering problems. In the present case we have made a careful examination of this procedure, based on an expression for M as the imaginary part of the fourth-order self-energy operator.[9] The result is that the procedure we follow here, and our subsequent neglect of terms such as $\langle i|\Sigma|i\rangle$ or $\langle f|\Sigma|f\rangle$ when they occur, is valid to the order we are calculating. Further aspects of this approach are given in Ref. 9.

We note next that the contribution of the vertex graph 1(e) can, through Eq. (2.1), be written as

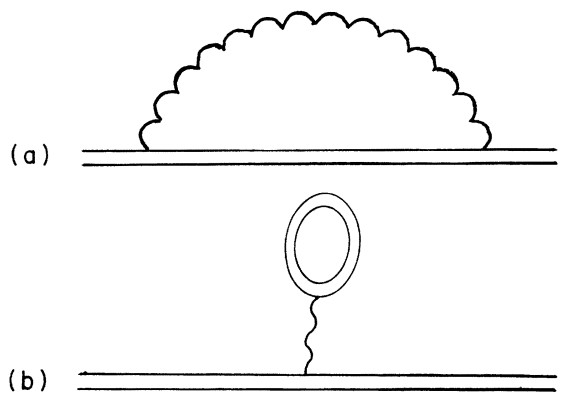

FIG. 2. (a) Electron self-energy operator Σ^{el}. (b) Vacuum polarization operator Σ^{γ}.

$$M^{\Lambda}=\langle f|\left(\frac{e}{\sqrt{2\omega_q}}\frac{d\Sigma_{LS}^{el}}{du}\bigg|_{u=0}-\frac{e\not{\epsilon}}{\sqrt{2\omega_q}}e^{iq\cdot x}\Sigma_R^{el}\right.$$
$$\left.-\Sigma_L^{el}\frac{e\not{\epsilon}}{\sqrt{2\omega_q}}e^{iq\cdot x}\right)|i\rangle\,. \qquad (2.5)$$

The terms involving Σ_R or Σ_L now cancel in $M=M^{\Lambda}+M^{\Sigma}$, as they should by gauge invariance, and we are left with the matrix element

$$M=\overline{M}^{\Sigma}+\overline{M}^{\Lambda}$$
$$=\frac{e}{\sqrt{2\omega_q}}\langle f|\left((\Sigma_{LS}^{el}+\Sigma^{\gamma})(\not{\pi}-m)^{-1}\not{\epsilon}e^{iq\cdot x}\right.$$
$$+\not{\epsilon}e^{iq\cdot x}(\not{\pi}-m)^{-1}(\Sigma_{LS}^{el}+\Sigma^{\gamma})$$
$$\left.+\frac{d}{du}[\Sigma_{LS}^{el}(\pi_\alpha-\epsilon_\alpha u e^{iq\cdot x})]_{u=0}\right)|i\rangle\,. \qquad (2.6)$$

The terms $\overline{M}^{\Sigma}$ in (2.6) are evaluated by inserting a complete set of eigenstates of the Dirac equation in the external Coulomb field, omitting, in accordance with the above remark, those terms corresponding to wave function renormalization effects:

$$\overline{M}_1^{\Sigma}=\frac{e}{\sqrt{2\omega_q}}\sum_n{}'u_f^\dagger\beta(\Sigma_{LS}^{el}+\Sigma^{\gamma})_f u_n(E_f-E_n)^{-1}u_n^\dagger\beta\not{\epsilon}e^{-i\vec{q}\cdot\vec{x}}u_i+\frac{e}{\sqrt{2\omega_q}}\sum_n{}''u_f^\dagger\beta\not{\epsilon}e^{-i\vec{q}\cdot\vec{x}}u_n(E_i-E_n)^{-1}u_n^\dagger\beta(\Sigma_{LS}^{el}+\Sigma^{\gamma})_i u_i\,. \qquad (2.7)$$

Here the sum $\sum_n'$ omits the state $n=f$ and the sum $\sum_n''$ omits the state $n=i$. The notation $(\Sigma_{LS}^{el}+\Sigma^{\gamma})_{i,f}$ implies that the operator, which is energy dependent, is to be evaluated at the energy of the state i,f. This rule follows from the fact that the external field is time independent, so the operator Σ carries the energy of the external line it acts upon.

The intermediate states that contribute to (2.7) include positive-energy states with the same orbital parity as i, and "negative-energy" states with opposite orbital parity to i. Of the positive-energy states, it is easy to see that the only ones that contribute to relative order α are the state $n=i$ in $\sum_n'$ and the state $n=f$ in $\sum_n''$. All other bound states, and the positive-energy continuum give contributions of higher order in $Z\alpha$. Hence the positive-energy states contribute

$$\overline{M}_+^{\Sigma}=\frac{e}{\sqrt{2\omega_q}}[u_f^\dagger\beta(\Sigma_{LS}^{el}+\Sigma^{\gamma})_f u_i\omega_q^{-1}u_i^\dagger\beta\not{\epsilon}e^{-i\vec{q}\cdot\vec{x}}u_i+u_f^\dagger\beta(\not{\epsilon}e^{-i\vec{q}\cdot\vec{x}})u_f(-\omega_q)^{-1}u_f^\dagger\beta(\Sigma_{LS}^{el}+\Sigma^{\gamma})_i u_i]+\text{higher-order terms}\,. \qquad (2.8)$$

Note that the energy denominators are equal and opposite ($\pm\omega_q$) in the two terms. Since the self-energy operator is rotation and reflection invariant, it can only link states with the same J, L. Therefore, to the order of interest, $\overline{M}_+^{\Sigma}$ only contributes to transitions between states of equal J. It is easy to prove that

$$u_f^\dagger\beta\not{\epsilon}e^{-i\vec{q}\cdot\vec{x}}u_f=u_i^\dagger\beta\not{\epsilon}e^{-i\vec{q}\cdot\vec{x}}u_i=(\omega_q/m)O(1) \qquad (2.9)$$

whatever the principal quantum number of f and i, provided that $J_i, L_i=J_f, L_f$. The corresponding off-diagonal matrix elements are order $(\omega_q/m)(Z\alpha)^2$, which is why those terms are neglected in (2.8). Therefore

$$\overline{M}_+^{\Sigma}\equiv\frac{e}{\sqrt{2\omega_q}}u_f^\dagger\beta[(\Sigma_{LS}^{el}+\Sigma^{\gamma})_f-(\Sigma_{LS}^{el}+\Sigma^{\gamma})_i]\,u_i\omega_q^{-1}u_f^\dagger\beta\not{\epsilon}e^{-i\vec{q}\cdot\vec{x}}u_f\,. \qquad (2.10)$$

In this expression, only terms in Σ that depend on energy will not cancel. To leading order, there are no such terms in Σ^{γ}, so that term cancels. Furthermore, the term proportional to $\alpha(Z\alpha)^4\ln Z\alpha$ in Σ_{LS}^{el} is independent of energy, and so also cancels. There are other terms in Σ_{LS}, of order $\alpha(Z\alpha)^4$, which produce the Bethe logarithm in the Lamb shift, and are energy dependent, and so survive. However, we shall see below that they cancel against part of the vertex contribution $\overline{M}^{\Lambda}$.

We consider next the contribution of negative-energy states to $\overline{M}^{\Sigma}$. For these states, the two energy denominators in 2.7 are approximately the same, and equal to $2m$. Therefore

$$\bar{M}_-^{\Sigma} \simeq \frac{e}{\sqrt{2\omega_q}} \sum_{n-} u_f^\dagger \beta(\Sigma_{LS}^{el} + \Sigma^\gamma) u_{n-}(1/2m) u_{n-}^\dagger \beta \not{\epsilon}\, e^{-i\vec{q}\cdot\vec{x}} u_i + \frac{e}{\sqrt{2\omega_q}} \sum_{n-} u_f^\dagger \beta \not{\epsilon} e^{-i\vec{q}\cdot\vec{x}} u_{n-}\,(1/2m) u_{n-}^\dagger \beta(\Sigma_{LS}^{el} + \Sigma^\gamma) u_i\,. \tag{2.11}$$

To the order of interest, we can approximate the sum over negative-energy states by the projection operator for a free negative-energy electron at rest. Therefore

$$\bar{M}_-^{\Sigma} \simeq \frac{e}{\sqrt{2\omega_q}} u_f^\dagger [\,\beta(\Sigma_{LS}^{el} + \Sigma^\gamma)\tfrac{1}{2}(1-\beta)(1/2m)\beta\not{\epsilon} e^{-i\vec{q}\cdot\vec{x}} + \beta\not{\epsilon} e^{-i\vec{q}\cdot\vec{x}}\tfrac{1}{2}(1-\beta)(1/2m)\beta(\Sigma_{LS}^{el} + \Sigma^\gamma)] u_i\,. \tag{2.12}$$

The vacuum polarization operator Σ^γ has a leading term proportional to $\beta\nabla^2 V$, an even operator, and therefore does not contribute to relative order α. However, the interaction of the external electric field with the "magnetic moment operator," i.e.,[10]

$$\Sigma_{mm}^{el} = ie\vec{\alpha}\cdot\vec{E}(\alpha/4\pi m)\,, \tag{2.13}$$

does contribute to this order:

$$\bar{M}_-^{\Sigma} = -\frac{i\alpha}{8\pi m^2}\frac{e}{\sqrt{2\omega_q}} u_f^\dagger [\not{\epsilon}\tfrac{1}{2}(1-\beta) e^{-i\vec{q}\cdot\vec{x}}\vec{\alpha}\cdot\vec{E} + \vec{\alpha}\cdot\vec{E}\tfrac{1}{2}(1-\beta)\not{\epsilon} e^{-i\vec{q}\cdot\vec{x}}] u_i \tag{2.14}$$

$$= \frac{i\alpha}{8\pi m^2}\frac{e}{\sqrt{2\omega_q}} u_f^\dagger [\vec{\sigma}\cdot\vec{\epsilon}\,\vec{\sigma}\cdot\vec{E} e^{-i\vec{q}\cdot\vec{x}} - \vec{\sigma}\cdot\vec{E}\,\vec{\sigma}\cdot\vec{\epsilon} e^{-i\vec{q}\cdot\vec{x}}] u_i$$

$$= \frac{i\alpha}{8\pi m^2}\frac{e}{\sqrt{2\omega_q}} u_f^\dagger [2i\vec{\sigma}\cdot\vec{\epsilon}\times\vec{E} e^{-i\vec{q}\cdot\vec{x}}] u_i\,. \tag{2.15}$$

The u_f and u_i in the last two lines are Pauli wave functions. To leading order, we expand the exponential in (2.15), and retain only the $-i\vec{q}\cdot\vec{x}$ term. Then

$$\bar{M}_-^{\Sigma} \simeq \frac{i\alpha}{4\pi m^2}\frac{e}{\sqrt{2\omega_q}} u_f^\dagger (\vec{\sigma}\cdot\vec{\epsilon}\times\vec{E})(\vec{q}\cdot\vec{x}) u_i\,. \tag{2.16}$$

This matrix element is easily evaluated for any specific states. If u_f and u_i are $S_{1/2}$ states, the angular integral is easily done, and gives

$$\bar{M}_-^{\Sigma}(nS, mS) \simeq \frac{i\alpha}{4\pi m^2}\frac{e}{\sqrt{2\omega_q}} \vec{\sigma}\cdot\vec{\epsilon}\times\vec{q}\,\tfrac{1}{3}\int g_{nS} V g_{mS} r^2\, dr\,.$$

From Eq. (2.6), the remaining vertex contribution $\bar{M}^\Lambda$ can be written as a derivative of Σ_{LS}^{el}. In our discussion above of Σ_{LS}^{el}, we have divided it into the magnetic-moment operator terms, and the Bethe terms. Correspondingly, the vertex operator will have two parts. The part coming from the derivative of the Σ_{mm}^{el} will give just the anomalous moment operator of a free electron, with corrections of relative order $(Z\alpha)$. This occurs because the deviations from the free-electron operator involve at least one extra power of the external Coulomb field E_i, which is of order $(Z\alpha)^3$, and this factor generates a correction of either $Z\alpha$ or $(Z\alpha)^3$, depending on whether the initial and final states are the same or different. The contribution of the anomalous magnetic moment to $\bar{M}^\Lambda$ is given by

$$\bar{M}_{mm}^{\Lambda} = \frac{e}{\sqrt{2\omega_q}} u_f^\dagger \beta \frac{\alpha}{8\pi m}[\not{\epsilon}, \not{q}] e^{-i\vec{q}\cdot\vec{x}} u_i\,. \tag{2.17}$$

As usual, this can be divided into two parts, by writing

$$[\not{\epsilon}, \not{q}] = 2(i\vec{\sigma}\cdot\hat{q}\times\hat{\epsilon} + \vec{\alpha}\cdot\hat{\epsilon})\omega_q\,.$$

Therefore,

$$\bar{M}_{mm}^{\Lambda} = \frac{e}{\sqrt{2\omega_q}}\frac{\alpha\omega_q}{4\pi m} u_f^\dagger \beta(i\vec{\sigma}\cdot\hat{q}\times\hat{\epsilon} + \vec{\alpha}\cdot\hat{\epsilon}) e^{-i\vec{q}\cdot\vec{x}} u_i$$

$$= \bar{M}_{mm,1} + \bar{M}_{mm,2}\,. \tag{2.18}$$

The second term, involving $\vec{\alpha}\cdot\hat{\epsilon}$, can be evaluated to leading order, by expressing the small components of u_i and u_f in terms of the large components. Again, for a transition between S states the angular integrals are simple, and yield

$$\bar{M}_{mm,2}^{\Lambda} \cong \frac{-i\alpha}{4\pi m^2}\frac{e}{\sqrt{2\omega_q}} \vec{\sigma}\cdot\vec{\epsilon}\times\vec{q}\,\tfrac{1}{3}\int g_{nS} V g_{mS} r^2\, dr\,, \tag{2.19}$$

which exactly cancels $\bar{M}_-^{\Sigma}$. It is possible to prove that this cancellation actually occurs for any transition between states of equal J, providing that the states i,f belong to different principal quantum numbers.

The other term in $\bar{M}_{mm}^{\Lambda}$ gets several contributions, from the product of large components multiplied by the first and third terms in the expansion of $e^{-i\vec{q}\cdot\vec{x}}$, and from the product of the small components. If the states i,f are S states belonging to the same principal quantum number, the term of relative order α comes only from the first of those contributions, and reduces to

$$\bar{M}_{mm,1}^{\Lambda}(nS, nS) = \frac{e}{\sqrt{2\omega_q}}\frac{\alpha}{4\pi}\frac{\omega_q}{m}\vec{\sigma}\cdot\hat{q}\times\hat{\epsilon}\,. \tag{2.20}$$

This is simply $\alpha/2\pi$ times the uncorrected matrix element for such a transition, corresponding just to the change in the total magnetic moment of the electron. These are the only terms of relative order α for $nS\to nS$ transitions.

On the other hand, for a transition between S states of different principal quantum number, all three terms are comparable, and add to zero. In other words, for the terms of relative order α,

$$\overline{M}^{\Lambda}_{mm,1}(nS, mS)=0 \quad \text{for } m\neq n\,. \tag{2.21}$$

This can be proven by direct evaluation of the matrix element to the required order. Alternatively, we can write $\overline{M}^{\Lambda}_{mm,1}$ for $S_{1/2}$ to $S_{1/2}$ transitions, as

$$\frac{ie}{\sqrt{2\omega_q}}\frac{\alpha\omega_q}{4\pi m}\vec{\sigma}\cdot\hat{q}\times\hat{\epsilon}\int r^2\,dr\,g_{mS}\times\left(1+\frac{p^2}{4m^2}-\frac{2}{3}\frac{p^2}{4m^2}-\tfrac{1}{8}q^2r^2\right)g_{nS}\,. \tag{2.22}$$

The sum of the first two terms gives exactly zero because of the orthogonality of the nS and mS states. The sum of the last two terms can be transformed, using a commutator identity, to a form involving only

$$\int r^2\,dr\,g_{mS}(H_{\rm NR}/m)g_{nS}\,.$$

This integral is of order $(\omega_{nS}/m)(Z\alpha)^2$, because of the approximate orthogonality of g_{nS}, g_{mS}, and the resultant contribution to $\overline{M}^{\Lambda}_{mm,1}$ is of relative order $\alpha(Z\alpha)^2$, and so is negligible to the order being considered. This vanishing of $\overline{M}^{\Lambda}_{mm,1}$ does not occur for transitions between $P_{1/2}$ states, because of the different angular wave functions involved.

We must finally consider the contribution to $\overline{M}^{\Lambda}$ coming from the even operator terms in $\Sigma^{\rm el}_{LS}$. These terms, which obtain contributions from low-energy virtual photons, have been analyzed by the methods of Erickson and Yennie.[5] The relevant terms in $\Sigma^{\rm el}_{LS}$ are those involving what these authors call I_{L_1} and I_{L_2}. Furthermore, the d/du operation required to calculate $\overline{M}^{\Lambda}$ from $\Sigma^{\rm el}$ need act only on the denominators in Erickson and Yennie's expressions for I_{L_1} and I_{L_2}. The result we get is that these terms exactly cancel the term $\overline{M}^{\Sigma}_{+}$, for all $nS_{1/2}$ to $mS_{1/2}$ transitions. That is, we obtain the result

$$\langle mS|\frac{d}{du}[\Sigma^{\rm el}(L_1+L_2)]_{u=0}|nS\rangle = -(1/\omega_q)\langle nS|\{[\Sigma^{\rm el}(L_1+L_2)]_{mS}-[\Sigma^{\rm el}(L_1+L_2)]_{nS}\}|mS\rangle \times u_{mj}\beta\not{\epsilon}e^{-i\vec{q}\cdot\vec{x}}u_{nj}\,, \tag{2.23}$$

where $\Sigma^{\rm el}(L_1+L_2)$ are the terms that give the self-energy contribution in (2.10), of relative order α. No simple reason for this exact cancellation is known to us. The details of the calculation and the results for transitions between other J states, will be published elsewhere.

The result of all of these considerations can be summarized simply. The total matrix element for a magnetic-dipole transition between $S_{1/2}$ states of hydrogenlike ions is given by

$$M(nS,nS)=M_0(nS,nS)(1+\alpha/2\pi) + \text{higher-order terms in } Z\alpha\,, \tag{2.24}$$

$$M(nS,mS)=M_0(nS,mS)[1+0\times\alpha/2\pi] + \text{higher-order terms in } Z\alpha\,. \tag{2.25}$$

Here, M_0 is the matrix element to lowest order in $Z\alpha$, without radiative corrections. M_0 is of order ω_q/m for $nS\to nS$ transitions, and of order $(Z\alpha)^2(\omega_q/m)$ for $nS\to mS$ transitions. The vanishing of the order-α correction for the $nS\to mS$ transitions appears fortuitous, and does not appear to occur for $nP_{1/2}$ to $mP_{1/2}$, or other transitions.

The experimentally more interesting cases of the decays $2\,^3S\to 1\,^1S+1\gamma$ in heliumlike ions have additional graphs to the hydrogenlike decays. Those of heliumlike decays will be discussed else where.

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[8]The usefulness of this identity was suggested to one of the authors by D. Yennie.

[9]J. Sucher and R. Barbieri (to be published).

[10]See Ref. 5, Eq. (3.13), p. 294; and Eq. (3.22), p. 297.