$$\omega_\perp^2(k) \to 2k^2 C_{44}(k)/mn, \quad \text{as } k \to \infty. \tag{B15}$$

For intermediate k, a simple interpolation formula was used:

$$\omega_\perp^2(k) = \frac{2k^2 C_{44}(k)}{mn} + \frac{\omega_\perp^2(0) - 2k^2[C_{44}(k) - n/\beta]/mn}{1 + k^2/k_0^2}, \tag{B16}$$

where the interpolation parameter was taken as $k_0 = 1.5$ Å^{-1}.

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Osmotic Pressure of Dilute Solutions of He^3 in He^4 †

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The osmotic pressure of dilute solutions of He^3 in liquid He^4 has been measured at 0.32, 0.65, and 1.16 °K for concentrations up to 2%. The technique does not require values of the He^4 fountain pressure and is substantially more precise than previous methods. The results are analyzed in terms of the effective-interaction model of Bardeen, Baym, and Pines as extended to nonzero temperatures by Ebner. The results do not agree well with the theory.

I. INTRODUCTION

Dilute solutions of He^3 in He^4 have been the subject of a great deal of experimental and theoretical effort in the past few years.[1] Bardeen, Baym, and Pines (BBP)[2] have successfully extended the Landau-Pomeranchuk[3] model of solutions to finite concentrations and low temperatures by means of an effective He^3 quasiparticle interaction V_k. Baym and Ebner[4] have calculated a V_k to fit the transport properties (spin diffusion[5] and thermal conductivities[6]) at low temperatures ($T \ll T_F$, where T_F is the Fermi temperature of the He^3 component of the solution). This V_k is in excellent agreement with that calculated from recently measured low-temperature static properties (heat of mixing,[7] phase separation curve,[8] and osmotic pressure[9]). Ebner has recently calculated a new V_k based upon exact solutions to the transport equations.[10] This V_k gives only fair agreement with the low temperature static properties.

The situation at high temperatures $(T > 2T_F)$ is much less complete. The interaction must be extended to the larger values of momentum transfer characteristic of the semiclassical He^3 quasiparticle system. Ebner[11,12] has used high temperature spin diffusion data to obtain a V_k which agrees well with the Baym and Ebner[4] and new Ebner[10] potentials at low k, and extends to larger k. He has used this V_k to calculate high temperature static properties such as the heat capacity, magnetic susceptibility, and osmotic pressure. Of these, the osmotic pressure is most sensitive to V_k.

The present work was undertaken to provide precise high temperature, $(T > 2T_F)$ low He^3 concentration $(X < 2\%)$ data on the osmotic pressure which could be compared with theory. Previous workers[13-15] have measured the osmotic pressure of solutions, but in a range of temperature and concentration unsuitable for meaningful comparison with Ebner's theory.

The osmotic pressure (π) was determined from measurements of the pressure of pure liquid He^4 in contact with a solution through a superleak. The solution and the pure He^4 were kept at the same temperature T. There was no vapor present in the pure He^4 chamber, so that it was free to assume a pressure $P(X, T) - \pi$ below the vapor pressure of the solution $P(X, T)$. In equilibrium, the chemical potentials of the pure He^4 and the He^4 component of the solution are equal[16,17]:

$$\mu_4(T, P, X) = \mu_4(T, P - \pi, 0). \tag{1}$$

Expanding the right-hand side of Eq. (1) gives

$$\mu_4(T, P, X) = \mu_4(T, P, 0) - \pi v_4^0 , \tag{2}$$

where v_4^0 is the molar volume of pure He^4, and where higher powers of π may be neglected in considering the present data. Eq. (2) serves as a definition of osmotic pressure, and reduces to van't Hoff's law:

$$\pi v_4^0 / XRT = 1 \tag{3}$$

for ideal solutions. Using the theoretical expressions[11,12] for $\mu_4(P, T, X) - \mu_4(P, T, 0)$, calculations of πv_4^0 were made and compared with experiment. The agreement between our data and calculations based on Ebner's V_k was poor. The data suggest that there is no form of V_k that will fit both the low and high temperature properties of solutions.

II. EXPERIMENT

The osmotic pressure of solutions of He^3 in He^4 was measured in the earlier experiments[14,15] by balancing it with the fountain pressure. Use of this technique would require a more accurate knowledge of the fountain pressure $(\int \rho s dT)$ than is currently available. In addition, the pure He^4 would have to be maintained at a slightly higher temperature than the solution, introducing uncertainties in the temperature of the solution at the point of contact with the pure He^4. Because of these problems, another method was used in these experiments.

The cell in which our measurements were made is shown in Fig. 1. Since it is constructed from a single piece of OFHC copper, both the pure He^4 (in the lower chamber) and the solution (in the upper chamber) are at the same temperature during the experiment. This prevents the generation of a fountain pressure between the two chambers. The osmotic pressure was determined from the pressure of the pure He^4 as measured with a commercial strain gauge transducer.[18]

The superleak connection between the upper and lower chambers was constructed[19] from a piece of 0.040-in. diam, $\frac{5}{16}$-in.-long Vycor[20] sealed into a brass tube with Stycast 2850GT Epoxy.[21] Calculations based upon Knudsen flow of the He^3 through the superleak indicate less than 1% of the He^3 in the solution chamber would diffuse into the lower

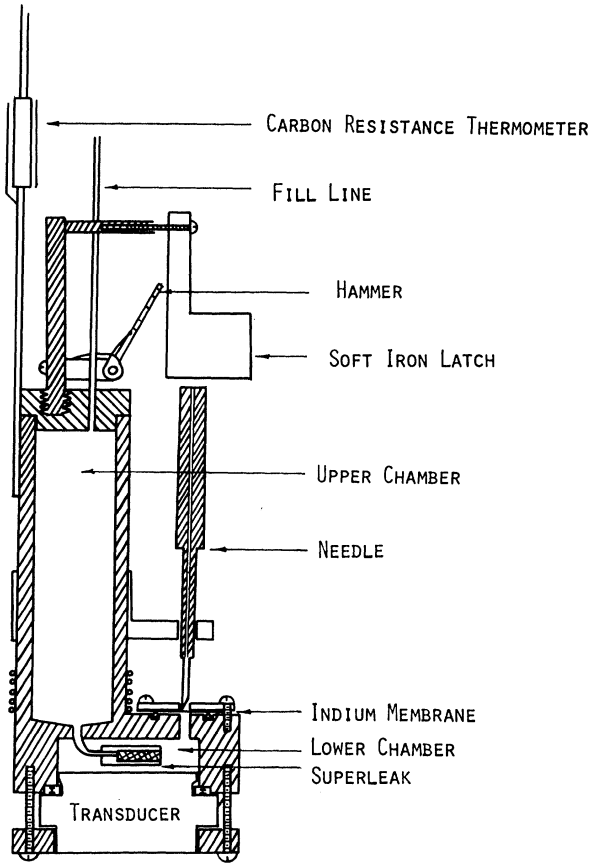

FIG. 1. Osmotic pressure cell.

chamber in 10 h. Since the solution was present for a maximum of 3 h, the effect was negligible.

The procedure for measuring osmotic pressure was as follows: A known quantity of He^4 from a room-temperature gas handling system was condensed into the cell when it was below 2°K. The lower chamber filled with liquid draining through the superleak. This process took approximately 45 min. A sufficient quantity of He^4 was condensed to fill both the lower chamber and approximately 80% of the upper chamber. After the He^4 was condensed, the transducer measured a pressure equal to

$$P_i = P(0, T) + h \, , \tag{4}$$

where $P(0, T)$ is the vapor pressure of pure He^4 and h is the hydrostatic head pressure of the He^4 above the transducer. Then, small amounts or "shots" of He^3 gas were allowed to condense into the upper chamber. The total amount of He^3 in the cell was determined after each shot from a room-temperature gas-handling system. After each shot, the pressure in the pure He^4 *decreased* to

$$P_f = P(X, T) + h' - \pi \, , \tag{5}$$

where $P(X, T)$ is the vapor pressure of the solution, h' is the hydrostatic head, and π is the osmotic pressure of the solution. Since very little He^3 (less than 2% of the He^4 in the upper chamber) was added in the course of an experiment, the cross-sectional area of the solution chamber was relatively large, and the density of the helium is small, $h' \approx h$. Therefore, when He^3 was added to the pure He^4 in the upper chamber, the transducer registered a change in the pressure of magnitude.

$$\pi' = P_i - P_f = P(0, T) - P(X, T) + \pi = -\Delta P + \pi \, . \tag{6}$$

The quantity $\Delta P = P(X, T) - P(0, T)$, the difference in vapor pressure between pure He^4 and the solution was determined from the vapor pressure measurements of Sydoriak and Roberts.[22] ΔP decreased from about 4% of π at 1.8°K to 0.5% of π at 0.65°K. Here π' is the magnitude of the stress applied to the pure He^4 by the osmotic pressure.

Depending upon the magnitude of X and T, the value of π was generally greater than $P(X, T) + h'$ in Eq. (5). The pressure in the pure He^4 was therefore negative. When the negative pressure reaches the tensile strength of pure He^4, the liquid will rupture, forming a bubble in the lower chamber. When this happens, the experiment must be terminated. The small tensile strength of liquid helium (less than 27 mm Hg) limited our measurements to $X < 2\%$.

The cell shown in Fig. 1 was used in two cryostats. One, a simple He^4 bath cryostat, was used to gather the data above 1°K. The cell was then placed in a He^3 cryostat, and two experiments above 1°K were run in order to confirm the earlier results. Then data were obtained at 0.32 and 0.65°K. The volume of both the upper and the lower chambers in the cell had to be accurately known in order to calculate the concentration of the solution from the known amounts of He^3 and He^4 condensed into it. The volume of both chambers was determined from room-temperature measurements, with the effect of thermal contraction at operating temperatures taken into account. The "insertion" of the transducer into the lower chamber was determined by measuring the position of the lower edge of the cell in relation to marks on the transducer body. These marks were a known distance from the sensitive diaphragm of the transducer. Since the thermal expansion and coefficient of the indium O-ring seal between the transducer and the lower chamber differed from that of the surrounding copper and stainless steel, the transducer "moved" when thermally cycled. This effect was minimized by measuring the position of the transducer before and after each run and averaging the results.

The volume of the upper chamber was checked experimentally by filling the cell with He^4 when it is above 2°K. The volume of the lower chamber could be determined experimentally by cooling the cell below 2°K and measuring the amount of additional He^4 necessary to fill both the upper and lower volumes. The quantity of He^4 necessary to fill the cell in each case was determined from the room-temperature gas handling system. Since the molar volume[23] of liquid He^4 is well known, the volume of each chamber could be determined. The volumes calculated from room-temperature measurements and the experimental values agreed to within 0.2%.

We define the term "upper volume" to include both the volume of upper chamber and the volume in the filling line which is at the same temperature as the cell. This definition is made because most of the dead volume (i.e., volume occupied by vapor), is at the temperature of the cell, since the volume of the fill line is small. This definition of the upper volume makes it easier to account for the amount of helium vapor in the dead volume and its effect upon the liquid concentration.

The amount of He^4 condensed into the cell was determined from the change in pressure in a known volume of He^4 at room temperature. The total amount of He^4 in the upper volume was determined from the total amount condensed by accounting for the amount of He^4 needed to fill the lower chamber. Small known quantities of He^3 were allowed to condense into the upper chamber after the cell had been stabilized at a predetermined run temperature. The total amount of He^3 in the upper chamber was determined from the change in pressure of He^3 in a known volume room-temperature gas handling system. The He^3 concentration in the liquid (X) could then be determined by accounting for the different amount of He^3 and He^4 vapor in the dead volume. The effect of the vapor was purposely minimized by allowing the upper volume to be 80% filled with He^4. This left plenty of room for the He^3 and did not allow the liquid level to enter the fill line [where h' would not equal h in Eqs. (4) and (5)].

The osmotic pressure is measured with a modified Consolidated Electrodynamics Corporation[18] strain gauge transducer, type 4-321-0002, 15 psia. This transducer measured the stress (π') applied to the pure He^4, from which the osmotic pressure (π) is determined from Eq. (6). The circuit used to observe the transducer output is shown in Fig. 2. The transducer is a four active arm strain gauge bridge. When pressure is applied to its sensitive surface, the resistance of two of the bridge arms increases and the resistance of the other two decreases. The output voltage of the transducer is a linear function of the pressure over the range of pressures used in this experiment. The output of the transducer was measured by balancing it with a known signal produced by a Leeds and Northrup Mueller bridge (using two 500 Ω standards as unknowns in that circuit). The off-balance signal was measured by a Gertsch TN1 null indicator. The Mueller bridge and the transducer were excited with identical secondaries on a 10 to 1 step-down transformer driven at 1 kHz by an audio oscillator. An attenuator on the primary of the transformer limited the power input of the transducer to $3\mu W$.

A No. 70 hole was drilled into the normally evacuated inner cavity of the transducer, permitting pressure calibration at 4.2 °K during each run. The calibration pressures were applied via He^4 gas in the exchange gas jacket surrounding the cell in the He^3 cryostat. This calibration technique was used because it forced the pressure sensitive diaphragm to move in the same direction (upward in Fig. 1) as does the osmotic pressure. Since the transducer had a temperature dependent output the data were gathered in a series of isothermal experiments in which changes in pressure were measured (i.e., π'), as the He^3 was added.

A problem which presents itself at the conclusion of an experiment is the removal of the He^4 in the lower chamber, before the pressure increase on warming could damage the transducer (maximum pressure ≈ 1 atm). It was impossible for the liquid to escape through the superleak once the cell is above 1.9 °K (the approximate λ point for He^4 in the fine channels of Vycor). The problem was solved by the use of the venting mechanism shown on the cell in Fig. 1. At the conclusion of the experiment, (that is, when the He^4 has ruptured) a large permanent magnet is brought up to the cryostat, causing a soft iron "latch" to rotate about its support screw. A spring loaded "hammer" then strikes a sharp "needle," driving it through a superleak-tight 0.005-in. thick indium membrane. It should be noted that this membrane was observed to anneal and become more difficult to break if allowed to remain at room temperature

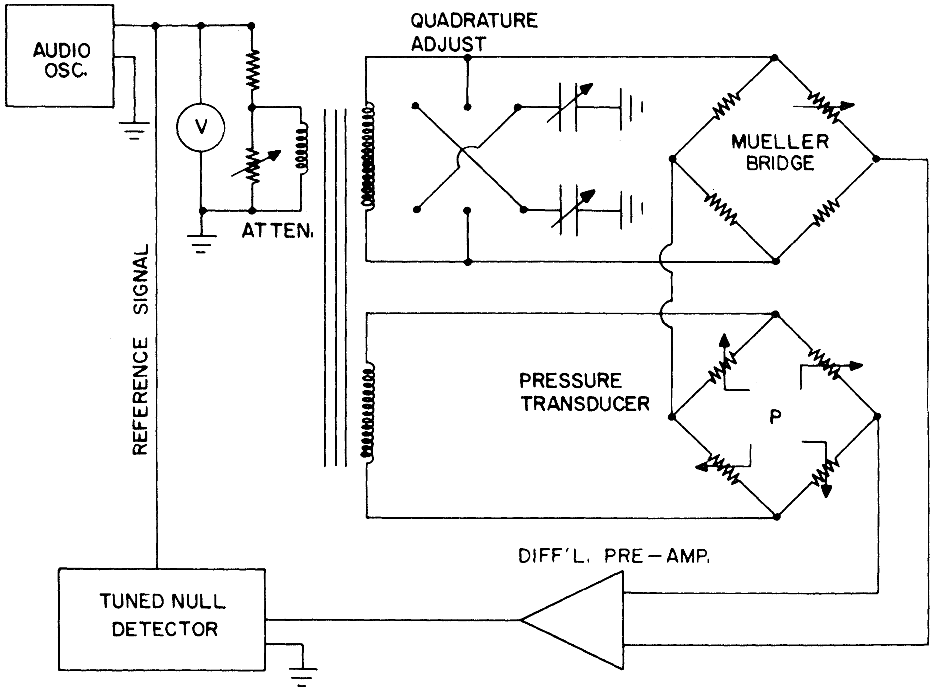

FIG. 2. System used to measure pressure transducer output signal.

for more than two weeks. The indium sheet was sealed to the cell with an indium O-ring and held in place with a copper covering plate, as shown in Fig. 1.

The temperature of the cell was determined from vapor pressure measurements on liquid He^3 and He^4 down to 0.6 °K. Below this temperature, a 220-Ω Speer carbon resistance thermometer was used. This carbon resistance thermometer was indium soldered to the cell as shown in Fig. 1, one electrical lead being the cryostat itself. The resistor was calibrated against the vapor pressure of He^3 and He^4 and extrapolated to 0.32 °K using the following form of the calibration:

$$\log_{10}R = A\left(\frac{\log_{10}R}{T+0.060}\right)^{1/2} + B. \qquad (7)$$

The validity of the above calibration at 0.32 °K was checked in a separate run in which the carbon resistance thermometer was compared with a chromium methylammonium sulphate magnetic thermometer.[24] The temperature of the cell was measured and regulated by means of a circuit similar to others discussed in the literature[25] and need not be discussed here.

III. DATA

The osmotic pressure data obtained in these experiments are displayed in Fig. 3. By plotting $\pi v_4{}^0/XRT$ versus X, the deviations from van't Hoff's law [Eq. (3)] are apparent. The data are grouped into four general temperatures: 1.80 °K, 1.16 °K, 0.65 °K, and 0.32 °K. The symbols at each temperature in Fig. 3 indicate individual isothermal experiments, the temperatures of which are within 10 m°K of the indicated general temperatures.

The first one or two osmotic pressure measurements in each run often appear several percent too high when compared with the ensuing data (see Fig. 3). We have not discovered the cause of this effect, but are convinced that these points are not representative of the solution osmotic pressure at the corresponding concentration. The basis for this statement is the fact that as $X \rightarrow 0$, van't Hoff's law must be true. Also, runs in which X is varied in smaller steps show $\pi v_0{}^4/XRT \rightarrow 1$ as $X \rightarrow 0$. A small nonlinear pressure region in the transducer characteristics was suspected as the source of the high initial points. A careful investigation of the pressure calibrations

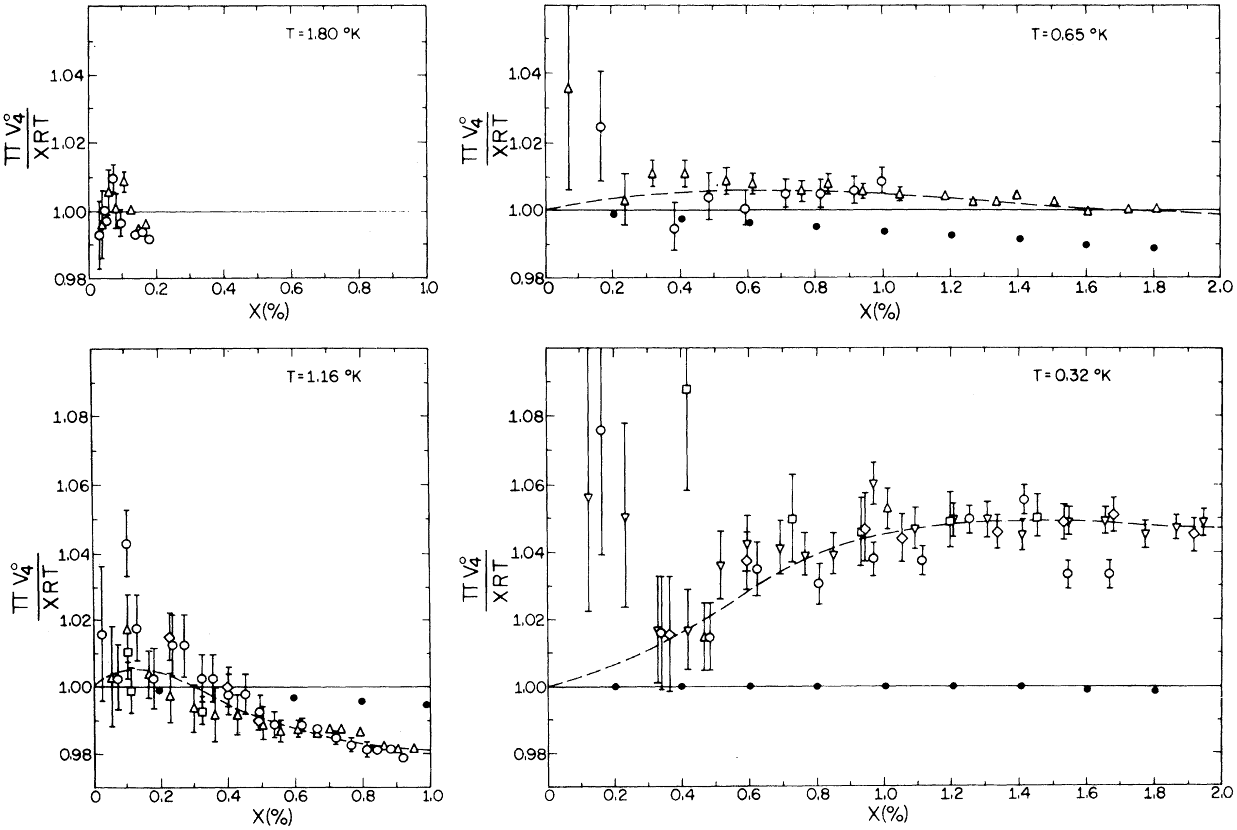

FIG. 3. Osmotic pressure π, displayed as $\pi v_4^0/XRT$ versus X, the He^3 concentration, at four temperatures. The solid circles are calculated from the theory of Ebner [Eq. (9)].

revealed that this was not the case. A check was made to determine the presence of any temperature dependence in the pressure calibration. No shift in the 4.2 °K calibration was observed at 2.4 °K. There could be a shift in the pressure calibration below 0.6 °K; this possibility could not be checked with the present apparatus.

IV. DISCUSSION

As was mentioned in the Introduction, the BBP theory has been quite successful in describing the properties of dilute solutions at low temperatures. According to BBP, the He^3 quasiparticle of effective mass m^* and of number density n_3 (equal to that of He^3 in solution) interact via a spin-and-velocity-independent effective potential $V(r)=\int V_k e^{i\vec{k}\cdot\vec{r}}\, d\vec{k}/(2\pi)^3$. Baym and Ebner[4] have used low-temperature spin diffusion[5] and thermal conductivity[6] data at 1.3% and 5.06% to fit V_k. In these experiments, $T \ll T_{F_0}$, where

$$k_B T_{F_0} = h^2 k_F{}^2/2m_0^* = (h^2/2m_0^*)\,(3\pi^2 n_3)^{2/3}, \tag{8}$$

and k_B is Boltzmann's constant, hk_F is the Fermi momentum of a quasiparticle, m_0^* is the effective mass for $X=0(m_0^*=2.34)$, and n_3 is the He^3 quasiparticle density. The low-temperature experiments determine V_k up to momentum transfer $k=2k_F(5\%)\approx 6\times 10^{+7}\ \mathrm{cm}^{-1}$.

At high temperatures $(T>2T_F)$, the statistics of the He^3 component of dilute solutions will be essentially classical, leading to quasiparticles with momenta $p\approx(3m_0^*kT)^{1/2}$. At $T=1$ °K one then has $k\approx 2p/\hbar \approx 10^8\ \mathrm{cm}^{-1}$ which is outside the Baym and Ebner V_k; Ebner has used high-temperature spin diffusion data[5] to extend V_k up to $k\approx 1.5\times 10^8\ \mathrm{cm}^{-1}$. The extended V_k also fits the high-temperature magnetic susceptibility[5] although this is not very sensitive to the interaction. Also, the susceptibility data is relative, and has been normalized to the theoretical susceptibility at $T=0$. Among the other high temperature static properties that may be compared with experiment, Ebner has calculated the osmotic pressure:

$$\frac{\pi v_4^0}{R} = TX\left\{1+\tfrac{1}{2}X(1-\alpha)+\tfrac{1}{3}X^2(1-\alpha+\alpha^2)+\frac{1}{6}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{T_{F_0}}{T}\right)^{3/2}\left[1+\tfrac{2}{3}X(1-\tfrac{1}{2}\alpha)\right]+\frac{2}{9\pi}\left(\frac{T_{F_0}}{T}\right)^{3}\left(1-\frac{16}{9\sqrt{3}}\right)\right\}$$
$$+\frac{n_4^0}{2k_B}\left(V_0-\tfrac{1}{2}\langle V\rangle\right)X^2\left[1+\tfrac{2}{3}X(1-2\alpha)\right]-\frac{1}{3}\left(\frac{2}{\pi}\right)^{1/2}X^2\left(\frac{T_{F_0}}{T}\right)^{3/2}\frac{n_4^0}{k_B}\left(\langle V\rangle-\langle V\rangle'\right). \tag{9}$$

Here, α is the BBP parameter defined by the empirical expression

$$n_3 = Xn_4^0(1+\alpha X)\ , \tag{10}$$

where $n_4{}^0$ is the number density of pure He^4. The quantities V_0, $\langle V\rangle$, and $\langle V\rangle'$ involve the interaction V_k. Here V_0 is the value of V_k at $k=0$, and is given by

$$V_0 = -\,0.081 m_4 s^2/n_4^0\ ; \tag{11}$$

whereas the Baym and Ebner[4] potential gives

$$V_0 = -\,0.064 m_4 s^2/n_4^0\ ; \tag{12}$$

where m_4 and s are the mass of a He^4 atom and the velocity of sound, respectively. This discrepancy at low k is a strictly accidental consequence of the parameters of the potential and is not necessitated by the observed behavior of the diffusion coefficient at high temperatures.[11]

The quantities $\langle V\rangle$ and $\langle V\rangle'$ in Eq. (9) are Boltzmann averages of V_k in momentum space, and

$$\langle V(T)\rangle = \langle V(\tfrac{4}{3}T)\rangle' = \frac{\int_0^\infty V_k k^2 dk \exp(-\hbar^2 k^2/4m_0^* k_B T)}{\int_0^\infty k^2 dk \exp(-\hbar^2 k^2/4m_0^* k_B T)}\ . \tag{13}$$

Equation (9) has been evaluated and is plotted in Fig. 3 as full circles. From this figure we see a significant discrepancy between theory and experiment. A more meaningful comparison can be made by noting that the first term on the right-hand side of Eq. (9) does not depend upon the effective interaction, and can be regarded as the osmotic pressure of an "ideal" noninteracting solution of Fermi quasiparticles. This term has been evaluated and subtracted from each data point, resulting in experimental values of the second and third terms. These terms are due to the presence of the He^3 quasiparticle interaction potential, and can be written

$$\pi_i V_4^0/R = X^2\left(\frac{n_4^0}{2k_B}\left(V_0-\tfrac{1}{2}\langle V\rangle\right)\right)$$
$$+X^3\left(\frac{n_4{}^0}{3k_B}\left(V_0-\tfrac{1}{2}\langle V\rangle\right)(1-2\alpha)\right)-\tfrac{1}{3}X^2\left(\frac{2}{\pi}\right)^{1/2}$$

$$\times \left(\frac{T_{F_0}}{T}\right)^{3/2} \frac{n_4^0}{k_B} \left(\langle V\rangle - \langle V\rangle'\right) . \qquad (14)$$

Note that since $(T_{F_0})^{3/2} \propto X$, the last term in Eq. (14) varies as X^3.

Equation (14) is evaluated and plotted in Fig. 4 along with the reduced data. The data differ from the theory in that it seems to contain a large X^3 term, whereas the X^3 term in Eq. (14) is quite negligible. This is demonstrated in Fig. 5 where $\pi_i V_4^0/X^2R$ is plotted against X. Assuming this discrepancy is within experimental error, we have taken a best $-X^2$ fit to the data in Fig. 5. This fit is shown in Fig. 4 and is in reasonable agreement with the data.

We have calculated values for $\langle V\rangle/|V_0|$, $\langle V\rangle'/|V_0|$, and $(V_0 - \frac{1}{2}\langle V\rangle)/|V_0|$ from the best $-X^2$ fit to the data by equating the coefficient of X^2 in Eq. (14) to the vertical intercept in Fig. 5, and the coefficient of X^3 to zero. These results are shown in Fig. 6. The discrepancy at 1.16 °K is perhaps not surprising since Ebner's theory does not account for any interactions between the He^3 quasiparticles and the He^4 excitations. The empirical temperature dependence of $\langle V\rangle$, $\langle V\rangle'$, and $(V_0 - \frac{1}{2}\langle V\rangle)$ is much stronger than predicted from Ebner's V_k. The values of $\langle V\rangle'$ given in Fig. 6 are not particularly meaningful since they are very sensitive to the X^3 dependence of $\pi_i V_4{}^0/R$ (the slope in Fig. 5), which has been taken as zero. The error bars on the $\langle V\rangle'$ data in Fig. 6 have been drawn considering only the estimated error in $\langle V\rangle$ used to evaluate $\langle V\rangle'$.

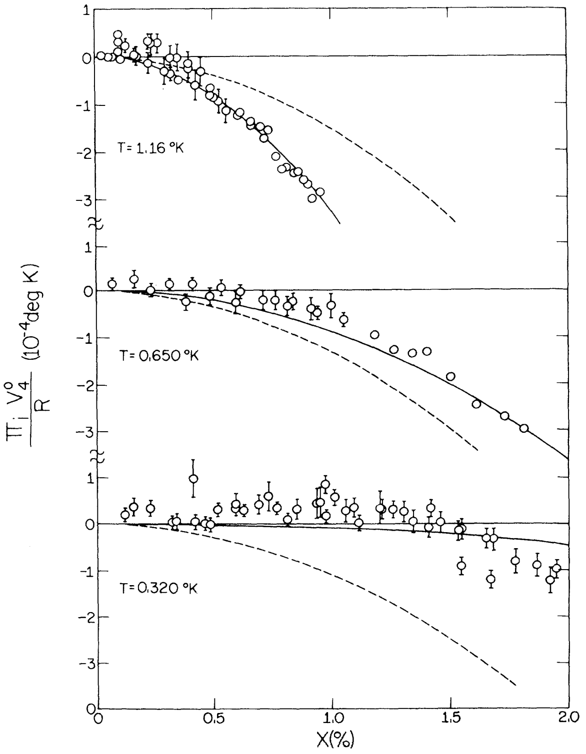

FIG. 4. Contribution to the osmotic pressure due to the effective interaction π_i, plotted as $\pi_i v_4^0/R$ versus X. The broken line represents Ebner's calculation [Eq. (14)]; the full line is the "best X^2 fit" to the data.

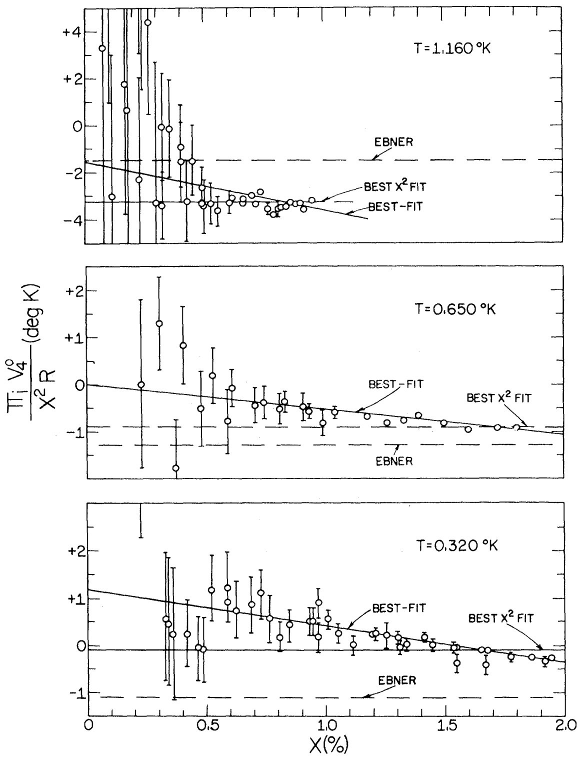

FIG. 5. Contribution to the osmotic pressure due to the effective interaction π_i, plotted as $\pi_i v_4^0/X^2R$ versus X. The broken line represents Ebner's calculation [Eq. (14)]; the full line is the "best X^2 fit" to the data.

Figure 6 demonstrates a rather drastic disagreement between theory and experiment. The significant fact, however, is that the agreement is not improved by a simple modification of V_k at high k (that is Ebner's extension of V_k). Although $\langle V\rangle$ is obtained from an integral of V_k over all k [Eq. (13)] a large fraction of this integral is determined by V_k in the low k region for the temperatures of interest here. Since V_k is quite well known in the low k region, our data suggest that the same V_k is not suitable for both the high- and low-temperature properties of dilute solutions of He^3 in He^4.

It is possible that the expression for the osmotic pressure of an "ideal" noninteracting solution [the first term on the right-hand side of Eq. (9)] given by Ebner is incorrect. Since this term is subtracted from the data to obtain π_i and then $\langle V\rangle$, an error here could produce large discrepancies

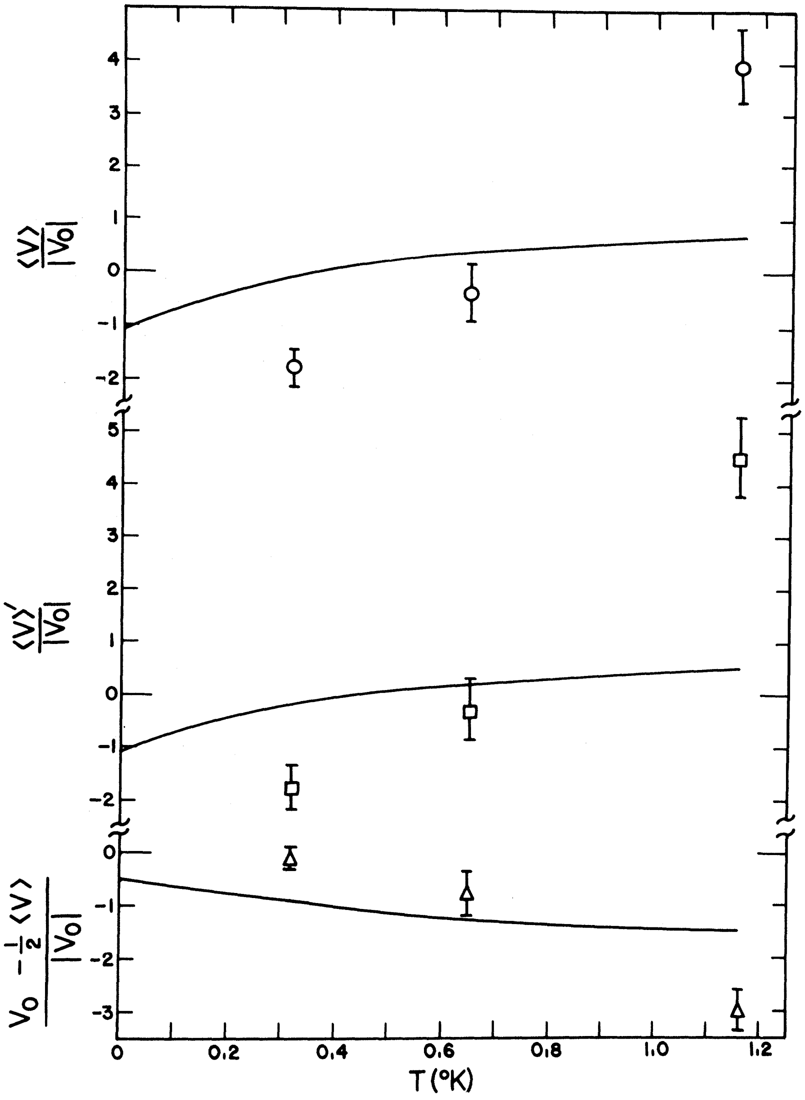

FIG. 6. Quantities $\langle V\rangle/|V_0|$, $\langle V\rangle'/|V_0|$, and $(V_0-\frac{1}{2}\langle V\rangle)/|V_0|$ defined in Eq. (13). The symbols ○, □ , and △ are the values of these quantities obtained from the "best X^2 fit" to the data and Eq. (14). The solid lines are values of these quantities obtained from Ebner's interaction V_k and Eq. (13).

between empirical and theoretical values of $\langle V\rangle$. It is not clear, however, what realistic modifications could be made to the "ideal" osmotic pressure term within the framework of the Landau-Pomeranchuk model.

The best agreement between the data and theory is at 0.65 °K. Since the data at 1.16 °K may be outside the range of validity of the theory (due to the He^4 excitations), it is possible that the only serious discrepancy is at 0.32 °K. We have attempted to determine the thermodynamic consistency of the 0.32 °K data with that at 0.65 °K using a simple model of solutions. Empirically[5,26] the specific heat of a dilute solution below about 0.4 °K is very close to that of an ideal Fermi gas $C_F(T, T_F)$, where T_F is as in Eq. (8) substituting $m^*(X)$ for m_0^*. If one then assumes that the molar entropy of the solution can be written

$$S=XS_F(T_F,T)+(1-X)S_4^0(T)\ , \tag{15}$$

where $S_F(T_F, T)$ is the ideal Fermi gas entropy, and $S_4^0(T)$ is the entropy per mole of pure He^4, then a purely thermodynamic calculation[27] leads to the temperature dependence of the osmotic pressure. Using $m^*(X)=m_0^*(1+X)$ for these low concentrations then gives

$$\pi(T_2,X)v_4^0(T_2)-\pi(T_1,X)v_4^0(T_1)$$
$$=B[U_F(T_F,T_1)-U_F(T_F,T_2)]\ , \tag{16}$$

where $U_F(T_F, T)$ is the energy of an ideal Fermi gas,[28] and

$$B=\frac{X^2}{1+X}-\frac{2}{3}\left(\frac{X}{1+\alpha X}\right)\ . \tag{17}$$

Using the dashed line in Fig. 3 to represent the data at 0.65 °K, Eq. (16) was used to compute the osmotic pressure at 0.32°K. The results were within 1% of the dashed line through the 0.32 °K data. Although this analysis is based on the assumption of Eq. (15), it does suggest that the data at 0.65 and 0.32 °K are mutually consistent.

We have also considered the consistency of our osmotic pressure data with the vapor pressure measurements of Sydoriak and Roberts,[22] which extend up from 0.6 °K. The vapor pressure data have been analyzed in terms of Ebner's[11] He^3 chemical potential to give empirical values of $\langle V(T)\rangle$. The results are quite sensitive to the value of E_3, the binding energy of a single He^3 atom in He^4. Using the value $(E_3 - L_3^0)/k_B = (0.312 \pm 0.007)$ °K obtained from the heat of mixing data[7] with $L_3^0/k_B = 2.472$ °K from the He^3 vapor pressure,[29] the values of $\langle V(T)\rangle$ determined between 0.6 and 0.9 °K are in excellent agreement with the osmotic pressure results in Fig. 6.

V. CONCLUSION

The static properties of dilute solutions of He^3 in He^4 at high temperatures are not consistent with an effective potential of the form proposed by BBP and extended by Ebner. The fact that the low-temperature effective potential can be successfully extended to high temperatures to describe the spin diffusion but not static properties suggests that the concept of the spin- and velocity-independent effective potential is not adequate to describe solutions completely.

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