Then $r_m^{(4,4)} \approx 2r_4$ and $r_m^{(4,3)} \approx r_3 + r_4$. Thus, $v_m^3/v_m^4 = (r_3/r_4)^3 [2r_m^{(4,3)} - r_m^{(4,4)}]/r_m^{(4,4)}$.

[14]C. Boghosian and H. Meyer, Phys. Letters 25A, 352 (1967); E. M. Ifft, D. O. Edwards, R. E. Sarwinski, and M. M. Skertic, Phys. Rev. Letters 19, 831 (1967); B. M. Abraham, O. G. Brandt, Y. Eckstein, J. Munarin, and G. Baym, Phys. Rev. (to be published).

[15]W. E. Massey and C.-W. Woo, Phys. Rev. Letters 19, 301 (1967).

[16]T. Davison and E. Feenberg, Phys. Rev. 178, 306 (1969).

[17]C.-W. Woo and W. E. Massey, Phys. Rev. 177, 272 (1969).

[18]H.-K. Sim and C.-W. Woo, Phys. Rev. 185, 401 (1969).

PHYSICAL REVIEW A VOLUME 1, NUMBER 2 FEBRUARY 1970

COMMENTS AND ADDENDA

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in Physical Review Letters and are not appropriate for regular Articles. It includes only the following types of communications: (1) comments on papers previously published in The Physical Review or Physical Review Letters; (2) addenda to papers previously published in The Physical Review or Physical Review Letters, in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section may be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.

Comments on the Calculation of Relativistic Bremsstrahlung Cross Sections*

H. K. Tseng and R. H. Pratt
Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania 15213
(Received 13 August 1969)

Recent attempts to calculate relativistic bremsstrahlung cross sections in the intermediate energy region are discussed, using results from an independent exact numerical calculation. The numerical work of Brysk, Zerby, and Penny gives quite good results for the bremsstrahlung cross section, differential only in photon energy, but significantly underestimates and overestimates the cross section, differential in photon energy and angle, in small-photon-angle regions and in large-photon-angle regions, respectively. The approximate analytical results of Elwert and Haug are quite good for low atomic number Z, but we can already see deviations for $Z=13$. For $Z=79$, their theory underestimates the cross section by large factors.

We wish to discuss two recent attempts[1,2] to calculate relativistic bremsstrahlung cross sections in the intermediate energy region. The difficulties in obtaining such results beyond Born approximation (the Bethe-Heitler formula) are well known and have been solved only for extreme relativistic energies, by Bethe and Maximon,[3] using analytical approximate high-energy electron wave functions. In the nonrelativistic case, results were obtained by Sommerfeld, Elwert, and others.[4] But a gap has remained in our predictions at intermediate energies (5 keV to 50 MeV) except for those cases in which Born approximation is valid ($\gamma_\beta \equiv 2\pi Z\alpha/\beta_2 \ll 1$). Recently, Brysk, Zerby, and

Penny[1] (BZP) have reported an attempt to fill the gap, calculating electron wave functions in a partial-wave expansion, while Elwert and Haug[2] (EH) have used approximate electron wave functions to obtain analytical results valid (neglecting screening) for all energies, at least for light Z elements. The natural question regarding BZP – as for any complicated numerical calculation – is whether the results can be verified in an independent calculation, whereas with regard to EH, one would like to ask for how high Z their formulas can be applied.

We wish to report answers to these questions based on an independent numerical calculation of bremsstrahlung cross sections which has been under way here. Our methods are similar to those used by Schmickley and Pratt[5] in the calculation of atomic photoeffect, and are based on the assumption that the atom is considered as a source of a static spherically symmetric field. Like BZP, the electron wave functions are calculated in partial-wave series. For simplicity, we confine our attention here to cases in which screening effects are small.

Figure 1 presents a comparison of our results (solid line) with the Born approximation[6] results (broken line); the results of EH[2] (crosses); and the results of BZP[1] (dotted broken line), for the case $Z=1$, $T_1 \equiv E_1 - m_e c^2 = 0.380$ MeV, $k = 0.228$ MeV, where E_1 is the energy of the incident electron and k the energy of the emitted photon. This case provides a check of our numerical calculations. The Coulomb parameter $\gamma_\beta = 7.2\times10^{-2}$ in this case; both our results and EH's results indicate that, as expected, the Born approximation is accurate with error of the order of 1%, the exact results larger than the Born approximation result. Define $\sigma(k) \equiv (k/Z^2)(d\sigma/dk)$. These results are presented in Table I. Our result is higher than that of Born approximation (as expected) by 0.8%, while the result of BZP is smaller than that of Born approximation by 1.2%. From Fig. 1, we conclude that our results are more accurate than those of BZP, particularly for radiation at small and large angles relative to the incident electron. For this low Z case, we find good agreement with the formula of Elwert and Haug (EH).

TABLE I. Comparison of bremsstrahlung differential cross-section values (differential in photon energy), for the case $Z=1$, $T_1=0.380$ MeV, $k=0.228$ MeV.

	Born	BZP	EH	Present theory
$\sigma(k)$ in mb	4.477	4.425	4.513	4.513

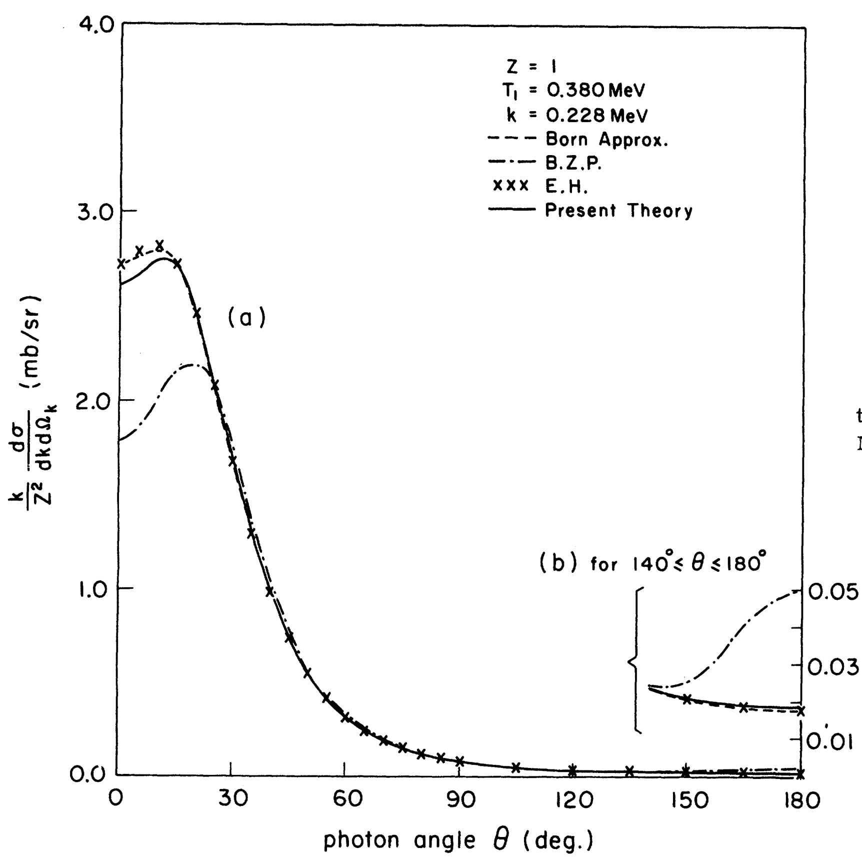

FIG. 1. Bremsstrahlung differential cross sections for $Z=1$, $T_1=0.380$ MeV, and $k=0.228$ MeV.

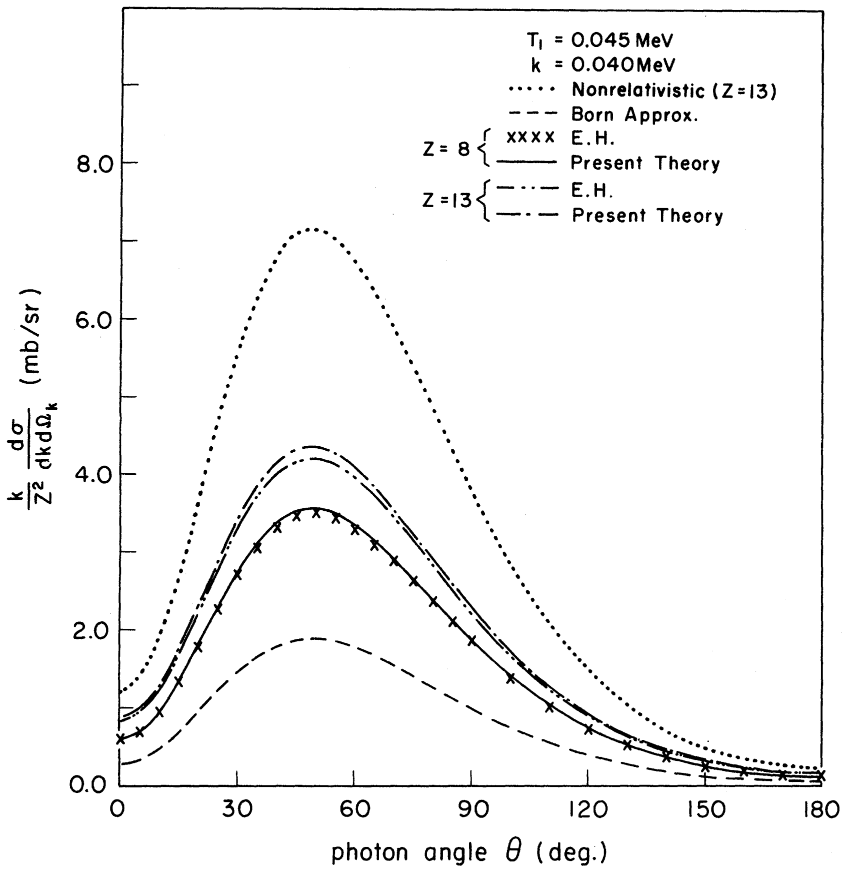

FIG. 2. Bremsstrahlung differential cross sections for $Z=8$ and $Z=13$, $T_1=0.045$ MeV, and $k=0.040$ MeV.

Figure 2 gives a comparison of our results (solid line for $Z=8$, single, dotted broken line for $Z=13$) with the Born-approximation results (broken line), the nonrelativistic results[2] for $Z=13$ (dotted line), and those of EH (crosses for $Z=8$, double dotted broken line for $Z=13$), for the cases $T_1=0.045$ MeV, $k=0.040$ MeV, with $Z=8$ and $Z=13$. The nonrelativistic results are poor since the initial electron velocity is quite relativistic ($\beta_1\approx 0.4$). The Born approximation is also poor since $\gamma_\beta=2.64$ for $Z=8$, and $\gamma_\beta=4.3$ for $Z=13$. For both $Z=8$ and $Z=13$, our results are quite close to those of EH but, as expected, agreement is better for $Z=8$. This shows the growing importance of the higher-order terms in Z, which the EH calculation neglects.

Figure 3 presents a comparison of our results (solid line) with the Born-approximation results (broken line), the results of BZP (dotted broken line), and the results of EH (crosses), for the case $Z=79$, $T_1=0.180$ MeV, $k=0.108$ MeV. The Born approximation is bad since $\gamma_\beta=7.5$. The results of EH are seen to be quite poor for high Z. Just as in the $Z=1$ case, we find that the BZP results are not good for small and large photon angles.

From these examples and from other similar cases which we have calculated, we conclude that the numerical calculation of Brysk, Zerby, and Penny gives reasonable results for the bremsstrahlung-cross-section differential in photon energy, but not for the bremsstrahlung-cross-section differential in photon energy and angle.[7] This type of difficulty suggests that important higher partial waves were either omitted or incorrectly calculated. We also conclude that the analytic expressions of Elwert and Haug, valid for low Z elements, unfortunately become quite poor for intermediate and high Z elements. We plan to present a more complete discussion of our results subsequently, and, at the same time, we hope to be able to discuss the effects of screening on bremsstrahlung cross sections.

We wish to thank the Computer Center of the University of Pittsburgh (partially supported by NIH Contract No. FR-00250), and Dr. M. J. Berger of the National Bureau of Standards for making computer time available to us.

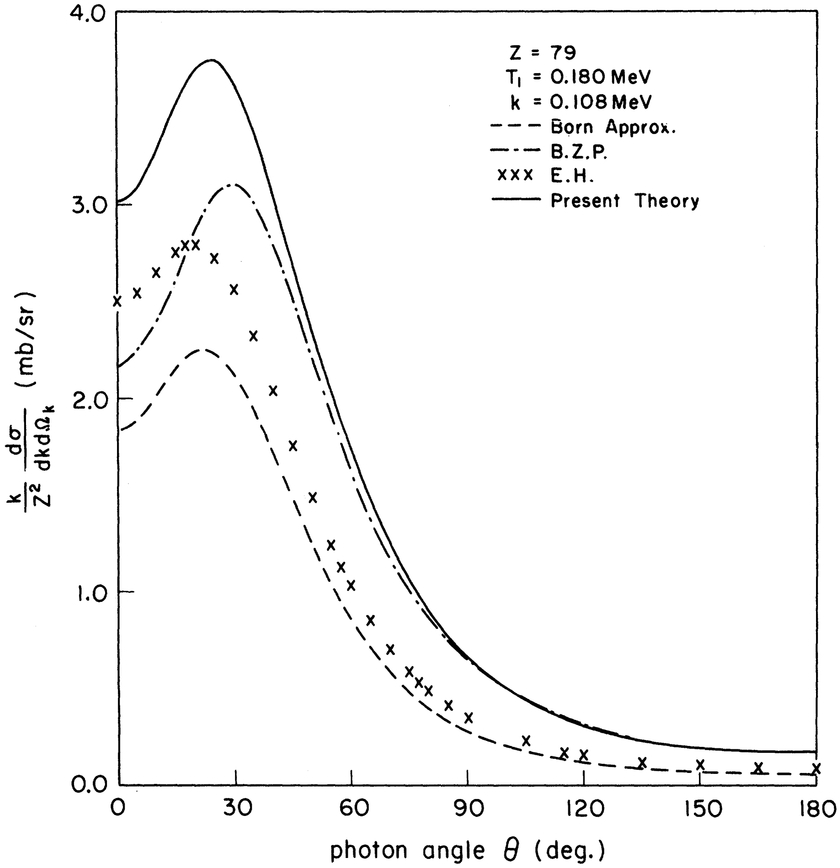

FIG. 3. Bremsstrahlung differential cross sections for $Z=79$, $T_1=0.180$ MeV, and $k=0.108$ MeV.

*Supported in part by the U.S. Atomic Energy Commission under Contract No. AT-(30-1)-3829.

[1]H. Brysk, C. D. Zerby, and S. K. Penny, Phys. Rev. 180, 104 (1969).

[2]G. Elwert and E. Haug, Phys. Rev. (to be published). We have written a computer code to obtain the needed cases from their formulas; it was verified that this code reproduces the results which EH presented.

[3]H. A. Bethe and L. C. Maximon, Phys. Rev. 93, 768 (1954).

[4]A. Sommerfeld, Ann. Phys. (N. Y.) 11, 257 (1931); G. Elwert, *ibid.* 34, 178 (1939); P. Kirkpatrick and L. Wiedmann, Phys. Rev. 67, 321 (1945).

[5]R. D. Schmickley and R. H. Pratt, Phys. Rev. 164, 104 (1967).

[6]H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) A146, 83 (1934); F. Sauter, Ann. Phys. (N. Y.) 20, 404 (1934); G. Racah, Nuovo Cimento 11, 461, 467 (1934).

[7]We understand that a similar conclusion has been reached by D. H. Rester and Q. Peasley, who have had access to the BZP codes (private communication).