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[1]R. S. Berry and S. E. Nielsen, preceding paper Phys. Rev. A 1, 383 (1970).
[2]S. E. Nielsen and R. S. Berry, Chem. Phys. Letters 2, 503 (1968).
[3]W. A. Chupka and J. Berkowitz, J. Chem. Phys. 48, 5726 (1968).
[4]W. A. Chupka and J. Berkowitz, J. Chem. Phys. (to be published).
[5]F. J. Comes and H. O. Wellern, Z. Naturforsch. 23a, 881 (1968).
[6]R. S. Berry, J. Chem. Phys. 45, 1228 (1966).
[7]J. N. Bardsley, Chem. Phys. Letters 1, 229 (1967).
[8]A. Russek, M. R. Patterson, and R. L. Becker, Phys. Rev. 167, 17 (1968).

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Mechanism of Ultrasonic Cavitation Nucleation in Liquid Helium by Quantized Vortices*

P. M. McConnell,† M. L. Chu, Jr.,‡ and R. D. Finch

University of Houston, Houston, Texas 77004

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The interaction between an ultrasonic field and one or a few quantized vortex lines in He II was investigated using the audible cavitation threshold as a means of detection. Experiments involved both rotating ultrasonic buckets and two shafts, each with a paddle attached, rotatable in the same or opposite sense near an ultrasonic field. Histograms of cavitation threshold measurements were not Gaussian, but appeared to be bimodal. The high threshold mode was most frequent in the quiescent state, but the lower mode was most frequent with rotation. The threshold values were found to fluctuate with time both with and without rotation. Slight and gradual reductions of audible threshold were observed in the rotating bucket sometimes, beginning at speeds less than that predicted by the Arkhipov-Vinen formula. Reductions at speeds higher than the Arkhipov-Vinen value were not directly proportional to the total number of vortices believed to be present in the bucket. Two shafts with paddles rotating in opposite senses caused larger reductions in threshold than a single shaft and paddle; but this was not the case when the shafts rotated in the same sense.

I. INTRODUCTION

Of the possible sources of cavitation nuclei (stabilized pockets of undissolved gas or vapor, cosmic rays or radioactivity, and free vortices), the most plausible one for He II is the vortex.[1-4] Finch and Chu[2] found that rotation of a shaft with an attached paddle above a critical speed near an ultrasonic field lowered the audible threshold of He II. This critical speed corresponded to that required for quantized circulation around the shaft, i.e.,

$$\omega_c = \hbar / m a^2 \quad , \tag{1}$$

where a is the shaft radius. It was concluded that a single free-vortex "tail" migrated into the sound field giving rise to the threshold reduction. An alternative suggestion by Hsieh[5] was that ring vortices peel off the end of the shaft and penetrate into the sound field.

Using Dean's[6] free-vortex model, Finch and Chu[2] pointed out that the tension required to rupture a singly quantized vortex core was not much less than that needed to break the molecular bonds. Thus, it was difficult to understand how a single core could act as the cavitation nucleus, and the nature of the vortex-ultrasound interaction remained in question. Recently, the suggestion has been made by McCloud[4] that the raveling of a vortex filament into a ball could result in energy concentrations sufficient to account for the observed values.

The primary objective of the present work was the elucidation of the vortex-ultrasound interaction. It was, therefore, necessary to know the vortex population within the sound field, as well as the core structure and behavior. Two schemes were devised to generate and sustain superfluid vorticity: (i) uniform rotation of a cylindrical bucket; and (ii) rotation of two shafts, each with an attached paddle, in the same and opposite sense. At the same time, the threshold reduction affords a method for detecting the speed or range of speeds at which the first vortices enter the superfluid in a rotating bucket. In performing these experiments it was decided that a detailed study of the statistical variation of the threshold, as well as its time dependence, should be made.

II. CRITICAL SPEEDS

In a rotating cylindrical bucket there is strong evidence that the vortices are singly quantized lines.[7] The speed at which the first line is thermodynamically favorable, ω_{c1}, is given by the Arkhipov-Vinen (AV) formula

$$\omega_{c1} = [\hbar/m(R^2 - a^2)] \ln(R/a) = \omega_0 \ln(R/a) \quad , \quad (2)$$

where R is the inside radius of the bucket, and a is the radius of the vortex singularity positioned along the bucket axis.[8,9] Measurements of angular momentum in He II by Hess and Fairbank[10] have confirmed Eq. (2) within experimental error. Since the core radius is usually much smaller than the bucket dimension,

$$\omega_0 \approx \hbar/mR^2 \quad .$$

If there were a vortex present in the bucket at this speed, then there would be synchronism between the bucket wall and an adjacent superfluid layer.

The following heuristic argument indicates that the critical speed found by Finch and Chu [Eq. (1)] is not inconsistent with the AV speed, since when a shaft is rotated uniformly in He II, it might be assumed that only a thin layer of superfluid adjacent to the shaft will, at first, participate in the rotation. The limiting form of Eq. (2), when R approaches a, may then be used to predict the lowest speed ω_s at which quantized circulation around the rotating shaft is thermodynamically favorable; i.e.,

$$\omega_s = \lim_{z \to 1} [\hbar/ma^2(z^2 - 1)] \ln z \ ,$$

where $z = R/a$. Applying L'Hopital's rule,

$$\omega_s = \hbar/2ma^2 \quad .$$

The condition of quantized circulation must also be satisfied around the shaft, and the lowest speed at which the circulation around the periphery of the shaft is *both favorable and quantized* is twice ω_s, namely, that given by Eq. (1). It does not follow that a vortex will necessarily be present at rotation speeds immediately above the thermodynamically favorable value, nor that vortices will be entirely absent at speeds below that value, depending on the experimental situation. The mechanism of creation of vortices *de novo* has been discussed by Vinen[11] and Iordanskii.[12]

III. APPARATUS AND TECHNIQUE

A. Ultrasonic Bucket

The general experimental scheme is shown in Fig. 1. A cylindrical PZT-4 driver (38.2 mm outer diam by 38.2 mm long) was slipped over a Pyrex cylinder, and the annular gap was packed with silicone grease which hardened at helium temperatures to provide good acoustical impedance matching. The inside radius, wall thickness, and length of the bucket were 14.1, 1.9, and 292 mm, respectively. A Teflon bearing located about 20

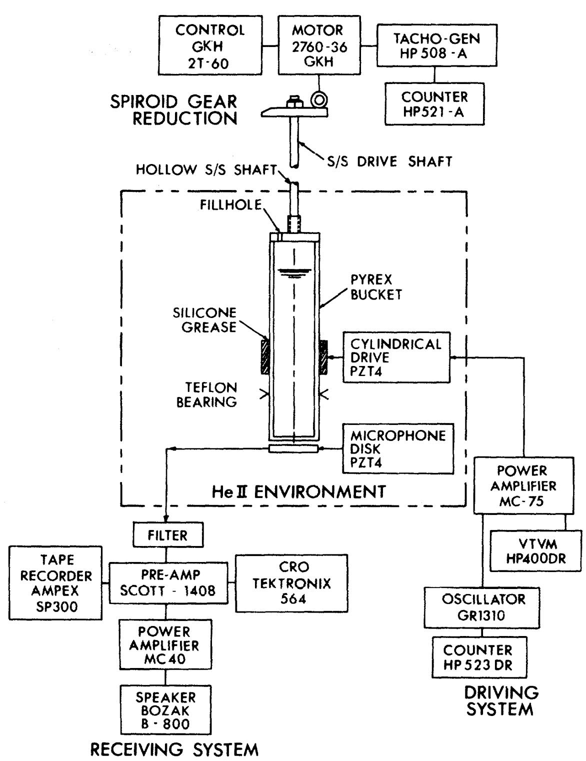

FIG. 1. Schematic diagram of ultrasonic bucket and circuitry.

mm above the bottom of the bucket reduced wobbling. To eliminate possible communication between the outside and inside baths, except through Rollin film flow, which was presumably small,[13] the top of the bucket extended into the helium vapor.

During all experiments using the glass bucket in He II, an unexpected difference between the levels inside and outside the bucket was observed, the inner being the higher. This difference was observable a few minutes after the outer level fell below the top of the bucket and increased to values which were typically about 50 and occasionally 80 mm. Since Rollin film flow would be in a direction out of the bucket (higher to lower level) with rates much too slow to account for the observed level difference,[13] the phenomenon was concluded to be caused by different rates of evaporation from the two volumes. The effect was beneficial in that the inner head remained nearly constant, reducing possible threshold variations due to changes of the static pressure.

The intensity of the acoustic field inside the bucket was investigated in liquid nitrogen by observing the response of a 3.18-mm-diam by 3.18-mm-long probe microphone[14] positioned at various points for each of seven horizontal planes. The maximum microphone response, located near an inner wall and about 25 mm from the top of the driving transducer, was found to be 1 order of magnitude higher than that of the outer bath. A transmission loss of about 12 dB through the bottom of the bucket should occur for a frequency of 10 kHz, but this loss would decrease at lower frequencies. At frequencies of 50 kHz and voltages of 3 to 10 V, displacements and velocities of the driving transducer wall were expected to be of the order 1 μ and a few mm/sec, respectively.

Two electrical leads for the cylindrical driver, both of which were soldered to its outer surface, passed through a hollow stainless-steel shaft to mercury dishes located outside the Dewar. This system proved to be very quiet since no noise other than motor noise at high armature speeds was detectable. A PZT-4 ceramic-disk microphone, whose resonant frequency was near 90 kHz and whose amplified response drove a Bozak 800 loudspeaker, was used in each experiment. Since only relative effects of various rotational speeds were of interest, the microphone voltage was not calibrated.

From Eq. (2), the critical speed for stability of the first vortex in the bucket ($R = 14.1$ mm) was 1.4×10^{-3} rad/sec. Such speeds were achieved by using a spiroid gear reduction of the motor. The armature speed of the motor could be monitored each second on a frequency counter whose input was provided by a tachogenerator. Vibration isolation was achieved by mounting the entire gear reduction system on a frame independent of the cryostat and coupling the vertical drive shaft to the loaded shaft with a short heavy piece of rubber hose.

B. Double Shafts

A plane-wave system, similar to that used by Finch and Chu and whose resonant frequency was about 89 kHz, was used in the rotating-shaft experiments. A paddle blade (10 by 50 mm rectangle, 1.0 mm thick) was epoxied directly to the bottom of each shaft, as shown in Fig. 2. A pulley and belt transmission located outside the Dewar was used, so that in one helium transfer it was possible to rotate a single shaft or two shafts in the same or opposite sense. Both the shaft and bucket arrangements were mounted on two thin-wall stainless-steel tubes and positioned in a standard-glass double Dewar.

C. Procedure

At the beginning of a typical experimental run, the Dewar was filled with liquid helium and the bath temperature was cooled from 4.2 to 2.16 ± 0.001 °K by pumping. Once the bath temperature stabi-

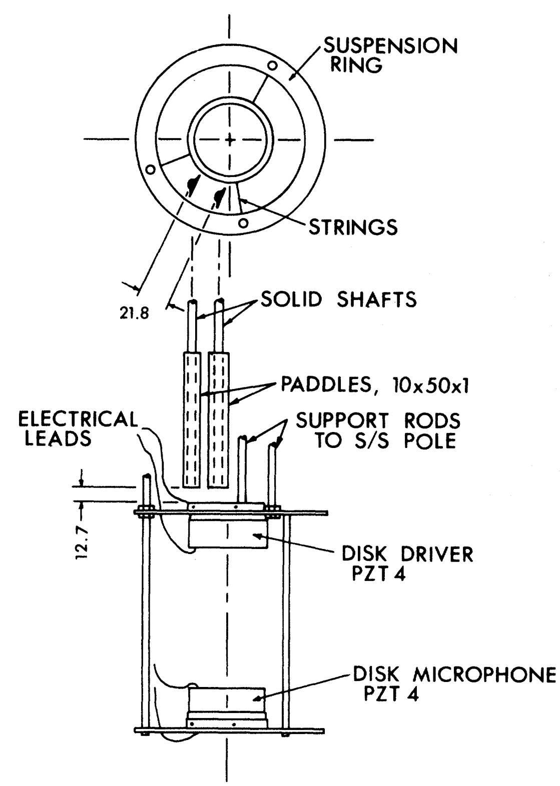

FIG. 2. Schematic diagram of the plane standing-wave system with double shafts and paddles. The resonant frequency of this system was about 89 kHz. Dimensions in mm.

lized it was necessary to select a working frequency for the driving transducer. The criterion for this selection was the most cavitation noise for a given driving voltage. This frequency was held constant to within 1 part in 10^5 throughout an experiment. As the voltage input to the driving transducer was increased (0.5 to 1.0 V per sec), a point was reached at which the audible cavitation threshold was detected using the loudspeaker. After this threshold voltage was observed, the driving voltage was quickly reduced to zero. The time between each data point for a fixed set of conditions was typically 5 to 10 sec. Sets of data were first taken in the absence of rotation. After the zero-rotation threshold was established, rotation was initiated, starting with lower angular velocities. In experiments to determine critical speeds, rotation rates were increased between each run of 6 to 35 readings. Waiting times of 15 to 45 min were allowed between each speed. It might be pointed out that according to Careri *et al.*,[15] vortices would decay within seconds at the temperatures used in these experiments. The zero-rotation threshold was occasionally taken after the rotational runs and after warming the bath above T_λ and then cooling back to the operating temperature.

IV. RESULTS

A. Critical Speeds in the Bucket

Experiments were performed to determine the critical speed of threshold lowering in the rotating cylindrical bucket. Variations of this experiment were repeated some 12 times, and a typical set of results is shown in Fig. 3, each point representing the arithmetic mean of a number of readings (shown at the right of each point). The dispersion in each set of readings is represented by a bar. The averaged data show a slight threshold reduction most clearly apparent at speeds above ω_{c1}. The general trend of this lowering near ω_{c1} was reproducible, but a distinct critical speed was not apparent. Frequently, reductions in threshold occurred at speeds between ω_0 and ω_{C1}. The effect on the critical speed of two thin blades protruding from the inner wall was investigated. Taking threshold measurements with increasing or decreasing voltages revealed hysteresis in the threshold but did not influence the "sharpness" of the reductions with respect to angular velocity. Data were taken for one run in which the bucket was rotated at a given speed for about 5 min and then stopped while the threshold measurements were made, and also for one run in which the bucket was rotated continuously while measurements were made. The protuberances inside the bucket had no obvious effect on the critical speed of threshold lowering, a result similar to that found by Finch and Chu for a rotating shaft and paddle. These results are shown in Fig. 4.

B. Large Sample Studies

The statistical variation of the audible threshold was studied by taking large samples of data; in some instances up to 200 threshold readings were taken for a fixed set of conditions. Results from the large sampling technique in the bucket with and without rotation, Fig. 5, clearly show a spread in the data from 6.2 to 9.0 V. However, with rotation, the frequency of occurrence of lower threshold values is enhanced. The distributions are skewed and appear to be roughly bimodal. The arithmetic mean of both sets of data is, however, nearly the same. The time variation of these data over a 3-h period is shown in Fig. 6, where each data point represents an average of a set of six readings with a 2-min waiting time allowed between each set. There was a cyclic raising and lowering of the audible threshold in time, with and without rotation, the period being roughly 120 to 3600 sec. Large samples of threshold data were taken at higher angular velocities of the bucket and similar results were obtained (see Fig. 7). At speeds 100 times ω_{c1}, the range of threshold values has been shifted and similar results were obtained (see Fig. 7). At speeds 100 times ω_{c1}, the range of threshold values has been

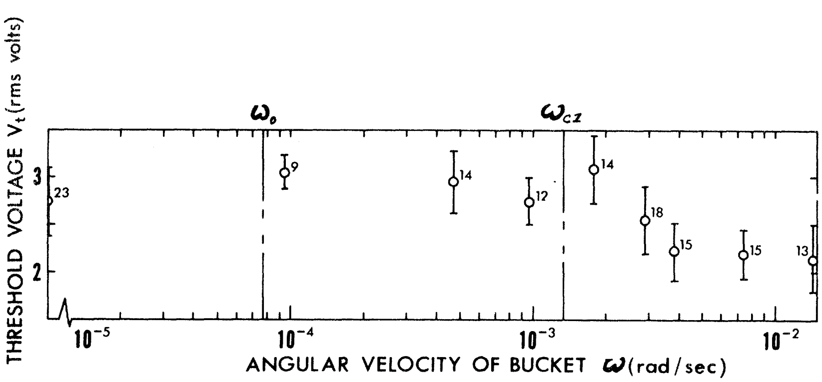

FIG. 3. Variation of threshold voltage with rotation speed of a cylindrical glass bucket (R = 14.1 mm; T = 2.08 °K; f_r = 52 088 Hz).

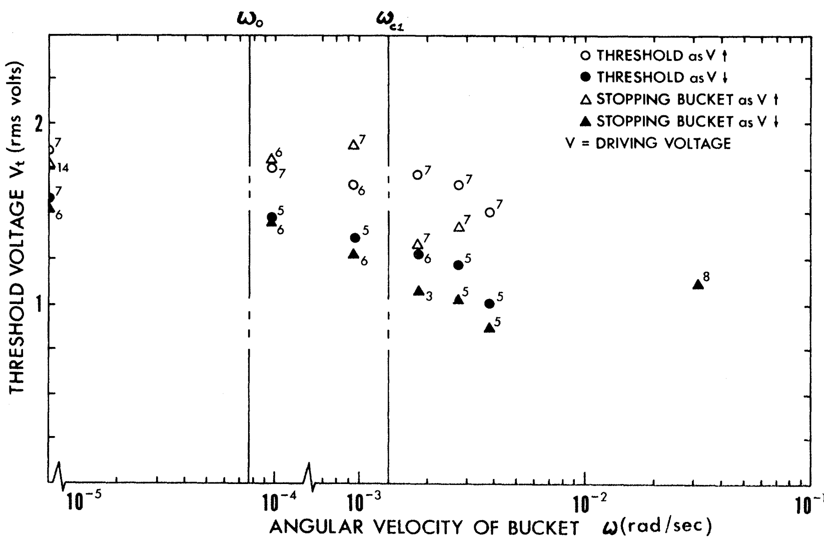

FIG. 4. Variation of threshold voltage with rotation speed of the cylindrical bucket ($R = 14.1$ mm) with two thin blade protuberances on the inner wall; blade width = 3.1 mm; $T = 2.09\,^\circ$K; $f_r = 50\,958$ Hz.

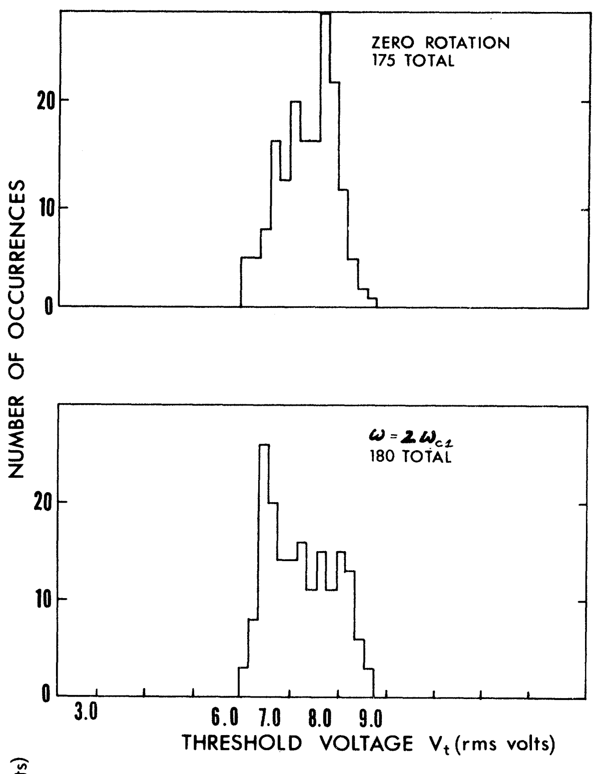

FIG. 5. Large samples of threshold taken in the glass bucket ($R = 14.1$ mm) with and without rotation; $T = 2.11\,^\circ$K; $f_r = 144\,247$ Hz.

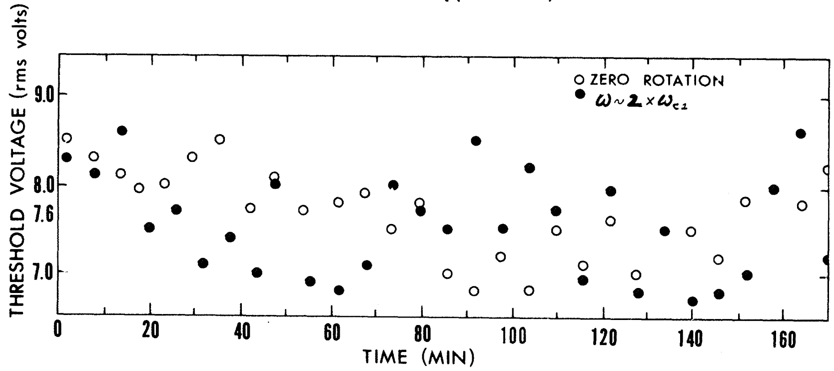

FIG. 6. Time variation of audible threshold taken in the glass bucket ($R = 14.1$ mm); $T = 2.11\,^\circ$K; $f_r = 144\,247$ Hz. Data points represent averages of six readings with a 2-min wait between each set.

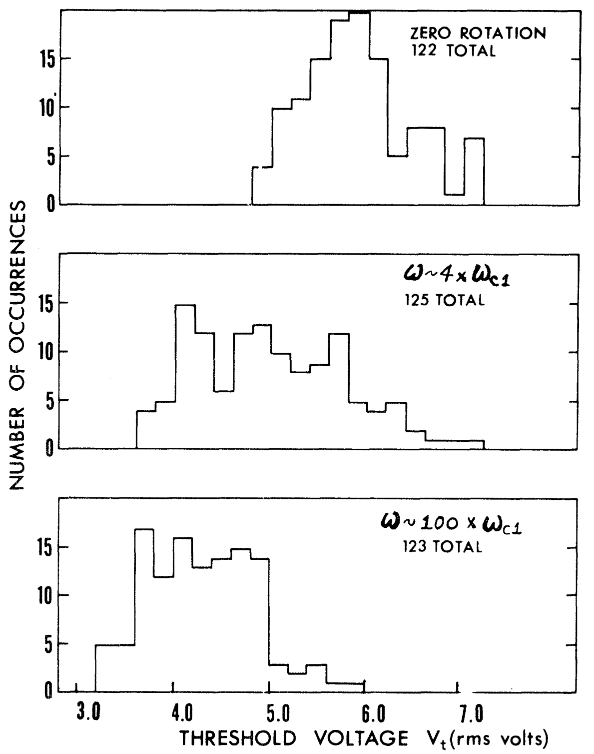

FIG. 7. Large samples of threshold taken in the glass bucket ($R = 14.1$ mm) with and without rotation; $T = 2.10\,°$K; $f_r = 143\,703$ Hz.

shifted to lower values, but the reductions are not directly proportional to the total number of lines believed to be present, since at this speed the bucket should contain approximately 1200 lines.[7] The large sampling technique was also applied in a repetition of Finch and Chu's original experiment with a plane acoustic field and single paddle. The histograms in this case (Fig. 8) are also skewed with secondary peaks occurring near 5.0 V both with and without rotation. Rotation above ω_c clearly has a marked influence on the generation of lower thresholds. This result supports the findings of Finch and Chu for the plane wave system, i.e., the occurrence of sharp threshold drops.

C. Rotation of Double Shafts

Data from the plane-wave system with two rotating shafts and paddles are shown in Fig. 9. The threshold values with this arrangement are higher than those in the previously mentioned experiment, probably due to a slight misalignment between the transducers, but since the objective was to determine relative changes in the threshold the conclusions are not affected. With shafts rotating in the same sense [Fig. 9(d)], the modal value is only slightly less than that for zero rotation. The data from the single shaft and two shafts rotating in the opposite sense [Figs. 9(b) and 9(c)], however, show a substantial increase in the number of occurrences of low thresholds. There is an indication that two shafts rotating in the opposite sense have the greatest ability to generate lower thresholds. The above results were reproducible. It should be noted that the helium head in the case of zero rotation was less than that in the runs with two shafts, which strengthens the conclusion that there is a marked drop between the two cases since the threshold tends to increase with static pressure.

V. DISCUSSION

The occurrence of threshold reductions in the cylindrical bucket rotating near the AV speed indicates that one or a few vortex lines are effective nucleation agents. By the same token, further evidence is added to that of Hess[10] and Packard and Sanders[16] as to the validity of the AV theory. From the experiments at high rotation speed (Fig. 7), it seems that one or a few vortices have nearly the same effect on the audible threshold as thousands with the same circulation. Consistent with these gradual threshold drops observed in

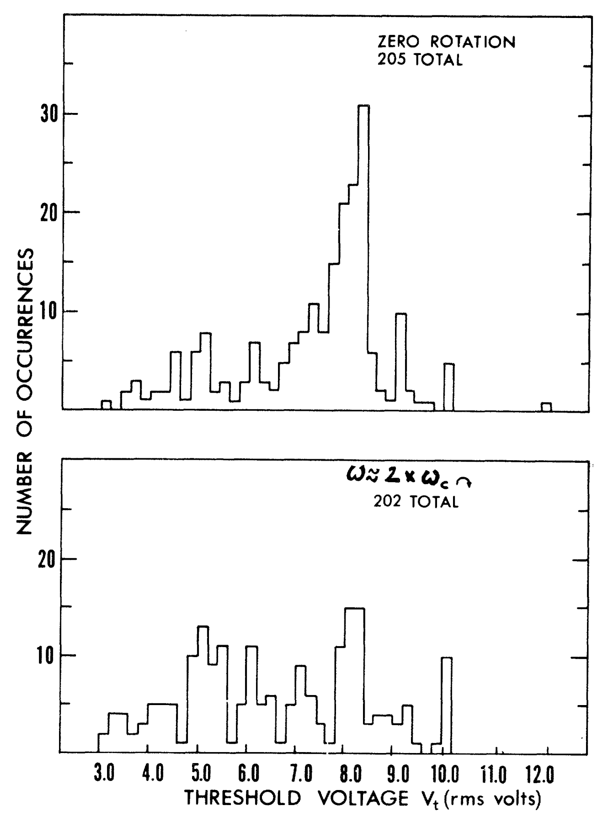

FIG. 8. Large samples of cavitation thresholds taken with the plane standing-wave system and single paddle; $T = 2.16\,°$K, $f_r = 89\,224$ Hz.

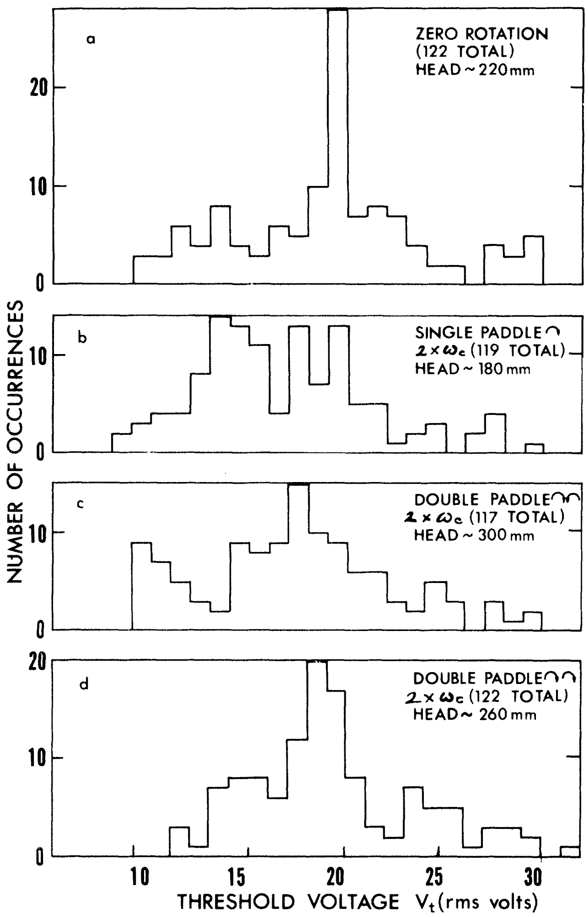

FIG. 9. Large samples of cavitation threshold taken with the plane standing-wave system with rotation of one and two paddles (shaft-center distance 21.8 mm); $T = 2.16$ °K; $f_r = 88\,735$ Hz.

the rotating bucket at higher speeds are the slight lowerings in threshold with two shafts rotating above the critical speed and in the same sense. If the response of a single quantized filament when excited ultrasonically is sharply nonlinear, then the lack of a proportional response with many lines is understandable. The slightly greater ability of two shafts and paddles rotating in opposite senses to produce greater threshold lowerings might be due to interactions (e.g., hydrodynamic forces of attraction) between singly quantized lines. These interactions might result in local increments of fluid velocity and, hence, lower Bernoulli pressures.

Because of the difficulty pointed out by Finch and Chu as to how a single vortex filament nucleates cavitation, it is inviting to speculate that the filament breaks up, yielding superfluid turbulence or becomes ravelled into a ball, as suggested by McCloud.[4] However, it is difficult to see how turbulence or ravelling could occur in these experiments and how such could be consistent with the statistical nature of the results, if it were present. Turbulent mixing or ravelling should presumably vary in extent, so that nuclei of various sizes should occur and, therefore, not give rise to a bimodal distribution. It is proposed that the modal value at the lower thresholds is due to vortices created by rotation. The modal value occurring at the higher thresholds might represent a species of nuclei different than those which give rise to the lower threshold mode, or it might be due to vortices created by the sound field itself on attaining a certain critical velocity. However, on either interpretation, the time dependence and bimodal nature of the results are attractively explained in terms of the coming and going of isolated filaments, unbroken and unravelled, in regions of maximum tension.

It is, thus, appropriate to review the assumptions of the simple illustration put forward by Finch and Chu.[2] First, it is questionable whether macroscopic laws of surface tension should apply to molecular dimensions. A surface-tension modification, similar to that proposed by Frenkel[17] for spherical holes, might be applied to the 1 Å radius vortex core. A second assumption in the simple model was that the vortex had unit-quantized circulation. Multiple circulations, up to 3 quantum units, could conceivably arise[18] and would clearly lower the maximum pressure reduction which depends on $(nh/m)^2$. If the vortex core is taken to be filled with normal fluid as proposed by Glaberson *et al.*,[19] the surface tension pressure σ/r would not be present, leaving only the Bernoulli pressure reduction. This reduction is of the order of 10^6 and 10^2 N/m^2, for cores of 1 Å and 100 Å, respectively. There are several theoretical arguments[19,20] for believing the core sizes in the liquid-filled model to be larger than estimated by Finch and Chu, and to be temperature dependent; e.g., a characteristic dimension of the Ginzburg-Pitaevskii[20] theory, corresponding to the vortex core radius, is $4.3(T_\lambda - T)^{-1/2}$ Å, which gives for $T = 2.1$ °K a core radius of 12 Å, a value in agreement with the theory of Glaberson *et al.* The core radius approaches infinity at the λ point, according to this theory. Bernoulli-pressure reductions around the liquid-filled core are still, however, much less than the cohesive forces between the liquid molecules. The simple model also neglects dynamical effects. For example, the harmonic variation of the applied pressure should be considered. However, a detailed examination of all these various points fails to yield results with any significant difference from the simple model, unless the radius were to becomes of the order of 10^{-7} m or larger.[21]

A proposal is now advanced that growth of the core might take place by rectified heat transfer, which has been investigated for spherical bubbles by Hsieh.[22] Heat is supplied to and removed from a pulsating core during its expansion and compression stages, respectively. However, since the surface area is greater during expansion, there will be a net flow of heat into the core over a cycle.[21] Thus, over a number of cycles, heat should be pumped into the pulsating core of a quantized vortex, resulting in its growth, since according to the theory of Glaberson *et al.*,[19] local heating causes the core to expand. Finally, such heating would result in vaporization occurring in the core, which constitutes a matrix of essentially normal fluid.

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†Present address: National Bureau of Standards, Boulder, Colo. 80302.

‡Present address: Department of Mechanical Engineering, University of Akron, Ohio.

[1]R. D. Finch, T. G. Wang, R. Kagiwada, M. Barmatz, and I. Rudnick, J. Acoust. Soc. Am. 40, 211 (1966).

[2]R. D. Finch and M. L. Chu, Jr., Phys. Rev. 161, 202 (1967).

[3]M. H. Edwards, R. M. Cleary, and W. M. Fairbank, in Quantum Fluids, edited by D. F. Brewer (North-Holland Publishing Co., Amsterdam, 1966), pp. 140–145.

[4]K. L. McCloud, PhD dissertation, Ohio State University, 1968 (unpublished). See also K. L. McCloud and C. F. Mate, Bull. Am. Phys. Soc. 12, 96 (1967).

[5]D. Y. Hsieh, California Institute Technology Report No. 85-44, 1967 (unpublished).

[6]R. B. Dean, J. Appl. Phys. 15, 446 (1944).

[7]Reviews of helium in rotation have been given by H. E. Hall, Phil. Mag. Suppl. 9, 89 (1960); and E. Andronikashvili and G. Mamaladze, Rev. Mod. Phys. 38, 567 (1966).

[8]R. G. Arkhipov, Zh. Esperim. i Teor. Fiz. 33, 116 (1957) [English transl.: Soviet Phys. – JETP 6, 90 (1957)].

[9]W. F. Vinen, Nature 181, 1524 (1958).

[10]G. B. Hess and W. M. Fairbank, Phys. Rev. Letters 19, 216 (1967).

[11]W. F. Vinen, in Quantum Fluids, edited by D. F. Brewer (North-Holland Publishing Co., Amsterdam, 1966), pp. 104–106.

[12]S. W. Iordanskii, Zh. Esperim. i Teor. Fiz. 48, 708 (1965) [English transl.: Soviet Phys. – JETP 21, 467 (1965)].

[13]K. R. Atkins, Liquid Helium (Cambridge University Press, Cambridge, England, 1959), p. 214.

[14]See, for instance, J. E. Barger, Technical Report No. 57, Harvard University, 1964 (unpublished), p. 163.

[15]G. Careri, W. D. McCormick, and F. Scaramuzzi, Phys. Letters 1, 61 (1962).

[16]R. E. Packard and T. M. Sanders, Jr., Phys. Rev. Letters 22, 823 (1969).

[17]J. Frenkel, Kinetic Theory of Liquids (Dover Publications, Inc., New York, 1959), p. 180.

[18]S. C. Whitmore and M. Zimmermann, Phys. Rev. 166, 181 (1968).

[19]W. I. Glaberson, D. Strayer, and R. J. Donnelly, Phys. Rev. Letters 1, 1740 (1968).

[20]V. L. Ginzburg and L. P. Pitaevskii, Zh. Esperim. i Teor. Fiz. 7, 34 (1958) [English transl.: Soviet Phys. – JETP 7, 858 (1958)].

[21]P. M. McConnell, PhD dissertation, University of Houston, 1969 (unpublished).

[22]D. Y. Hsieh, Division of Engineering and Applied Science, California Institute of Technology, Office of Naval Research Report No. 85–23 (unpublished).