Velocity of Sound, Density, and Grüneisen Constant in Liquid ^{4}He†

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By measuring the pressure dependence of the velocity of sound, we have determined both the pressure dependence of the density and the Grüneisen constant u of liquid ^{4}He. Measurements were made below 0.1 K and in the vicinity of 0.5 K. Our determinations of the pressure dependence of the density agree quite well with that determined by Boghosian and Meyer, who used a capacitance bridge. Since the latter results rely on the validity of the Clausius-Mossotti relation and a pressure-independent electric polarizability, the present work can be interpreted as supporting both of these assumptions. We found that $u(\rho_0) \equiv (\rho/c)dc/d\rho = 2.84$ under the vapor pressure at 0.1 K. Using this value of u to calculate the attenuation of sound according to a three-phonon mechanism, we obtain an attenuation of less than half the measured value. Thus, the present theory of sound attenuation must be incomplete.

INTRODUCTION

In Landau's quantum hydrodynamical model for liquid ^{4}He[1,2] the density ρ and the density-dependent sound velocity $c(\rho)$ [together with its derivatives $\partial c(\rho)/\partial\rho$, etc.] play a central role. Since the pressure P is the usual laboratory variable, it is important to know $\rho(P)$ and $c(P)$. The most accurate measurements to date of $\rho(P)$ have been obtained by measuring the pressure dependence of the dielectric constant $\epsilon(P)$ and relating this to the density through the Clausius-Mossotti equation.[3] Since this involves assuming the pressure independence of the molar polarizability (together with the validity of the Clausius-Mossotti equation), it is of interest to have an independent determination of $\rho(P)$. A knowledge of $c(P)$ enables us to determine $\rho(P)$ by integration, since $c^2 = (\partial P/\partial\rho)_T$. There have been measurements of $c(P)$ by a number of techniques,[4-6] but the temperatures at which the measurements were made were not low enough to satisfy the condition for isothermal propagation ($\omega\tau \gg 1$, where τ is the appropriate phonon-phonon relaxation time). Thus, it is of interest to measure $c(P)$ at very low temperatures. With the development of the phase-comparison technique, the accuracy with which the change in velocity with pressure may be determined has been increased significantly. With this improvement the pressure dependence of the density may be determined with an accuracy not previously possible.

The Grüneisen constant $u = (\rho/c)\partial c/\partial\rho$ sets the strength of the phonon-phonon interaction in ^{4}He. This constant also appears in the expressions for the attenuation of sound. Thus, it is also of interest to know u with precision sufficient to allow a meaningful comparison between the theoretical and measured sound attenuation.

Since the results of the present experiment will allow us to determine a number of quantities relating to phonon-phonon interactions in liquid helium, it is worthwhile to include a brief review of the theory. Consider first the additional potential energy per unit volume $E(\rho)$ on increasing the density of the liquid at absolute zero, from the equilibrium density $\rho_0(V_0)$ to a final density $\rho(V)$; i.e.,

$$E(\rho) = -\frac{1}{V}\int_{V_0}^{V} P\,dV = \rho\int_{\rho_0}^{\rho} \frac{P(\rho')}{\rho'^2}\,d\rho' . \tag{1}$$

The kinetic energy per unit volume is given by

$$\tfrac{1}{2}\vec{v}\cdot\rho\vec{v} , \tag{2}$$

where $\vec{v}$ is the local velocity of the liquid. Thus, the total energy per unit volume, which is the Hamiltonian density, will be

$$H = \tfrac{1}{2}\vec{v}\cdot\rho\vec{v} + \rho\int_{\rho_0}^{\rho} \frac{P(\rho')}{\rho'^2}\,d\rho' . \tag{3}$$

Making use of the fact that $P(\rho_0) = 0$ (at absolute zero the pressure is zero at the equilibrium density), we may expand Eq. (3) in a Taylor series about ρ_0, obtaining

$$H = H_0 + V_3 + V_4 + \cdots , \tag{4}$$

where

$$H_0 = \tfrac{1}{2}\vec{v}\cdot\rho_0\vec{v} + [c^2(\rho_0)/2\rho_0](\rho-\rho_0)^2 , \tag{5}$$

$$V_3 = \tfrac{1}{2}[\vec{v}\cdot(\rho-\rho_0)\vec{v}] + \frac{1}{3!}\frac{d}{d\rho}\left(\frac{c^2}{\rho}\right)\Bigg|_{\rho=\rho_0}(\rho-\rho_0)^3 , \tag{6}$$

and

$$V_4 = \frac{1}{4!}\frac{d^2}{d\rho^2}\left(\frac{c^2}{\rho}\right)\Bigg|_{\rho=\rho_0}(\rho-\rho_0)^4 . \qquad (7)$$

If we terminate the expansion after H_0, the elementary excitations of the system will be noninteracting phonons. An excellent account of the mathematical details can be found in London's[7] and Khalatnikov's[8] books. The term V_3, being third order, leads to a three-phonon coupling in first-order perturbation theory and to a four-phonon coupling in second order. V_4 couples four phonons in first order. Thus, an accurate knowledge of ρ and $c^2(\rho)$ is essential to the theory of phonon-phonon interactions. It was with this goal that the present investigation was initiated.

EXPERIMENT

In this experiment we determined the change with pressure of the transit time of a sound wave traveling through a known distance D in the liquid. The change in transit time is given by

$$t(P) - t(P=0) \equiv \Delta t = D/c(P) - D/c(0) . \qquad (8)$$

Thus, if we know the velocity at zero pressure, we can determine the velocity at any other pressure using (8). The absolute value of the velocity of sound in liquid ^{4}He at the vapor pressure has been measured by a number of workers,[9–18] using a variety of methods. The change in velocity with temperature has been measured, commencing at very low temperatures, using the phase comparison method, by Whitney and Chase[19,20] and also by Abraham *et al.*[21,22] These data are necessary for extrapolating the high-temperature measurements of the absolute value of the velocity to zero temperature. Whitney and Chase,[20] having done a detailed statistical analysis of the available data, have determined a value of the zero-pressure velocity at absolute zero. They found $c = (2.383 \pm 0.001)\times 10^4$ cm/sec; we adopt this value in the analysis which follows. The absolute value of the sound velocity could not be accurately determined in our work because unloaded transducers, which have a relatively slow rise time (~ 10 μsec), were employed. This limits the accuracy with which the time of arrival of the pulse can be determined. The transducers were operated unloaded, because it had previously been determined that an exceptionally high signal-to-noise ratio could be achieved using this approach.[23]

The pressure dependence of the sound velocity was first studied by Findlay, Pitt, Grayson-Smith, and Wilhelm[4] using an interferometric technique. It was later studied in greater detail by Atkins and Stasior,[5] as well as by Vignos and Fairbank.[6] In both of these latter measurements the time-of-flight pulse-echo technique was employed. With the development of the phase-comparison technique, the change in velocity with pressure could be determined with much greater accuracy; it was therefore worthwhile to repeat these measurements, especially since the value of u could not be determined accurately from the existing data.

When a sound wave is propagated under the condition $\omega\tau \gg 1$, where τ is a characteristic relaxation time in the liquid, the propagation is isothermal since the liquid does not come into local thermal equilibrium with the sound wave. The isothermal sound velocity is given by

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_T . \qquad (9)$$

Thus, from a knowledge of the pressure dependence of this velocity, one can, on integrating Eq. (9), determine the pressure dependence of the density:

$$\rho(P) - \rho(0) = \int_0^P \frac{dP}{c^2(P)} . \qquad (10)$$

For the temperatures used in this experiment, $\omega\tau \gg 1$. At higher temperatures, where $\omega\tau \ll 1$ (typically greater than 1 K for the ultrasonic frequencies usually employed), the situation is more complicated. In a classical liquid, the nd sound velocity in this limit would be given by $c^2 = (\partial P/\partial\rho)_S$. In liquid He II this is not, in general, correct due to a coupling between first and second sound,[7,8] but the sound velocity approaches this value asymptotically as T approaches zero. For these reasons we have not attempted to determine the high-temperature equation of state in this experiment.

A simplified diagram of the ultrasonic comparator used in these measurements is shown in Fig. 1. The technique is basically an interferometric one.[23] Energy from a continuously running oscillator is divided into two branches: the signal or helium branch and the reference branch. The time difference between the pulses which activate switches A and B is adjusted to be equal to the transit time through the helium. This is done to ensure that the signals arrive at the receiver at the same time. The two branches are recombined at the input of the amplifier and, since both are derived from the same source, interfere coherently with one another. To obtain the initial reading, only enough He was condensed to fill the cell, and the delay line was adjusted so that a null was obtained at the receiver input. This point was taken as the zero pressure point. Were a larger amount added, such that the liquid partially filled the capillary, two pressure contributions could arise. First, there would be the hydrostatic

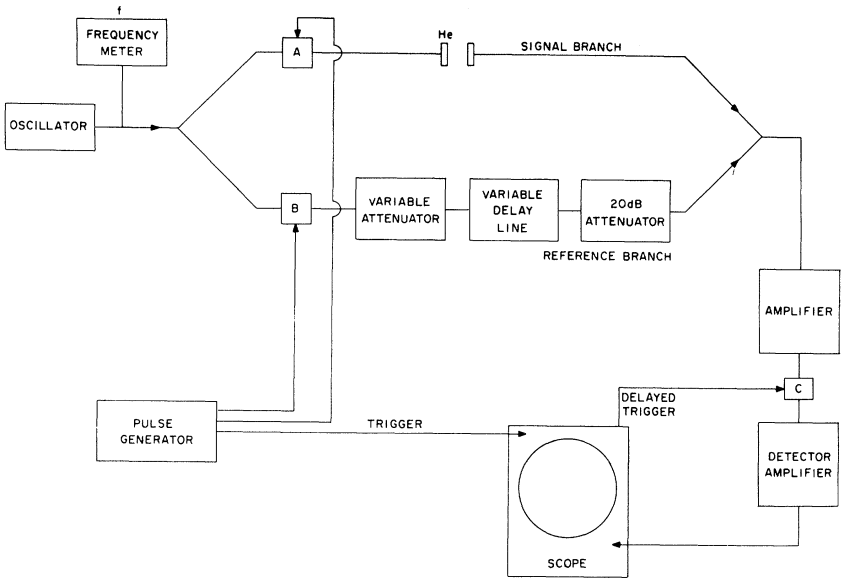

FIG. 1. Simplified diagram of the ultrasonic comparator.

pressure head in the capillary; second, the pressure would be determined by the liquid in the warmest part of the capillary.

The pressure was then raised to a value where the first data point was to be taken, and the attenuator and delay line were adjusted so that the null was reestablished. The pressure increments were such that the transit time changed by several periods. At pressures less than 1 atm, where equilibrium times were determined by gas flow through a narrow capillary, the number of nodes occurring during a pressure increment was recorded and the delay line readjusted so that a null was observed. A finite time is also required for condensation of the liquid. Typical equilibrium times were on the order of 15 min. At pressures above 1 atm, when the capillary was filled with liquid above the bath level, the equilibrium times were of the order of $\frac{1}{2}$ min. Under these circumstances the pressure increments could be adjusted so that the transit time changed by an integral number of periods, which eliminated the necessity of adjusting the delay line. Over very large pressure changes it was necessary to readjust the time difference between the A and B switches to account for gross changes in the sound velocity. The frequency of the oscillator must be unusually stable since a frequency shift $\Delta f/f$ causes an apparent velocity shift $\Delta c/c$. The frequency of the oscillator was continuously monitored during the course of the experiment and was maintained at 12 014 665 ± 10 Hz.

In order to isolate the effect of temperature on the quantities determined in these experiments, it was decided to make runs below 0.1 K. Refrigeration was achieved using adiabatic demagnetization of 500 g of potassium chromium alum. The refrigerator salt pill was fabricated from single-crystal slabs and coil foil. Thermal isolation was improved by the use of a ferric ammonium alum guard pill. Both pills were attached to a ^{3}He evaporation refrigerator via Pb superconducting switches. The above components were surrounded by a shield which was cooled by a second ^{3}He refrigerator. The magnet used was a 20-kG Westinghouse superconducting solenoid. The cerium magnesium nitrate (CMN) magnetic thermometer was also fabricated from single-crystal slabs and coil foil. The sonic cell employed was the same as that used in previous investigations,[21–24] with the exception that the reservoir at the top was removed. Coil foils from both the refrigerator and magnetic thermometer were in thermal contact with the cell. Resistance thermometers were attached to the refrigerator and guard pill and were useful in determining thermal equilibrium. The magnetic thermometer was calibrated against the vapor pressure of ^{3}He. Temperatures as low as 16 mK were reached on demagnetization from 1.0 K. Since the temperature dependence of the velocity is negligible below 0.1 K, there was no need for any lower temperature.

Due to the unusually large thermal conductivity of superfluid ^{4}He, special precautions had to be taken to ensure thermal isolation of the low-temperature portion of the cryostat. Thermal isolation was achieved by using appropriate lengths of Cu-Ni capillary (0.025 cm i. d., 0.005-cm wall) between the various stages in the cryostat. These lengths were, between the main ^{4}He bath and the ^{4}He refrigerator, 30 cm; separating the ^{4}He refrigerator from the ^{3}He refrigerator, 144 cm; between the ^{3}He refrigerator and the guard pill, 87 cm; and between the guard pill and the refrigerator pill, 40 cm. In order to ensure good thermal grounding of the liquid to the refrigerator pill, the

liquid passed through a 76 cm length of 0.041-cm-i.d. capillary which was in good thermal contact with the salt. A 0.33-cm-diam copper wire was threaded through this last section of capillary to increase the area and consequently to reduce the Kapitza boundary resistance. A second thermal shield, made out of coil foil and kept at the potassium chromium alum temperature, surrounded the cell and the CMN thermometer.

DATA AND ANALYSIS

Rewriting Eq. (8) in terms of the zero-pressure sound velocity of Whitney and Chase,[20] we have

$$c(P)=\frac{D}{\Delta t+[D/c(0)]}=\frac{1.0100}{\Delta t+0.4238\times10^{-4}}\ \text{cm sec}^{-1}, \tag{11}$$

where the path length D is 1.0100 cm and Δt is in seconds. Our results for the pressure dependence of the sound velocity at 0.1 K are given in Table I. Table II gives our experimental values for $T=0.45$ K, where for the vapor pressure velocity we have used the value 238.34 m sec^{-1}.

In order to determine various derived quantities from our data it is helpful to describe the data in analytic form. By least-squares fit procedures we determined several expressions that represented our data fairly well; one of them expressed the time delay as a seven-term power series in the pressure; a slightly better fit was obtained by expressing the pressure as a six-term power series in the time delay; but by far the simplest expression that fit the data well was a three-term virial expansion of the pressure in powers of the density:

$$P(\text{atm})=A_1(\rho-\rho_0)+A_2(\rho-\rho_0)^2+A_3(\rho-\rho_0)^3, \tag{12}$$

where ρ_0 is the density at zero pressure. Because of its simplicity, the theoretical significance of its coefficients, and the fact that it gives a very good fit to our data, Eq. (12) was chosen as the analytic description of our data.

Since Eq. (12) does not contain the velocity directly, one must use the relation

$$\frac{c^2(\text{m sec}^{-1})}{1.01325\times10^2}=\frac{dP}{d\rho}$$

$$=A_1+2A_2(\rho-\rho_0)+3A_3(\rho-\rho_0)^2 \tag{13}$$

to obtain an implicit relationship between P and c. The numerical factor converts atmospheres to dyn/cm^2. For any value of P, Eq. (12) can be numerically solved for the corresponding value of ρ. When this value of ρ is put into Eq. (13), one gets a calculated value of $c^2(\rho)$ to be compared with the experimental value. It is not difficult to calculate the derivatives of Eq. (13) with respect to the parameters A_1, A_2, and A_3 [including the derivative of ρ with respect to A_1, A_2, and A_3 from Eq. (12)], so that one has all the necessary information to least-squares fit the data to Eq. (12). Since neither the time delay nor pressure is known with unlimited precision, it is desirable to use a generalized least-squares procedure, developed by Deming,[25] which takes into account the uncertainties present in both variables. In this method the function S is minimized where

TABLE I. Experimental values of sound velocity in ^{4}He under pressure for $T<0.1$ K.

P (atm)	c (m sec^{-1})	P (atm)	c (m sec^{-1})
0.0699	238.841	6.4889	283.351
0.1120	239.215	7.0666	286.698
0.1680	239.673	7.5452	289.433
0.2250	240.137	8.0416	292.221
0.2667	240.444	8.5563	295.064
0.3370	241.038	9.0937	297.962
0.4560	242.064	9.6515	300.917
0.7588	244.445	10.2293	303.932
1.0789	247.014	10.8351	307.008
1.3431	249.041	12.1198	313.350
1.6160	251.103	13.3325	319.118
1.9008	253.199	14.6381	325.103
2.1884	255.330	15.8355	330.414
2.7220	259.146	17.1184	335.902
3.2897	263.079	18.4921	341.575
3.8071	266.546	19.9553	347.441
4.3475	270.106	21.5239	353.516
4.9170	273.762	23.2085	359.805
5.4192	276.885	24.3862	364.124
5.9410	280.081		

TABLE II. Experimental values of sound velocity in ^{4}He under pressure for $T\sim0.5$ K.

P (atm)	c (m sec^{-1})	P (atm)	c (m sec^{-1})
0.0580	238.764	3.4669	264.240
0.1104	239.234	4.3551	270.122
0.1680	239.707	5.3263	276.272
0.2822	240.657	6.3881	282.708
0.3996	241.616	7.5548	289.452
0.5792	243.067	8.8359	296.525
0.7627	244.536	10.2455	303.952
0.9528	246.024	11.8042	311.761
1.2110	248.036	13.5338	319.982
1.4798	250.079	15.4544	328.648
1.7593	252.160	17.5973	337.797
2.0461	254.274	20.2478	348.467
2.4951	257.511	22.9545	358.770
2.9673	260.832		

$$S=\sum_{i=1}^{N}\left[\left(\frac{V_{ti}}{\sigma_t}\right)^2+\left(\frac{V_{Pi}}{\sigma_P}\right)^2\right]. \qquad (14)$$

Here, N is the number of experimental points; σ_t and σ_P are the expected variances of the time delay and of the pressure variables, respectively; and V_{ti} and V_{Pi} are the t and P components, respectively, of a line segment from the experimental point to the calculated curve. The selection of this line segment affects the statistical weight given to the P and t variables; the optimum weighting occurs when the line segment is normal to the curve in a plot of P/σ_P against t/σ_t. The values $\sigma_t = 10^{-9}$ sec and $\sigma_P = 10^{-3}$ atm were used here.

Figure 2(a) shows the values of c from Table I plotted versus P. The solid curve is the parametric relationship obtained from Eqs. (12) and (13). The quality of the agreement between our data and these equations is given in Fig. 2(b), where the deviations of our velocity values from the calculated curve are plotted. The fit is remarkably good for such a small number of terms. The value of A_1 follows from the measured sound velocity at zero pressure[20] and, in the units used here, is given by

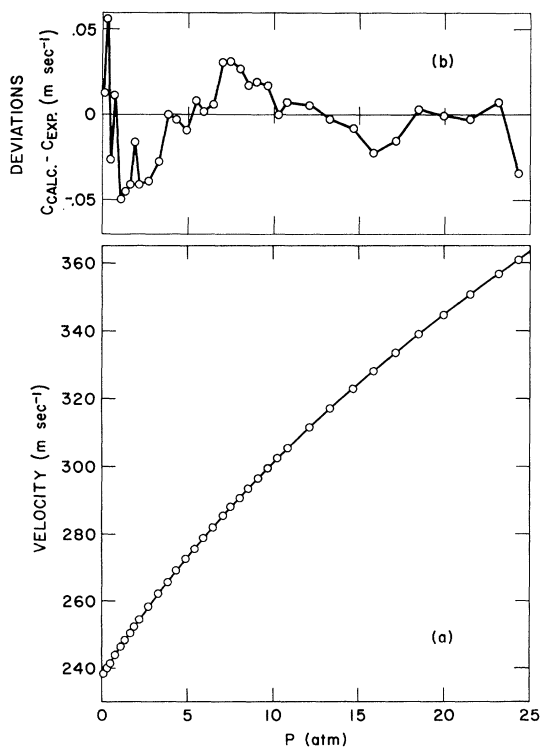

FIG. 2. (a) Pressure dependence of the velocity of sound in liquid ^{4}He for T less than 0.1 K. Solid curve is our fit using Eqs. (12) and (13). (b) Deviations between the measured and fitted velocity.

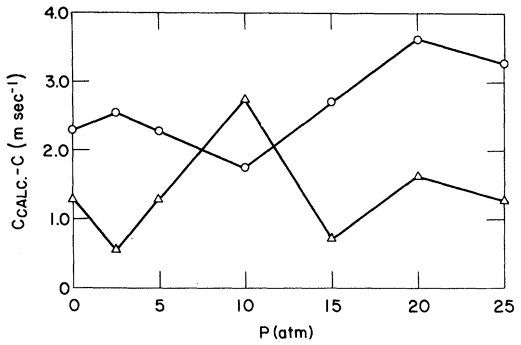

FIG. 3. Comparison of the fitted values of $c(P)$ from Table III and the measurements of Vignos and Fairbanks (Ref. 6) (open circle) and Atkins and Stasior (Ref. 5) (open triangle).

$$A_1 = c^2(0)/(1.01325\times10^2)$$

$$= (5.60\pm0.01)\times10^2 \text{ atm cm}^3\text{g}^{-1}.$$

The values of A_2 and A_3, determined from the least-squares fit, are

$$A_2 = (1.0970\pm0.0007)\times10^4 \text{ atm cm}^6\text{g}^{-2},$$

$$A_3 = (7.33\pm0.01)\times10^4 \text{ atm cm}^9\text{g}^{-3},$$

where the errors are standard deviations obtained from the least-squares fit and do not include possible systematic errors.

The values of the parameters derived from the data of Table II are

$$A_1' = (5.61\pm0.01)\times10^2 \text{ atm cm}^3\text{g}^{-1},$$

$$A_2' = (1.0952\pm0.0007)\times10^4 \text{ atm cm}^6\text{g}^{-2},$$

$$A_3' = (7.31\pm0.02)\times10^4 \text{ atm cm}^9\text{gm}^{-3}.$$

Figure 3 shows the difference between our calculated values of c from Table III and the values of c at 1.0 K of Vignos and Fairbank[6] and those at 1.25 K of Atkins and Stasior.[5] Note that our deviation plot in Fig. 2(b) is on a much expanded scale.

Figure 4 shows a similar comparison of our values of ρ (using[26] $\rho_0 = 0.14513$) from Table III with the values obtained by Keesom and Keesom[27] at 1.25 K and by Boghosian and Meyer[3] at 0 K (extrapolated from 0.5 K). The agreement with the values of Boghosian and Meyer is excellent; only at one point is the difference more than 10^{-4} g/cm^3, the accuracy to which their data are given. This experimentally establishes that the molar polarizability of liquid He is independent of pressure. This was an assumption made by Boghosian and Meyer in obtaining densities from

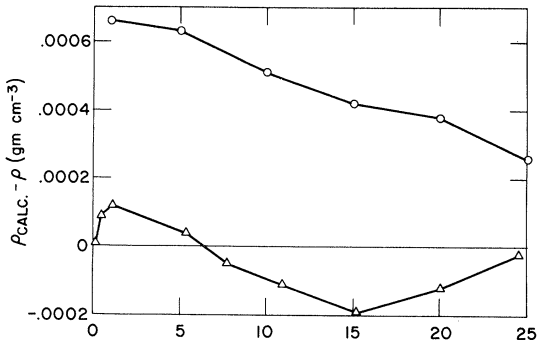

FIG. 4. Comparison of the fitted values of $\rho(P) - \rho(0)$ with the measurements of Keesom and Keesom (Ref. 27) (open circle) and Boghosian and Meyer (Ref. 3) (open triangle).

their measurements of the dielectric constant ϵ of He using the Clausius-Mossotti relation

$$\rho = (3M/4\pi\alpha_P)(\epsilon - 1)/(\epsilon + 2), \tag{15}$$

where M is the molecular weight, and α_P is the polarizability. Since our values of ρ agree with theirs to within 0.1%, the polarizability is independent of pressure (below 25 atm) to within 0.1%.

The values of $\rho(P)$ presented here rely on a knowledge of c (0): the zero-temperature and -pressure velocity. Should some future measurement improve the precision of $c(0)$, the full precision of these data could be realized. The values of $(\rho - \rho_0)$ given in Table III may be corrected by multiplying the values by the factor $1 + 2.4[238.30 - c(0)]/c(0)$.

Table III contains values of $P, c, \rho - \rho_0$, $u \equiv (\rho/c)(dc/d\rho)$, and $w \equiv (\rho^2/c)(d^2c/d\rho^2)$ calculated from Eqs. (12) and (13) using the values of A_1, A_2, and A_3 above. In calculating u and w we have used the value of $\rho_0 = 0.14513$ determined by Kerr and Taylor.[26] We have made an attempt to find out how much the results of Table III depend on the form of the expression used to fit the data. We have made detailed comparisons of the numbers obtained from all the expressions which give a reasonably good description of the data. From these we have concluded that the results of Table III are largely independent of the analytic description, except perhaps in the last digit given. The results at 0.5 K yield values which are the same as those listed in Table III to within our estimated

TABLE III. Pressure, sound velocity, density change, Grüneisen constant, $u = (\rho/c)(dc/d\rho)$, and $w = (\rho^2/c)(d^2c/d\rho^2)$ calculated from Eqs. (12) and (13) in the text at 0.1 K.

P (atm)	c (m sec^{-1})	$\rho - \rho_0$ (g cm^{-3})	u	w
0.0	238.30	0.0	2.84	8.26
1.00	246.35	0.00173	2.78	7.91
2.00	253.91	0.00334	2.73	7.61
3.00	261.05	0.00487	2.69	7.35
4.00	267.83	0.00632	2.64	7.12
5.00	274.28	0.00770	2.61	6.91
6.00	280.44	0.00902	2.57	6.73
7.00	286.34	0.01028	2.54	6.56
8.00	292.02	0.01149	2.51	6.41
9.00	297.48	0.01266	2.48	6.26
10.00	302.75	0.01378	2.46	6.14
11.00	307.84	0.01487	2.44	6.02
12.00	312.78	0.01592	2.41	5.90
13.00	317.56	0.01694	2.39	5.80
14.00	322.20	0.01793	2.37	5.70
15.00	326.71	0.01889	2.35	5.61
16.00	331.11	0.01983	2.34	5.53
17.00	335.39	0.02074	2.32	5.45
18.00	339.56	0.02163	2.30	5.37
19.00	343.64	0.02250	2.29	5.30
20.00	347.62	0.02335	2.27	5.23
21.00	351.51	0.02418	2.26	5.17
22.00	355.32	0.02499	2.25	5.10
23.00	359.05	0.02578	2.23	5.05
24.00	362.70	0.02656	2.22	4.99
25.00	366.28	0.02733	2.21	4.94

accuracy.

As stated earlier, the main motivation for carrying out the present experiment was to obtain an accurate value for u, since this quantity is a crucial parameter in theories of the sound attenuation. According to theory,[28-32] the attenuation coefficient α of a sound wave of frequency ω arising from a three-phonon process is given by

$$\alpha = \frac{\pi^2}{30}\frac{(u+1)^2}{\rho}\frac{\kappa^4}{\hbar^3 c^6} \times \omega T^4[\tan^{-1}(2\omega\tau) - \tan^{-1}(3\gamma\bar{p}^2\omega\tau)], \quad (16)$$

where $\bar{p}^2$ is the average thermal phonon momentum $3\kappa T/c$, and τ is the thermal phonon lifetime. The coefficient γ accounts for dispersion in the phonon spectrum at long wavelengths as defined in the expression $\epsilon = cp(1-\gamma p^2)$, where ϵ and p are the energy and momentum of a phonon, respectively. The same theory predicts a temperature dependence of the sound velocity, given by

$$c(T) - c(0) = \frac{\pi^2}{60}\frac{(u+1)^2}{\rho\hbar^3}\left(\frac{\kappa T}{c}\right)^4 \ln\frac{1+(2\omega\tau)^2}{1+(3\gamma\bar{p}^2\omega\tau)^2}. \quad (17)$$

If we assume γ to be small enough so that the second term in Eq. (16) may be neglected, but at the same time not so small as to make the four-phonon process dominate, then Eq. (16) takes the form

$$\alpha = \frac{\pi^3}{60}\frac{(u+1)^2}{\rho}\frac{\kappa^4}{\hbar^3 c^6}\omega T^4, \quad (18)$$

where we have approximated $\tan^{-1}(2\omega\tau) \simeq \frac{1}{2}\pi$ in the limit $\omega t \gg 1$. Alternatively, if a frequency is chosen such that $3\gamma\bar{p}^2\omega\tau \ll 1 \ll \omega\tau$, we again arrive at Eq. (18). The attenuation of sound in ^{4}He at low temperature, where Eq. (16) should be valid, has been reported by a number of investigators.[21,22,24,33-38] Although the recent work reported in Ref. 22 was quite exhaustive and precise, nonetheless, most of the measurements agree within experimental error where they overlap. The perplexing thing is that the measured attenuation is usually larger than that calculated from Eq. (18). Using the measured values of c_0 and ρ_0 together with the values of u determined in this experiment, we find

$$\alpha = 4.82_4 f T^4 \times 10^{-6}\,\text{dB/cm Hz K}^4, \quad \text{(vapor pressure)}$$
$$\alpha = 0.215_1 f T^4 \times 10^{-6}\,\text{dB/cm Hz K}^4, \quad \text{(25 atm)} \quad (19)$$

and

$$c(T) - c(0) = 16.23 T^4\,\text{cm/sec K}^4, \quad \text{(vapor pressure)} \quad (20)$$

where f is the frequency. The recent work of Abraham *et al.*[22] shows that the attenuation is given approximately by

$$\alpha = 11.6 \times 10^{-6} f T^4\,\text{dB/cm Hz K}^4 \quad \text{(vapor pressure)} \quad (21)$$

in the frequency range 12–36 MHz and temperature range 100–500 mK. There appears to be no way to account for this discrepancy at the present time, and it is concluded that the present theory of sound attenuation is incomplete. We believe that some essential aspect has not been included in the theory.

The attenuation arising from a four-phonon process has been recalculated by Eckstein,[39] who finds

$$\alpha = 8.66[(u+1)^4/192\pi^3\gamma c^5\rho^2](\kappa/\hbar c)^4\omega^3 T^4 = 38.8\times10^{-8}(f^3T^4/\gamma)\,\text{g}^2\,\text{cm dB/sec}^2\,\text{Hz}^3\,\text{K}^4 \quad (22)$$

in the limit $\hbar\omega \ll \kappa T$. Note that this differs from an earlier incorrect calculation by Landau and Khalatnikov.[2] The experimental value of the dispersion constant γ is somewhat uncertain at the present time. Eckstein and Varga[40] have shown, however, that it is possible to derive γ from the hydrodynamic Hamiltonian given in Eq. (4). They find the expression for γ

$$\gamma = \tfrac{2}{3}(\kappa/\hbar)^2(u^2 - 0.6)/32\pi^2\rho\hbar c, \quad (23)$$

which yields the value 5.15×10^{37} in cgs units at zero pressure. This value is in reasonable agreement with that estimated[22] from inelastic neutron scattering data.

CONCLUSIONS

The pressure dependence of both the velocity of sound and the density in liquid ^{4}He has been determined with a much higher precision than in previous work. Our values of $\rho(P)$ deduced from the measurement of $c(P)$ agree well with $\rho(P)$ as determined from dielectric constant measurements within the accuracy of these latter measurements. Barring happenstance, this agreement justifies the use of the Clausius-Mossotti relation and establishes the pressure independence of the molar polarizability α_P of liquid ^{4}He. Our value of u definitely establishes the conclusion[22] that the theory of sound propagation in ^{4}He is incomplete. This value of u when substituted into the expression developed by Eckstein and Varga[40] yields a value of γ, the Landau-Khalatnikov dispersion constant, in reasonable agreement with neutron scattering results.

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