$$
(2\pi)^{-3/2}e^{i\vec{k}_{21}\cdot\vec{r}_{21}}=(2\pi)^{-3/2}e^{i[\vec{k}_{21}\cdot(\vec{r}_{23}-\vec{r}_{13})]};
$$

 $r_{\rm 21}^{\rm -1}$ is replaced by – $r_{\rm 23}^{\rm -1}$; and $dr_{\rm 21}$ is replaced by $-dr_{13}$. The integrals are then easily evaluated and we find that the matrix element of r_{12} ⁻¹ is

$$
(1/\pi^2)(1/K^2)\sqrt{2}\,\mu^{5/2}/(\mu^2+\kappa_{23}^2)^2\,,\qquad \qquad (26)
$$

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Structures in the Low-Energy e^- -He Scattering Cross Sections*

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A cylindrical, retarding potential difference type of electron spectrometer with high-energy resolution has been used to study the structures in the e^- -He scattering cross section in a transmission experiment. Twenty-four structures have been observed between the transmission maximum due to He $1s2s^2S_{1/2}$ at 19.30 ± 0.01 eV and He⁺ 1s ${}^2S_{1/2}$ at 24.60 \pm 0.02 eV, 11 of which have been observed previously. The agreement with previous measurements for the positions of the structures is good where comparison is possible. Four of the new structures have been observed below the $n=2$ states of He. The excitation onsets for $2^{3}S_{1}$, 2^1S_0 , 2^3P^0 , and 2^1P^0 have been observed, as well as some excitation onsets at higher energies. No structures have been observed below He^{-2}S_{1/2}. Two structures have been observed above He^{+ 2}S_{1/2} which were observed previously.

INTRODUCTION

The first e^- -He scattering cross-section measurements were reported by Ramsauer.¹ In these experiments, electrons from a photoelectric source were momentum selected by a magnetic selector, then passed through a gas cell and collected. The total cross section was directly determined by a study of the electron beam attenuation as a function of the gas pressure in the interaction region.

The energy resolution of a spectrometer can be defined in terms of ΔE , the full energy width at half-maximum current (FWHM). In a magnetic selector of fixed geometry, ΔE increases with the energy. The large resonance at 19.3 eV first observed by Schulz² was not detected by Ramsauer, $¹$ </sup> because of poor energy resolution at that energy.

The recognition of the need for high-energy resolution in electron-scattering experiments was olution in electron-scattering experiments was
made by Fox and co-workers.³ In 1957, Schulz and $\overline{\text{Fox}}$ used a retarding-potential-differe (RPD) electron gun,³ with FWHM of about 0.1 eV to study the cross section for excitation of helium metastable levels near threshold. ⁴ Baranger and Gerjuoy were able to fit the resulting cross-section measurements for excitation to the $1s2s³S$, level of helium near threshold, ⁴ with a one-level Breit-Wigner formula.⁵ Thus, they postulated the presence of a short-lived compound state of $He⁻$ in the vicinity. Furthermore, they have urged that the elastic scattering of e^- -He be reexamined with better energy resolution, since the existence of structure in the elastic cross section would also support the idea of a compound model. In 1963, Schulz, using an electrostatic energy se-In 1905, Schurz, using an electrostatic energy se-
lector,⁷ found an elastic resonance in the scattere current to a fixed scattering angle for e^- -He scattering at about 0.5 eV below the $1s2s³S$, threshold. ' The resolution of the resonance was limited to about 0. 1 eV by the instrumental resolution. Fleming and Higginson also reported structure in the e^- -He scattering cross section at about the same energy as Schulz, 8 but with poor energy resolution due to the Maier-Leibnitz technique used in their measurement.⁹ Simpson and Fano, using a spherical electrostatic monochromator with about 0.05 eV energy resolution in a transmission experiment, confirmed the position of the resonance established by Sehulz and also estimated the width of the resonance to be less than 0. 01 the width of the resonance to be less than 0.01
eV.¹⁰ Golden and Bandel, using a magnetic momentum selector similar to that of Ramsauer, ' but with better energy resolution at 19 eV, found the elastic resonance in helium in the total cross
section.¹¹ They observed the resonance to be section.¹¹ They observed the resonance to be limited by instrumental resolution to about 0. 1 eV.

Agreement as to the position of this resonance is very good $(19.3 \pm 0.01 \text{ eV})$, with a width of between 0. 004 and 0.010 eV. This resonance may be attributed to the temporary formation of the ionic state He⁻ $1s2s^2S_{1/2}$.
Kuyatt *et al*, ¹² working w

External extending $\frac{1}{2}$ working with the spherical electrical elecontation monochromator of Simpson and Fano, 10 trostatic monochromator of Simpson and Fano, but with an improved signal-to-noise ratio, have found 10 additional structures between 19.3 and the onset of He⁺, and two resonances near the $n=2$ thresholds of doubly-excited He.

In this work, the results are given for the structures found in the e^- -He scattering cross section, as seen in transmission. The present experiment has been performed with a cylindrical RPD gun, which is capable of an energy resolution of better than 0. 01 eV and an over-all signal-to-noise ratio of a few parts in $10⁴$.

APPARATUS

Several devices have been used in past electronatom scattering experiments to obtain beams of electrons of more or less sharply defined energy. Schulz has used both a cylindrical electrostatic,² Schulz has used both a cylindrical electrostatic,
and an RPD monochromator.¹³ Simpson has developed very refined spherical electrostatic monochromators. $^{10, 12}$ Golden has used both magnetic¹¹ and cylindrical electrostatic 14 monochromators. More or less, all of these systems have given energy resolutions ranging upwards from about 0. 05 eV. The apparatus described here uses the RPD technique to make an energy-selecting element. The RPD principle was first described by Fox '*et al.*,³ and was used in a slightly different way by *et al.*,³ and was used in a slightly different way $\frac{1}{3}$ Schulz.¹³ Our electron gun uses the same princi ple as was used by Schulz with the following two exceptions: In the present case the electron optics is more refined and no axial magnetic field is used. The earth's magnetic field is compensated for within a few percent by a Helmholtz coil.

Figure 1 shows a schematic diagram of the present experimental arrangement. The gun uses a

system of cylindrical electrostatic lenses. The oxide-coated cathode is in a Pierce geometry. This has been done in order to have a good approximation to a parallel beam of electrons at the front of the gun. The lenses which follow were designed according to the electron lens design curves given according to the electron lens design curves given
by Spangenberg.¹⁵ The first two lenses have been designed to obtain a parallel beam of electrons in the equipotential zone of the RPD electrode. At the third electrode (the RPD electrode), the lowerenergy electrons of the energy distribution in the beam are repelled by the retarding field while the higher energy ones are just able to pass the electrode. Those which pass the RPD electrode are collected by the optics which follows and focussed through a scattering cell 25 mm long with entrance and exit circular apertures of 1 mm diam. At the same time, the energy is increased so that a narrow, slightly converging beam at the scattering energy passes through the scattering cell to the collector. If the retarding voltage V_R is made more positive by a small quantity ΔV , some of the electrons repelled at the previous setting are allowed to pass. 'These electrons lie in a range of energy $\Delta E = e \Delta V$ in first approximation. If the voltage ΔV is applied as a square wave, the electrons in the range ΔE are transmitted during one-half of the cycle and are not transmitted during the other half. If the collector is connected to an ac detector, only the current due to electrons in the energy range ΔE will be detected. However, the preceding discussion is valid only if the electrons travel perpendicular to the retarding plane at the RPD electrode. In other words, since the retarding plane has been made peryendicular to the axis of the gun, only the axial component of velocity is selected. The spread in the transverse direction is unaffected by the retarding field. The limit of the energy resolution in the RPD method is set for the most part by the degree to which a parallel beam can be part by the degree to which a parallel beam can k
made at the retarding plane.¹⁶ In the present arrangement, the ratio of beam current I and FWHM ΔE is approximately a constant, such that $I/\Delta E$ $\simeq 10^{-7}$ A/eV. Other factors limiting the energy resolution have proved to have an effect at least 2 orders of magnitude smaller than this.¹⁶ In the present work, the resolution is limited by the Doppler broadening due to the thermal motion of .
Doppler broadening due to the thermal motio
the target atoms.¹⁷ The FWHM which can be achieved is less than 0. 01 eV, as estimated by observations of the 19.3-eV resonance with various servations of the 19.3-eV resonance with vari-
pulse heights on the RPD electrode.¹⁶ For the present work, the energy resolution was fixed to 0. 05 eV.

The vacuum system is of all stainless-steel construction. An oil diffusion pump of 3000-liter/sec pumping speed is used in connection with a zeolite trap. This combination allows a differentialpumping ratio between vacuum chamber and gas cell of more than 3000 to be maintained. The system reaches, after 300° C baking, a base pressure of 10^{-9} Torr.

The gas pressure was read with a high-pressure The gas pressure was read with a high-pre
ion gauge, ¹⁸ which was corrected for the iongauge calibration constant as specified by the manufacturer. Several runs at various pressures were made to check the pressure independence of the observed structures.

The energy scale was calibrated from the onset of excitation to the ${}^{3}S_1$ state as 19.818 eV and the of excitation to the ${}^{3}S_{1}$ state as 19.818 eV and the ${}^{1}S_{0}$ state as 20.614 eV.¹⁹ This energy scale was found to be in agreement with that obtained by subtracting the contact potential as determined by the voltage on the RPD electrode from the voltage difference between the scattering cell and the cathode. The positions of the features observed are believed to be precise to \pm 0.02 eV except for the maximum at 19.3 eV, which is believed to be precise to 0. 01 eV.

The detector used was a lock-in amplifier (phase sensitive detector) connected to the collector. A reference signal from the lock-in amplifier was used to drive a variable amplitude square-wave pulser whose output modulated the retarding voltage. The output of the lock-in amplifier was connected directly to the Y axis of an $X-Y$ recorder whose X axis was driven by a voltage proportional to the voltage between the scattering cell and cathode. Alternatively, the output of the lock-in amplifier was fed to an averaging computer. This made the signal-to-noise ratio better by a factor equal to the square root of the number of sweeps stored by the averager, within the limitation imposed by the stability of the whole apparatus.

Changes in contact potential during the run were carefully monitored by looking at the width of the first helium resonance. This width seen by the averager should increase if the position of the resonance moves as a function of time. The upper limit to the resonance broadening going from 1 sweep to about 100 was about 0.055 eV. The data output from the averaging computer was plotted and smoothed, regarding as noise any signal lower than a factor of 3 of the rms noise and with an energy dependence sharper than the beam FWHM.

RESULTS

Figure 2 shows a plot of the energy range from about 56. 3 eV to about 58. ⁷ eV. The energy scale was obtained from the position of the ${}^{2}S_{1/2}$ state of He⁻ at 19.3 eV. The solid line is drawn to represent the background current. Two structures may be observed on the plot which have been first ob-
served by Kuyatt, Simpson, and Mielczarek.¹² served by Kuyatt, Simpson, and Mielczarek.¹² The first is centered at about 56. 8 eV. This structure consists of a dip in the current with minimum at 56. 71 eV followed by a rise in the current with maximum at 56.93. In the case of this struc-

ELECTRON ENERGY {eV)

FIG. 2. Transmitted current for the energy range of 56.23-58.63 eV (single sweep). (Pressure \simeq 0.5 Torr.)

ture, the area enclosed between the maximum and the background line is approximately the same as that enclosed between the minimum and the background line. The other resonance is centered at about 58 eV and is composed of a minimum at 57. 87 eV, followed by a maximum at 58.08 eV. In this case, the area enclosed between the maximum and the background is about twice that between the minimum and the background. These two structures have been assigned the configurations $2s^22p^2P_0$ and $2s2p^2D$, respectively, by Fano tions $2s^22p^2P_0$ and $2s2p^2D$, respectively, by Fano
and Cooper.²⁰ Recently, Burrow and Schulz²¹ have used the trapped-electron method 22 to study the decay of these two compound states by two-electron emission. The positions of the observed structures are summarized in Table I.

Since there were so many structures observed between 19.3 eV and the onset of ionization, it was decided to try to improve the signal-to-noise ratio

in order to better identify them. In order to achieve this, the output of the phase-sensitive detector was connected to the input of an averaging computer. Figures 3-5 show the resulting output of the averaging computer for the sum of 80 sweeps in the energy range from about 19.1 to 24.6 eV. The sweep rate on the X axis was 40 sec/eV with a response time of 300 msec on the Y axis. Roughly this gives an improvement of about a factor of 5 in the signal-to-noise ratio over that of the data of a single sweep. This type of data was obtained about IO times with about the same result. It was not found practical to store data for much longer periods of time. The signal-to-noise ratio (S/N) was found to increase substantially for periods of the order of 6 or 7 h. For longer running times, not much improvement was realized due to detuning effects. Since the increase in S/N goes as the square root of the number of sweeps, to increase S/N by another factor of 5, one would have to have about 2000 sweeps $(150-175h)$.

Figure 3 shows the energy range from 19.1 to 20. 8 eV. Also shown are the two states of neutral helium in this energy range (the solid vertical lines). The onset of the excitation to these two states (labeled A and B on the figure) was used to
calibrate the energy scale.¹⁹ Thus, the first resonance was found to have a maximum at 19.30 \pm 0.01 eV (labeled 1') and a minimum at 19.40 \pm 0.02 eV (labeled 1). Between 1', 1, and He 1s2s ${}^{3}S_{1}$ at 19.818 eV two maxima (and two minima) in the transmitted current may be seen on the plot. Between He ${}^{3}S_{1}$ and He ${}^{1}S_{0}$ there are three additional maxima (and minima). We will attempt to classify the associated five compound states ac-
cording to the procedure of Fano and Cooper.²⁰ cording to the procedure of Fano and Cooper.²⁰ That is, all quartet negative-ion states will not be considered since they require a spin flip and all negative-ion states which don't have a parity -1^l (where l is the orbital quantum number of the incoming or extra electron) will not be considered. This leaves only the following possible states for $n = 2$: He^{-1s2s2p²P°: He^{-1s2p²²D, He^{-1s2p²²S.²³}}} $n=2$: He^{-1s2s2p²P⁰: He^{-1s2p²²D, He^{-1s2p²²S.²³}}} It should be noted that each of the above He configurations can give rise to two separate states because there can be an energy difference de-

aReference 12.

b_{Reference 21.}

pending on whether the two outermost electrons of each configuration have their spins parallel or antiparallel.²⁴ However, some of the possible antiparallel.²⁴ However, some of the possible states may not exist.²⁵ A rough estimate of the separation of the two $^{2}P^{0}$ states can be obtained by making a hydrogenic analog between the energy separation of the lowest ${}^{3}P$ and ${}^{1}P$ states of doubly excited He and the $^{2}P^{0}$ states of He⁻. Thus, to get the He⁻²P[°] separation, we divide the ${}^{3}P$ -¹P separation of doubly excited He by 4. (For He^- , the two outer electrons move in approximately a $Z = 1$ field, whereas for He, the two outer electrons move in approximately a $Z = 2$ field.) From the data of in approximately a $Z = 2$ field.) From the data of
Simpson, Mielczarek, and Cooper,²⁶ we obtain a separation of 0.4 eV for the $2p^0$ states of He⁻ and from the calculations of O' Malley and Geltman, 2^7 from the calculations of O' Malley and Geltman,

a separation of 0. 47 eV. Following the assignment given by Fano and Cooper for the doubly excited states of He, we assign 2', ² the designation $({}^{2}P^{0})$ + \star 28 From the arguments presented above we might expect the second ${}^{2}P^{0}$ level to lie near, but slightly below, 20 eV. The nearest level is the 4', ⁴ level, which, on the basis of the above argument, we should assign the designation He^{-2po} \star . We are thus left with three levels unassigned, 3', 3, 5', 5, and 6', 6. If the state of He⁻⁽²D) $_{1}$ exists it would most likely be above the first ${}^{2}P_{0}$ level and therefore we would assign it to the 3', ³ state. One might reasonably expect the $He^{-} ({}^{2}P^{0})_{A}^{A} + {}^{2}P^{0}_{A}^{A}$ difference to be larger than the $({}^2D)$ ++- $({}^2D)$ ++ difference and therefore we migh possibly assign ${}^{2}P^{0}$ + to the 5', 5 level and $({}^{2}D)$ +

FIG. 4. Transmitted current for the energy range of 20.8-22.7 eV (total of 80 sweeps). (Pres-

to the $4'$, 4 level. In either case, the $6'$, 6 level should be assigned the configuration $(2S)$ ++ if it exists. We expect the (^{2}S) $_{\uparrow\uparrow}$ - (^{2}S) $_{\uparrow\downarrow}$ separation to be larger than the $({}^2P^0)_{\text{A}^{\text{A}}-}({}^2P^0)_{\text{A}^{\text{A}}}$ separation, and therefore the (^{2}S) $_{\textbf{+}}$ level if it exists should be at 21 eV or slightly higher.

I I I I ^I I

Burke, Cooper, and Ormonde have used the close-coupling method to calculate the electron
impact excitation of the $n=2$ states of He.²⁹ T impact excitation of the $n=2$ states of He.²⁹ Their results give a large broad peak in the total metastable production at 20.2 eV (P wave) and another at about 21.0 eV $(D$ wave). Their results are in substantial agreement with the experiments which looked at the total production of metastables. $4,30$ The peaks in the excitation cross section should appear as minima in a transmission experiment, and indeed there is a minimum at 20. 2 eV.

Turning to Fig. 4,we see that the break due to the onset of excitation to the ${}^{3}P_{0}$ 20.962 eV actu-

ally appears at 20.93 eV (labeled C). This is prob-
ably due to the *D*-wave resonance at 21.0 eV, 29 ably due to the D-wave resonance at 21.0 eV . ²⁹ which serves to depress the transmitted current. In fact, the second (^2S) $_{\uparrow\downarrow}$ resonance might also be lost in the complicated structure near 21 eV. The break due to the onset of excitation to the ${}^{1}P$ at 21. 22 is labeled D on Fig. 4. The structures below the $n = 3$ states of He labeled 7-14 (13 and 14 are shown on Fig. 5) are more difficult to discuss in terms of single-level resonances since the largest separation for the $n = 3$ states of He ${}^{3}S$ -1S is only about 0. 2 eV. These and the other higherenergy structures would have to be discussed in terms of multilevel resonances. However, we venture to say that there is probably a resonance in the excitation cross section for the $n=3$, ${}^{3}S$ state of He at an energy near 22. 76 eV which is evidenced by the minimum in the transmission at that energy shown on Fig. 5. The step labeled E

for the energy range of 22.7-24.65 eV (total of 80 sweeps), (Pres-

is found in the close neighborhood of the correct position of the ionization onset. This structure is probably due to the presence of a series of unresolved resonances with He⁺ as a limit. The result is a sharp increase in the transmitted current at the onset of ionization as has been discussed by
Kuyatt *et al*. ¹² Kuyatt et al.¹²

Table II shows a list of the positions of all of the features observed up to ionization in He. The table also shows the results of the experiment of ble also shows the results of the experiment of
Kuyatt *et al*., ¹² for comparison. About twice as many structures have been found in the present experiment as have been seen by Kuyatt et al. However, the agreement is excellent for those features where comparison is possible. A search of the energy range from essentially 0 to 19.3

eV has revealed no structure.

Finally, it should be stated that additional work on this problem towards identifying the structures should be done. Experimentally, this entails argular distribution experiments which include provision for energy-loss measurements.

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 24.64 ± 0.1

^cCalibration point.

He $1s5s\,{}^{3\!}S_1$ 23.969 eV

onset of ${\rm He}^+$ 24.585 ${\rm eV}$

TABLE II. Helium transmission features below ionization.

Electron transmission experiment, Ref. 12.

18' 18 19' 19

 $\mathbf E$

23.96 24.06 24.24 24.36 24.60

Energies in eV obtained from Ref. 19.

Min Max Min Step

 $_{\rm{Max}}$

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