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Theory of the Forward Peak in the Angular Distribution of Electrons Ejected by Fast Protons*

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The cross section for ejecting electrons by 300-keV protons is calculated using the first term in the Neuman expansion of Faddeev's equation for the final state of the electron-proton-residual ion system. This approximation predicts a peak at 0° in the angular distribution of electrons ejected with a velocity approximately equal to the velocity of outgoing protons. Numerical results for He and H_2 target gases are given and compared with the experiments of Rudd and co-workers. The qualitative behavior of the forward peak in the experimental angular distributions is well accounted for.

I. INTRODUCTION

Experimental techniques have reached the stage where the energy and angular distribution of electrons ejected by heavy ion impact can be measured. Because cross sections that are differential in both the energy and angle of the ejected electron are obtained, these experiments produce much valuable information about the ionization process. In particular, Rudd, elaborating on a suggestion by Oldham,[1] has proposed a new mechanism for ionization to explain the forward peaking in the published data[2] on the ionization of He and H_2 by proton impact. Since the forward peaking is most pronounced when the electron is ejected with a velocity nearly equal to the velocity of the proton, Rudd argues that some electrons are carried along with the proton for a short time and then move away from the proton as free electrons. This paper discusses the theoretical basis for Rudd's mechanism, and shows in just what sense an electron in continuum state is carried along by a proton.

We seek a first-order approximation for the final-state wave function of the electron, proton, and residual ion which can describe an electron being carried along by the proton. Of equal importance is the requirement that the approximate wave function give a reasonably good indication of what a more accurate theory would predict. The first term in the Neumann expansion of Faddeev's[3] equation for the final-state wave function exactly satisfies these two requirements. Since our approximation is only first order, its range of validity is considerably restricted at the outset. The highest energy for the proton in the experiments of Rudd *et al.* was 300 keV. Since a 300-keV proton is only moving with a velocity of about 3.5 times the velocity of an electron in the first Bohr orbit of the hydrogen atom, a first-order approximation generally does not adequately describe the detailed distribution of electrons in energy and angle. A plane wave accurately describes the motion of the proton, but the wave function of the ejected electron is distorted by both the field of the residual ion and the field of the proton. Since

all particles in the final state are charged, the distortion extends over the entire region occupied by the atomic electron; and no simple wave function can be expected to predict all features of the doubly differential cross section. However, electrons ejected with an energy greater than 2 or 3 a.u. must have come from close collisions with the incident proton. The ejection of these more energetic electrons, which are ejected in an essentially binary encounter, can be reasonably well treated by first-order approximations. For example, the Born-approximation calculation by Massey and Mohr[4] predicts a sharp peak in the angular distribution of energetic electrons at an angle such that energy and momentum of the proton-electron subsystem are conserved. The data of Rudd *et al.*[2] indeed shows this peak, indicating that electrons with 2 or 3 a.u. of energy are produced by binary collisions. However, the angular distribution of electrons of a definite energy shows another peak not predicted by the Born approximation. This peak occurs at 0° and at electron energies such that the velocity of the outgoing electron approximately equals the velocity of the proton. Since most protons are scattered forward, this suggests that some electrons are carried along by the proton. This "carrying along of the electron" is described by a Coulomb plane wave centered about the proton. Such a final-state wave function does not go beyond the binary collision model for the ionization process; consequently, the theory only describes the angular distribution of the faster electrons.

The Born approximation of Massey and Mohr[4] employs a Coulomb plane wave centered at the residual ion as the final-state wave function. Such a wave function takes account of the distortion of the electron distribution by the electrostatic attraction of the residual ion, but neglects distortion due to the proton. For the faster electrons, the latter distortion is at least as important as the former, since the relative velocity of the proton and electron may actually be less than the relative velocity of the electron and the residual ion. This additional attraction will affect the energy and angular distribution of the faster electrons; consequently, we seek an approximate wave function which takes this distortion into account.

Since the final state of the system involves three free particles, we will base our discussion on Faddeev's integral equation for the final-state wave function. For our purposes, two properties of Faddeev's equations are important. First, all particles are treated on an equal footing. No pair of particles is singled out for special treatment. This is not only a property of the equations and their exact solutions, but it is also a property of each term of the Neumann expansion of the solutions, in contrast to the Born approximation of Massey and Mohr where the electron-residual ion pair is singled out for special treatment. Secondly, Faddeev's equations are written in terms of the two-body T matrix elements rather than the two-body potentials; consequently, the first term of the Neumann expansion correctly describes (a) an electron moving slowly away from the residual ion while the proton moves rapidly away, and (b) an electron and a proton moving slowly away from one another, but both moving rapidly away from the residual ion. The Born approximation describes only the first configuration of particles in the final state.

It is useful to note the connection between excitation, charge exchange, and ionization. Excitation of the target atom and production of slow electrons by ionization differ only in the amount of energy the atomic electron receives. In both cases, the final state of the electron is essentially an eigenstate of the target. However, electrons ejected in the forward direction with a velocity approximately equal to the velocity of the scattered proton are never in an eigenstate of the target. Here, the final state is more nearly an eigenstate of the electron-proton system, that is, an eigenstate of the hydrogen atom. This component of the electron spectrum is an extrapolation in energy of charge exchange to an excited state of hydrogen, and the mechanism proposed by Rudd could be termed "charge exchange to a continuum state." But Rudd's proposed mechanism and true charge exchange differ in one fundamental respect: To determine the total number of excited atoms produced in a proton-atom collision, we add the cross sections for producing excited states of the target (direct excitation) and the cross sections for producing excited states of the proton-atom system (charge exchange to excited states). This procedure is correct, since charge exchange and excitation are physically distinct processes. There is no physical distinction between ionization and "charge exchange to a continuum state"; indeed the latter is only one mechanism for ionization. One therefore expects to add the amplitudes for the two mechanisms (a) and (b) rather than the cross sections. This is not exactly correct, but the correct prescription for combining the amplitudes follows directly from Faddeev's equations, and is discussed in Sec. II.

It is well known that first-order theories of charge exchange are less satisfactory than the Born theory of excitation, primarily because initial and final states are not orthogonal.[5] We will find that the same difficulty occurs in our treatment of ionization; nonetheless, the theory gives a reasonable qualitative and semiquantitative interpretation of the experimental results. These aspects are discussed further in Sec. IV.

The general theory is presented in Sec. II, numerical results are given in Sec. III, and the validity of the conclusions is discussed in Sec

IV. Atomic units are used throughout, except in the presentation of numerical results for comparison with experiment. Then MKS units are used.

II. GENERAL THEORY

Our system consists of three particles of masses m_1, m_2, and m_3 with coordinates r_1, r_2, and r_3 relative to a space-fixed system. Faddeev's expression for the wave function of three asymptotically free particles is[3]

$$\Psi = \Phi + \psi^{(1)} + \psi^{(2)} + \psi^{(3)} , \tag{1}$$

where Φ is the plane wave:

$$\Phi = (2\pi)^{-9/2} e^{i(\vec{k}_1\cdot\vec{r}_1 + \vec{k}_2\cdot\vec{r}_2 + \vec{k}_3\cdot\vec{r}_3)} , \tag{2}$$

and $\psi^{(i)} (i = 1, 2, 3)$ satisfies

$$\begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \end{pmatrix} = \begin{pmatrix} \Phi^{(23)} - \Phi \\ \Phi^{(31)} - \Phi \\ \Phi^{(12)} - \Phi \end{pmatrix} - G_0 \begin{pmatrix} 0 & T_{23} & T_{23} \\ T_{31} & 0 & T_{31} \\ T_{12} & T_{12} & 0 \end{pmatrix} \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \end{pmatrix} . \tag{3}$$

There, T_{ij} is the two-body T matrix element for a system with particles i and j interacting via a potential V_{ij}, and the third particle is free. $\Phi^{(ij)}$ is the corresponding wave function satisfying the Schrödinger equation

$$\left(-\frac{1}{2m_1}\nabla_1^2 - \frac{1}{2m_2}\nabla_2^2 - \frac{1}{2m_3}\nabla_3^2 + V_{ij} \right) \Phi^{(ij)} = E\Phi^{(ij)} , \tag{4}$$

with the boundary condition that $\Phi^{(ij)}$ approaches the plane wave (2) for large separations of all the particles. For example, $\Phi^{(23)}$ is

$$(2\pi)^{-3} e^{-i(\vec{k}_1\cdot\vec{r}_1 + \vec{K}_{23}\cdot\vec{R}_{23})} \psi^{-}_{\vec{\kappa}_{23}}(\vec{r}_{23}) , \tag{5}$$

where $\vec{\kappa}_{ij}$ is the relative momentum of particles i and j with velocities $\vec{v}_i$ and $\vec{v}_j$,

$$\vec{\kappa}_{ij} = m_i m_j (\vec{v}_i - \vec{v}_j)/(m_i + m_j) , \tag{6}$$

$\vec{r}_{ij}$ is the relative coordinates of particles i and j,

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j , \tag{7}$$

$\vec{K}_{ij}$ is the momentum of the c. m. of particles i and j,

$$\vec{K}_{ij} = m_i\vec{v}_i + m_j\vec{v}_j , \tag{8}$$

and $\vec{R}_{ij}$ is the coordinate of the c. m. of particles i and j:

$$\vec{R}_{ij} = (m_i\vec{r}_i + m_j\vec{r}_j)/(m_i + m_j) . \tag{9}$$

$\psi^{-}_{\vec{\kappa}_{ij}}$ is a continuum wave function of particles i and j, and G_0 is the Green's functions for three noninteracting particles. Faddeev has shown that the solutions of (1) are unique, that is, the boundary conditions on Φ are completely incorporated in the inhomogeneous term. For short-range potentials, this ensures that Ψ is correct in the asymptotic region where at least one of the particles is well separated from the other two. A wave function obtained by truncating the Neumann expansion of (3) also has the correct asymptotic form. For potentials of infinite range, such as the Coulomb potential, one cannot be sure that Ψ is correct in the asymptotic region; however, the form of the inhomogeneous term in (3) strongly suggests that when one particle is far from the other two, the exact wave function and lower-order approximations obtained by truncating the Neumann series for ψ are correct. Because we are interested in the effect of the proton-electron attraction when the proton and electron are close together but far from the residual ion, we will use a wave function which consists of the first term in the Neumann expansion of (3):

$$\psi_f \approx \Phi^{(23)} + \Phi^{(31)} + \Phi^{(12)} - 2\Phi . \tag{10}$$

The wave function for the initial state, in which particles 2 and 3 are bound together and 1 is free, is

$$\chi_i = (2\pi)^{-3} e^{i(\vec{k}_1'\cdot\vec{r}_1 + \vec{K}_{23}'\cdot\vec{R}_{23})} \psi_{23}(\vec{r}_{23}) , \tag{11}$$

where $\psi_{23}(\vec{r}_{23})$ is the wave function of the bound system. Primes on the momentum vectors $\vec{k}_1'$ and $\vec{K}_{23}'$ denote initial values.

The initial- and final-state wave functions are substituted in the matrix element

$$\tau_{fi} = (\Psi^{-}_{f'} V_i \chi_i) \tag{12}$$

to give the transition amplitude for the transition of the system from the initial to the final state. It is important to note that a δ function describing momentum conservation of the c. m. of all three particles has not been factored out of τ_{fi}. It is left in (12) for convenience in transforming between various coordinate systems.

In applying (10)–(12) to the ejection of an electron from an atom by a proton, we let subscript 1 refer to the proton, 2 to the electron, and 3 to the residual ion. Then V_i is just

$$-1/r_{12}+1/r_{13}\ . \tag{13}$$

Two approximations, in addition to the neglect of higher-order terms in the expansion of (3), will be made. We will (a) replace $\Phi^{(31)}$ by Φ, and (b) make the Brinkman-Kramers approximation, i.e., we will drop $1/r_{13}$ in (13). Since the momentum of the incoming proton is several thousand atomic units for 300-keV protons, the Coulomb plane wave of the proton differs little from an undistorted plane wave; hence (a) is well justified. Jackson and Schiff[6] have shown for charge exchange collisions that, in the Born approximation, the matrix element of $1/r_{13}$ is of the same order of magnitude as the matrix element of $1/r_{12}$, thus one cannot neglect the $1/r_{13}$ term on the grounds that it is small in a first-order approximation. Rather, the ratio of the matrix element of $1/r_{13}$ to the matrix element of $1/r_{12}$ is of the order of the mass of the electron divided by the mass of the proton when the exact wave function is used.[6] On this basis, we set

$$V_i=-1/r_{12} \tag{14}$$

in (12). In Sec. IV, we will show that the inclusion of $1/r_{13}$ gives a completely unrealistic angular distribution of ejected electrons, as one would expect on the basis of Wick's[6] argument.

With the additional approximations, we then have, for ψ_f^-,

$$\Psi_f^- = \Phi^{(23)}+\Phi^{(12)}-\Phi\ . \tag{15}$$

Usually the hydrogen atom continuum functions are given in terms of the coordinates $\vec{r}_{21}=\vec{r}_2-\vec{r}_1$ and the momentum

$$\vec{\kappa}_{21}=m_1m_2(\vec{v}_2-\vec{v}_1)/(m_1+m_2)\ ,$$

therefore we shall denote the hydrogenic function $\psi_{\vec{\kappa}_{12}}(\vec{r}_{12})$ by $\psi_{\vec{\kappa}_{21}}(\vec{r}_{21})$ to avoid confusion later on.

Substituting (14) and (15) into (12) gives

$$\tau_{fi}=(\Phi^{(23)},\ V_i\chi_i)+(\Phi^{(12)},\ V_i\chi_i)-(\Phi,\ V_i\chi_i)$$

$$=a_{23}+a_{12}-a\ , \tag{16}$$

which defines a_{nm} and a.

Expression (16) for τ_{fi} is a sum of three amplitudes. a_{23} is the usual Born amplitude, a_{12} is the amplitude for exchange into a continuum state, and a is a counter term arising because the term with all outgoing particles in a plane wave has been counted twice. Equation (16) is the correct prescription for adding the amplitude for exchange into a continuum state to the Born amplitude.

To evaluate τ_{fi}, we need only carry out the integrations in (16). a_{23} is just the Born amplitude (multiplied by a momentum conservation δ function), and has been evaluated by Massey and Mohr[4] for s electrons. Using their result, with some corrections in notation,[7] we find

$$a_{23}=-\frac{2\sqrt{2}\mu^{5/2}\exp(\pi\mu/\kappa_{23})\Gamma(1-i\mu/\kappa_{23})(\kappa-\kappa_{23}\cos\gamma-i\mu\cos\gamma)}{\pi^3K[\mu^2+(\vec{K}-\vec{\kappa}_{23})^2]^2[K^2-(\kappa_{23}+i\mu)^2]}\left(\frac{K^2-(\kappa_{23}+i\mu)^2}{\mu^2+(\vec{K}-\vec{\kappa}_{23})^2}\right)^{-i\mu/\kappa_{23}}, \tag{17}$$

where γ is the angle between $\vec{K}$ and $\vec{\kappa}_{23}$, and μ is the effective charge.

To evaluate a_{12} we first express the initial wave functions in terms of the coordinates $\vec{r}_{21}$, $\vec{r}_{23}$, and $\vec{R}$, where $\vec{R}$ is the coordinate of the c.m. of all three particles. We find, after some manipulation,

$$a_{12}=-(2\pi)^{-3}\int dR\, e^{i(\vec{k}_1'+\vec{K}_{23}'-\vec{k}_3-\vec{K}_{12})\cdot\vec{R}}$$

$$\times(2\pi)^{-3/2}\int dr_{21}\, r_{21}^{-1} e^{i\vec{A}\cdot\vec{r}_{21}}\psi_{\vec{\kappa}_{21}}^{-*}(\vec{r}_{21})$$

$$\times(2\pi)^{-3/2}\int dr_{23} e^{i\vec{B}\cdot\vec{r}_{23}}\psi_{23}(\vec{r}_{23})\ , \tag{18}$$

where $\vec{A}=m_1m_2(\vec{v}_2-\vec{v}_1)/(m_1+m_2)-m_1(\vec{v}_1'-\vec{v}_1)$;

$\vec{B}=m_1m_3(\vec{v}_1'-\vec{v}_1)/(m_1+m_3)-m_2m_3(\vec{v}_2-\vec{v}_3)/(m_2+m_3)$.

Recognizing that $m_1(\vec{v}_1'-\vec{v}_1)$ is the momentum transfer $\vec{K}$, and approximating $m_1m_3/(m_2+m_3)\approx m_1$, we have for $\vec{A}$ and $\vec{B}$,

$$\vec{A}=\vec{\kappa}_{21}-\vec{K},\quad \vec{B}=\vec{K}-\kappa_{23}\ . \tag{19}$$

The first integral in (22) is just the momentum conservation δ function, $\delta(\vec{k}_1'+\vec{K}_{23}'-\vec{k}_3-\vec{k}_{12})$, and we have used momentum conservation to simplify the expressions for A and B. The third integral is just the Fourier transform of the bound-state wave function and is easily evaluated. The second integral has been evaluated by Landau and Lifshitz.[8]

Note first that the integral does not converge since the potential drops off only as r_{21}^{-1} as $r_{21}\to\infty$. This difficulty is also encountered when the Fourier transform of r_{21}^{-1} is evaluated, and is easily circumvented by replacing r_{21}^{-1} by

$r_{21}^{-1}e^{-\lambda r_{21}}$ and taking the limit $\lambda \to 0$ after the integration has been performed. Using Landau and Lifshitz's[8] expression for the second integral I, then taking the limit as $\lambda \to 0$, and using the convention that all complex quantities raised to a power are taken with the argument whose absolute values is least,[8] we have

$$I = -\tfrac{1}{2}\pi^{-2}e^{\pi/2\kappa_{21}}\Gamma(1 - i/\kappa_{21})(K^2)^{-1-i/\kappa_{21}} \times [(\vec{K}+\vec{\kappa}_{21})^2 - \kappa_{21}^2]^{-i/\kappa_{21}} , \quad (20)$$

where the argument of the quantity inside square brackets is zero if the quantity is positive and $-\pi$ if the quantity is negative. For electrons in the forward peak, $\kappa_{21} \approx 0$, and the argument is zero.

We are interested in the ejection of electrons from 1S states, so we take $\psi_i = \mu^{3/2}e^{-\mu r/\sqrt{\pi}}$, where μ is an effective charge. The third integral is then just

$$2\sqrt{2}\,\mu^{5/2}/\pi[\mu^2 + (\vec{K} - \vec{\kappa}_{23})^2]^2 . \quad (21)$$

Substituting (20) and (21) into (18), and dropping the momentum-conservation δ function, we get

$$a_{12} = -\frac{\exp(\pi/2\kappa_{21})\Gamma(1 - i/\kappa_{21})\sqrt{2}\,\mu^{5/2}}{\pi^3 K^2[\mu^2 + (\vec{K} - \vec{\kappa})^2]^2} \times \left(\frac{(\vec{K}+\vec{\kappa}_{21}) - \kappa_{21}^2}{K^2}\right)^{-i/\kappa_{21}} . \quad (22)$$

The third amplitude only involves integrals over the plane-wave continuum functions and is easily evaluated in the set of coordinates R, r_{21}, r_{23}. The required amplitude is the same as (18) except that $\psi_{\vec{\kappa}_{21}}$ is replaced by a plane wave. We find

$$a = -\sqrt{2}\,\mu^{5/2}/\pi^3 K^2[\mu^2 + (\vec{K} - \vec{\kappa}_{23})^2]^2 . \quad (23)$$

Note that the absolute values $|a_{12}|$ and $|a|$ differ only in that $|a_{12}|$ is multiplied by

$$\exp(\pi/2\kappa_{21})|\Gamma(1 - i/\kappa_{21})| = \{2\pi/\kappa_{21}[1 - \exp(-2\pi/\kappa_{21})]\}^{1/2} . \quad (24)$$

From (24), it is apparent that a_{12} becomes large and dominates the cross section as $\kappa_{21} \to 0$, as is found experimentally. Since (24) is just the enhancement factor describing the increased electron density near the proton due to the electrostatic attraction between the electron and the proton, the forward peak does correspond to an electron being carried along by the proton. Thus, the forward peaking can be qualitatively described using just the first term of the Neumann expansion of Faddeev's equation. In Sec. III, we shall make a quantitative comparison with experiment.

III. NUMERICAL RESULTS

With wave functions normalized according to (11), the cross section for scattering a proton into a solid angle $d\Omega_p$ and ejecting an electron in the momentum range $d\vec{\kappa}_{23} = \kappa_{23}\,dE\,d\Omega_e$, where E is the energy of the electron $\frac{1}{2}\kappa_{23}^2$, is[8]

$$d\sigma = (4\pi^2 M)^2|\tau_{fi}|^2\kappa_{23}\,dE\,d\Omega_e\,d\Omega_p . \quad (25)$$

Equation (25) is integrated over $d\Omega_p$ numerically to obtain the cross-section differential in the energy and angle of the ejected electron. Since the experimental data are for the ionization of He and H_2, approximate wave functions for $\psi_{23}(\vec{r}_{23})$ and $\psi_{\vec{\kappa}_{23}^-}(\vec{r}_{23})$ are needed. We follow the approach of Bates and Griffing[9] in that we take $\psi_{23}(\vec{r}_{23})$ and $\psi_{\vec{\kappa}_{23}}(\vec{r}_{23})$ to be hydrogenic wave functions with an effective charge μ equal to $(2I)^{-1}$, where I is the ionization potential of He and H_2. The cross section is then obtained by multiplying (25) by the number of electrons, in this case, two. Bates *et al.*[9] discuss the validity of this procedure. Here we need only remark on one minor difference between our choice of $\psi_{\vec{\kappa}_{23}^-}(\vec{r}_{23})$ and that of Bates. The wave function $\psi_{\vec{\kappa}_{23}^-}$ is the produce of two factors: a function $e^{i\vec{\kappa}_{23}\cdot\vec{r}_{23}}F$ normalized to 1 at $r_{23} = 0$, and a normalization constant $e^{\mu\pi/2\kappa_{23}}\Gamma \times (1 - i\mu/\kappa_{23})$. The use of an effective charge in F is appropriate where r_{23} is small and the electron feels a potential of an effective charge. It is not appropriate when the electron is far from the He^+ ion. This region does not contribute directly to the matrix element, but it does contribute indirectly through the normalization constant, since the normalization constant is primarily determined by the wave function at large r_{23}. Here the electron sees a charge of 1, therefore we set $\mu = 1$ in the normalization constant, but not in the confluent hypergeometric function. At most, this changes the Born cross section by 30% for He and 3% for H_2.

Figures 1 and 2 compare the data of Rudd *et al.*[2] with the Born approximation and with this approximation. The forward peak is reproduced, but is overestimated by a factor of 2 in this approximation. More importantly, the variation of the shape of the angular distribution curves with the ionization potential of the target and with the energy of the outgoing electron is well reproduced. Since the shape of the angular distribution curves is primarily determined by the peak resulting from the increase of $1/\kappa_{21}$ as $\kappa_{21} \to 0$, and also by the peak resulting from the increase of $[\mu^2 + (\vec{K} - \vec{\kappa}_{23})^2]^{-2}$ when $\vec{K} = \vec{\kappa}_{23}$, this indicates that the enhancement factor (24) is indeed responsible for the forward peak. The presence of the enhance-

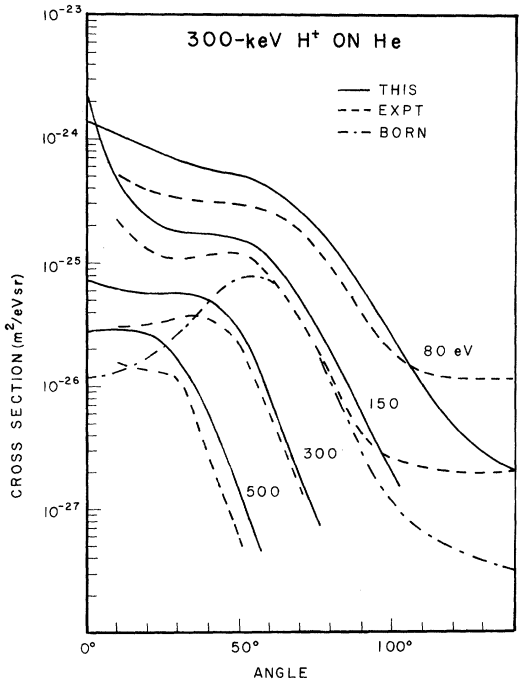

FIG. 1. Angular distribution of electrons ejected from He by 300-keV protons. Experimental curves are from Rudd *et al.* (Ref. 2).

ment factor in our wave function depends only on the assumption that (10) is a good approximation in the asymptotic region. Thus, our results indicate that the normalization constant of the exact final-state wave function will contain the factor (24). This is quite reasonable in view of the electron-proton attraction in the final state. Our explanation of the forward peak is better founded than the actual calculation, which assumes that (10) is a good wave function everywhere. Furthermore, our use of hydrogenic wave functions for the initial state does not affect this conclusion, since a more accurate wave function must still give a cross section with a peak at $\vec{K}=\vec{\kappa}_{23}$, the condition for conservation of momentum of the proton and electron alone. The sharpness of the momentum conservation peak mainly depends upon the spatial extent of the electron distribution, and this is reasonably well approximated by $\psi_i \alpha e^{-\mu r}$, with μ equal to the square root of twice the ionization potential.

Recently, Salin[10] has obtained the forward peak by introducing an effective charge

$$Z = 1 + \kappa_{23}/\kappa_{21} \ ,$$

in the final-state wave function. This charge appears in the normalization constant of the wave function, consequently the cross section becomes large as κ_{21} approaches zero. Salin's effective charge is determined by requiring the wave function to have an asymptotic form similar to ours, but otherwise the approximations are quite different. Both methods obtain the forward peaking because the normalization constants become large as $\kappa_{21} \to 0$.

IV. BRINKMAN-KRAMERS APPROXIMATION

The number of electrons ejected by protons is overestimated in this approximation, especially for the low-energy electrons. The Brinkman-Kramers approximation similarly overestimates charge-exchange cross sections. In both cases the overestimate is caused, at least in part, by the nonorthogonality of initial- and final-state wave functions.[5] The charge-exchange cross section is reduced when $1/r_{13}$ or the modification of $1/r_{13}$ proposed by Bessel and Gerjouy[5] is kept in V_i, and one may ask if $1/r_{13}$ should be kept in our calculation. We may determine the qualitative effect of including V_i on the angular distribution by evaluating the matrix element of $1/r_{13}$ with all final-state wave functions replaced by plane waves. This matrix element is conveniently evaluated in the system of coordinates $\vec{R}$, $\vec{r}_{23}$, and $\vec{r}_{13}$. It is identical to (18) except that $\psi_{\vec{\kappa}_{21}}$ is replaced by the plane wave:

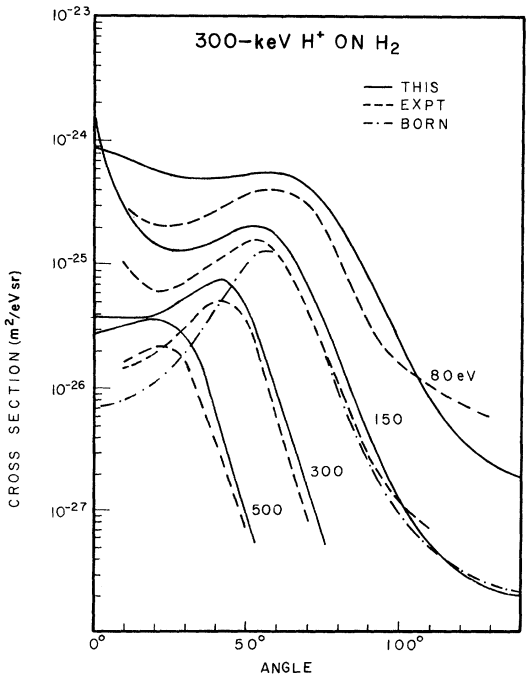

FIG. 2. Angular distribution of electrons ejected from H_2 by 300-keV protons. Experimental curves are from Rudd *et al.* (Ref. 2).

$$(2\pi)^{-3/2} e^{i\vec{K}_{21}\cdot\vec{r}_{21}} = (2\pi)^{-3/2} e^{i[\vec{K}_{21}\cdot(\vec{r}_{23}-\vec{r}_{13})]};$$

r_{21}^{-1} is replaced by $-r_{23}^{-1}$; and dr_{21} is replaced by $-dr_{13}$. The integrals are then easily evaluated, and we find that the matrix element of r_{12}^{-1} is

$$(1/\pi^2)(1/K^2)\sqrt{2}\,\mu^{5/2}/(\mu^2+\kappa_{23}^2)^2 , \qquad (26)$$

which is as large as the matrix element of r_{21}^{-1}, but does not depend upon the angle at which the electrons are ejected. This type of angular distribution is quite at variance with any reasonable model for the ionization process, and is not in accord with the data. Terms in V_i depending only upon r_{13} should therefore be rejected in accordance with Wick's[6] arguments.

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Structures in the Low-Energy e^--He Scattering Cross Sections*

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A cylindrical, retarding potential difference type of electron spectrometer with high-energy resolution has been used to study the structures in the e^--He scattering cross section in a transmission experiment. Twenty-four structures have been observed between the transmission maximum due to He^- $1s2s^2\,^2S_{1/2}$ at 19.30 ± 0.01 eV and He^+ $1s\,^2S_{1/2}$ at 24.60 ± 0.02 eV, 11 of which have been observed previously. The agreement with previous measurements for the positions of the structures is good where comparison is possible. Four of the new structures have been observed below the $n=2$ states of He. The excitation onsets for 2^3S_1, 2^1S_0, 2^3P^0, and 2^1P^0 have been observed, as well as some excitation onsets at higher energies. No structures have been observed below He^- $^2S_{1/2}$. Two structures have been observed above He^+ $^2S_{1/2}$ which were observed previously.

INTRODUCTION

The first e^--He scattering cross-section measurements were reported by Ramsauer.[1] In these experiments, electrons from a photoelectric source were momentum selected by a magnetic selector, then passed through a gas cell and collected. The total cross section was directly determined by a study of the electron beam attenuation as a function of the gas pressure in the interaction region.

The energy resolution of a spectrometer can be defined in terms of ΔE, the full energy width at half-maximum current (FWHM). In a magnetic selector of fixed geometry, ΔE increases with the energy. The large resonance at 19.3 eV first observed by Schulz[2] was not detected by Ramsauer,[1] because of poor energy resolution at that energy.