

Comments on Ion Mobility in Very Dilute Solutions of He³ in He⁴ at Low Temperatures*

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A realistic model potential is used to explain the low-temperature low-concentration positive-ion mobility. It is suggested that a similar potential could explain the negative-ion mobility.

Recently, Meyer and Neeper¹ have measured the mobilities of positive and negative ions in very dilute He³ - He⁴ mixtures in the temperature range 0.05–0.5 K under a variety of conditions of electric field and pressure. Other measurements at higher temperatures and concentrations have been made by Meyer and Reif² and Kovdria and Esel'son.³ The observed behavior of the positive ion at low temperatures¹ is as follows: (i) the zero-field mobility $\mu_+(0, T)$ decreases with decreasing temperature; (ii) the drift velocity is a supralinear function of electric field; and (iii) $\mu_+(0, T)$ increases with increasing pressure. The behavior of the negative ion is similar except that $\mu_-(0, T)$ increases quite rapidly for decreasing $T \lesssim 0.08$ K. Heretofore, a hard-core interaction between ion and He³ atom has been assumed in discussing the measurements.^{2, 3} It is shown in this paper that a more realistic potential can be used to explain the observations.

It is generally assumed⁴ that the positive ion is a molecular ion (He₂⁺) surrounded by some small number of effectively bound He atoms – a “cluster” model for the ion, in other words. Since the zero-point energy of a He³ atom in He⁴ liquid is ~ 4 K, it is unlikely that a He³ atom would replace a He⁴ atom in the cluster or that He³ atoms will congregate in the neighborhood of the ion, so that a physical growth or change in mass of the positive ion is not probable. We adopt the following model potential for the ion-He³ atom interaction: An attractive term proportional to $1/r^4$ due to polarization of the He³ atom; an attractive term proportional to $1/r^6$ due to dipole-dipole interactions; and a repulsive term which we represent as proportional to $1/r^{12}$. Following Mason and Schamp,⁵ we write the potential in the following way:

$$V(r) = \frac{1}{2} \epsilon \left[(1 + \gamma) \left(\frac{r_m}{r} \right)^{1/2} - 4\gamma \left(\frac{r_m}{r} \right)^6 - 3(1 - \gamma) \left(\frac{r_m}{r} \right)^4 \right] \quad (1)$$

In (1), γ represents the relative strength of the two attractive terms, ϵ is the minimum value of $V(r)$, and r_m is the radius at which this minimum occurs.

We expect that $r_m \approx R_+ + R_3$ which is ~ 8 Å and, therefore, from the polarizability of a He³ atom, that $\epsilon \sim 2$ K. (Note that for $\gamma = 1$ we recover the standard van der Waals potential.)

In order to use the phenomenological potential (1) to calculate the ion mobility, one has to decide whether a semiclassical or quantum-mechanical calculation is appropriate. The usual criterion⁶ for the applicability of a classical calculation is that the uncertainty in momentum be small compared to the momentum transfer, i.e., $\hbar^2 k^2 / 2m_3 \ll k r_m V(r_m)$, where m_3 is the He³ effective mass and k is the He³-atom wave vector. From what is known⁷ about the positive ion, we estimate that a classical calculation will be satisfactory down to 0.1 K. For lower temperatures it becomes necessary to perform a quantum-mechanical calculation, and many partial waves must be included. (For very low temperatures such that $k r_m \lesssim 1$ the calculation again becomes simplified, as only s - and p -wave scattering is important, and one can use the results of O'Malley, Spruch and Rosenberg⁸ for the cross section.) Further, density corrections must be included as the He³ concentration is increased. These corrections will be required when the volume occupied by a He³ atom is comparable to the effective volume of the ion, i.e., in order to ignore density corrections we must have $\frac{4}{3} \pi R_+^3 n_3 \ll 1$, where n_3 is the He³ number density. Thus we must consider only concentrations of He₃, C_3 , such that $n_3 \sim (\frac{4}{3} \pi R_+^3)^{-1} \times 10^{-2}$ or $C_3 \sim 3 \times 10^{-4}$. (This number is about a factor of 10 smaller for the negative ion.) This condition is violated in most of the available data.

Applying the results of Mason and Schamp⁵ we find, for $C_3 \lesssim 3 \times 10^{-4}$,

$$C_3 \mu_+(0, T) = \frac{3e}{16N_4} \left(\frac{2}{\pi \epsilon m_3 r_m^4} \right)^{1/2} \times [T^{*1/2} \Omega^{(1,1)}(T^*)]^{-1}, \quad (2)$$

where e is the electronic charge, N_4 is the number density of the solution, $T^* = k_B T / \epsilon$, and the $\Omega^{(l,n)}(T^*)$ integrals are tabulated in Ref. 5. Figure 1 shows (2) plotted down to 0.05 K [at which temperature

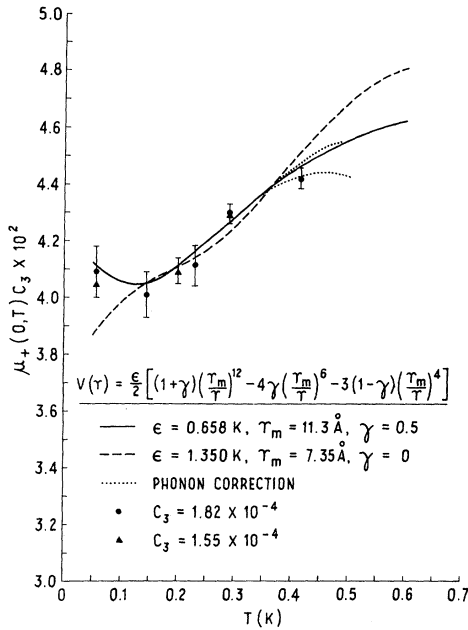


FIG. 1. Theoretical equation (2) plotted versus temperature, using $V(r)$ as in (1), with the two sets of values for the parameters shown in the figure. The experimental points, shown as a closed circle and a closed triangle, are from Ref. 1. The phonon correction to $\mu_+(0, T)$ from Ref. 9 is shown as a dotted line. The lower correction applies to the solid line; the upper to the dashed line.

(2) should be breaking down] for two sets of values of the parameters in (1), together with data points from Ref. 1. Included also in Fig. 1 are phonon corrections from the data of Schwarz and Stark.⁹ The fit is not unambiguous but the values found for the parameters in (1) compare favorably with what we estimated. The cross section is $\sim \pi r_m^2$ which is about three times bigger than the cross section for roton and phonon scattering.^{7,9} In the high-temperature limit, (2) gives $\mu_+(0, T)C_3 \propto T^{-1/3}$, in general agreement with Refs. 2, 3, and 9. Note the critical dependence of the shape of (2) on $V(r)$. To fit the negative-ion data, a weaker repulsive potential is needed, but it is not clear what model to adopt for the attractive part. ($\Omega(l, n)^*$ integrals for other forms of potential can be found in Ref. 10.)

The dependence of drift velocity $v_D(E, T)$ on electric field E can be explained in terms of a general result given by Kihara,¹¹ which may be expressed as a series in powers of $[\mu(0, T)E]^2$. For the positive ion, using $V(r)$ given in (1), we find, for $T \sim 0.15$ K,

$$v_D(E, T) \approx \mu_+(0, T)E \left(1 + \frac{M_3}{k_B T} [\mu_+(0, T)E]^2 \times 10^{-2} \right). \quad (3)$$

Putting in numbers, (3) yields $E \approx 5$ V/cm for the value of the field at which nonlinear behavior should be observable, and $v_D(E, T) < \mu_+(0, T)E$. Meyer and Neep¹ find $E \approx 6$ V/cm and $v_D(E, T) > \mu_+(0, T)E$. The sign of the leading term in the expansion for $v_D(E, T)$ is positive if $\mu(0, T)$ is an increasing function of T , and vice versa. Thus, over the temperature range covered in Ref. 1, the negative ion's drift velocity should vary with field in precisely the manner observed.

We suggest that the variation of mobility with pressure involves changes in the parameters of the model potential chosen, as well as changes resulting from an increase in the Fermi energy with pressure. That the former is the case is obvious for the case of the negative ion, since its effective radius changes drastically upon the application of pressure.¹² The evidence of Ref. 1 suggests further that the major changes occur in the attractive part of $V(r)$.

The variation of $\mu_+(0, T)C_3$ with C_3 is quite complex. There are two distinct contributions: (i) the effect of statistics, and (ii) the effect of density corrections. For $C_3 \sim 3 \times 10^{-3}$, statistical corrections amount to about a 0.2% increase in $\mu(0, T)C_3$. The density corrections are hard to estimate but we note that at this concentration $\frac{4}{3}\pi R_+^3 n_3$ is already ~ 0.1 . The data of Ref. 1 indicate a 10% increase in $\mu_+(0, T)C_3$ at $C_3 \sim 3 \times 10^{-3}$ and a subsequent decrease as C_3 is increased still further. Similar behavior was observed for $\mu_-(0, T)C_3$. We suggest that density effects are responsible in large measure for this behavior since experiments on the viscosity of dense gases show similar behavior.¹³

From the foregoing, it is obvious that at low temperatures the treatment of an ion as a hard sphere of well-defined radius breaks down. In this connection, we note that in calculations of mobility in pure He³ such as those of Josephson and Lekner, and Gould and Ma,¹⁴ it is necessary to take into account the energy dependence of the momentum-transfer cross section in order to get a more accurate temperature dependence. In order to obtain model potentials for the positive-(negative-) ion-He₃-atom interaction, further data is needed. (The negative ion is presumably much softer than the positive ion.) Also, in order to evaluate data at concentrations of about 1%, calculations of the density-dependent effects upon mobility are needed. It seems, too, that the problem of explaining the temperature and concentration-dependent^{1, 15} velocity at which vortex rings are created is related to the problem of calculating ion-He³-atom cross sections.

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Electric Dipole Moment of the Thallium Atom*

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Results of a search for the electric dipole moment of the thallium atom using previously described techniques and apparatus are reported.

Previously described^{1,2} atomic-beam techniques and apparatus were used to search for an electric dipole moment (EDM) of the Tl atom. The motivation for the Tl experiments was twofold: (1) Such experiments are sensitive to $\vec{I} \cdot \vec{E}$ type parity and time-reversal violating interactions³ because Tl has a *P* ground state whereas alkali atoms used in the previous work have *S* ground states, and (2) the relativistic enhancement of an assumed electron EDM in Tl may be comparable to that for Cs because of thallium's high atomic number ($Z=81$)⁴.

Experiments were performed in the same way as those for alkali atoms^{1,2}. The flop-in transition ($F=1, m_F=0 \rightarrow F=1, -1$) in the $6^2P_{1/2}$ ground state of Tl was monitored. The transition was induced in a Ramsey double-hairpin structure in a uniform magnetic field (\vec{H}) of a few gauss. Intense beams of Tl, which lasted for more than 24 h of running time, were produced by a high-temperature stainless-steel oven.⁵ Tl was detected by surface ionization on a hot platinum strip.

The ions were collected by a negatively biased copper cylinder which surrounded the Pt wire. A quadrupole mass spectrometer⁶ was used to identify Tl. The spectrometer was not used, however, during the electric field experiments. In spite of thallium's low ionization efficiency on Pt (~ 1 to 2%)⁷, we were able to detect Tl with a signal-to-noise ratio comparable to that achieved in the previously reported^{1,2} alkali-atom experiments. This is attributed to our ability to form Tl beams of sufficient intensity to compensate for the low ionization efficiency. At resonance, the recovered beam was $\sim 25 \times 10^{-10}$ A. The width at half-intensity of the central Ramsey peak was ~ 1.5 kc. The details of the apparatus have been published elsewhere in the literature.⁸

To detect a possible EDM in Tl, a large ac voltage was applied to parallel metal electric field plates situated between the rf loops of the Ramsey double-hairpin structure. The shift δf in the Bohr resonance frequency due to the applied elec-