nitrate (CMN). Data extend over the range 22–110 m°K.

Assuming an effective spin Hamiltonian

$$\mathcal{H} = -J \sum_{\{ij\}} \vec{I}_i \cdot \vec{I}_j \; , \qquad (1)$$

a bcc lattice, and nearest-neighbor interactions only, the susceptibility $\chi$ is expected[6] to be given by

$$\chi = (C/T)[1 - \theta/T + \tfrac{3}{4}(\theta/T)^2 + \cdots] \; , \qquad (2)$$

where C is the Curie constant and $\theta \equiv -2\,J/k$, $k$ being Boltzmann's constant.

At high enough temperatures, one has

$$C/\chi \simeq T + \theta \; . \qquad (3)$$

The straight line shown on Fig. 1 is a fit of the data to this equation, in which case we find $\theta = +(3.5 \pm 0.4)$ m°K. The data are somewhat better fitted by the expression

$$\chi T = C\,[1 - a(1/T) + b(1/T)^2] \qquad (4)$$

obtained from Eq. (2) with $a = \theta$ and $b = \frac{3}{4}\,\theta^2$. The fit gives

$$a = \theta = +(4.6 \pm 0.2)\ \text{m°K and } b = (35.2 \pm 12.7)\ \text{m °K}^2 = \tfrac{3}{4}((6.9 \pm 1.3)\,\text{m °K})^2 \; ,$$

where the indicated errors represent the rms deviation of $\chi T^2/C$ for $a$ and $\chi T^3/C$ for $b$. Since these rms deviations represent only the relationship of the data to a smooth curve they no doubt do not correctly assess the accuracy of the value of $\theta$ for the solid He$^3$ sample. The above values of $|\theta|$ are somewhat greater than the value of 2.9 m°K expected for a molar volume of 24.1 cm$^3$/mole from measurements of $(\partial P/\partial T)_V$ by Panczyk, Scribner, Straty, and Adams.[7] However, the determination of the sign of $\theta$ to be positive appears to be definitive. Thus at low enough temperatures a state of nuclear antiferromagnetism is to be expected in the solid.

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# Fine Structure of the $2^3P$, $3^3P$, and $4^3P$ States of Li$^+$

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The fine structure of the $2^3P$, $3^3P$, and $4^3P$ states of Li$^+$ has been computed, and the results are given in a table.

In view of experiments which are being planned to measure the fine structure of the triplet levels of Li$^+$, we have computed theoretical values for the fine-structure splitting of the $2^3P$, $3^3P$, and $4^3P$ levels of the Li$^7$ positive ion. The methods used in the calculation are basically the same as those which were employed for the case of helium,[1] and the results are given in Table I. The factor $\alpha^2 R_{\text{Li}^7} = 5.843099$ has been used in converting to cm$^{-1}$, and the values in the table are estimated to be subject to an error of not more than ±1 in the last figure quoted. This estimate of the error

TABLE I. Fine-structure splitting of the $^3P$ levels of $Li^+$.

| Level | $\Delta\nu_{01}$ $cm^{-1}$ | $\Delta\nu_{12}$ $cm^{-1}$ | $\Delta\nu_{02}$ $cm^{-1}$ |
|---|---|---|---|
| $2^3P$ | 5.194 43 | 2.089 88 | 3.104 55 |
| $3^3P$ | 1.347 3 | 0.627 0 | 0.720 2 |
| $4^3P$ | 0.540 | 0.265 | 0.275 |

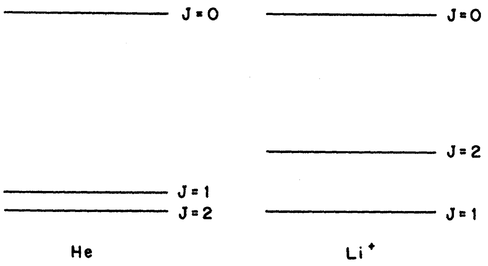

FIG. 1. Relative positions of the three $2^3P$ levels in He and in the $Li^+$ ion.

is based on the rate of convergence of the results as the number of terms included in the wave-function expansion is increased (expansions using up to 364 terms were used).

The values obtained for the $2^3P$ level agree with the early experimental results of Schüler.[2,3] Our results also verify the Bethe and Salpeter prediction,[3] based on the use of hydrogenic wave functions, that for the $^3P$ levels of the $Li^+$ ion, the $J=1$ level lies deepest, followed by the $J=2$ level, in contradistinction to the case of helium, where the order of these two levels is reversed, as shown in Fig. 1 for the $2^3P$ state.

The computations include the effect of the $\alpha^3$ quantum electrodynamic correction,[1] and also the correction arising from the depression of the $^3P$ $J=1$ level due to the close-lying $^1P_1$ level. This latter correction, which amounts to 0.004 65 $cm^{-1}$ for the $2^3P$ level and to 0.001 26 $cm^{-1}$ for the $3^3P$ level, has to be added to $\Delta\nu_{01}$ and to $\Delta\nu_{12}$, and has already been included in the results of Table I.

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# Width of the Surface Layer of Liquid $He^4$ †

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We note that a divergence predicted for the contribution of capillary waves to the surface-layer thickness disappears if the effect of gravity is included. The resulting finite value for $He^4$ is computed as a function of temperature, and a comparison is made with the static contribution to the thickness. Application to other liquid systems is discussed.

Widom has recently asserted[1] that one consequence of Atkins's ripplon theory of surface tension[2] is the divergence of $\delta^2$, the mean square displacement from equilibrium of a liquid surface. In this paper, we remove this divergence by including the gravitational term in the ripplon dispersion relation and are thus able to calculate $\delta^2$.

The full Hamiltonian for ripple waves on the surface of a semiinfinite fluid of mass density $\rho$ and surface tension $\gamma$ is of the form[3]

$$H=\sum_{\vec{k}}\left(v(k)\,Q_{\vec{k}}Q_{-\vec{k}}+t(k)P_{\vec{k}}P_{-\vec{k}}\right)\ , \tag{1}$$

$$v(k)=\tfrac{1}{2}(\rho g+\gamma k^2)\ , \tag{2a}$$

$$t(k)=\tfrac{1}{2}k/\rho\ , \tag{2b}$$

giving the dispersion relation[3,4]

$$\omega^2(k)=(\gamma/\rho)\,k^3+gk\ . \tag{3}$$