shown that if $\epsilon(\vec{k}, 0)$ is negative for some value of $\vec{k}$, the positive background will be unstable against the development of spontaneous density fluctuations of this wave vector. In the long-wave-length limit $\epsilon(\vec{k}, 0) \leq 0$ corresponds to a negative compressibility of the system electrons plus background. The fact that the low-density electron lattice state is one of negative compressibility has been pointed out by H. M. Van Horn [Phys. Rev. 157, 342 (1967)]. The conclusion reached by Pines and Nozières is that the condition $\epsilon(\vec{k}, 0) > 0$ [which implies the analyticity of $\epsilon(\vec{k}, \omega)$ in the upper half of the complex ω plane] follows from the requirement that the positive background be stable under the influence of the electron gas.

[27]For arbitrary k, the relations (4.3) and (4.4) in the classical limit imply $S(k) \geq k^2/(k_D^2 + 3k^2)$, which is a weaker statement than (4.13).

[28]This relation is verified in the noninteracting case at $T=0$, where the sound velocity $s_0 = \hbar k_F/(\sqrt{3}m)$. Similarly, in the case of a low-density weakly interacting classical plasma, s^2 is given by the value $k_B T/m$, again in agreement with (4.14).

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[31]R. P. Feynman, Phys. Rev. 94, 262 (1954). Although the Bijl-Feynman trial excited state $\rho(\vec{k})|\Phi_0\rangle$ (where $|\Phi_0\rangle$ is the exact ground state) was applied originally to He^4, this state works equally well as a trial excited state for a system of fermions. This follows from the fact that $\rho(\vec{k})|\Phi_0\rangle$ has the same symmetry property (symmetric or antisymmetric under exchange of particles) as $|\Phi_0\rangle$, and is orthogonal to it.

[32]The fact that the Feynman result is exact for Bose systems at $T=0$ in the limit $k \to 0$ has been shown by A. Miller, D. Pines, and P. Nozières [Phys. Rev. 127, 1452 (1962)]. For the electron gas this follows from the form (4.10) of $\chi'(k, \omega)$.

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Anticorrelation in Two-Photon Attenuated Laser Beam

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The density matrix in the P representation of a beam of radiation amplified by a two-photon amplifier has been derived up to the lowest order in the time-dependent perturbation theory without placing any restriction on the population of the state of the atom. It is shown that a laser beam containing noise in addition to the harmonic signal exhibits anticorrelation after being passed through such an amplifier, if less than one-sixth of the total number of atoms are maintained in the excited state.

INTRODUCTION

Recently, there has been a good deal of discussion[1–6] on the correspondence between the newly developing quantum theory[6–9] of optical coherence and the older semiclassical theory.[3,4,8–12] The classical definition of coherence functions is identical with the quantum definition, if the weight functional $P(\{v_{\vec{k}}\})$ in the diagonal phase-space representation of the density operator is real, nonsingular, and non-negative. There also exist fields for which $P(\{v_{\vec{k}}\})$ takes negative values in some regions of the complex $v_{\vec{k}}$ planes. These fields do not have classical analogs. With such fields, lesser photon coincidences than the random background may be recorded in a Hanbury Brown–Twiss detector.[13] Radiation in a pure Fock state is an example of such fields. This effect, referred to as anticorrelation, has not so far been observed experimentally, because it is very unusual in practice to have well-defined numbers of photons and because the conventional sources of optical fields have non-negative values of $P(\{v_{\vec{k}}\})$ throughout the complex $v_{\vec{k}}$ planes.

Recently, the authors have shown[14] that the statistical nature of photons is changed after interaction with a one-photon oscillator. Photon oscillators can thus be used for producing optical fields with photon statistics different from those of conventional sources. In this paper, we shall show that it is possible to obtain an optical field which can exhibit anticorrelation from a laser beam, by passing it through a two-photon oscillator. This gives a practical method of observing anticorrelation with the help of ideal photodetection.[15]

DENSITY MATRIX OF OUTPUT RADIATION

Let us consider an atomic system interacting with a single-mode radiation field. The Hamiltonian of this system in Heisenberg representation can be expressed as $H=H_0+H_I$, where[16]

$$H_0=\sum_b E_b|b\rangle\langle b|+\omega(a^\dagger a+\tfrac{1}{2})$$

and $H_I=\sum_{b,c}\zeta_{bc}(a^\dagger+a)|b\rangle\langle c|$

in the dipole approximation.[17] Here, E_b is the eigenvalue of the Hamiltonian of the free atom corresponding to the eigenstate $|b\rangle$, the $a^\dagger$ and a are creation and annihilation operators for radiation which has the frequency ω, and

$$\zeta_{bc}=e(2\pi/\omega V)^{1/2}\langle b|\vec{\epsilon}\cdot\vec{p}|c\rangle \quad ,$$

where $\vec{\epsilon}$ is the unit vector along the direction of polarization of radiation, $\vec{p}$ is the momentum operator for the atom, and V is the interaction volume. Let us also assume that only two states, $|u\rangle$ and $|l\rangle$, are initially significantly populated and are connected through a two-photon resonance, i.e., $E_u-E_l=2\omega_0\simeq 2\omega$, and that no other pair of states is connected through a two-photon or a one-photon resonance. Then, at no time will states other than $|u\rangle$ and $|l\rangle$ be appreciably populated.

If the interaction starts at time t, the density operator of the system can be written as $\rho(t)=\rho^A(t)\rho^R(t)$, where the superscripts A and R refer to the free atom and radiation, respectively. After a time τ, we have

$$\rho(t+\tau)=u(\tau)\rho(t)u^\dagger(\tau),$$

where
$$u(\tau)=u^{(0)}(\tau)+\sum_{n=1}^{\infty}(-i)^n\int_0^\tau d\tau_1 \times\int_0^{\tau_1}d\tau_2\cdots\int_0^{\tau_{n-1}}d\tau_n u^{(0)}(\tau-\tau_1)H_I \times u^{(0)}(\tau_1-\tau_2)H_I\cdots u^{(0)}(\tau_{n-1}-\tau_n)$$

and $u^{(0)}(\tau)\equiv e^{-iH_0\tau}$. If we write the reduced density operator of the field as

$$\rho^R(t+\tau)\equiv\sum_b\langle b|\rho(t+\tau)|b\rangle = \int d^2v P^R(v,t+\tau)|v\rangle\langle v| \quad ,$$

then $P^R(v,t+\tau)$ is given by[18]

$$P^R(v,t+\tau)=\pi^{-2}\int d^2\alpha \times[\exp(|v|^2+|\alpha|^2+\alpha^* v-\alpha v^*)]\langle-\alpha|\rho^R(t+\tau)|\alpha\rangle \quad .$$

Lengthy but direct calculations lead, on including only the lowest-order contribution in ζ, to

$$\begin{aligned}P^R(v,t+\tau) &= P^R(V,t)+A\{N_u[H(\partial^2\partial^{*2}-2V\partial^2\partial^*-2V^*\partial\partial^{*2} \\ &-4(1-VV^*)\partial\partial^*+4V\partial+4V^*\partial^*+2(1-2VV^*)) \\ &-G(V^2\partial^2-2V^2V^*\partial)-G^*(V^{*2}\partial^{*2}-2VV^{*2}\partial^*)] \\ &-N_l[2H(1-2VV^*)-G^*(V^2\partial^2+2(2-VV^*)V\partial) \\ &-G(V^{*2}\partial^{*2}-2(2-VV^*)V^*\partial^*)]\}P^R(V,t) \quad , \qquad (1)\end{aligned}$$

where $V=ve^{i\omega\tau}$, $\partial\equiv\partial/\partial V$, $\partial^*\equiv\partial/\partial V^*$

$$A=\sum_i(\omega_{il}-\omega)^{-2}|\zeta_{ui}\zeta_{il}|^2 \quad ,$$

$$N_u=\rho^A_{uu}(t),\quad N_l=\rho^A_{ll}(t),$$

$$G=(\omega_0-\omega)^{-2}[e^{i(\omega_0-\omega)\tau}-1],$$

and $H=-(G+G^*)$

if we take $|\omega_0-\omega|^{-1}\ll\tau$.

The variation of m, the number of photoelectrons ejected in a time interval T, is[11]

$$\Delta_m^2=\langle m\rangle_{av}+(KT)^2(\Delta_N^2-\langle\hat{N}\rangle) \ ,$$

where $\Delta_m^2=\langle m^2\rangle_{av}-\langle m\rangle_{av}^2$

and $\Delta_N^2=\langle\hat{N}^2\rangle-\langle\hat{N}\rangle^2$

are the variance of number of photons. Here, $\langle\hat{O}\rangle$ stands for the expectation value of the operator $\hat{O}$, and

$$\langle m^2\rangle_{av}=\sum_m m^2 p(m,t,T) \ ,$$

$$\langle m\rangle_{av}=\sum_m m\, p(m,t,T) \ ,$$

where $p(m,t,T)$ is the probability for ejection of m photoelectrons in the time interval t to $t+\tau$. Mandel, Sudarshan and Wolf[11] have interpreted the two terms of Δ_m^2 as being due to (i) the variation in the number of particles obeying Poisson distribution ($\langle m\rangle_{av}$), and (ii) the fluctuations in the classical wave field

$$(\Delta_N^2-\langle\hat{N}\rangle=\int d^2v\,P^R(v)(|v|^2-\langle|v|^2\rangle)^2,$$

where $\langle|v|^2\rangle=\int d^2v\,P^R(v)|v|^2)$,

respectively. When $\Delta_N^2-\langle N\rangle$ is <0 and therefore Δ_m^2 is $<\langle m\rangle_{av}$, the field is said to be exhibiting anticorrelation.[6,19]

ANTICORRELATION IN THE OUTPUT RADIATION

For a laser beam, we take[20]

$$P^R(v,t)=Ce^{-u^2/2\sigma^2} \quad ,$$

where $u=|v|^2-|v_0|^2$,

$$C=4(2\pi)^{-3/2}\sigma^{-1}[1+\mathrm{erf}(|v_0|^2/2^{1/2}\sigma)]^{-1} \ .$$

Here σ^2 is a measure of the noise and $|v_0|^2=\lim\langle\hat{N}\rangle$ as $\sigma\to 0$ is a measure of the content of harmonic signal. For this distribution, Eq. (1) leads to

$$\begin{aligned}[\Delta_N^2-\langle\hat{N}\rangle]_{t+\tau} &= \sigma^2(1-\pi C|v_0|^2e^{-B}-\pi^2C^2\sigma^2e^{-2B}) \\ &+AH[N_u\{10|v_0|^4+24|v_0|^2+4+8\sigma^2|v_0|^2+26\sigma^2 \\ &-\pi C\sigma^2(2|v_0|^4-2|v_0|^2-28+2\sigma^2)e^{-B}\end{aligned}$$

$$- \pi^2 C^2 \sigma^4 (2|v_0|^2 + 8) e^{-2B}\} - N_l \{2|v_0|^4 + 8\sigma^2 |v_0|^2 + 2\sigma^2 - \pi C \sigma^2 (2|v_0|^4 - 2\sigma^2 |v_0|^2 + 2\sigma^2 - 8\sigma^4) e^{-B} - 2\pi^2 C^2 |v_0|^2 \sigma^4 e^{-2B}\}] , \quad (2)$$

where $B = |v_0|^4/2\sigma^2$.

Since $|v_0|^2 \gg 1$, the above equation simplifies to $\Delta_{\hat{N}}^2 - \langle \hat{N} \rangle = 10AH(N_u - N_{\rm th})$, where for low noise, the threshold value of N_u is

$$N_{\rm th} \simeq \tfrac{1}{6} N_0 - (\sigma^2/12AH), \text{ and } N_0 (= N_u + N_l)$$

is the total number of atoms in the oscillator. These relations make it evident that if a laser beam with low noise is incident on a two-photon oscillator having less than one-sixth of the total number of atoms in the excited state, the output field will exhibit anticorrelation.

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