# Measurements of the Energy-Loss Factor of an Electron in a Cesium Discharge*

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This paper reports the determination of the loss factor of electron energy lost per collision moving in a cesium discharge. Measurements of the electric field $E$ and the electron temperature $U_e$ are used to determine the energy-loss factor of the electron. Those values are measured using Langmuir probes. The energy-loss factor of the electron in a cesium discharge has been obtained for $E/n_a$ (V cm$^2$) values between $8\times10^{-17}$ and $6.2\times10^{-15}$.

## INTRODUCTION

Current interest in alkali vapors has stimulated numerous experimental and theoretical investigations. In particular numerous experimental and theoretical studies have been performed on collisions between electrons and cesium atoms. Measurements have been reported for the total,[1] momentum transfer,[2–8] excitation,[9] and ionization[10,11] cross sections.

On the other hand, in order to understand the energy relations of the electrons in a positive column of a discharge, the energy-loss factor of the electron has been measured in various gases.[12] However, in view of the difficulties associated with experimental studies in cesium, no measurement has been reported of the energy-loss factor of the electron. The present study was initiated in an attempt to obtain energy-loss factors of the electron moving in cesium discharges, using the principle similar to that discussed by Bickerton and von Engel.[13]

## PRINCIPLE

The average electron energy $\frac{3}{2}eU_e$ in a positive column of a steady discharge is governed by the energy conservation equation:

$$\tfrac{3}{2}e\vec{\nabla}_r\cdot\langle J_e\vec{v}_e\rangle = e\langle\vec{E}\cdot\vec{v}_e\rangle + \tfrac{3}{2}e\langle\Delta U_e\rangle, \qquad (1)$$

where $\vec{v}_e$ and $\frac{3}{2}e\langle\Delta U_e\rangle$ are the electron velocity and the mean energy change of an electron per unit time due to collisions with other particles. By taking into account the energy loss caused by elastic and inelastic collisions with atoms, Coulomb collisions with ions, and the energy gain due to collisions of the second kind with excited atoms, the following equations are deduced[14]:

$$\frac{dU_e}{dx} = \tfrac{2}{3}E\left[1-\frac{8}{\pi}\left(\frac{U_e}{E\lambda_e}\right)^2\kappa_e\right] \qquad (2)$$

and

$$\kappa_e = \kappa_{el}\left(1+\frac{n_e}{n_g}\frac{\bar{q}_{ei}}{\bar{q}_m}\right)+\sum_k\frac{\bar{q}_k}{\bar{q}_m}\frac{U_k}{U_e} - \sum_j\frac{n_{aj}}{n_g}\frac{\bar{q}_{aj}}{\bar{q}_m}\frac{U_{aj}}{U_e}, \qquad (3)$$

where the meaning of the symbols is as follows: $\kappa_{el}$ is the elastic energy-loss factor of the electron; $\bar{q}_{ei}$ is the mean cross section of Coulomb collisions; $\bar{q}_k$ is the mean cross section of excitation collisions for the transition to the $k$th excited state; $\bar{q}_{aj}$ is the mean cross section ($j$th excited state) of collisions of the second kind with electrons; $\bar{q}_m$ is the mean cross section of the electron-atom momentum transfer collisions; $n_e$ is the number density of electrons; $n_{aj}$ is the number density of the $j$th excited atoms; $n_g$ is the number density of the atoms in the ground state; $eU_k$ is the excitation energy of the $k$th excited state; and $eU_{aj}$ is the energy gain due to a collision of the second kind with $j$th excited atom.

Each value has been averaged with respect to the velocity distribution of electrons. In deriving Eq. (2), the relation for the drift velocity $v_{ed} = (e/m_e)(\bar{c}_e/\lambda_e)E$ has been adopted, where $m_e$, $\lambda_e$, and $\bar{c}_e$ are the electronic mass, the mean free path of electrons, and the mean velocity of electrons, respectively.

It has been pointed out experimentally[8] that even in the $10^{-4}$ range of degree of ionization, electron-ion collisions caused a noticeable effect on transport properties of cesium plasmas in the range of $U_e$ of 0.25–0.5 eV. This is simply because the cross section $q_{ei}$ increases as $U_e^{-2}$. Consequently, the low electron temperature gives rise to a significantly high $q_{ei}$. Therefore, the formula based on the binary collision of electron-ion for the energy-loss expression[15] has been included in Eq. (2). For a homogeneous plasma, we obtain $dU_e/dx = 0$, and Eq. (2) becomes

$$U_e = (\tfrac{1}{8}\pi)^{1/2}\frac{E\lambda_e}{\sqrt{\kappa_e}} = 0.63\,\frac{E}{p_0}\,\frac{\lambda_{el}}{\sqrt{\kappa_e}}, \qquad (4)$$

where $\lambda_{el}$ is the value of $\lambda_e$ at $p_0 = 1$ Torr. Equation (4) represents a relationship between the electron temperature and $E/p_0$ and was used to determine $\kappa_e$ experimentally from measurements of $U_e$ and $E/p_0$.

## EXPERIMENTAL

The experiments were carried out with a cylindrical Pyrex glass discharge tube of inner diameter 3 cm having a hot tungsten cathode and a tungsten anode of 2.5 cm diam with a spacing of 40 cm between the two electrodes. The probe is made of a tungsten rod of 1 mm diam enclosed in a glass sheath except for one flat end which acts as a plane probe. Two probes with the same dimensions are positioned on the tube axis and are separated by 3.4 cm. The nearest probe is located about 20 cm from the cathode. Measurements were made by using the two probes with cesium pressures varying from 6.6 to 74 mTorr. The cesium used was 99.99% pure. The temperature of the discharge tube was maintained around 300 °C by a temperature-controlled oven. This temperature was about 100–160 °C higher than the temperature of the cesium reservoir. The part of the discharge tube containing the two probes was heated up to about 350 °C using a separate resistor wire wrapped around the tube. This avoided condensation of cesium vapor on the surface of the probe. The temperature of the Cs reservoir was monitored and kept at the desired value by a copper-constantan thermocouple. The vapor pressure of Cs was calculated from the expression found by Taylor and Langmuir.[16] The temperature variation of the Cs reservoir is believed to be within 2 °C.

In order to ensure the validity of the Langmuir probe measurements, special attention was paid to the change of probe surface coverage by cesium condensation. In order to determine the electron temperature, electron density, and space potential, the work function of the probe surface must be constant during the measurement of each probe characteristic. In a cesium plasma the work function varies because the fraction of coverage of the probe surface changes with the temperature of the probe and because the probe is heated by the probe current during the measurement. Therefore, in order to keep a substantially constant coverage, the probe surface was heated by drawing a higher probe current than that used during the measurements. This technique was applied before each measurement. The probe current and heating time were increased until good reproducibility of the probe characteristics was obtained. This indicates that the probe surface is at a high enough temperature to keep the tungsten surface clean. The heating time and current were varied with the pressure of Cs. The probe current-voltage characteristic was drawn on an $X$-$Y$ recorder with a logarithmic amplifier in the probe current branch. From the semilog plots of the current-voltage characteristics, an example of which is shown in Fig. 1, the electron temperature and plasma potential were determined. The linear behavior of the characteristics over 2 orders of magnitude of the probe current was taken as experimental verification of the existence of a Maxwellian distribution of velocities. Deviations from linearity at low probe currents, which are seen in Fig. 1, simply indicate that because the probe current is the sum of the electron and ion current, contribution of the ion current cannot be neglected compared to the electron current. An experimental verification of the validity of the probe characteristics has been performed by means of measurements of the ratio of the electron saturation current $I_{es}$ to the ion saturation current $I_{is}$. Ratios obtained from experiments agreed well with the theoretical prediction of $I_{es}/I_{is} \doteq (2M/m_e)^{1/2} = 680$.[17] For example, for $p_0 = 74$ mTorr, the ratio is 660, as shown in Fig. 1.

The electric field was determined from potential measurements made with probes positioned axially

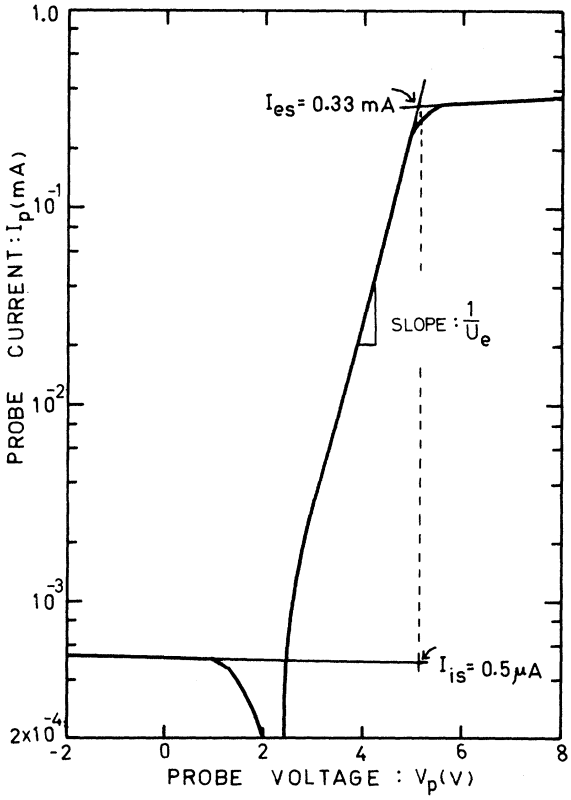


FIG. 1. Typical Langmuir probe characteristic taken at $p_0 = 74$ mTorr and a discharge current of 20 mA. The probe voltage was applied relative to the cathode. $I_{es}$ and $I_{is}$ are the electron saturation current and ion saturation current, respectively. From the slope of the probe current, $U_e = 0.45$ eV is obtained.

along the positive column at 3.4-cm intervals at a constant discharge current of 20 mA. Measurements of $U_e$ and $E/p_0$ ($p_0$, reduced pressure at 0 °C) taken at 20 mA are shown in Fig. 2. Using a set of the measured values of $U_e$ and $E/p_0$, and using $\lambda_{el}$ calculated from Brode's ($U_e > 0.6$ eV)[1] and Nighan's ($U_e < 0.6$ eV) data,[8] we can calculate $\kappa_e$ from Eq. (4). Values of $\kappa_e$ are shown in Fig. 3 against $E/n_a$($n_a$, atomic number density in $cm^{-3}$), where the elastic energy loss factor $\kappa_{el} = 2m_e/M$ ($M$, atomic mass of Cs) is shown by a horizontal line.

## DISCUSSION

In order to verify our measurements of $U_e$, we compared the measurements of $U_e$ with Schottky's theory, which is based on a balance between (a) ionization by electron collisions, and (b) charge loss by ambipolar diffusion to the wall. According to the theory,[12] $U_e$ is determined as a function of $p_0R$ ($R$, tube radius) and gas:

$$p_0R = \frac{1}{c}\left(\frac{e^y}{1.2\times10^7 y^{1/2}}\right)^{1/2}, \qquad (5)$$

where $y = V_i/U_e$ and $c = (aV_i^{1/2}/\mu_{+1})^{1/2}$, and $a$, $V_i$, and $\mu_{+1}$ are the gas constant in $V^{-1}$, the ionization potential of the gas in V and the mobility of ions at 1 Torr of the gas in $cm^2V^{-1}sec^{-1}$, respectively. Equation (5) is the relation between $U_e$ and $p_0R$ for all gases. Since the dissociation energy of $Cs_2^+$ is 0.35 eV, most ions are considered to be atomic $Cs^+$ ions for the range of electron temperature in this experiment. The mobilities of $Cs^+$ in cesium vapor have been measured by various methods[18–21] with data for $\mu_{+1}$ ranging 30.4–91.2 $cm^2\,V^{-1}\,sec^{-1}$. Lee and Mahan[20] found good agreement between the charge transfer cross section derived from their measured mobility for $Cs^+$ of $\mu_{+1} = 91.2$ and a prediction from a simple theory of resonance charge transfer. However, it must be noted that their theoretical value of the cross section is half the experimental value of Palyukh and Savchin.[22] Therefore, in this case, the value[18] of $\mu_{+1} = 49.4$ is chosen in order to calculate $c$ in Eq. (5). Using $a = 2.8$, $V_i = 3.89$, and $\mu_{+1} = 49.4$, we obtain $c = 0.33$. Hence, substituting $c = 0.33$ in Eq. (5), we can calculate the relation of $p_0$ and $U_e$ for Cs, which is shown in Fig. 2 with a fixed $R = 1.5$ cm. Measurements of $U_e$ made at a constant discharge current of 20 mA agree well with Schottky's theory at higher pressures, which is based on direct ionization from the ground state. This good agreement indicates that at this current the ionization process is mainly controlled by direct ionization. Over lower pressures, electron temperatures are higher than that of the Schottky theory, because of failure of the ambipolar diffusion theory.[12] It is known that collisional ionization in cesium vapor takes place in one- or two-step processes, depending on the magnitude of electron density. At higher electron densities, the ionization process is mainly due to a two-step process:

$$\begin{aligned} &Cs + e \rightarrow Cs^* + e, \quad \Delta E \doteqdot 1.4 \text{ eV}, \\ &Cs^* + e \rightarrow Cs^+ + 2e, \quad \Delta E \doteqdot 2.5 \text{ eV}, \end{aligned} \qquad (6)$$

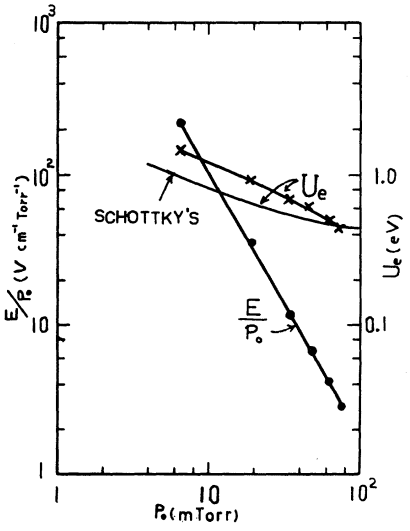


FIG. 2. Measurements of $U_e$ and $E/p_0$, cesium, tube current 20 mA, $R = 1.5$ cm. A comparison with Schottky's theory for the electron temperature is shown.

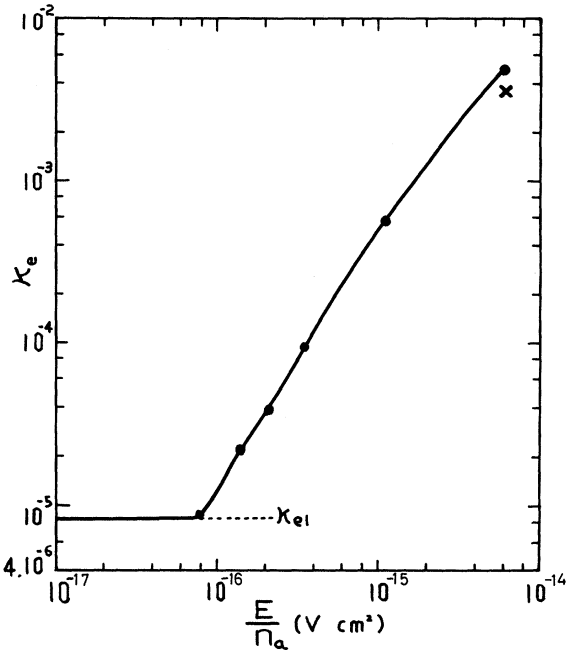


FIG. 3. Values of $\kappa_e$ against $E/n_a$, where × denotes the calculated value from Eq. (3).

where $Cs^*$ denotes the excited cesium atom. Also, a process $Cs^*+Cs^* \rightarrow Cs_2^+ + e$ is possible.

Unfortunately, the probability of ionization for a cesium atom, taking into account the stepwise process, is not known. When the ionization takes place by the stepwise process, the electron temperature is not independent of the electron density, but is a function of both the electron density and of $p_0R$. The ionization rate per unit volume is proportional to the electron density and the density of excited atoms (which is also proportional to the electron density). We can calculate the density of atoms in $k$ state ($n_k$) assuming the presence of excitation only from the ground state:

$$n_k=\left(\frac{2e}{m_e}\right)^{1/2}\tau_k n_e n_g \int_{U_k}^{\infty} q_k(U)F(U)U^{1/2}dU, \quad (7)$$

where $\tau_k$ is the lifetime of the $k$ state, $n_e$ is the electron density, and $F(U)$ is the electron energy distribution function in $U$(eV). The lifetime of the excited state is of the order of $10^{-8}$ sec, which is comparable with the electron-atom collision time. In addition, it is necessary to take into account the imprisonment of resonant radiation, since the absorption probability for the resonance lines of atomic cesium is high. According to Holstein[23] the lifetime of an imprisoned light quantum in cesium vapor over a pressure range 0.01–1 Torr is $1.2\times10^{-5}R^{1/2}$sec, where $R$ is the tube radius in cm. Therefore, the lifetime of the excited states becomes $1.5\times10^{-5}$ sec using $R=1.5$ cm. For simplicity, the excitation cross section is simulated by a step function given by $q_k=0$ for $U<U_k$ and $q_k=q_{k\,\max}$ for $U\geq U_k$. Assuming a Maxwellian energy distribution and using $q_{k\,\max}=9\times10^{-15}$ cm$^2$ (which is the combined excitation cross section of $6P$ states), and measured values of $U_e=0.45$ eV and $n_e=2.4\times10^{10}/\text{cm}^3$ at $p_0=74$ mTorr, we estimate $n_k \doteq 2\times10^{-2}n_g$. Also, according to Klarfeld,[24] the probability of ionization for excited atoms may be about 10 times the probability for ionization from the ground state. Even considering this factor for the ionization probability of excited atoms and bearing in mind that the value of $n_k$ is over-estimated, it can be seen that the stepwise ionization is less important than the direct ionization at this current.

It should be pointed out that the energy-loss factor over the $E/n_a$ region in this experiment is mainly due to inelastic collisions. This is quite reasonable because the excitation energy for the $6P_{1/2}$ and $6P_{3/2}$ states are 1.39 and 1.45 eV, respectively. An estimate is made to verify the measurement at $p_0=6.6$ mTorr using Eq. (3). Employing the measured values of $n_e=4\times10^9/\text{cm}^3$ and $U_e=1.41$ eV, we can obtain for $n_e q_{ei}$ the value $8.7\times10^{-4}$ cm$^{-1}$ and a ratio of $n_e q_{ei}/n_g q_m$ of $9.1\times10^{-5}$ using Brode's datum of $q_m=4\times10^{-14}$ cm$^2$. Therefore, $n_e q_{ei}/n_g q_m$ is neglected compared to 1. Cross sections for collisions of the second kind are not known for the $6P$ states. It is known for mercury[25] that the cross section for collisions of the second kind for $Hg\cdot{}^3P_2$ to $Hg\cdot{}^3P_1$ possesses a maximum at an electron energy of about one-third of the threshold energy between these two states, and that the cross-section maximum for the process ${}^3P_2+e \rightarrow {}^3P_1+e$ is about 2.2 times that of the process ${}^3P_1+e \rightarrow {}^3P_2+e$. In our case, considering that the electron temperature of 1.41 eV is close to the threshold energy of 1.39 eV, the ratio of the cross section for collisions of the second kind, averaged with respect to the electron energy distribution to the averaged excitation collision cross section may be less than the ratio of these maximum values, depending on the velocity distribution function of electrons. Substituting $n_e=4\times10^9/\text{cm}^3$, $U_e=1.41$ eV, the lifetime and excitation cross section for the $6P$ states mentioned in Eq. (7) then yield the value $2.1\times10^{-2}$ for the ratio of the densities of excited to ground-state atoms $n_{aj}/n_g$. In order to determine the relative importance of the third and fourth terms in Eq. (3), we must take the two factors mentioned above into consideration. It is found that the predominance of the average cross section for collisions of the second kind over that for excitation collision is counteracted by the fact that the ratio of the excited to ground-state atomic number densities is so small. The net result is that the third term in Eq. (3) may be at least 10 times larger than the fourth. Neglecting for the moment the contribution from the fourth term in the estimate of the measured value of $\kappa_e$, using a slope of $7.1\times10^{-15}$ cm/eV for the linear form[9] of the excitation cross section ($6P$ state) with the electron energy and setting $U_e=1.41$ eV, we obtain the value of $\kappa_e=3.6\times10^{-3}$ at $E/n_a=6.2\times10^{-15}$, from Eq. (3). This is fairly good agreement with the measurement shown in Fig. 3. No calculations were made including any higher excited states since the cross sections for excitation to higher states are expected to be small compared to the cross section for excitation to the resonance states. At high energies, where the Born approximation is valid, the relative magnitude of the cross sections for excitation to various levels varies as the oscillator strength for the transition involved. In cesium the oscillator strength for the transitions between the resonance $6P$ states and the ground state are very large compared to those for other states.[26] Although the Born approximation predictions are not expected to be valid at the low energies involved in this case, calculations by Seaton[27] indicate that the oscillator strength can

be used as a rough guide to the magnitude of the excitation cross section at low energies.

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# Wave Coupling across a Shock Wave in a Viscous Plasma

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This paper presents a theory of the coupling between electron-acoustic, ion-acoustic, and electromagnetic plane waves at an idealized shock front in a two-fluid, fully ionized viscous plasma. The viscous-fluid equations and Maxwell's equations are used to derive the dispersion equation relating the frequency and propagation constant of possible modes of propagation in a viscous plasma. Boundary conditions which must hold at the shock front in the presence of an incoming perturbation are then derived, and the amplitudes and energies of predicted wave modes generated at the boundary are calculated. In the limit of small viscosity, the energy-coupling ratios reduce to the values found previously in the inviscid problem, and for all values of viscosity we find enhanced wave transmission as previously found in the inviscid problem. Additionally, maxima in the energy-coupling ratios are found to exist, depending upon the viscosity of the medium.